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Stability and Stabilization of Markovian Jump Linear Systems with Partly Unknown Transition Probabilities ⋆

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Abstract: In this paper, the stability and stabilization problems of a class of continuous-time and discrete-time Markovian jump linear system (MJLS) with partly unknown transition probabilities are investigated. The system under consideration is more general, which covers the systems with completely known and completely unknown transition probabilities as two special cases, the latter is hereby the switched linear systems under arbitrary switching. Moreover, in contrast with the uncertain transition probabilities studied recently, the concept of partly unknown transition probabilities proposed in this paper does not require any knowledge of the unknown elements. The sufficient conditions for stochastic stability and stabilization of the underlying systems are derived via LMIs formulation, and the relation between the stability criteria currently obtained for the usual MJLS and switched linear systems under arbitrary switching are exposed by the proposed class of hybrid systems. Two numerical examples are given to show the validness and potential of the developed results.

1. INTRODUCTION

In past decades, Markovian jump systems (MJS) have been widely investigated and many useful results have been obtained, see for example, Boukas [2005], Costa et al. [2005]. The motivation on the study of the class of systems is the fact that many dynamical systems subject to random abrupt variations can be modeled by MJS such as manufacturing system, networked control system (NCS), etc. Typically, MJS are described by a set of classical differential (or difference) equations and a Markov stochastic process (or Markov chain) governing the jumps among them. As a dominant factor, the transition probabilities in the jumping process determine the system behavior to a large extent, and so far, many analysis and synthesis results have been reported assuming the complete knowledge of the transition probabilities. Recently, an interesting extension is to consider the uncertain transition probabilities, which aims to utilize robust methodologies to deal with the norm-bounded or polytopic uncertainties presumed in the transition probabilities, see for example, Karan et al. [2006], Xiong et al. [2005]. Unfortunately, the structure and "nominal" terms of the considered uncertain transition probabilities have to be known *a priori* in these burgeoning references.

The ideal knowledge on the transition probabilities are definitely expected to simplify the system analysis and design, however, the likelihood to obtain such available knowledge are actually questionable and the cost are probably expensive. A typical example can be found in NCS, where the packet dropouts and channel delays are well-known to be modeled by Markov Chains with the usual assumption that all the transition probabilities are completely accessible, Krtolica et al. [1994], Seiler and Sengupta [2005], Zhang et al. [2005]. However, in almost all types of communication networks, either the variation of delays or the packet dropouts can be vague and random in different running period of networks, all or part of the elements in the desired transition probabilities matrix are hardly or costly to obtain. The same problems may arise in other practical systems with jumps. Therefore, rather than the large complexity to measure or estimate all the transition probabilities, it is significant and necessary from control perspectives to further study more general jump systems with partly unknown transition probabilities.

In this paper, the basic stability and stabilization problems of a class of continuous-time and discrete-time Markovian jump linear system (MJLS) with partly unknown transition probabilities are investigated. The considered systems are more general than the systems with completely known or completely unknown transition probabilities, which can be viewed as two special cases of the ones tackled here. Moreover, in contrast with the recent research on uncertain transition probabilities, our proposed concept of the partly unknown transition probabilities does not require any knowledge of the unknown elements, such as the bounds or structures of uncertainties. In addition, the relation between the stability criteria currently obtained for the usual MJLS and switched linear system under arbitrary switching are exposed by our proposed systems. The remainder of the paper is organized as follows. In Section 2, the considered systems are formulated and the purposes of the paper are stated. In Section 3, the stochastic stability and stabilization conditions for the underlying systems are derived via LMIs formulation in the continuous-time and

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discrete-time cases, respectively. Two numerical examples are provided to illustrate the validness and applicability of the developed results in Section 4, and Section 5 concludes the paper.

Notation: The notation used in this paper is fairly standard. The superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n dimensional Euclidean space; $\mathbb{N}^+$ represents the sets of positive integers, respectively. For notation $(\Omega, \mathcal{F}, \mathcal{P})$, Ω represents the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $E[\cdot]$ stands for the mathematical expectation. In addition, in symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for the terms that are introduced by symmetry and $diag\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The notation $P > 0$ (≥ 0) means P is real symmetric positive (semi-positive) definite, and M_i is adopted to denote $M(i)$ for brevity. I and 0 represent respectively, identity matrix and zero matrix.

2. PROBLEM FORMULATION AND PRELIMINARIES

Fix the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and consider the following continuous-time and discrete-time Markovian jump linear systems, respectively:

$$\dot{x}(t) = A(r_t)x(t) + B(r_t)u(t) \tag{1}$$

$$x(k+1) = A(r_k)x(k) + B(r_k)u(k) \tag{2}$$

where $x(t) \in \mathbb{R}^n$ (respectively, $x(k)$) is the state vector and $u(t) \in \mathbb{R}^l$ (respectively, $u(k)$) is the control input. The jumping process $\{r_t, t \geq 0\}$ (respectively, $\{r_k, k \geq 0\}$), taking values in a finite set $\mathcal{I} \triangleq \{1, ..., N\}$, governs the switching among the different system modes. For continuous-time, $\{r_t, t \geq 0\}$ is a continuous-time, discrete-state homogeneous Markov process and has the following mode transition probabilities:

$$\Pr(r_{t+h} = j | r_t = i) = \begin{cases} \lambda_{ij}h + o(h), & \text{if } j \neq i \\ 1 + \lambda_{ii}h + o(h), & \text{if } j = i \end{cases}$$

where $h > 0$, $\lim_{h\to 0}(o(h)/h) = 0$ and $\lambda_{ij} \geq 0$ ($i, j \in \mathcal{I}$, $j \neq i$) denotes the switching rate from mode i at time t to mode j at time $t+h$, and $\lambda_{ii} = -\sum_{j=1, j\neq i} \lambda_{ij}$ for all $i \in \mathcal{I}$. Furthermore, the Markov process transition rates matrix $\mathbf{\Lambda}$ is defined by:

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1N} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2N} \\ & & \ddots & \\ \lambda_{N1} & \lambda_{N2} & \cdots & \lambda_{NN} \end{bmatrix}$$

For discrete-time case, the process $\{r_k, k \geq 0\}$ is described by a discrete-time homogeneous Markov chain, which takes values in finite set $\mathcal{I}$ with mode transition probabilities:

$$\Pr(r_{k+1} = j | r_k = i) = \pi_{ij}$$

where $\pi_{ij} \geq 0 \; \forall \; i, j \in \mathcal{I}$, and $\sum_{j=1}^{N} \pi_{ij} = 1$. Likewise, the transition probabilities matrix is defined by:

$$\boldsymbol{\pi} = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1N} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2N} \\ & & \ddots & \\ \pi_{N1} & \pi_{N2} & \cdots & \pi_{NN} \end{bmatrix}$$

The set $\mathcal{I}$ contains N modes of system (1) (or system (2)) and for $r_t = i \in \mathcal{I}$ (respectively, $r_k = i$), the system matrices of the ith mode are denoted by (A_i, B_i), which are real known with appropriate dimensions. In addition, the transition rates or probabilities of the jumping process in this paper are considered to be partly accessed, i.e., some elements in matrix $\mathbf{\Lambda}$ or $\boldsymbol{\pi}$ are unknown. For instance, for system (1) or system (2) with 4 operation modes, the transition rates or probabilities matrix $\mathbf{\Lambda}$ or $\boldsymbol{\pi}$ may be as:

$$\begin{bmatrix} \lambda_{11} & ? & \lambda_{13} & ? \\ ? & ? & ? & \lambda_{24} \\ ? & \lambda_{32} & \lambda_{33} & ? \\ ? & ? & \lambda_{43} & \lambda_{44} \end{bmatrix}, \quad \begin{bmatrix} \pi_{11} & ? & \pi_{13} & ? \\ ? & ? & ? & \pi_{24} \\ \pi_{31} & ? & \pi_{33} & ? \\ ? & ? & \pi_{43} & \pi_{44} \end{bmatrix}$$

where "?" represents the unaccessible elements. For notation clarity, $\forall i \in \mathcal{I}$, we denote $\mathcal{I} = \mathcal{I}_{\mathcal{K}}^i + \mathcal{I}_{\mathcal{UK}}^i$ with

$$\begin{aligned} \mathcal{I}_{\mathcal{K}}^i &\triangleq \{j : \lambda_{ij} \text{ (or } \pi_{ij}) \text{ is known}\}, \\ \mathcal{I}_{\mathcal{UK}}^i &\triangleq \{j : \lambda_{ij} \text{ (or } \pi_{ij}) \text{ is unknown}\}, \end{aligned} \tag{3}$$

Moreover, if $\mathcal{I}_{\mathcal{K}}^i \neq \emptyset$, it is further described as

$$\mathcal{I}_{\mathcal{K}}^i = (\mathcal{K}_1^i, ..., \mathcal{K}_m^i), \;\; \forall 1 \leq m \leq N \tag{4}$$

where $\mathcal{K}_m^i \in \mathbb{N}^+$ represent the m**th** known element with the index $\mathcal{K}_m^i$ in the i**th** row of matrix $\mathbf{\Lambda}$ or $\boldsymbol{\pi}$. Also, we denote $\lambda_{\mathcal{K}}^i \triangleq \sum_{j\in\mathcal{I}_k^i} \lambda_{ij}$, $\pi_{\mathcal{K}}^i \triangleq \sum_{j\in\mathcal{I}_k^i} \pi_{ij}$ throughout the paper.

Remark 1. The accessibility of the jumping process $\{r_t, t \geq 0\}$ (or $\{r_k, k \geq 0\}$) in the existing literature is commonly assumed to be completely accessible ($\mathcal{I}_{\mathcal{UK}}^i = \emptyset$, $\mathcal{I}_{\mathcal{K}}^i = \mathcal{I}$) or completely unaccessible ($\mathcal{I}_{\mathcal{K}}^i = \emptyset$, $\mathcal{I}_{\mathcal{UK}}^i = \mathcal{I}$). Moreover, the transition rates or probabilities with polytopic or norm-bounded uncertainties require the knowledge of bounds or structure of uncertainties, which can still be viewed as accessible in the sense of this paper. Therefore, our transition rates or probabilities matrix considered in the sequel is a more natural assumption to the Markovian jump systems and hence covers the existing ones.

For the underlying systems, the following definitions will be adopted in the rest of this paper. The more details can be referred to Boukas [2005], Costa et al. [2005] and the references therein.

Definition 1. System (1) is said to be stochastically stable if for $u(t) \equiv 0$ and every initial condition $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathcal{I}$, the following holds,

$$E\left\{\int_0^\infty \|x(t)\|^2 \, |x_0, r_0\right\} < \infty$$

Definition 2. System (2) is said to be stochastically stable if for $u(k) \equiv 0$ and every initial condition $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathcal{I}$, the following holds,

$$E\left\{\sum\nolimits_{k=0}^{\infty} \|x(k)\|^2 \, |x_0, r_0\right\} < \infty)$$

The purposes of this paper are to derive the stochastic stability criteria for system (1) and system (2) when the transition rates or probabilities are partly unknown, and to design a state-feedback stabilizing controller such that the resulting closed-loop systems are stochastically stable. The mode-dependent controller is considered here with the form:

$$u(t) = K(r_t)x(t) \text{ (respectively, } u(k) = K(r_k)x(k)) \tag{5}$$

where K_i ($\forall r_t = i \in \mathcal{I}$, or $r_k = i \in \mathcal{I}$) is the controller gain to be determined. To this end, the following Lemmas on the stochastic stability of systems (1) and (2) are firstly recalled and their proofs can be found in the cited references.

Lemma 1. Boukas [2005] System (1) is stochastically stable if and only if there exists a set of symmetric and positive-definite matrices $P_i, i \in \mathcal{I}$ satisfying

$$A_i^T P_i + P_i A_i + \mathcal{P}^i < 0 \tag{6}$$

where $\mathcal{P}^i \triangleq \sum_{j\in\mathcal{I}} \lambda_{ij} P_j$.

Lemma 2. Costa et al. [2005] System (2) is stochastically stable if and only if there exists a set of symmetric and positive-definite matrices $P_i, i \in \mathcal{I}$ satisfying

$$A_i^T \mathcal{P}^i A_i - P_i < 0 \tag{7}$$

where $\mathcal{P}^i \triangleq \sum_{j\in\mathcal{I}} \pi_{ij} P_j$.

3. MAIN RESULTS

In this section, we will develop the stability and stabilization results based on Lemmas 1 and 2, for the underlying systems in the continuous-time and discrete-time context, respectively.

3.1 Continuous-time case:

Let us first give stability analysis for the unforced system (1) with $u(t) \equiv 0$. The following theorem presents a sufficient condition on the stochastic stability of the considered system with partly unknown transition probabilities (3).

Theorem 3. Consider unforced system (1) with partly unknown transition probabilities (3). The corresponding system is stochastically stable if there exist matrix $P_i > 0, i \in \mathcal{I}$ such that

$$(1+\lambda_{\mathcal{K}}^i)(A_i^T P_i + P_i A_i) + \mathcal{P}_{\mathcal{K}}^i < 0, \forall j \in \mathcal{I}_{\mathcal{K}}^i \tag{8}$$

$$A_i^T P_i + P_i A_i + P_j \geq 0, \forall j \in \mathcal{I}_{\mathcal{UK}}^i, j = i \tag{9}$$

$$A_i^T P_i + P_i A_i + P_j \leq 0, \forall j \in \mathcal{I}_{\mathcal{UK}}^i, j \neq i \tag{10}$$

where $\mathcal{P}_{\mathcal{K}}^i \triangleq \sum_{j\in\mathcal{I}_{\mathcal{K}}^i} \lambda_{ij} P_j$.

Proof. Based on Lemma 1, we know that the system (1) is stochastically stable if (6) holds. Since one always has $\sum_{j\in\mathcal{I}} \lambda_{ij} = 0$, we can rewrite the left-hand side of (6) as:

$$\Theta_i \triangleq A_i^T P_i + P_i A_i + \mathcal{P}^i + \sum_{j\in\mathcal{I}} \lambda_{ij}\left(A_i^T P_i + P_i A_i\right)$$

Thus, from (3), we have

$$\begin{aligned}\Theta_i &= (1 + \sum_{j\in\mathcal{I}_{\mathcal{K}}^i} \lambda_{ij})\left(A_i^T P_i + P_i A_i\right) + \sum_{j\in\mathcal{I}_{\mathcal{K}}^i} \lambda_{ij} P_j \\ &\quad + \sum_{j\in\mathcal{I}_{\mathcal{UK}}^i} \lambda_{ij}\left(A_i^T P_i + P_i A_i\right) + \sum_{j\in\mathcal{I}_{\mathcal{UK}}^i} \lambda_{ij} P_j \\ &= (1+\lambda_{\mathcal{K}}^i)\left(A_i^T P_i + P_i A_i\right) + \mathcal{P}_{\mathcal{K}}^i \\ &\quad + \sum_{j\in\mathcal{I}_{\mathcal{UK}}^i} \lambda_{ij}\left(A_i^T P_i + P_i A_i + P_j\right)\end{aligned}$$

Then, $\forall j \in \mathcal{I}_{\mathcal{UK}}^i$ and if $i \in \mathcal{I}_{\mathcal{K}}^i$, it is straightforward that $\Theta_i < 0$ by (8), (10) and $\lambda_{ij} \geq 0$ ($\forall i, j \in \mathcal{I}$, $j \neq i$). On the other hand, $\forall j \in \mathcal{I}_{\mathcal{UK}}^i$ and if $i \notin \mathcal{I}_{\mathcal{K}}^i$, one can further obtain

$$\begin{aligned}\Theta_i &= (1+\lambda_{\mathcal{K}}^i)\left(A_i^T P_i + P_i A_i\right) + \mathcal{P}_{\mathcal{K}}^i \\ &\quad + \lambda_{ii}\left(A_i^T P_i + P_i A_i + P_i\right) \\ &\quad + \sum_{j\in\mathcal{I}_{\mathcal{UK}}^i, j\neq i} \lambda_{ij}\left(A_i^T P_i + P_i A_i + P_j\right)\end{aligned}$$

Since we have $\lambda_{ii} = -\sum_{j=1, j\neq i} \lambda_{ij} < 0$, then according to (8)-(10), one can also readily obtain $\Theta_i < 0$. Therefore, if (8)-(10) hold (obviously, no knowledge on $\lambda_{ij}, \forall j \in \mathcal{I}_{\mathcal{UK}}^i$ is needed therein), we conclude that the system (1) is stochastically stable against the partly unknown transition probabilities (3), which completes the proof. □

Remark 2. Note that if $\mathcal{I}_{\mathcal{UK}}^i = \emptyset$, $\forall i \in \mathcal{I}$, the underlying system is the one with completely known transition probabilities, which becomes the MJLS in the usual sense. Consequently, the conditions (8)-(10) are reduced to (8), which is equivalent to (6). Also, if $\mathcal{I}_{\mathcal{K}}^i = \emptyset$, $\forall i \in \mathcal{I}$, i.e., the transition probabilities are completely unknown, then the system can be viewed as a switched linear system under arbitrary switching. Correspondingly, the condition (8)-(10) are reduced to $-P_i \leq A_i^T P_i + P_i A_i \leq -P_j$, which implies $P_i = P_j = P > 0$, $A_i^T P + P A_i = -P < 0$, namely, a latent quadratic common Lyapunov function will be shared among all the modes. Therefore, in the continuous-time context, the condition is such that the resulting switched linear system is globally uniformly asymptotically stable Liberzon [2003].

Now let us consider the stabilization problem of system (1) with control input $u(t)$. The following theorem presents sufficient conditions for the existence of a mode-dependent stabilizing controller with the form (5).

Theorem 4. Consider system (1) with partly unknown transition probabilities (3). If there exist matrices $X_i > 0$ and Y_i, $\forall i \in \mathcal{I}$ such that

$$\begin{bmatrix} (1+\lambda_{\mathcal{K}}^i)(A_i X_i + X_i A_i^T + B_i Y_i + Y_i^T B_i^T) + \lambda_{ii} X_i & \mathcal{S}_{\mathcal{K}}^i \\ * & -\mathcal{X}_{\mathcal{K}}^i \end{bmatrix} < 0,$$
$$\forall (j \in \mathcal{I}_{\mathcal{K}}^i,\ j = i) \tag{11}$$

$$\begin{bmatrix} (1+\lambda_{\mathcal{K}}^i)(A_i X_i + X_i A_i^T + B_i Y_i + Y_i^T B_i^T) & \mathcal{S}_{\mathcal{K}}^i \\ * & -\mathcal{X}_{\mathcal{K}}^i \end{bmatrix} < 0,$$
$$\forall (j \in \mathcal{I}_{\mathcal{K}}^i,\ j \neq i) \tag{12}$$

$$A_i X_i + X_i A_i^T + B_i Y_i + Y_i^T B_i^T + X_j \geq 0,$$
$$\forall \left(j \in \mathcal{I}_{\mathcal{UK}}^i,\ j = i\right) \tag{13}$$

$$\begin{bmatrix} A_i X_i + X_i A_i^T + B_i Y_i + Y_i^T B_i^T & X_i \\ * & -X_j \end{bmatrix} \leq 0,$$
$$\forall \left(j \in \mathcal{I}_{\mathcal{UK}}^i,\ j \neq i\right) \tag{14}$$

where

$$\mathcal{S}_{\mathcal{K}}^i \triangleq \left[\sqrt{\lambda_{i\mathcal{K}_1^i}} X_i, ..., \sqrt{\lambda_{i\mathcal{K}_m^i}} X_i\right] \tag{15}$$

$$\mathcal{X}_{\mathcal{K}}^i \triangleq diag\left[X_{\mathcal{K}_1^i}, ..., X_{\mathcal{K}_m^i}\right] \tag{16}$$

with $\mathcal{K}_1^i$,...,$\mathcal{K}_m^i$ described in (4), then there exists a mode-dependent stabilizing controller of the form (5) such that the resulting system is stochastically stable. Moreover, if the LMIs (11)-(14) have a solution, an admissible controller gain is given by

$$K_i = Y_i X_i^{-1} \tag{17}$$

Proof. Consider system (1) with the control input (5) and replace A_i by $A_i + B_iK_i$ in (8)-(10), respectively. Then, performing a congruence transformation to (8) by P_i^{-1}, we can obtain

$$(1+\lambda_{\mathcal{K}}^i)\left[(A_i+B_iK_i)P_i^{-1}+P_i^{-1}(A_i+B_iK_i)^T\right] +P_i^{-1}\mathcal{P}_{\mathcal{K}}^iP_i^{-1}<0$$

Setting $X_i \triangleq P_i^{-1}$, $Y_i \triangleq K_iX_i$ and considering (15) and (16), by Schur complement, one can obtain that the above inequality is equivalent to (11) for $j \in \mathcal{I}_{\mathcal{K}}^i$, $j = i$, and (12) for $j \in \mathcal{I}_{\mathcal{K}}^i$, $j \neq i$, respectively. Similarly, (13) and (14) can be worked out from (9) and (10). Therefore, if (11)-(14) hold, (8)-(10) will be satisfied in Theorem 1 such that the underlying system is stochastically stable. Moreover, the desired controller gain is given by (17). This completes the proof. □

Remark 3. It is worth noting that (11) and (13) in Theorem 2 will not be checked simultaneously due to the fact $\mathcal{I}_{\mathcal{K}}^i \cap \mathcal{I}_{\mathcal{UK}}^i = \emptyset$.

3.2 Discrete-time case:

The following theorem presents a sufficient condition on the stochastic stability of the unforced system (2) with partly unknown transition probabilities (3).

Theorem 5. Consider the unforced system (2) with partly unknown transition probabilities (3). The corresponding system is stochastically stable if there exists matrix $P_i > 0, i \in \mathcal{I}$ such that

$$A_i^T\mathcal{P}_{\mathcal{K}}^iA_i - \pi_{\mathcal{K}}^iP_i < 0,\ \forall j \in \mathcal{I}_{\mathcal{K}}^i, \quad (18)$$

$$A_i^TP_jA_i - P_i < 0,\ \forall j \in \mathcal{I}_{\mathcal{UK}}^i \quad (19)$$

where $\mathcal{P}_{\mathcal{K}}^i \triangleq \sum_{j\in\mathcal{I}_{\mathcal{K}}^i}\pi_{ij}P_j$.

Proof. Based on Lemma 2, we know that the system (1) is stochastically stable if (6) holds. Now due to $\sum_{j\in\mathcal{I}}\pi_{ij} = 1$, we rewrite the left-hand side of (7) as

$$\Psi_i \triangleq A_i^T\left(\sum\nolimits_{j\in\mathcal{I}}\pi_{ij}P_j\right)A_i - (\sum\nolimits_{j\in\mathcal{I}}\pi_{ij})P_i$$

Thus, from (3), we have

$$\begin{aligned}\Psi_i &= A_i^T\left(\sum\nolimits_{j\in\mathcal{I}_{\mathcal{K}}^i}\pi_{ij}P_j\right)A_i - (\sum\nolimits_{j\in\mathcal{I}_{\mathcal{K}}^i}\pi_{ij})P_i\\ &\quad + A_i^T\left(\sum\nolimits_{j\in\mathcal{I}_{\mathcal{UK}}^i}\pi_{ij}P_j\right)A_i - (\sum\nolimits_{j\in\mathcal{I}_{\mathcal{UK}}^i}\pi_{ij})P_i\\ &= A_i^T\mathcal{P}_{\mathcal{K}}^iA_i - \pi_{\mathcal{K}}^iP_i + \sum\nolimits_{j\in\mathcal{I}_{\mathcal{UK}}^i}\pi_{ij}\left(A_i^TP_jA_i - P_i\right)\end{aligned}$$

Then, since one always has $\pi_{ij} \geq 0, \forall j \in \mathcal{I}$, it is straightforward that $\Psi_i < 0$ if (18) and (19) hold. Obviously, no knowledge on $\pi_{ij}, \forall j \in \mathcal{I}_{\mathcal{UK}}^i$ is needed in (18) and (19), we can hereby conclude that the system (1) is stochastically stable against the partly unknown transition probabilities (3), which completes the proof. □

Remark 4. Analogous to Remark 2 for continuous-time case, if $\mathcal{I}_{\mathcal{UK}}^i = \emptyset$, $\forall i \in \mathcal{I}$, the conditions are reduced to (7), the classical criterion to check the stochastic stability for the usual discrete-time MJLS. Also, if $\mathcal{I}_{\mathcal{K}}^i = \emptyset$, $\forall i \in \mathcal{I}$, the system becomes a discrete-time switched linear system under arbitrary switching. The conditions (18) and (19) are reduced to $A_i^TP_jA_i - P_i < 0$, which is the criterion obtained in Daafouz et al. [2002] by a switched Lyapunov function approach to guarantee the system is globally uniformly asymptotically stable in discrete-time context.

Now consider the system (2) with control input $u(k)$, the following theorem presents sufficient conditions for the existence of a mode-dependent stabilizing controller with the form (5).

Theorem 6. Consider system (2) with the partly unknown transition probabilities (3). If there exist matrices $X_i > 0$ and Y_i, $\forall i \in \mathcal{I}$ such that

$$\begin{bmatrix} -\mathcal{X}_{\mathcal{K}}^i & \mathcal{L}_{\mathcal{K}}^i(A_iX_i+B_iY_i)\\ * & -\pi_{\mathcal{K}}^iX_i\end{bmatrix} < 0,\ \forall j \in \mathcal{I}_{\mathcal{K}}^i, \quad (20)$$

$$\begin{bmatrix} -X_j & A_iX_i+B_iY_i\\ * & -X_i\end{bmatrix} < 0,\ \forall j \in \mathcal{I}_{\mathcal{UK}}^i, \quad (21)$$

where

$$\mathcal{L}_{\mathcal{K}}^i \triangleq \left[\sqrt{\pi_{i\mathcal{K}_1^i}}I, ..., \sqrt{\pi_{i\mathcal{K}_m^i}}I\right]^T,\ \forall j \in \mathcal{I}_{\mathcal{K}}^i \quad (22)$$

$$\mathcal{X}_{\mathcal{K}}^i \triangleq diag\left[X_{\mathcal{K}_1^i}, ..., X_{\mathcal{K}_m^i}\right],\quad \forall j \in \mathcal{I}_{\mathcal{K}}^i \quad (23)$$

with $\mathcal{K}_1^i$,...,$\mathcal{K}_m^i$ described in (4), then there exists a mode-dependent stabilizing controller of the form (5) such that the resulting system is stochastically stable. Moreover, if the LMIs (20)-(21) have a solution, an admissible controller gain is given by

$$K_i = Y_iX_i^{-1} \quad (24)$$

Proof. First of all, by Theorem 3, we know that system (2) is stochastically stable with the partly unknown transition probabilities (3) if the inequalities (18) and (19) hold. By Schur complement, (18) and (19) are respectively equivalent to:

$$\begin{bmatrix} -P_{\mathcal{K}_1^i} & 0 & \cdots & 0 & \sqrt{\pi_{i\mathcal{K}_1^i}}P_{\mathcal{K}_1^i}A_i\\ * & -P_{\mathcal{K}_2^i} & & \vdots & \sqrt{\pi_{i\mathcal{K}_2^i}}P_{\mathcal{K}_2^i}A_i\\ * & * & \ddots & 0 & \vdots\\ * & * & * & -P_{\mathcal{K}_m^i} & \sqrt{\pi_{i\mathcal{K}_m^i}}P_{\mathcal{K}_m^i}A_i\\ * & * & * & * & -\pi_{\mathcal{K}}^iP_i\end{bmatrix} < 0, \quad (25)$$

$$\begin{bmatrix} -P_j & P_jA_i\\ * & -P_i\end{bmatrix} < 0. \quad (26)$$

Now, consider the system with the control input (5) and replace A_i by $A_i + B_iK_i$ in (25) and (26), respectively. Setting $X_i \triangleq P_i^{-1}$, performing a congruence transformation to (25) by $diag[\mathcal{X}_{\mathcal{K}}^i, X_i]$ and applying the change of variable $Y_i \triangleq K_iX_i$, we can readily obtain (20). Also, (21) can be worked out from (26) in the same way. Therefore, if (20) and (21) hold, (18) and (19) will be satisfied in Theorem 3, i.e. the underlying system is stochastically stable. Moreover, the desired controller gain is given by (24). This completes the proof. □

Remark 5. It is seen from the above theorems that the stochastic stability for the underlying system is actually guaranteed by the two aspects, i.e., efficiently utilizing the partly known transition probabilities (see (8) and (18)), together with some requirements on the latent quadratic Lyapunov function $V_i(x_t,t) = x_t^T P_i x_t, \forall i \in \mathcal{I}$ (respectively, $V_i(x_k,k) = x_k^T P_i x_k, \forall i \in \mathcal{I}$). For continuous-time case, the requirements are $V_j(x_t,t) \le -\dot{V}_i(x_t,t)$, $\forall \left(j \in \mathcal{I}^i_{\mathcal{UK}},\ j \neq i\right)$ and $-\dot{V}_i(x_t,t) \le V_i(x_t,t)$, $\forall (j \in \mathcal{I}^i_{\mathcal{UK}},\ j = i)$ (from (8) and (9) respectively), which implies $\dot{V}_i(x_t,t) < 0$ and $V_j(x_t,t) \le V_i(x_t,t)$. For discrete-time case, the requirements are $\triangle V_i(x_k,k) \triangleq V_i(x_{k+1},k+1) - V_i(x_k,k) < 0, \forall \left(j \in \mathcal{I}^i_{\mathcal{UK}},\ j = i\right)$ and $V_j(x_{k+1},k+1) - V_i(x_k,k) < 0, \forall \left(j \in \mathcal{I}^i_{\mathcal{UK}},\ j \neq i\right)$, which can be easily deduced by (19).

From the development in the above theorems, one can clearly see that our obtained stability and stabilization conditions actually cover the results for the usual MJLS and the switched linear systems under arbitrary switching (all the transition probabilities are unknown). Therefore, the systems considered and corresponding criteria explored in the paper are more general in hybrid systems field.

4. NUMERICAL EXAMPLES

In this section, two numerical examples will be given to show the validness and potential of our developed theoretical results, respectively, in the continuous-time and discrete-time cases.

Example 1. Consider the MJLS (1) with four operation modes and the following data:

$$A_1 = \begin{bmatrix} -0.75 & -0.75 \\ 1.50 & -1.50 \end{bmatrix},\ A_2 = \begin{bmatrix} -0.15 & -0.49 \\ 1.50 & -2.10 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.30 & -0.15 \\ 1.50 & -1.80 \end{bmatrix},\ A_4 = \begin{bmatrix} -0.90 & -0.34 \\ 1.50 & -1.65 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 5 \\ -1 \end{bmatrix},\ B_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix},\ B_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix},\ B_4 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}.$$

The two cases for the transition probabilities matrix are considered in Table 1:

case I

	1	2	3	4
1	−1.3	0.2	?	?
2	?	?	0.3	0.3
3	0.6	?	−1.5	?
4	0.4	?	?	?

case II

	1	2	3	4
1	?	?	0.8	0.3
2	0.3	?	0.3	?
3	?	0.1	−1.5	?
4	?	0.2	?	?

Table 1. Different transition rates matrices.

Our purpose here is to design a mode-dependent stabilizing controller of the form of (5) such that the resulting closed-loop system is stochastically stable with the partly unknown transition rate probabilities (3). By solving (11)-(17) in Theorem 2, the controller gains are solved as:

$$\begin{aligned} \text{Case I: } & K_1 = [\,-0.11\ \ -0.25\,], K_2 = [\,0.02\ \ -1.31\,], \\ & K_3 = [\,-0.81\ \ -0.70\,], K_4 = [\,-0.09\ \ -0.38\,] \\ \text{Case II: } & K_1 = [\,0.20\ \ -0.33\,],\ K_2 = [\,0.12\ \ -1.10\,], \\ & K_3 = [\,-0.62\ \ -0.38\,], K_4 = [\,-0.04\ \ -0.03\,] \end{aligned}$$

Furthermore, applying the above controllers and giving two possible system modes evolution, the state response of the closed-loop system are shown in Figures 1-2 under given initial condition $x_0 = [-1.2\ 0.6]^T$.

Now, the following example gives the verification on the results for the discrete-time counterpart.

Example 2. Consider the MJLS (2) with four operation modes and the following data:

$$A_1 = \begin{bmatrix} 0.32 & -0.40 \\ 0.8 & -0.80 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.08 & -0.26 \\ 0.80 & -1.12 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0.16 & -0.08 \\ 0.80 & -0.96 \end{bmatrix},\ A_4 = \begin{bmatrix} 0.48 & -0.18 \\ 0.80 & -0.88 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix},\ B_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ B_4 = \begin{bmatrix} 0.8 \\ -1 \end{bmatrix}.$$

The two cases of the transition probabilities matrix are considered as in Table 2:

case I

	1	2	3	4
1	0.3	?	0.1	?
2	?	?	0.3	0.2
3	?	0.1	?	0.3
4	0.2	?	?	?

case II

	1	2	3	4
1	0.3	?	?	0.4
2	?	0.2	0.3	?
3	?	?	0.5	0.3
4	?	?	0.1	?

Table 2. Different transition probabilities matrices.

Analogous to the continuous-time case, an admissible controller can be solved by (20)-(24) in Theorem 4 with the following control gains:

$$\begin{aligned} \text{Case I: } & K_1 = [\,-0.28\ 0.32\,], K_2 = [\,0.36\ \ -0.42\,], \\ & K_3 = [\,-0.48\ 0.52\,], K_4 = [\,0.25\ \ -0.45\,] \\ \text{Case II: } & K_1 = [\,-0.21\ 0.24\,], K_2 = [\,0.35\ \ -0.41\,], \\ & K_3 = [\,-0.47\ 0.51\,], K_4 = [\,0.22\ \ -0.42\,] \end{aligned}$$

Figures 3-4 show the state response of the corresponding closed-loop system for given initial condition $x_0 = [-0.3\ 0.4]^T$ under two different modes evolution.

It is seen from the curves in Figures 1-4 that, despite the partly unknown transition probabilities, the designed controllers are feasible and effective ensuring the resulting closed-loop systems are stable, in the continuous-time or in discrete-time cases, respectively.

5. CONCLUSION

The stability and stabilization problems for a class of continuous-time and discrete-time Markovian jump linear system (MJLS) with partly unknown transition probabilities are investigated in this paper. The considered systems are more general than the systems with completely known or completely unknown transition probabilities, which can be viewed as two special cases of the ones we tackled here. The LMI-based stochastic stability and stabilization conditions for the underlying systems are derived for both continuous-time and discrete-time context. Numerical examples are presented to show the validness and applicability of the developed results. As the proposed conditions are LMI based, they can be easily extended to other control or state estimation problems for the underlying systems, such as H_∞ control, filtering, etc.

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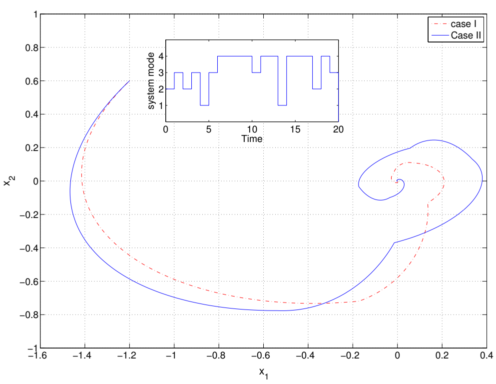

Fig. 1. State response of the closed-loop system under mode evolution r_t^1

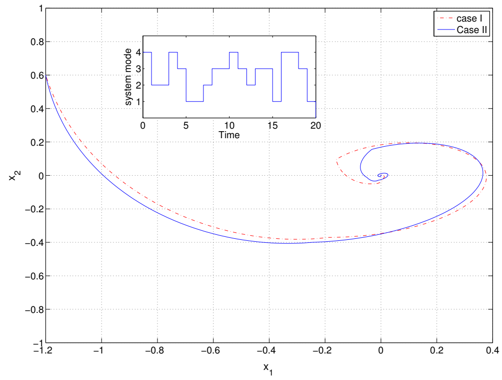

Fig. 2. State response of the closed-loop system under mode evolution r_t^2

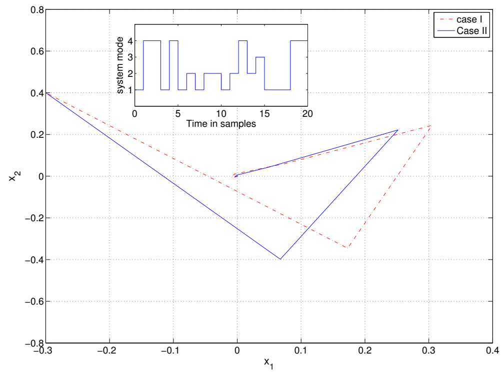

Fig. 3. State response of the closed-loop system under mode evolution r_k^1

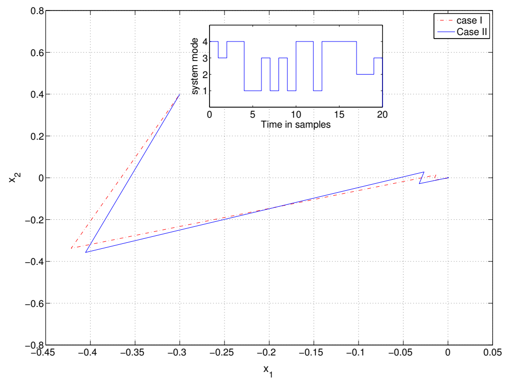

Fig. 4. State response of the closed-loop system under mode evolution r_k^2