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Observer-Based Residual Generation for Linear Differential-Algebraic Equation Systems ⋆

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Abstract: Residual generation for linear differential-algebraic systems is considered. A new systematic method for observer-based residual generation is presented. The proposed design method places no restrictions on the system to be diagnosed. If the fault of interest can be detected in the system, the output from the design method is a residual generator in state-space form that is sensitive to the fault of interest. The method is iterative and relies only on constant matrix operations such as multiplications, null-space calculations and equivalence transformations, and thereby straightforward to implement. An illustrative numerical example is included, where the design method is applied to a non-observable model of a robot manipulator.

Keywords: FDI, fault diagnosis, residual generation, observer, differential-algebraic equation, unknown input

1. INTRODUCTION

The aim of fault diagnosis is to detect and isolate faults present in a system. With the rising demand for reliability and safety of technical systems, fault diagnosis has become increasingly important. One approach is to generate a set of residuals where different subsets of residuals respond to different subsets of faults. For this reason decoupling of faults in residuals is fundamental. Furthermore, decoupling can also be used to handle disturbances or unknown inputs.

Differential-algebraic equation (DAE) systems, or descriptor systems, are important in the residual generation context since DAE-systems appear in large classes of technical systems like mechanical-, electrical-, and chemical systems. Further, DAE-systems are also the result when using physically based object-oriented modelling tools, e.g. Modelica, Mattson et al. (1998).

For the class of linear state-space systems, residual generation is an extensively studied area. Main approaches are for example the parity-space method, Chow and Willsky (1984), the factorization approach e.g. Frank and Ding (1994), and different observer-based methods, Chen and Patton (1999), Massoumnia et al. (1989), Hou and Müller (1994). For the more general class of linear DAE-systems, the list of previous works is not as extensive but includes parity-space approaches, Sauter et al. (1996), Maquin et al. (1993), parity-space-like approaches, Nyberg and Frisk (2006), Varga (2003), a parametric approach, (Duan et al. (2002)), and several observer-based methods, Hou (2000), Shields (1994), Marx et al. (2003).

Several of the above mentioned residual generation approaches for DAE-systems have limitations since they have restrictions on the system to be diagnosed. The observer-based methods Shields (1994) and Marx et al. (2003), both assumes observability and so does the parity-space method, Sauter et al. (1996). In addition, Marx et al. (2003), does not handle decoupling in the measurement equation. Observability is not assumed in Maquin et al. (1993), but instead decoupling is not considered.

The main contribution in this paper is a new observer-based method for residual generation in linear DAE-systems. In contrast to the above mentioned methods, no restrictions are placed on the system to be diagnosed. This means that if the fault of interest is possible to detect, a residual generator can be designed with the proposed method. The method is based only on constant matrix operations such as multiplications, null-space calculations and equivalence transformations, and thereby straightforward to implement.

The paper is organized as follows. Section 2 presents preliminaries and states the problem formulation and objective. In Section 3, the principles of the design method is preseneted. Section 4 verifies, in two theorems, that the objective is met with the proposed design method. In Section 5 the method is applied to a non-observable DAE model of a robot manipulator and Section 6 concludes the paper. An appendix summarizes the design method as a ready-to-implement algorithm.

2. PRELIMINARIES AND PROBLEM FORMULATION

Consider the linear time-invariant differential-algebraic equation (DAE) system described by

$$E\dot{x} = Ax + Bu + Fd + Hf \tag{1a}$$

$$y = Cx + Du + Gd + Jf \tag{1b}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, $y \in \mathbb{R}^m$, $d \in \mathbb{R}^q$, and $f \in \mathbb{R}^s$ are vectors of the states, inputs, outputs, disturbances, and faults of interest respectively. The inputs and outputs are

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considered as known variables and the states, disturbances and faults as unknowns. The matrix $E \in \mathbb{R}^{k\times n}$ may be singular and the disturbance-vector d consists of faults and unknown inputs that are to be decoupled. The matrices A, B, F, H, C, D, G, and J are all constant real-coefficient matrices of appropriate dimensions.

Before stating the main objective, the notion of fault detectability is needed. First, let $\mathcal{O}_{NF}$ denote the set of all known trajectories u and y consistent with the DAE-system (1) under the presence of no faults, i.e.

$$\mathcal{O}_{NF} = \{[u, y] | \exists x, d; E\dot{x} = Ax + Bu + Fd, \\ y = Cx + Du + Gd\}\,. \tag{2}$$

Note that u, y, x, and d are here considered to be trajectories. In a similiar way, $\mathcal{O}_f$ is defined as the corresponding set when the fault f is allowed to be non-zero. Or formally,

$$\mathcal{O}_f = \{[u, y] | \exists x, d, f; E\dot{x} = Ax + Bu + Fd + Hf, \\ y = Cx + Du + Gd + Jf\}\,. \tag{3}$$

The sets $\mathcal{O}_{NF}$ and $\mathcal{O}_f$ will in the sequel be referred to as observation sets. With $\mathcal{O}_{NF}$ and $\mathcal{O}_f$ defined, fault detectability can now be defined, see also Nyberg and Frisk (2006).

Definition 1. (Fault Detectability). Fault f is detectable in (1) if $\mathcal{O}_f \not\subseteq \mathcal{O}_{NF}$.

It may be noted that fault detectability is a system property.

To check if given trajectories of u and y belongs to the observation set $\mathcal{O}_{NF}$ or not, i.e if a fault is present in the system, residuals can be used. In this work, only residuals that are outputs from state-space systems are considered, leading to the following definition.

Definition 2. (Residual Generator). The linear time-invariant state-space system

$$\dot{\xi} = \bar{A}\xi + \bar{B}u + \bar{M}y \tag{4a}$$
$$r = \bar{C}\xi + \bar{D}u + \bar{N}y \tag{4b}$$

is a residual generator for (1) and r is a residual if

$$[u, y] \in \mathcal{O}_{NF} \Rightarrow \lim_{t\to\infty} r = 0. \tag{5}$$

Note that r may here be multi-dimensional.

The problem can now be formulated as follows. Given the system (1), where it is assumed that the fault f is detectable, the objective is to create a residual generator for (1) where the residual is sensitive to f, that is, the transfer function from fault to residual is non-zero.

3. PRINCIPLES OF THE DESIGN METHOD

As stated in the problem formulation, the input to the design method is assumed to be a DAE-system on the form (1), where f is detectable. The design method consists of two main parts. First, a system in state-space form with no disturbances present is extracted from the input system. This is done iteratively, where disurbances are decoupled and the dimension of the system is reduced in each step. Second, a residual generator based on the decoupled system is designed. The principles of the design method are presented below.

Step 1: Write the system on the form

$$\begin{bmatrix} E \\ 0 \end{bmatrix} \dot{x} = \begin{bmatrix} A \\ C \end{bmatrix} x + \begin{bmatrix} B \\ D \end{bmatrix} u + \begin{bmatrix} M \\ N \end{bmatrix} y + \\ \begin{bmatrix} F \\ G \end{bmatrix} d + \begin{bmatrix} H \\ J \end{bmatrix} f \tag{6}$$

Step 2: Let

$$r = \text{rank} \begin{bmatrix} F \\ G \end{bmatrix}, \tag{7}$$

and

$$P = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}, \tag{8}$$

with $P_1 \in \mathbb{R}^{(k+m-r)\times k}$, $P_2 \in \mathbb{R}^{(k+m-r)\times m}$, $P_3 \in \mathbb{R}^{r\times k}$, $P_4 \in \mathbb{R}^{r\times m}$ chosen such that the rows of $[\,P_1\ P_2\,]$ form a basis for the left null-space of $\begin{bmatrix} F \\ G \end{bmatrix}$, and the rows of $[\,P_3\ P_4\,]$ form a basis for the image of $\begin{bmatrix} F \\ G \end{bmatrix}$. This implies that

$$\text{rank } P = k + m, \tag{9}$$
$$P_1F + P_2G = 0, \tag{10}$$
$$\text{rank } (P_3F + P_4G) = r. \tag{11}$$

Step 3: Pre-multiply (6) with the full-rank matrix P. Since (10) holds, the result becomes

$$P_1E\dot{x} = (P_1A + P_2C)x + (P_1B + P_2D)u + \\ (P_1M + P_2N)y + (P_1H + P_2J)f \tag{12a}$$
$$P_3E\dot{x} = (P_3A + P_4C)x + (P_3B + P_4D)u + \\ (P_3M + P_4N)y + (P_3F + P_4G)d + \\ (P_3H + P_4J)f. \tag{12b}$$

Step 4: Due to (11), the matrix $(P_3F + P_4G)$ has full row-rank. Therefore, (12b) does not contain any usable information and is discarded.

Step 5: Let t = rank (P_1E). If $t = n$, go to step 8, otherwise continue to step 6.

Step 6: Find, by e.g. singular-value decomposition, non-singular matrices U and V such that

$$U\,(P_1E)\,V = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix}, \tag{13}$$

where $\Sigma \in \mathbb{R}^{t\times t}$ is a non-singular matrix.

Step 7: Pre-multiply (12a) with U, then introduce the non-singular state-transformation

$$w = V^{-1}x, \quad w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \tag{14}$$

where $w_1 \in \mathbb{R}^t$ and $w_2 \in \mathbb{R}^{(n-t)}$ to obtain

$$\begin{bmatrix} \Sigma \\ 0 \end{bmatrix} \dot{w}_1 = \begin{bmatrix} A_1 \\ A_3 \end{bmatrix} w_1 + \begin{bmatrix} A_2 \\ A_4 \end{bmatrix} w_2 + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u + \\ \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} y + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} f, \tag{15}$$

where

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} = U(P_1A + P_2C)V, \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = U(P_1B + P_2D), \\ \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = U(P_1M + P_2N), \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = U(P_1H + P_2J), \tag{16}$$

and $A_1 \in \mathbb{R}^{t\times t}$, $A_4 \in \mathbb{R}^{(k+m-r-t)\times(n-t)}$, $B_1 \in \mathbb{R}^{t\times p}$, $M_1 \in \mathbb{R}^{t\times m}$, and $H_1 \in \mathbb{R}^{t\times s}$. The variable w_2 is now seen as a disturbance, and hence the system (15) is on the same form as (6). Return to step 1 with the system (15) as input.

Step 8: Find a non-singular matrix U such that

$$U(P_1E) = \begin{bmatrix} \Pi \\ 0 \end{bmatrix}, \tag{17}$$

where $\Pi \in \mathbb{R}^{n\times n}$ is non-singular.

Step 9: Pre-multiply (12a) with U, then multiply the dynamic part of the result with Π^{-1}, to obtain

$$\dot{x} = \bar{A}_1 x + \bar{B}_1 u + \bar{M}_1 y + \bar{H}_1 f \tag{18a}$$

$$0 = A_2 x + B_2 u + M_2 y + H_2 f \tag{18b}$$

where

$$\bar{A}_1 = \Pi^{-1}A_1, \quad \bar{B}_1 = \Pi^{-1}B_1,$$
$$\bar{M}_1 = \Pi^{-1}M_1, \quad \bar{H}_1 = \Pi^{-1}H_1,$$
$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = U(P_1A + P_2C), \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = U(P_1B + P_2D),$$
$$\begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = U(P_1H + P_2J), \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = U(P_1M + P_2N). \tag{19}$$

Step 10: Find a matrix $L \in \mathbb{R}^{n\times m}$ such that all eigenvalues of the matrix $(\bar{A}_1 + LA_2)$ have negative real-parts. Pre-multiply (18) with the non-singular matrix

$$Q = \begin{bmatrix} I & L \\ 0 & I \end{bmatrix} \tag{20}$$

to obtain

$$\dot{x} = (\bar{A}_1 + LA_2)x + (\bar{B}_1 + LB_2)u + (\bar{M}_1 + LM_2)y + (\bar{H}_1 + LH_2)f \tag{21a}$$

$$0 = A_2 x + B_2 u + M_2 y + H_2 f. \tag{21b}$$

Step 11: Design the residual generator as

$$\dot{\xi} = (\bar{A}_1 + LA_2)\xi + (\bar{B}_1 + LB_2)u + (\bar{M}_1 + LM_2)y \tag{22a}$$

$$r = A_2\xi + B_2 u + M_2 y. \tag{22b}$$

The design method is summarized as a ready-to-implement algorithm in Appendix A.

Remark 3. Step 10 requires that (18) is observable or at least detectable, see e.g. Rugh (1996). If this is not the case, the canonical structure theorem, e.g. Gilbert (1963), can be used to extract the observable subsystem from (18), which instead is used in step 10.

Remark 4. The states ξ in (22) is actually an estimate of a linear combination of the states x in (1), and (22) is sometimes referred to as a FDI (Fault Detection and Isolation) observer, see e.g. Hou and Müller (1994). It may also be noted that observer-based residual generation has strong connections with the design of unknown-input observers, see e.g. Müller and Hou (1994) for state-space systems and e.g. Sun and Cheng (2004) for DAE-systems. The aim in these works is to estimate the states of the system and not generate a residual suitable for fault detection. However, if an observer for a system can be designed, a residual can be created as the difference between measurements and estimated states.

Remark 5. Throughout this work, it assumed that the system to be diagnosed is a DAE-system and the design method is described in this framework. Still, the method can likewise be applied to a state-space system, i.e. a system where $E = I$.

4. CORRECTNESS OF THE DESIGN METHOD

In this section it is verified that the objective stated in Section 2 is met. That is, that the output from the proposed design method is a residual generator for (1), and that the corresponding residual is sensitive to the fault f.

Since the design method (or algorithm, as in Appendix A) is iterative, the following result is needed.

Lemma 6. With (1) as input, the design method terminates.

Proof. The system (1) has $k + m$ equations. In step 4 at least one equation is removed. Since $t \geq 0$ in step 5, the algorithm will terminate, if not earlier, after at most $k + m$ iterations when the remaining system is of zero dimension. □

4.1 Residual Generator Property

The output from the design method is (22) which is based on the system (21), obviously different from (1). A key property for (22) to be a residual generator for (1) is that the systems (1) and (21) have equal observations sets. This means that designing a residual generator for (1) is equivalent to designing a residual generator for (21). This property is the result of the following lemma.

Lemma 7. Let (1) be the input to the design method, $\mathcal{O}_{NF}$ defined by (2) and $\mathcal{O}_f$ by (3). Let $\mathcal{O}'_{NF}$ be the set of trajectories u and y consistent with (21) when $f = 0$ and $\mathcal{O}'_f$ the corresponding sets when f is allowed to be non-zero. It holds that $\mathcal{O}_{NF} = \mathcal{O}'_{NF}$ and $\mathcal{O}_f = \mathcal{O}'_f$.

Proof. Given (1) as input to the design method, Lemma 6 states that the method will terminate. Two scenarios of execution of the steps 1 to 11 are possible. Either steps 1-5 followed by steps 8-11 is performed directly, else steps 1-7 will be iterated until the condition in step 5 holds, and then steps 8-11 will be performed. In both cases, steps 3, 9, and 10 consist of multiplication with non-singular matrices and does not change the sets $\mathcal{O}_f$ and $\mathcal{O}_{NF}$ in any of the execution cases. The same holds for step 7 in the second case. Hence, the critical part is step 4, where equation (12b) is discarded. For the first execution case it must be shown that the observation sets are equal for (12) and (15) and for the second case that the same holds for (12) and (18). Or in other words for both cases, that (12b) can be discarded without loosing any usable information. Here, the second case is considered and the first case can be shown in the same manner. Since it is trivial that the observation sets for (12) are subsets of the observation sets for (18), only the reverse inclusion is shown. Let $\tilde{x}$, $\tilde{u}$, and $\tilde{y}$ be trajectiories satisfying (18) when $f = 0$. Since (12a) and (18) are related by a non-singular transformation, $\tilde{x}$, $\tilde{u}$, and $\tilde{y}$ also satisfies (12a). As a consequence of step 2, (11) holds. This implies that the matrix $(P_3F + P_4G)$ has full row-rank and hence the matrix has a right-inverse. Denote this right-inverse R and choose

$$\tilde{d} = RP_3E\dot{\tilde{x}} - R(P_3A + P_4C)\tilde{x} - R(P_3B + P_4D)\tilde{u} - R(P_3M + P_4N)\tilde{y}. \tag{23}$$

With $\tilde{d}$ and the previously defined $\tilde{x}$, $\tilde{u}$, and $\tilde{y}$, the equation (12b) is satisfied. This shows that (12b) does not contain any usable information and can be discraded. The reasoning can be repeated for the case when f is allowed to be non-zero to show that the observation sets are equal for (12) and (18), which completes the proof. □

With help of Lemma 7, it can be shown that the first part of the stated objective is met.

Theorem 8. Let (1) be the input to the design method and (22) the output. The system (22) is a residual generator for (1) and r in (22b) is a residual.

Proof. Assume $f = 0$ and let $[u, y] \in \mathcal{O}_{NF}$, where $\mathcal{O}_{NF}$ is the set defined in (2). Lemma 7 then implies that u and y also satisfy (21). By subtracting (21a) from (22a) and (21b) from (22b) the autonomous system

$$\dot{\lambda} = (\bar{A}_1 + LA_2)\lambda \tag{24a}$$
$$r = A_2\lambda, \tag{24b}$$

is obtained, where $\lambda = \xi - x$. Since, according to step 10, the matrix L is chosen such that all eigenvalues of $(\bar{A}_1 + LA_2)$ have negative real-parts, it follows directly that $\lim_{t\to\infty} r = 0$ and hence (22) is a residual generator for (1) and (22b) is a residual. □

4.2 Fault Sensitivity

The aim of this section is to show that the residual generator (22) is sensitive to the fault f, i.e. that the transfer function from f to the residual r is non-zero. However, the residual generator (22) is written on a form without faults. By again, as in the proof to Theorem 8, subtracting (21a) from (22a) and (21b) from (22b), the relation between f and r can be described as

$$\dot{\lambda} = (\bar{A}_1 + LA_2)\lambda - (\bar{H}_1 + LH_2)f \tag{25a}$$
$$r = A_2\lambda - H_2 f, \tag{25b}$$

where $\lambda = \xi - x$.

From (25), the transfer function from fault to residual can be written as

$$G_{rf}(s) = A_2\left(-sI + \bar{A}_1 + LA_2\right)^{-1}\left(\bar{H}_1 + LH_2\right) - H_2. \tag{26}$$

The result that verifies that the second part of the objective is met with the design method here follows.

Theorem 9. Let (1) be the input to the design method and (25) the output. If f is detectable in (1), the transfer function from fault to residual (26) is non-zero.

Proof. The transfer function (26) can by power-series expansion of $\left(-sI + \bar{A}_1 + LA_2\right)^{-1}$ be written as

$$G_{rf}(s) = A_2\left(-sI + \bar{A}_1 + LA_2\right)^{-1}\left(\bar{H}_1 + LH_2\right) - H_2 = -\sum_{i=1}^{\infty} A_2\left(\bar{A}_1 + LA_2\right)^{i-1}\left(\bar{H}_1 + LH_2\right)s^{-i} - H_2. \tag{27}$$

To show the contrary of the claim, i.e. that $G_{rf}(s) = 0$ implies $\mathcal{O}_f \subseteq \mathcal{O}_{NF}$, assume $G_{rf}(s) = 0$. Using (27), $G_{rf}(s) = 0$ is equivalent to

$$H_2 = 0, \tag{28}$$
$$A_2\left(\bar{A}_1 + LA_2\right)^{i-1}\left(\bar{H}_1 + LH_2\right) = 0, \quad i = 1, \ldots, \infty. \tag{29}$$

As a consequence of the Cayley-Hamilton theorem, $A_2\left(\bar{A}_1 + LA_2\right)^{i-1}$, for $i \geq n+1$, can be written as a linear combination of

$$A_2, A_2\left(\bar{A}_1 + LA_2\right), \ldots, A_2\left(\bar{A}_1 + LA_2\right)^{n-1}, \tag{30}$$

therefore it is sufficient to consider the matrix

$$\Omega = \begin{bmatrix} A_2 \\ A_2\left(\bar{A}_1 + LA_2\right) \\ \vdots \\ A_2\left(\bar{A}_1 + LA_2\right)^{n-1} \end{bmatrix}. \tag{31}$$

The condition (29) clearly implies $\left(\bar{H}_1 + LH_2\right) \in \operatorname{Ker}\Omega$ and the two cases $\operatorname{rank}\Omega = n$ and $\operatorname{rank}\Omega < n$ will now be studied separately.

For the first case, i.e. when (22) and (25) are both observable, $\dim \operatorname{Ker}\Omega = 0$ which implies $\left(\bar{H}_1 + LH_2\right) = 0$. From (28), $H_2 = 0$ and it must hold that $\mathcal{O}_f = \mathcal{O}_{NF}$.

For the second case, let $[\tilde{u}, \tilde{y}] \in \mathcal{O}_f$. This means that there exist trajectories, say $\tilde{f}$ and $\tilde{x}$ with $\tilde{x}(t_0) = \tilde{x}_0$, such that

$$\dot{\tilde{x}} = (\bar{A}_1 + LA_2)\tilde{x} + (\bar{B}_1 + LB_2)\tilde{u} + (\bar{M}_1 + LM_2)\tilde{y} + (\bar{H}_1 + LH_2)\tilde{f} \tag{32a}$$
$$0 = A_2\tilde{x} + B_2\tilde{u} + M_2\tilde{y} + H_2\tilde{f}. \tag{32b}$$

It will now be shown that there exists a trajectory ζ that along with the trajectories $\tilde{u}$ and $\tilde{y}$ satisifies (32) when $\tilde{f} = 0$. Consider the residual generator (25) and let $\tilde{\lambda}(t_0) = \tilde{\lambda}_0 \in \operatorname{Ker}\Omega$. This implies that $\tilde{\lambda}$ will be a trajectory in the non-empty unobservable subspace of (25). Evaluation of (25) with the initial state $\tilde{\lambda}_0$, together with (28) and (29), yields $r \equiv 0$ independent of f. In particular, this holds for $f = \tilde{f}$ and hence

$$\dot{\tilde{\lambda}} = (\bar{A}_1 + LA_2)\tilde{\lambda} - (\bar{H}_1 + LH_2)\tilde{f} \tag{33a}$$
$$0 = A_2\tilde{\lambda} - H_2\tilde{f}. \tag{33b}$$

Now form $\zeta = \tilde{x} + \tilde{\lambda}$, $\zeta(t_0) = \tilde{x}_0 + \tilde{\lambda}_0$, and combine (32) with (33) to obtain

$$\dot{\zeta} = (\bar{A}_1 + LA_2)\zeta + (\bar{B}_1 + LB_2)\tilde{u} + (\bar{M}_1 + LM_2)\tilde{y} \tag{34a}$$
$$0 = A_2\zeta + B_2\tilde{u} + M_2\tilde{y}. \tag{34b}$$

Thus, there exists a trajectory ζ satisfying the fault-free system (34) so that $[\tilde{u}, \tilde{y}] \in \mathcal{O}_{NF}$, implying $\mathcal{O}_f \subseteq \mathcal{O}_{NF}$ and the proof is complete. □

Remark 10. The two systems (22) and (25) are two ways of writing a residual generator. The form (22) is the so called computational form, and (25) is usually referred to as internal form.

5. EXAMPLE

To illustrate the design method, it is applied to a DAE model of a three-link planar manipulator from Hou (2000) and Hou and Müller (1996), see Figure 1. The objective of the manipulator is to apply a constant horizontal force in the region between point A and B, e.g. for cleaning the region. The manipulator consists of an end-effector, three rods, and three joints. Via actuators at every joint, a torque can be applied to move the effector repeatedly between A and B. The manipulator is equipped with four sensors measuring the height of the end-effector, the contact force in the horizontal direction, and tracking signals.

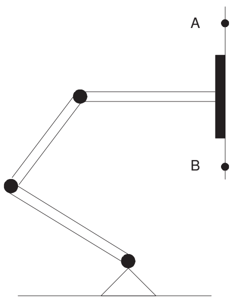


Fig. 1. The three-link planar manipulator

The DAE model has three states for the Cartesian coordinates of the end-effector, three states for the derivatives of the Cartesian coordinates, two states for Lagrangian multipliers, and three states for the controller, altogether 11 states. In this example, the original fault model has been extended with a sensor fault. The process is subjected to 3 faults. Fault f_1 represents a fault in actuator 1, f_2 a fault in the tracking reference signal, and f_3 a fault in sensor 4. Hence, the form of the DAE is

$$E\dot{x} = Ax + Bu + Hf \tag{35a}$$
$$y = Cx + Jf, \tag{35b}$$

where $x \in \mathbb{R}^{11}$, $u \in \mathbb{R}^3$, $y \in \mathbb{R}^4$, and $f \in \mathbb{R}^3$. Numerical values of the matrices E, A, B, and C can be found in Hou (2000) or Hou and Müller (1996). The matrix E is square with rank $E = 9$ and (35) is regular. Further, the system (35) is not impulse observable (Dai (1989)), since

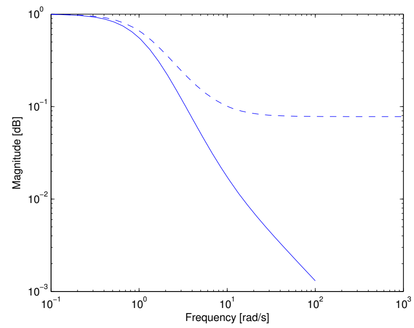
$$\text{rank} \begin{bmatrix} E & A \\ 0 & E \\ 0 & C \end{bmatrix} = 18 \neq \text{rank } E + n = 9 + 11 = 20. \tag{36}$$

This means that methods assuming observability, for example Shields (1994), Marx et al. (2003), and Sauter et al. (1996), can not be applied to the system.

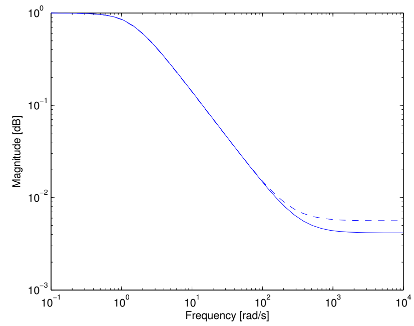
The design objective is to create three residual generators for (35), each monitoring one fault. In residual generator 1, the transfer function from fault f_1 should be non-zero, and the same should hold for fault f_2 in residual generator 2 and for fault f_3 in residual generator 3. Since each residual generator should monitor only one fault, two faults need to be decoupled in each residual generator. This means that f_2 and f_3 are seen as disturbances in residual generator 1 and the matrix $F_1 = [\, H_2 \; H_3 \,]$, and $G_1 = [\, J_2 \; J_3 \,]$ can be formed, where H_i and J_i denotes the i:th column of the matrices H and J respectively. In the same way the matrices F_2, G_2, F_3, and G_3, with the columns from H and J corresponding to the faults to be decoupled in each residual generator, are created.

Performing the design according to the method in Section 3 with the three different configurations of system (35) as input, three disturbance decoupled systems on the form (18) with 6, 5, and 5 states respectively are obtained. For all three input systems, the algorithm terminates after 3 iterations. The three systems are all observable, and hence it is straightforward to perform step 10. For all three residual genereators, the poles are placed in -1.

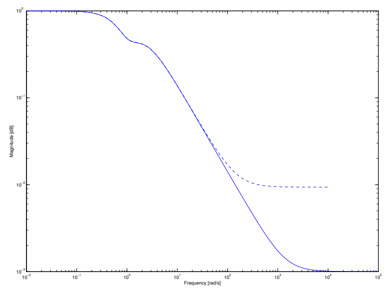
All three residual generators have two-dimensional residuals. By calculating the transfer functions from fault to residual for each residual generator, it can be verified that $G_{rf_i}(s) = 0$, when $i = 2, 3$ for residual generator 1, when $i = 1, 3$ for residual generator 2, and when $i = 1, 2$ for residual generator 3. To verify that the design objective is met, the transfer functions from the monitored faults to the residual for each residual generator is shown in Figures 2(a), 2(b), and 2(c). It is clear that all transfer functions are non-zero.



(a) Transfer functions from fault f_1 to the two residuals in residual generator 1



(b) Transfer functions from fault f_2 to the two residuals in residual generator 2



(c) Transfer functions from fault f_3 to the two residuals in residual generator 3

Fig. 2. Transfer functions from monitored faults to residuals in the obtained residual generators

6. CONCLUSIONS

Residual generation for linear DAE-systems has been considered. A new systematic method for observer-based residual generation has been presented. In contrast to several previous methods, no restrictions such as observability is placed on the system to be diagnosed. This means that if the fault of interest is detectable in the system to be diagnosed, a residual generator can be designed with the design method in this paper. It has been verified in Theorem 8 and 9 that the output from the design method is indeed a residual generator, and that the corresponding transfer function from fault to residual is non-zero. Finally note that even though the design method has been described in the framework of DAE-systems, it can likewise be applied to state-space systems.

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Appendix A. DESIGN ALGORITHM

The design method is here summarized as an algorithm in which the following functions has been used

- `null` computes a basis for the null-space of a matrix.
- `svd` performs a singular-value decomposition.
- `stabilize` computes a feedback gain such that all eigenvalues of the resulting matrix have negative real-parts.

Algorithm 1

Input: Matrices E, A, B, F, H, C, D, G, and J corresponding to a system on the form (1) where $E \in \mathbb{R}^{k\times n}$.

Output: Matrices $\bar{A}$, $\bar{B}$,$\bar{M}$, $\bar{C}$, $\bar{D}$, and $\bar{N}$ corresponding to a residual generator on the form (4).

$N := I$

$t := 0$

while $t \neq n$ **do**

$[\, P_1 \; P_2 \,] := \texttt{null}(\begin{bmatrix} F \\ G \end{bmatrix}^T)^T$

$t := \text{rank } P_1 E$

$(U, \Sigma, V) := \texttt{svd}(P_1 E)$

$\begin{bmatrix} B \\ D \end{bmatrix} := U(P_1 B + P_2 D)$

$\begin{bmatrix} H \\ J \end{bmatrix} := U(P_1 H + P_2 J)$

$\begin{bmatrix} M \\ N \end{bmatrix} := U(P_1 M + P_2 N)$

if $t \neq n$ **then**

$E := \Sigma$

$\begin{bmatrix} A & F \\ C & G \end{bmatrix} := U(P_1 A + P_2 C)V$

end if

end while

$\begin{bmatrix} A \\ C \end{bmatrix} := U(P_1 A + P_2 C)$

$L := \texttt{stabilize}(\Sigma^{-1} A, C)$

$\bar{A} := \Sigma^{-1} A + LC$

$\bar{B} := \Sigma^{-1} B + LD$

$\bar{M} := \Sigma^{-1} M + LN$

$\bar{C} := C$

$\bar{D} := D$

$\bar{N} := N$