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Stable Target Tracking using Observer Based Velocity Estimation

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Abstract: In applications where mobile robots are used to track non-cooperative moving objects it is often required that not only the position but also the velocity of the moving target can be measured. In this paper, we consider a problem in 2D where the tracking robots are equipped only with vision and position sensors and are unable to measure target velocity directly. Instead, two separate observers for target velocity are proposed and shown to stabilize the two tracking controls used by the robots. To evaluate the observers, results from simulations with observer based velocity estimates are compared to corresponding results where the velocity estimates are given by the standard Extended Kalman Filter algorithm.

1. INTRODUCTION

The problem of designing motion controls for mobile robots following either a leader or a moving target is well studied. In Kumar et al. (2002) for instance, leader-following formation controls based on a *unicycle model* are proposed. Other related work, based on the same model, can be found in Beard et al. (2003); Egerstedt and Hu (2001); Kang et al. (2004); Tanner et al. (2004); Sastry et al. (2003); Pappas et al. (2005). A common assumption in much of the work done in this area is that the velocity of the leader/target is directly accessible to the followers, either because the leader robot has some means of communicating its velocity to its followers or because a central processing unit can compute the target velocity using measurements from a distributed network of sensors and send the information back to the individual robots. In many robotic systems this is not the case and a reoccurring problem in robot control is therefore to estimate the velocity of a neighbor using only local sensor data. For example, image based velocity estimation has been subject to much attention within the field of computer vision. Lately, the problem of velocity estimation has received renewed topicality in the development of intelligent safety and navigation system in the automotive industry, described for instance by Chang et al. (2004).

In this paper we consider a leader-follower system where the follower is equipped only with range sensors (for instance IR) and a vision system. No radio communication between the leader and the follower is possible. The goal of the follower is to track the moving leader with a fixed relative distance and bearing angle. Depending on the desired bearing angle, the motion of the follower is decided by one of two control algorithms presented in Section 3. In both algorithms, and in many similar algorithms in literature, the velocity of the target/leader is required as input. The fact that the estimated speed needs to be incorporated into the robot's own motion control makes the velocity estimation more critical. It is well-known that with this sort of feedback in the system, direct computation of the target speed using unfiltered measurements on distance and bearing angle tend to increase measurement errors and induce instability in the system. The standard method for dealing with noisy data in nonlinear systems is Extended Kalman Filtering (EKF). In this paper we, instead, use the fact that the considered system is locally observable and we solve the problem of stabilizing the leader-follower controls by, for each of the two control algorithms, designing a separate observer for target velocity that stabilizes the system.

The paper is organized as follows. In Section 2 and 3 we describe the system and the control algorithms. In Section 4 we treat the subject of target velocity estimation and state the main results on stability. In Section 5 the theoretical results are verified in simulations and, for comparison, the performances of the observer based control algorithms are compared to the corresponding results given by EKF. Finally, in Section 6, the results are discussed and evaluated.

2. THE SYSTEM

In the application studied in this paper, the objective is to control the velocity, v, and angular velocity, ω, of a mobile robot so that it will follow a moving target with a given distance, d_0, and bearing angle, β_0. In the paper we only treat the case with one tracking robot, but the results can easily be extended to multi-agent systems. The robot is assumed to have unicycle dynamics, *i.e.*, it can be modeled as

$$\begin{aligned}\dot{x} &= v\cos\phi \\ \dot{y} &= v\sin\phi \\ \dot{\phi} &= \omega.\end{aligned} \tag{1}$$

The mathematical model for the target is not specified, but it is assumed that the target has a well defined orientation and that the motion of the target is restricted to be along the axis of orientation. It is also assumed that the motion of the target is smooth and that the velocity and angular velocity are bounded.

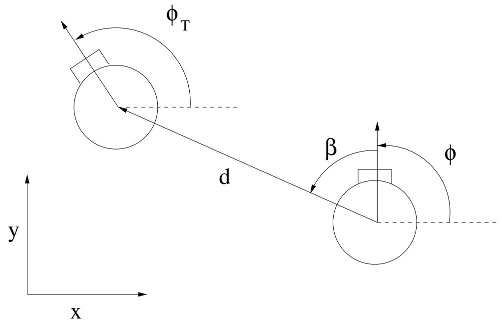

Fig. 1. Mobile robot tracking a moving target.

In order to navigate, the robot completely relies on information obtained from its sensors. In this case, the tracking robot is assumed to be equipped with on-board range sensors, making it possible to measure relative angle and distance to target, and a vision system that enables visual estimation of the difference in orientation between the tracking robot and the target. Orientation estimation based on camera images is a well studied area in computer vision/robotics (see for instance DeMenthon and Davis (1995), Ekvall (2007)) and the reader is therefore referred to literature for further details on this topic.

Let d and β denote the *actual* distance and bearing angle to target, as measured by the sensors, in oppose to d_0 and β_0 which denote the *desired* values on d and β. Then the tracking errors can be defined to be $\Delta d = d - d_0$, $\Delta\beta = \beta - \beta_0$ and $\gamma = \phi - \phi_T$, where γ is the difference in orientation between the tracking robot and the target. The error dynamics for the system are obtained by taking the time derivatives of the tracking errors. By using the connection between global and relative coordinates it is possible to express the error dynamics as a function of angles and distances.

$$\begin{aligned}
\Delta\dot{d} &= -v\cos(\Delta\beta + \beta_0) + v_T\cos(\Delta\beta + \gamma + \beta_0) \\
\dot{\gamma} &= \omega - \omega_T \\
\Delta\dot{\beta} &= -\omega + \tfrac{v\sin(\Delta\beta+\beta_0) - v_T\sin(\Delta\beta+\gamma+\beta_0)}{d_0+\Delta d}
\end{aligned} \tag{2}$$

In the equations above, v_T and ω_T denote the velocity and angular velocity of the target. The objective of the tracking control is to drive the robot as close as possible to the desired state where the tracking errors equals zero, or in other words to stabilize the system (2) in ($\Delta d = 0, \gamma = 0, \Delta\beta = 0$).

3. TRACKING CONTROL

We assume for this application that the desired distance between the tracking robot and the moving target is $d_0 > 0$ and that the desired bearing angle to target is $\beta_0 \in [0, \frac{\pi}{2}]$ rad. (the case $\beta_0 \in [-\frac{\pi}{2}, 0]$ can of course be treated equivalently). To obtain the desired tracking for the whole range of possible values of β_0 we use the two different control algorithms proposed by Gustavi and Hu (2006). For the serial case ($\beta_0 \in [0, \frac{\pi}{2})$) we use a simple proportional controller

$$\begin{aligned}
v &= \frac{c_0 d\cos(\beta - \beta_0) - c_0 d_0 + v_T\cos(\gamma + \beta_0)}{\cos(\beta_0)} \\
\omega &= \frac{c_0 d\sin(\beta) - c_0 d_0\sin(\beta_0) - v_T\sin(\gamma)}{d_0\cos(\beta_0)},
\end{aligned} \tag{3}$$

where $c_0 > 0$. The derivation of (3) is quite straightforward and intuitive and the result is a robust control that globally drives Δd and $\Delta\beta$ to zero. However, there is no upper bound on the magnitude of the control actions as β_0 approaches $\pi/2$. Parallel tracking ($\beta_0 = \frac{\pi}{2}$) is generally more difficult to achieve than serial tracking since it has to be at least partially based on a prediction on the motion of the target. For the parallel case we use the following control

$$\begin{aligned}
v &= v_T + c_1(d - d_0)\cos(\beta) - c_2(\beta - \beta_0) \\
\omega &= c_3(\beta - \beta_0) - c_4(d_0 - d\sin(\beta)) - c_5\gamma,
\end{aligned} \tag{4}$$

where $c_2 \geq 0$, $c_j > 0$, $j = 1, 3, 4, 5$. Because of the different approach, parallel tracking is more sensitive to noise than serial tracking. Also, the velocity of the tracking robot must constantly be adjusted depending on the curvature of the trajectory, even if the velocity of the target is constant. With control (4), $c_2 = 0$ gives sufficiently good tracking if $\omega_T \approx 0$. With more challenging trajectories, stability may be increased by setting $c_2 > 0$.

Implementation of the control algorithms above requires access to measurements or estimates of d, β, γ, and v_T. If the true value of v_T is known, both control algorithms can be shown to stabilize the system (2). In fact, for the serial tracking control (3), access to an estimate of the target speed, v_T, is not necessary for stability, the control is stable even if v_T is set to zero (see Gustavi and Hu (2006)). However, setting $v_T = 0$ results in a static positioning error relative the target, so even a rough estimate of the speed of the target could significantly improve performance of the tracking control. For the parallel tracking control (4), a good estimate of the target speed is essential in case the true v_T is not known.

4. ESTIMATION OF TARGET SPEED

There exist a number of methods that could be used to obtain an estimate of v_T based on sensor output. The most commonly used method for nonlinear systems, such as the one at hand, is probably Extended Kalman Filtering (EKF). EKF is an ad hoc extension of the linear Kalman filter. It is well documented and often gives a good result, but it is not guaranteed to converge and is known to fail sometimes. In this paper we therefore suggest another approach, based on the observability of the system. We show that it is possible to construct observers for target velocity that stabilize the system (2). If a sufficiently good observer can be found, this method for velocity estimation is very efficient as it requires a minimum of computations. In Section 4.2 we propose two observers and state some results on stability, but for comparison we first review the EKF algorithm, and show how it can be applied to this problem.

4.1 State Estimation using EKF

In order to use the EKF algorithm to estimate the velocity of a moving target, v_T, the system equations (2) have to be modified and expressed in discrete time and v_T must be considered as an extra state variable. On the other hand, the state variable γ in (2) is directly measurable by the robot's vision system and not depending on any of the other state variables so it can be considered as a known input to the system. Thus, the modified system (2) now has three state variables; Δd, $\Delta\beta$ and v_T. The system equations for Δd and $\Delta\beta$ are known from (2), but the time dependency of v_T is unknown and therefore modeled as white noise with standard variation b. With

$$\xi_{1,i} = \Delta\beta_i + \beta_0$$
$$\xi_{2,i} = \Delta\beta_i + \beta_0 + u_{3,i},$$

the discrete time system equations for the modified system can be written as

$$\begin{aligned}\Delta d_i &= \Delta d_{i-1} + \Delta t(-u_{1,i}\cos\xi_{1,i} + v_{T,i-1}\cos\xi_{2,i})\\ \Delta\beta_i &= \Delta\beta_{i-1} + \Delta t(-u_{2,i} + \tfrac{u_{1,i}\sin\xi_{1,i} - v_{T,i-1}\sin\xi_{2,i}}{d_0+\Delta d_{i-1}})\\ v_{T,i} &= v_{T,i-1} + bw_i,\end{aligned}$$

with output

$$z_{1,i} = \Delta d_{i-1} + c_1\nu_{1,i}$$
$$z_{2,i} = \Delta\beta_{i-1} + c_2\nu_{2,i}$$

and control input

$$u_{1,i} = v_i$$
$$u_{2,i} = \omega_i$$
$$u_{3,i} = \gamma_i.$$

In the above equations, w, ν_1 and ν_2 represent normalized white noise. The control input $u_3 = \gamma$ is assumed to be measurable, while $u_1 = v$, and $u_2 = \omega$ are computed from the tracking control using measured output from the system and the latest estimate of v_T.

After adapting the system to the required mathematical form, the EKF algorithm can be applied to obtain the desired state estimates (see for instance Anderson and Moore (1979)). The EKF algorithm is not guaranteed to converge, and in this case simulations show that the problem is ill-conditioned. The Kalman gain easily grows to cause large fluctuations in the estimated state vector. Analyzability of the system is complicated by the fact that the control input, u, is implicitly depending on the current state of the system, so the underlying reason for the stability problem is hard to trace. To improve the behavior of the filter, the Kalman gain was multiplied by a factor 0.05. The introduction of a constant factor < 1 improved robustness significantly, but unfortunately also caused an increased convergence time.

4.2 Observer Based Estimation of Target Velocity

If either (3) or (4) is plugged into (2) and v_T is considered as an extra state variable, then one can easily show that the augmented system is locally observable if Δd, γ and $\Delta\beta$ are considered as output. Thus, it is at least possible to design a local, possibly even global, observer for v_T, provided that some assumptions can be made on the target dynamics.

We first consider the case $\beta_0 \in [0, \frac{\pi}{2})$. We know that it is at least possible to design a local observer, but in this case our aim is to design a nonlocal and reduced dimension observer that, together with tracking control (3), stabilizes the error dynamics (2). We propose the following dynamical equation for the observer (with $c_{z1} > 0$):

$$\dot{z}_1 = c_{z1}\cos(\gamma+\beta_0)(d\cos(\beta-\beta_0) - d_0). \tag{5}$$

Theorem 1. Suppose the motion of the target satisfies the following condition:

$$v_T(t) \geq v_0 > 0,\ \dot{v}_T(t) \in L_2[0,\infty),\ \omega_T(t) \in L_2[0,\infty).$$

Then, using control (3) in the system (2) with v_T replaced by the observer (5) and $c_0^2 > c_{z1}/d_0$, we have as $t \to \infty$ globally

$$\Delta d \to 0, \quad \Delta\beta \to 0.$$

Further more, $\gamma \to 0$ from almost everywhere.

Proof. Let the desired position coordinates of the tracking robot be denoted (x_0, y_0) and define

$$x_e = x_0 - x_T$$
$$y_e = y_0 - y_T$$
$$\Delta z_1 = z_1 - v_T,$$

where (x_T, y_T) are the coordinates of the target. Then, after plugging in the control (3), where v_T is replaced by z_1, the error dynamics (2) can be rewritten as

$$\begin{aligned}\dot{x}_e &= -c_0x_e + \Delta z_1\cos\phi_T\\ \dot{y}_e &= -c_0y_e + \Delta z_1\sin\phi_T\\ \Delta\dot{z}_1 &= -c_{z1}\cos(\gamma+\beta_0)[x_e\cos(\phi+\beta_0)\\ &\quad +y_e\sin(\phi+\beta_0)] - \dot{v}_T\\ \dot{\gamma} &= \omega - \omega_T.\end{aligned} \tag{6}$$

Now let

$$\bar{x}_e = \sin\phi_T x_e - \cos\phi_T y_e$$
$$\bar{y}_e = \cos\phi_T x_e + \sin\phi_T y_e.$$

Expressed in new coordinates, system (6) becomes

$$\begin{aligned}\dot{\bar{x}}_e &= -c_0\bar{x}_e + \omega_T\bar{y}_e\\ \dot{\bar{y}}_e &= -c_0\bar{y}_e + \Delta z_1 - \omega_T\bar{x}_e\\ \Delta\dot{z}_1 &= c_{z1}(\tfrac{\sin(2\gamma+2\beta_0)}{2}\bar{x}_e - \cos^2(\gamma+\beta_0)\bar{y}_e) - \dot{v}_T\\ \dot{\gamma} &= \frac{c_0}{d_0\cos(\beta_0)}(\bar{x}_e\cos\gamma + \bar{y}_e\sin\gamma - v_T\sin\gamma) - \omega_T.\end{aligned} \tag{7}$$

The system is defined on $R^3 \times S^1$. When setting $\omega_T = 0$, $\dot{v}_T = 0$, there are two equilibria for the system:

$$(\bar{x}_e, \bar{y}_e, \Delta z_1, \gamma) = (0,0,0,0) \text{ and}$$
$$(\bar{x}_e, \bar{y}_e, \Delta z, \gamma) = (0,0,0,\pi),$$

provided that we define γ to be in the interval $(-\pi, \pi]$. It is easy to show that the first equilibrium is locally exponentially stable and the second one is unstable. We shall now show that the domain of attraction for the first equilibrium is $R^3 \times S^1 \backslash \{0,0,0\} \times \{\pi\}$.

Let us first treat $\gamma(t)$ as a time-varying function in the first three equations of (7). Define the Lyapunov function

$$V = L(\bar{x}_e)^2 + (\bar{y}_e - \frac{1}{c_0}\Delta z_1)^2 + \frac{1}{c_0^2}(\Delta z_1)^2,$$

where $L > 0$ is a constant. If L is chosen to be sufficiently large, it is possible to show that $\dot{V} \le 0$ and that $(\bar{x}_e(t), \bar{y}_e(t), \Delta z_1(t))$ will converge to zero exponentially if, for some constant $a > 0$,

$$\int_0^t \cos^2(\gamma(s) + \beta_0) ds \ge at \tag{8}$$

when t is sufficiently large.

Letting $\dot{V} = 0$ implies that $\bar{x}_e(t) = 0$, $c_0 \bar{y}_e(t) = \Delta z_1(t)$ and $\cos(\gamma(t) + \beta_0)\Delta z_1(t) = 0$. Suppose $\Delta z_1(t) \neq 0$, then it follows from (7) that

$$\begin{aligned} \Delta \dot{z}_1 &= -\frac{c_{z1}}{c_0}\cos^2(\gamma + \beta_0)\Delta z_1 \\ \dot{\gamma} &= \frac{c_0}{d_0 \cos(\beta_0)}(\frac{1}{c_0}\Delta z_1 - v_T)\sin\gamma. \end{aligned} \tag{9}$$

Provided that $v_T > 0$, it is easy to see from (9) that $(\Delta z_1(t), \gamma(t)) \to (0,0)$ as long as $\gamma(0) \neq \pi$. However, if $\gamma(0) = \pi$, then $(\Delta z_1(t), \gamma(t)) \to (0, \pi)$. Thus, the first three state variables in (7), *i.e.*, $(\bar{x}_e(t), \bar{y}_e(t), \Delta z_1(t))$, globally converge to zero exponentially for all values of $\gamma(0)$ if $v_T > 0$. As for γ, it will converge to zero from all initial values except from $\gamma(0) = \pi$.

By the well known classical results on input to state stability (see for example Khalil (1996)), we know that for any given initial condition, $(\bar{x}_e(t), \bar{y}_e(t), \Delta z_1(t))$ will remain to be L_2 if the "inputs" ω_T and $\dot{v}_T$ are L_2, which is our assumption. This implies that Δd and $\Delta\beta$ are L_2. The additional assumption that $v_T(t) \ge v_0 > 0$ assures that $\sin\gamma$ is also L_2. By recursion we can show that in a cascaded system, where robot j is set to follow robot $j-1$, all $\dot{v}_j$, ω_j, $j \ge 1$ are L_2, and $v_j(t) \ge \frac{1}{2}v_0$ when t is sufficiently large.

Now we consider the case $\beta_0 = \frac{\pi}{2}$. In this case we propose an observer that, in combination with tracking control (4), locally stabilizes the error dynamics (2). The new observer is defined by

$$\dot{z}_2 = -c_{z2}(\beta - \beta_0)\cos(\gamma). \tag{10}$$

Theorem 2. Suppose the motion of the target satisfies the following condition:

$$v_T(t) \ge v_0 > 0,\ \dot{v}_T(t) \in L_2[0,\infty),\ \omega_T(t) \in L_2[0,\infty).$$

If the control for the tracking robot is given by (4) with $c_2 = 0$ and with v_T replaced by the observer z_2 defined by eq. (10), then for some choice of constants $c_1, c_3, c_4, c_5 > 0$, the equilibrium $(d, \gamma, \beta) = (d_0, 0, \beta_0)$ is locally exponentially stable.

Proof. Let v_T in control (4) be replaced by the observer z_2. Insert the control (4) in (2), define $\Delta z_2 = z_2 - v_T$ and linearize the system around the equilibrium $(\Delta d, \gamma, \Delta\beta, \Delta z_2) = (0,0,0,0)$. This gives

$$\begin{aligned} \Delta\dot{d} &= -v_T\gamma \\ \dot{\gamma} &= c_4\Delta d - c_5\gamma + c_3\Delta\beta + \omega_T \\ \Delta\dot{\beta} &= -c_4\Delta d + c_5\gamma - c_3\Delta\beta + \frac{1}{d_0}\Delta z_2 + \omega_T \\ \Delta\dot{z}_2 &= -c_{z2}\Delta\beta - \dot{v}_T. \end{aligned} \tag{11}$$

The associated system matrix is

$$A = \begin{bmatrix} 0 & -v_T & 0 & 0 \\ c_4 & -c_5 & c_3 & 0 \\ -c_4 & c_5 & -c_3 & \frac{1}{d_0} \\ 0 & 0 & -c_{z2} & 0 \end{bmatrix}. \tag{12}$$

The matrix A is, in general, time-varying (v_T is time-varying). The following lemma gives us a sufficient condition for uniformly asymptotic stability. The lemma can be proved in the same spirit as that of Theorem 2 in Brockett (1970)(p. 206).

Lemma 3. Considering a time varying matrix A. Assume that A is bounded and the real part of all the eigenvalues of A satisfy $Re\lambda(t) \le \gamma < 0$ for all t, and $\|\dot{A}\| \in L_2(0,\infty)$. Then $\dot{x} = Ax$ is exponentially stable.

By constructing the characteristic polynomial of A and using Routh-Hurwitz criterion, one can show that all eigenvalues of A have negative real part if and only if

$$\frac{c_5 c_{z2}}{(c_3 + c_5)} > v_T c_4 d_0. \tag{13}$$

Thus, if an upper bound on v_T is known, the control parameters can be designed in order to give stable tracking for all possible v_T. Under the given assumptions, we can show recursively (analogous to the previous theorem) that in a cascaded system $\dot{v}_j$, ω_j are in $L_2[0,\infty)$ for all robots $j \ge 1$.

For the case $c_2 > 0$, an inequality corresponding to that of eq. (13) can be derived. This inequality becomes more complex than for the case $c_2 = 0$ and is therefore not as useful in the process of choosing the constants. It can, however, be used to verify stability for a given set of constants. A good rule when choosing the constants is to set $c_1 \approx c_4 \approx c_5$, while c_2 and c_3 should be slightly smaller.

5. SIMULATIONS

The simulations in this section are made in order to evaluate the tracking performance of the control algorithms (3) and (4) when combined with the proposed observers (5) and (10). For comparison, corresponding simulation results for EKF based tracking are also presented. The performances of the observers, in terms of deviations from true target speed, are studied separately.

To simulate measurement errors, white noise was added to the "measurements" of d, β and γ. The noise had standard deviation $0.1d_0$ for distance measurements and $\frac{\pi}{16}$ for angular measurements. Pre-filtering of data is likely to improve the results but was not used here.

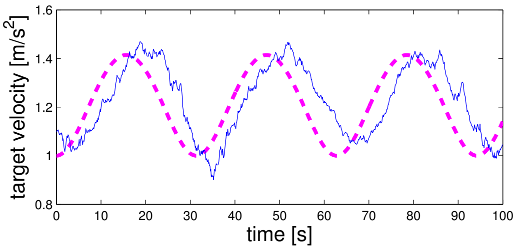

Fig. 3. EKF-estimate of target speed (solid line) and true target speed (dotted line) from a simulation with tracking control (3), $\beta_0 = 0$ and sinusoidal target trajectory.

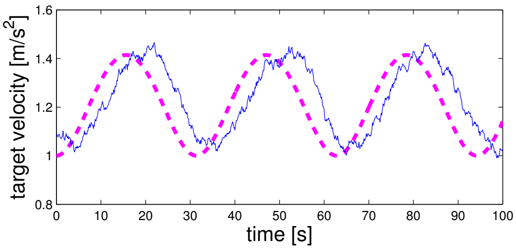

Fig. 4. Observer estimate of target speed (solid line) and true target speed (dotted line) from a simulation with tracking control (3), $\beta_0 = 0$ and sinusoidal target trajectory.

	$\beta_0 = 0$		$\beta_0 = \frac{\pi}{4}$	
	Observer	EKF	Observer	EKF
mean $\Delta d/d_0$	0.0795	0.0805	0.0819	0.0805
std $\Delta d/d_0$	0.1773	0.2680	0.2182	0.1995
mean $\Delta\beta$	-8.814e-04	-6.347e-4	0.0015	0.0036
std $\Delta\beta$	0.0453	0.0461	0.0659	0.0637
mean Δz_1	0.0282	0.0289	0.0322	0.0342
std Δz_1	0.1331	0.1923	0.2042	0.1863

Table 1. Error data obtained from a set of simulations with control (3) running over 25 periods of a sinusoidal reference trajectory.

We first study control (3) and the corresponding observer (5). The allowed reference angles for this control are $\beta_0 \in [0\ \frac{\pi}{2})$. In the simulations, the target was set to follow a sinusoidal path (see fig. 2) with a slowly varying speed. The simulations using the EKF-estimate and the simulations using the observer (5) showed that the positioning and estimation errors were, more or less, of the same magnitude for the two approaches (see table 5). The observer based control showed a slightly better performance at reference angles close to $\beta_0 = 0$ while the EKF based control gave a slightly smaller spread at large values on β_0. Fig. 3 and 4 show the estimated and real target speed for two simulations with $\beta_0 = 0$, using the EKF-approach (fig. 3) and the observer based-approach (fig. 4).

Let us now consider control (4) and the corresponding observer (10). This control is only valid for $\beta_0 = \frac{\pi}{2}$. Contrary to the case $\beta_0 = 0$, the desired speed of the tracking robot in this case is strongly depending on the curvature of the trajectory. Not only does this make tracking more difficult on curved trajectories, it also affects the possibilities to correctly estimate target speed with a simple observer. We shall therefore study the two cases $\omega_T = 0$ and $|\omega_T| > 0$ separately (although convergence can only be shown for $\omega_T \in L_2$).

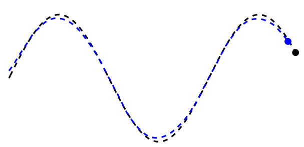

Fig. 2. Target and tracking robot and their trajectories after a simulation with serial tracking control, $\beta_0 = 0$.

In fig. 5 and 6, estimated and real target velocity for the EKF-approach and the observer-approach are shown for two simulations where the target was moving on a straight trajectory ($\omega_T = 0$) with varying speed. Even if velocity estimates are worse for these simulations than for the simulations with $\beta_0 = 0$ they are still sufficiently good to obtain stable tracking. The mean and standard deviation of the positioning errors Δd and $\Delta\beta$ from the simulations corresponding to fig. 5 and 6 are shown in table 5.

Finally we consider the case $\beta_0 = \frac{\pi}{2}$ and $|\omega_T| > 0$. The target was set to move with varying velocity on a sinusoidal trajectory similar to the trajectory shown in fig. 2 but with ω_T restricted to a smaller interval. In this very challenging test, the EKF-estimate fails to reliably detect changes in target velocity and presents an almost random behavior. The observer, on the other hand, clearly shows a periodic behavior. This behavior, however, is depending rather on the shape of the target trajectory than on the target velocity. Additional simulations with constant $\omega_T > 0$ (target turning *away* from the follower) has shown that the observer (10) consequently overestimates target speed, while setting $\omega_T < 0$ (target turning *against* the follower) results in an underestimation of v_T. If (10) is considered mainly as a state estimate for target velocity, then this is indeed a poor result. If, on the other hand, the main purpose of the observer is to stabilize the corresponding tracking control, it turns out that the suggested parallel control (4) in practice works better with the proposed observer than with the true target velocity plugged into the control equations. As long as the curvature of the target trajectory is sufficiently small, the observer based control is quite robust. The EKF based control on the other hand suffers more from the inability to estimate target speed. Compared to the observer based control it shows a significantly increased risk for looping when ω_T

	$\omega_T = 0$		ω_T varying	
	Observer	EKF	Observer	EKF
mean $\Delta d/d_0$	0.1082	0.1079	0.1104	0.1486
std $\Delta d/d_0$	0.1054	0.1154	0.1142	0.1465
mean $\Delta\beta$	-8.2850e-4	0.0387	0.0011	-0.0061
std $\Delta\beta$	0.1423	0.1812	0.1503	0.4321
mean Δz_2	0.0035	0.0292	0.0059	0.0223
std Δz_2	0.0730	0.1390	0.1779	0.1508

Table 2. Error data obtained from a set of simulations with control (4) running over 25 periods of a sinusoidal reference trajectory.

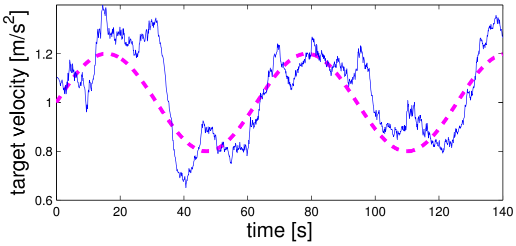

Fig. 5. EKF-estimate of target speed (solid line) and true target speed (dotted line) from a simulation with tracking control (4), $\beta_0 = \frac{\pi}{2}$ and straight target trajectory ($\omega_T = 0$).

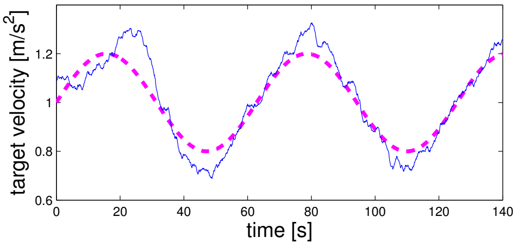

Fig. 6. Observer estimate of target speed (solid line) and true target speed (dotted line) from a simulation with tracking control (4), $\beta_0 = \frac{\pi}{2}$ and straight target trajectory ($\omega_T = 0$).

takes on large values (see fig. 5). In a simulation that ran over 25 periods of the sinusoidal reference trajectory, the follower made 13 loops when using the EKF based control while it avoided making one single loop when using the observer based control during the same period of time.

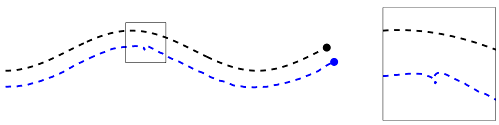

Fig. 7. Target and tracking robot and their trajectories after a simulation with EKF based control (4). Note the loop made by the follower when $|\omega_T|$ is large.

6. CONCLUSIONS

Simulations have shown that both the EKF-approach and the observer-approach produce good estimates of target velocity and give stable tracking with small tracking errors when β_0 is close to zero. As expected, results were not quite as good for the more difficult case $\beta_0 = \frac{\pi}{2}$. As long as the target was following a straight trajectory, both methods were able to produce good estimates of target velocity, but in the simulation with curved trajectories, the estimation of target velocity was poor for both methods. Despite this, the tracking performance of the observer based tracking control was still good as long as the curvature of the reference path was sufficiently small.

If computation time is compared for the observer based approach and the simple EKF-approach used in these simulations, it is found that the observer-approach is more time efficient. With the observer-approach, the new estimate for target speed is computed directly from available sensor data, while in the EKF-method, several computations, including the inverse computation of a matrix, must be made in each step of the algorithm. In real time applications computation time is often critical so this is a strong motivation for using the observer based approach instead of the EKF-approach. Also, stability for the observer based method is not only verified in simulations, but for some special, but important, cases it has been proved mathematically. The results in this paper suggest that, for the application described here, the observer based estimation method would in most cases be the most suitable.

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