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Robust nonlinear controllers for bioprocesses

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Abstract: In this paper, we propose to track a pre-described profile for the substrate concentration inside a continuous and well mixed bioreactor where a single bioreaction takes place. The proposed approach uses a high gain based controller in order to achieve the control objective. Two main characteristics of the proposed controller are worth to be mentioned. The first one lies in the fact that its gain involves a design function that has to satisfy a mild condition which is given. Many expressions of such a function are proposed and it is shown that some of these expressions lead to many variants of sliding mode like controllers. In addition, the proposed controller incorporates a filtered integral action which allows to carry out a robust compensation of the state and output step disturbances, as well as a significant reduction of unavoidable measurements noises. The second main property of the proposed controller lies in the fact that its design does not require any model for the reaction rates. Indeed, the time evolution of these rates are estimated on-line through appropriate nonlinear observers. The so provided estimates are then used by the proposed control scheme. Simulation results are given is order to highlight the performance of the proposed approach.

Keywords: Process control, bioreactor, High gain state feedback control, Sliding mode control, High gain observer.

1. INTRODUCTION

The control of biotechnological processes has been extensively investigated over the last decades. However, the design of controllers for bioreactors has been hampered by important obstacles. One of them is the lack of understanding of microbial metabolic pathways and cellular control mechanisms, which is needed in order to model the process and formulate meaningful process control algorithms so that the final objective of process optimization can be achieved. A way to circumvent this difficulty is to use a mass balance based model: the biological lacks of knowledge are located in dedicated terms, namely the reaction rates.

In this paper, we focus on a standard control problem in continuous bioreactors which consists in tracking a pre-described component concentration (usually the substrate one) using the dilution rate (or equivalently the input flow rate) as the manipulated variable. Among the several approaches proposed for this problem, the most known one is unmistakably the linearizing nonlinear feedback control first proposed in Hoo and Kantor [1986] and later popularized by Bastin and Dochain [1990], Harmand et al. [2006]. However, a major drawback for this approach lies in the fact that the underlying controller requires the knowledge of the reaction rates. To overcome this difficulty, a number of alternative control feedback laws, assuming model uncertainties, have been proposed. Some of these variants use interval observers Rapaport and Harmand [2002], Gouzé et al. [2000] and they assume a partial knowledge of the reaction rates. Other ones do not suppose any analytical expression for the reaction rates. However, these rates are either calculated by differentiating some available measurements Alvarez-Ramirez and Femat [1999], or assumed to be correlated to the available outputs through unknown constants which are updated through appropriate adaptive control laws Mailleret et al. [2004].

The purpose of this work is to propose an efficient and systematic design of a nonlinear controller to track a desired pre-described profile of a component concentration inside a continuous bioreactor. The proposed control scheme consists in a high gain like state feedback controller that has been naturally suggested by judiciously exploring the duality with the high gain observer design proposed in Farza et al. [2005]. Three features of the proposed high gain like control design framework are worth to be emphasized. The first feature consists in the fact that its needs only available measurements without using their differentiations. The second feature is a unified high gain control design framework through an appropriate design function. This allows to recover all commonly used high gain controllers from the usual high gain ones to the sliding modes based ones. The third feature is a high gain based closed loop estimation of the reaction rates to improve the control performances while providing an appropriate monitoring if needed Farza et al. [1998, 2004].

This paper is organized as follows. In the next section, one introduces a typical bioprocess model which shall be used throughout this paper for illustration purposes. In section 3, one considers a class of nonlinear systems including the considered bioprocess model and the design of the proposed state feedback controller is detailed with a full convergence analysis of the tracking error in a free disturbances case. The main characteristics of the proposed controller lies in the fact that its gain involves a design function which specification allows to recover all commonly used high gain controllers from the usual high gain controllers to the sliding modes based ones. Different expressions of this function are proposed in this section. Moreover, a high observer is introduced in order to estimate the reaction rates. The so provided estimates are used in the proposed control scheme. In section 4, the proposed nonlinear controller is used to track a pre-described substrate profile in the bioreactor introduced in section 2. Simulation results are given and discussed throughout this section.

2. A TYPICAL BIOREACTOR MODEL FOR CONTROL PURPOSES

The description of the dynamic behaviour of bioreactors might be fairly complex and involves a large set of algebraic-differential equations. However, for control purposes, a reduced order model that adequately describes the dynamics of the relevant variables, biomass and substrate concentrations, is sufficient. Indeed, let us consider a typical bioprocess dealing with a simple microbial culture involving a single biomass x_2 which is growing by consuming a single substrate x_1. It should be emphasized that the considered process is chosen mainly for its simplicity and illustrative properties. Nevertheless, the reader can refer for example to Bailey and Ollis [1986], Bastin and Dochain [1990] where a panorama of bioprocess models corresponding to simulation and real experiments and to which the entire theory presented here can be applied are described.
The mass balance model of the considered bioprocess can be described as follows:

$$\begin{cases} \dot{x}_2 = r - Dx_2 \\ \dot{x}_1 = -kr + D(S_{in} - x_1) \end{cases} \tag{1}$$

where x_2 and x_1 respectively denote the biomass and substrate concentrations (mg/l), r is the reaction rate $(mg/(l.day))$, S_{in} is the input substrate concentration (mg/l), D (1/day) is the dilution rate and finally k is a yield coefficient. The control objective consists in tracking a pre-described substrate profile $S^\star(t)$. The input variable is the dilution rate. In order to achieve the control objective, one shall synthesize a state feedback controller whose implementation does not require any model for the reaction rate. Moreover, such a controller must incorporate a filtered integral action. The filtering is mainly motivated by measurement noise sensitivity reduction while the integral action allows to achieve a robust offset free performance in the presence of step like disturbances.

To tackle the tracking problem, one introduces the following notations :

- Let $\tilde{x}_1 = x_1 - S^\star$, be the substrate tracking error.
- Let $\tilde{x}_1^f$ be a filtered version of $\tilde{x}_1$ which is obtained as the output of a first order filter with a unitary static gain and a time constant equal to τ, and which entry is $\tilde{x}_1$.
- Let σ^f be the integral of $\tilde{x}_1^f$, i.e. $\dot{\sigma}^f = \tilde{x}_1^f$.

Using the above notations, the original tracking problem can be interpreted as a regulation problem for the tracking error system which can be written as follows:

$$\begin{cases} \dot{\sigma}^f(t) = \tilde{x}_1^f(t) \\ \dot{\tilde{x}}_1^f(t) = -\frac{1}{\tau}\tilde{x}_1^f(t) + \frac{1}{\tau}\tilde{x}_1(t) \\ \dot{\tilde{x}}_1(t) = -\alpha(t) + D(t)(S_{in}(t) - x_1(t)) - \dot{S}^\star(t) \end{cases} \tag{2}$$

where $\alpha = kr$ denotes a normalized reaction rate.
One shall perform a change of variables to bring the equations of the control design model (2) into coordinates that will be easier to work with. Indeed, let $\Phi : \mathbb{R}^3 \to \mathbb{R}^3$, $x = \begin{pmatrix} \sigma^f \\ \tilde{x}_1^f \\ \tilde{x}_1 \end{pmatrix} \mapsto z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \Lambda x$ where

$$\Lambda = diag(1, 1, \frac{1}{\tau}) \tag{3}$$

One can check that the map Φ puts system (2) under the following form:

$$\begin{cases} \dot{z} = Az + B(b(z)u - z^\star(t) + g(t)) + \varphi(z) \\ y = z \end{cases} \tag{4}$$

$$\text{where } A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{5}$$

$$b(z) = \frac{1}{\tau}(S_{in} - S^\star - z_3), \ u = D$$

$$z^\star(t) = \frac{\dot{S}^\star(t)}{\tau}, \ g(t) = -\frac{\alpha(t)}{\tau}, \ \varphi(z) = \begin{pmatrix} 0 \\ -\frac{1}{\tau}z_2 \\ 0 \end{pmatrix}$$

In the next section, one shall synthesize a state feed-back control for a class of nonlinear systems including system (4).

3. HIGH-GAIN FEEDBACK CONTROL LAWS DESIGN

Consider the following strict-feedback form:

$$\begin{cases} \dot{z} = Az + B(b(z)u + g(t) - z^\star(t)) + \varphi(z) \\ y = z \end{cases} \tag{6}$$

with $z = (z_1 \ z_2 \ z_3)^T$, the matrices A and B are as in (5) and φ is a smooth function which assumes a triangular structure with respect to z, i.e. $\varphi(z) = (\varphi_1(z_1) \ \ \varphi_2(z_1, z_2) \ \ \varphi_3(z_1, z_2, z_3))^T$ where the state $z \in \vartheta$ an open subset R^3 and it is assumed to be measured, the input $u \in U$ a connected set of $\mathbb{R}$, $b(z)$ is a non vanishing function on $\mathbb{R}$; $g(t)$ is an unknown function et $z^\star(t)$ is a known signal. The control task we address is the problem of state-feedback regulation for system (6).

The synthesis of the control law is made under the following hypotheses:

$\mathcal{H}1$. the function $b(z)$ does not vanish, i.e.
$$\exists \beta > 0; \forall z \in \mathbb{R}^3 : |b(z)| > \beta$$
$\mathcal{H}2$. The function $\varphi(z)$ is Lipschitz in its arguments over the domain of interest ϑ and one has $\varphi(0) = 0$.

$\mathcal{H}3$. The function $g(t)$ is unknown and its first time derivative is uniformly bounded.

As we shall see later, the expression of the control law we shall propose does depend on the function $g(t)$. However, since the time evolution of such a function is not available, one shall use an observer which provides on-line estimate of such a function. The equations of such observer can be written as follows Farza et al. [1998, 1999]:

$$\left\{ \begin{array}{ll} \dot{\hat{z}}_3 & = \hat{g}(t) + b(z)u(z) - z^\star(t) + \varphi_3(z) - 2\theta(\hat{z}_3 - z_3) \\ \dot{\hat{g}}(t) & = -\theta^2(\hat{z}_3 - z_3) \end{array} \right. \quad (7)$$

where $\hat{z}_3, \hat{g} \in \mathbb{R}$ are the respective estimates of z_3 and g, and $\theta > 0$ is the observer design parameter. The main characteristic of observer (7) is stated by the following Theorem.

Theorem *3.1.* Under hypothesis $\mathcal{H}3$, the trajectories of observer (7) satisfy the following property:

$$\exists \theta_0 > 0; \forall \theta \geq \theta_0; \exists \mu_o(\theta), \eta_o(\theta) > 0 \text{ such that } \forall t \geq 0 :$$

$$|\hat{g}(t) - g(t)| \leq \eta_o(\theta) e^{-\mu_o(\theta)t} |\hat{g}(0) - g(0)| + m\frac{\delta}{\theta} \quad (8)$$

where $m > 0$ is a real constant and δ is the upper bound of the first time derivative of $g(t)$. Moreover, one has: $\lim_{\theta \to \infty} \mu_o(\theta) = +\infty$.

Statement of Theorem 3.1 means that in the case where g is constant, the estimation error converges exponentially to zero. Otherwise, if the first time derivative of $g(t)$ is bounded by a constant δ, the estimation error remains in a ball whose radius can be made as small as desired by choosing θ high enough.
The considered control design framework borrows from the high gain observer design proposed in Farza et al. [2005] thanks to the duality with respect to the high gain observation Gauthier and Kupka [2001], Hammouri and Farza [2003], Farza et al. [2005] . This leads to the following state feedback control law

$$u(z) = \frac{1}{b(z)}(-\hat{g}(t) + z^\star + \nu(z))$$
$$\text{with } \ \nu(z) = -B^T K_c(\bar{S}\Gamma_\lambda z) \quad (9)$$

where Γ_λ is a diagonal matrix defined by
$$\Gamma_\lambda = diag\left(\lambda^3, \lambda^2, \lambda\right) \quad (10)$$

with $\lambda > 0$ a real number, $\bar{S}$ is the unique symmetric positive definite solution of the the following algebraic Lyapunov equation :
$$\bar{S} + A^T\bar{S} + \bar{S}A = \bar{S}BB^T\bar{S} \quad (11)$$

and $K_c : \mathbb{R}^3 \mapsto \mathbb{R}^3$ is a bounded design function satisfying the following property

$$\forall \xi \in \Omega \ one\ has\ \xi^T BB^T K_c(\xi) \ \geq \ \frac{1}{2}\ \xi^T BB^T \xi \quad (12)$$

where Ω is any compact of $\mathbb{R}^3$.
Remark *3.1.* Let $C = B^T$. From the fact that the following algebraic Lyapunov equation
$$S + A^T S + SA = C^T C \quad (13)$$

has a unique Symmetric Positive Definite solution S Gauthier et al. [1992], one can deduce that equation (11) has a unique symmetric positive definite solution $\bar{S}$ which can be expressed as follows

$$\bar{S} = TS^{-1}T \ \ with \ \ T = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad (14)$$

Furthermore and from the expression of $S^{-1}C^T$ given in Farza et al. [2004], one obtains:

$$B^T\bar{S} = CS^{-1}T = [3 \ \ 3 \ \ 1]$$

The control law (9) achieves the control objective as stated by the following theorem.
Theorem *3.2.* The trajectories of (6) under hypotheses $\mathcal{H}1$ - $\mathcal{H}3$ where the input u is given by (9) satisfy the following property:
$$\exists \lambda_0 > 0; \ \forall \lambda \geq \lambda_0; \ \forall \theta \geq \theta_0;$$
$$\exists M > 0; \ \exists \mu_c(\lambda), \eta_1(\theta, \lambda), \eta_2(\lambda) \geq 0,$$
such that $\forall t \geq 0$:

$$\|z(t)\| \leq \lambda^3 \sqrt{\frac{\lambda_{max}(\bar{S})}{\lambda_{min}(\bar{S})}} e^{-\mu_c(\lambda)t} \|z(0)\| + \eta_1(\theta, \lambda)e^{-\mu_o(\lambda)t} + \eta_2(\lambda)\frac{M\delta}{\theta}$$

where $\lambda_{max}(\cdot)$ (resp. $\lambda_{min}(\cdot)$) denotes the largest (resp. smallest) eigenvalue of $(\cdot)$; θ_0, $\mu_o(\theta)$ and δ are given by Theorem 3.1 . Moreover, one has:
$$\lim_{\lambda \to \infty} \mu_c(\lambda) = \lim_{\lambda \to \infty} \mu_o(\lambda) = +\infty \ \text{ and } \ \lim_{\lambda \to \infty} \eta_2(\lambda) = 1$$
Remark *3.2.* Statement of Theorem 3.2 means that: if the function g is constant, $z(t)$ converges globally exponentially to zero for relatively high values of λ. In the case where $\dot{g}(t)$ is bounded, $\|z(t)\|$ can be made aritrarily small by choosing high values of the observer design parameter θ.

Proof of Theorem 3.2: Using (9), system (6) can be written as follows:

$$\dot{z} = Az - BB^T K_c(\bar{S}\Gamma_\lambda z) + \varphi(z) + B\tilde{g}(t)$$

where $\tilde{g}(t) = \hat{g}(t) - g(t)$. Set $\bar{z} = \Gamma_\lambda z$. Since $\Gamma_\lambda A \Gamma_\lambda^{-1} = \lambda A$ and $\Gamma_\lambda B = \lambda B$, one has:

$$\dot{\bar{z}} = \lambda A\bar{z} - \lambda BB^T K_c(\bar{S}\bar{z}) + \Gamma_\lambda\varphi(z) + \lambda B\tilde{g}(t)$$

Now, let $V(\bar{z}) = \bar{z}^T S\bar{z}$ be the candidate Lyapunov function. Using (11), one obtains:

$$\begin{aligned}
\dot{V} &= 2\bar{z}^T\bar{S}\dot{\bar{z}} \\
&= -\lambda V + \lambda\bar{z}^T\bar{S}BB^T\bar{S}\bar{z} - 2\lambda\bar{z}^T\bar{S}BB^T K_c(\bar{S}\bar{z}) \\
&\quad + 2\bar{z}^T\bar{S}\Gamma_\lambda\varphi(z) + 2\lambda\bar{z}^T\bar{S}B\tilde{g}(t) \\
&= -\lambda V - 2\lambda\left(\xi^T BB^T K_c(\xi) - \frac{1}{2}\xi^T BB^T\xi\right) \\
&\quad + 2\bar{z}^T\bar{S}\Gamma_\lambda\varphi(z) + 2\lambda\bar{z}^T\bar{S}B\tilde{g}(t)
\end{aligned}$$

where $\xi = \bar{S}\bar{z}$. Using inequality (12), one obtains

$$\dot{V} \ \leq \ -\lambda V + 2\|\bar{S}\bar{z}\|\left(\|\Gamma_\lambda\varphi(z)\| + \|\lambda B\tilde{g}(t)\|\right) \qquad (15)$$

Since the function φ assumes a triangular structure and is Lipschitz, one can show that for $\lambda \geq 1$:

$$2\|\bar{S}\bar{z}\|\|\Gamma_\lambda\varphi(z)\| \leq c_1 V \qquad (16)$$

where $c_1 > 0$ is a constant which does not depend on λ. Using (8), one obtains:

$$\|\lambda B\tilde{g}(t)\| \leq \lambda\left(\eta_0(\theta)\|\tilde{g}(0)\|e^{-\mu_o(\theta)t} + \frac{m\delta}{\theta}\right)$$

and hence

$$\|2\bar{S}\bar{z}\|\|\lambda B\tilde{g}(t)\| \leq \lambda\left(k_1 e^{-\mu_o(\theta)t} + \frac{k_2\delta}{\theta}\right)\sqrt{V} \qquad (17)$$

where $k_1, k_2 > 0$ are some constants which do not depend on θ, nor λ.
Combining (15), (16) and (17), one gets

$$\dot{V} \leq -(\lambda - c_1)V + \lambda\left(k_1 e^{-\mu_o(\theta)t} + \frac{k_2\delta}{\theta}\right)\sqrt{V} \qquad (18)$$

this implies that for $\lambda \geq c_1$, one has:

$$\begin{aligned}
\sqrt{V(t)} &\leq e^{-\frac{\lambda - c_1}{2}t}\sqrt{V(0)} + \frac{k_1\lambda}{\lambda - c_1 + 2\mu_o(\theta)}e^{-\mu_o(\theta)t} \\
&\quad + \frac{\lambda}{\lambda - c_1}\frac{k_2\delta}{\theta}
\end{aligned}$$

Using the fact that, for $\lambda \geq 1$, one has:

$$\|z(t)\| \leq \|\bar{z}(t)\| \leq \lambda^3\|z(t)\|$$

one obtains:

$$\begin{aligned}
\|z(t)\| &\leq \sqrt{\frac{\lambda_{max}(\bar{S})}{\lambda_{min}(\bar{S})}}\lambda^3 e^{-\frac{\lambda - c_1}{2}t}\|z(0)\| \\
&\quad + \frac{1}{\sqrt{\lambda_{min}(\bar{S})}}\frac{k_1\lambda}{\lambda - c_1 + 2\mu_o(\theta)}e^{-\mu_o(\theta)t} \\
&\quad + \frac{1}{\sqrt{\lambda_{min}(\bar{S})}}\frac{\lambda}{\lambda - c_1}\frac{k_2\delta}{\theta} \qquad (19)
\end{aligned}$$

The parameters $\lambda_0, M, \mu_c, \eta_1$ and η_2 required by Theorem 3.2 are;

$$\lambda_0 = \max(1, c_1); \mu_c(\lambda) = \frac{\lambda - c_1}{2}; \quad M = k_2;$$

$$\eta_1(\theta, \lambda) = \frac{1}{\sqrt{\lambda_{min}(\bar{S})}}\frac{k_1\lambda}{\lambda - c_1 + 2\mu_o(\theta)};$$

$$\eta_2(\lambda) = \frac{1}{\sqrt{\lambda_{min}(\bar{S})}}\frac{\lambda}{\lambda - c_1}$$

Remark *3.3.* Consider the following system:

$$\begin{cases} \dot{x} = Ax + B(b(x)u - x^\star + g(t)) + \varphi(x) \\ y = x \end{cases} \qquad (20)$$

where$A = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ 0 & 0 & 0 \end{pmatrix}$ with $a_1, a_2 \neq 0$ are real constants; $B^T = [0 \ \ 0 \ \ 1]$ and the function φ assumes a triangular structure with respect to x. One can easily show that the corresponding control law (9) is then given by

$$u(x) = \frac{1}{b(x)}(-\hat{g}(t) + x^\star + \nu(x)) \qquad (21)$$

$$\nu(x) = -\frac{1}{a_1a_2}B^T K_c\left(\bar{S}\Gamma_\lambda\Lambda x\right) \text{ and } \Lambda = diag\left(1, a_1, a_1a_2\right)$$

Indeed, let us consider the change of coordinates $z = \Lambda x$. Then, system (20) can be rewritten as follows

$$\begin{cases} \dot{z} \ = \ \Lambda A\Lambda^{-1}z + \Lambda B(b(\Lambda^{-1}z)u - x^\star + g(t)) \\ \quad + \ \Lambda\varphi(\Lambda^{-1}z) \\ y \ = \ x = \Lambda^{-1}z \end{cases} \qquad (22)$$

Taking into account the structure of the the system state realization as well as the transformation matrix, one gets

$$\Lambda A\Lambda^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \Lambda B = \frac{1}{a_1a_2}B$$

One hence recovers the structure of the considered class of systems, i.e. system (6), and naturally deduces the expression of the state feedback control law (21).

3.1 Some particular design functions

The control law involves a gain depending on the bounded design function K_c which is completely characterized by the fundamental property (12). Some useful design functions are given below to emphasize the unifying feature of the proposed high gain concept.

- The usual high gain design function given by
$$K_c(\xi) \ = \ k_c\ \xi \qquad (23)$$
where k_c is a positif scalar satisfying $k_c \ \geq \ \frac{1}{2}$. Notice that the required property is fulfilled over R^3.

- The design function involved in the actual sliding mode framework
$$K_c(\xi) \ = \ k_c\ sign(\xi) \qquad (24)$$
where k_c is a positif scalar and 'sign' is the usual signum function. It is worth mentioning that the required property (12) holds in the case of bounded input bounded state systems. However, this design function induces a chattering phenomena which is by no means suitable in practical situations.

- The design function that are commonly used in the sliding mode practice, namely

$$K_c(\xi) \; = \; k_c \; tanh(k_o \xi) \qquad (25)$$

where $tanh$ denotes the hyperbolic tangent function and k_c and k_o are positif scalars. One can easily shows that the design function (25) satisfies the property (12) for relatively great values of k_o. More particularly, recall that one has $\lim\limits_{k_o \longrightarrow +\infty} tanh(k_o \tilde{z}) = sign(\tilde{z})$.

4. APPLICATION TO BIOREACTOR

In this section, the control scheme described above shall be illustrated through the bioprocess introduced in section 2. In this application, the control objective consists in tracking the following pre-described substrate profile $S^\star$: the substrate concentration has to be maintained at a desired set point which values changes at 175 and 275 days as shown in figure 1 Lopez et al. [2006]. On the basis on the bioreactor model (2) and according to Remark 3.3, the expression of the control law can be written as follows:

$$\begin{cases} D \;\; = \dfrac{1}{S_{in} - x_1} \left(\dot{S^\star}(t) + \hat{\alpha}(t) + \nu(x) \right) \\ \nu(x) \; = \; -\tau [0 \;\; 0 \;\; 1] \; K_c(\bar{S} \Gamma_\lambda \Lambda x) \end{cases} \qquad (26)$$

where the dynamics of $x = \begin{pmatrix} \sigma^f \\ \tilde{x}_1^f \\ \tilde{x}_1 \end{pmatrix}$ is governed by (2) and Λ is given by (3).

In the case where the function K_c specializes as in (25), the expression of ν becomes:

$$\nu(x) = -k_c \tau \tanh(k_o(\lambda^3 \sigma^f + 3\lambda^2 \tilde{x}_1^f + 3\frac{\lambda}{\tau}\tilde{x}_1))$$

Notice that the expression of the dilution rate in (26) does depend on the estimation of the normalized reaction rate $\alpha(t) = kr(t)$. Such estimate is provided by an observer of the form (7) which equations can be written as follows :

$$\begin{cases} \dot{\hat{x}}_1 \;\; = -\hat{\alpha}(t) + D(S_{in} - \hat{x}_1) - 2\theta(\hat{x}_1 - x_1) \\ \dot{\hat{\alpha}}(t) \; = \; \theta^2(\hat{x}_1 - x_1) \end{cases} \qquad (27)$$

where $\hat{x}_1$ and $\hat{\alpha}$ are the respective estimates of x_1 and α. Notice that, for simplicity purposes, observer (27) uses x_1 rather than $\tilde{x}^1 = x_1 - S^\star$ to estimate α.

4.1 Simulation results

Controller (26) has been simulated using the bioreactor balance model (1). However, for simulation purposes, the reaction rate has to be expressed. Indeed, the specific growth rate has been supposed to follow the well known Monod law, i.e.: $r = \dfrac{\mu_{max} x_1 x_2}{K_S + x_1}$ where μ_{max} and K_S respectively denote the maximum specific growth rate and the saturation constant.

In order to simulate practical situations, the measurements of x_1 issued from the model simulation have been corrupted by a measurement noise with a zero mean value and a standard deviation equal to 0.223. Moreover, a step disturbance with a magnitude of 10 $(mg/(l.day))$

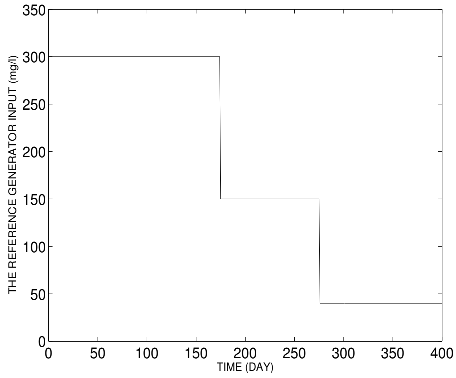

Fig. 1. The reference generator input sequence

intervening in the dynamics of x_1 and occurring in the interval [150 250], has been considered. Several simulation studies have been carried out to emphasize the applicability of the proposed control strategy using all the design functions K_c that have been described above. As the performance were almost comparable, one will present only those obtained with the design function given by expression (25). A particular emphasis has been put on the design parameter specification to deal with the control system performance requirements. A satisfactory shaping of the control system input-output performance has been achieved with the following specifications.

- The parameters involved in the control design have been specified as follows

$$k_c = 1, \; k_o = \; 5 \;, \; \lambda = 5, \; \tau = 500, \; \theta = 10$$

- The desired profile for the substrate has been generated bearing in mind the input saturations as well as the input sequence shown in figure 1. A first order reference generator with unitary static gain and a pole $p = -5$ has been used to provide the substrate reference together with its first time derivative which is required for the control law implementation.

The values of the physical constants used in simulation are:

$$\mu_{max} = 1.064 day^{-1}; \quad K_S = 43.9 mg/day$$
$$k = 2.686 mg/mg$$

Notice that the above three values of the physical parameters as well as the expression of the reaction rate are not known by the controller and they are only used to simulate the bioreactor model in order to generate the substrate pseudo-measurements.

Simulation results are reported in figures 2 to 4. Figures 2 and 4 respectively show the input-output performance of the control system and the observer performance. Since the curves corresponding to the desired and real substrate concentrations, which are given in figure 2 are superimposed, we have reproduced in figure 3 the difference between these curves, i.e. the substrate tracking error. There are three main features that are worth to be mentioned. Firstly, the servo requirements have been achieved by the proposed control approach, namely improved regulation dynamics and a robust offset free performances. Secondly, the input performance are relatively well filtered thanks to the considered design functions. Thirdly, an accurate estimate of the reaction rate has been provided by the nonlinear observer. Such estimate which has been used in the proposed control scheme can also be exploited for engineering monitoring purposes.

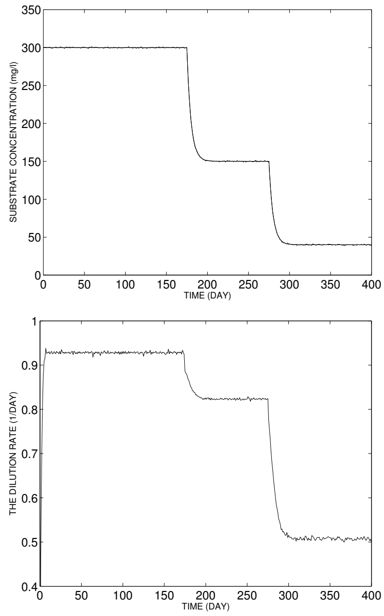

Fig. 2. Output and input time evolution

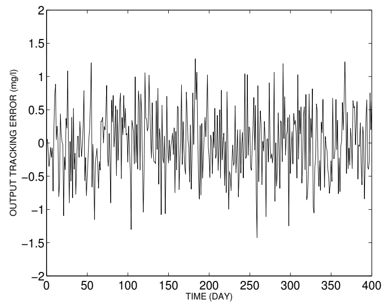

Fig. 3. Output tracking error

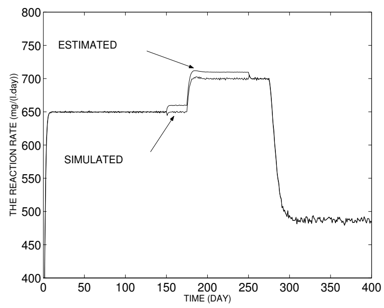

Fig. 4. Estimate of the normalized reaction rate

5. CONCLUSION:

A systematic approach to design a nonlinear controller for the robust tracking of a pre-described substrate concentration in a continuous bioreactor has been presented presented. It has been shown that the control objective can be appropriately handled using a high gain state feedback controller with a filtered integral action. The latter provided a suitable tracking capability and offset-free performance. Of primary importance, an adequate high gain observer has been designed to provide accurate on-line estimates of the reaction rates which were exploited by the controller. The proposed approach can be used for other control objectives such as the tracking of biomass or product pre-described profiles. Numerical simulations were carried out and the obtained results clearly confirmed the good performance of the control scheme proposed.

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