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Active fault detection and dual control in multiple model framework ⋆

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Abstract: The paper deals with active fault detection and dual control in the multiple model framework. A monitored and controlled system is described by a discrete-time linear stochastic model at each step of a finite time horizon. The model belongs to an a priori given set of models, and known transient probabilities describe switching between the models. The goal is to design an active detector and controller that processes all available information and generates decisions and inputs. The decisions inform whether a fault has occurred in the system, and the inputs should simultaneously control and excite the system. As the control is in conflict with the excitation, the dual control problem arises. It is shown that both active fault detection and dual control can be solved using Bellman's principle of optimality, and a corresponding backward recursive equation is derived. The approximative solution of the backward recursive equation is discussed, and an algorithm based on an application of rolling horizon and nonlinear filtering techniques is presented. The presented approach is illustrated in a simple numerical example.

1. INTRODUCTION

The fault detection problem has received a lot of attention during recent years because of increasing requirements on safety, reliability, and low maintenance costs. The main goal is to design a detector that processes measurements and generates decisions about faults in a system. The very earliest detectors were based on using redundant sensors to detect their failures [Daley et al., 1979]. These quite simple detectors are still used in safety-critical systems, but they suffer from issues such as higher cost, increased weight and the need of additional space for redundant sensors. To avoid these disadvantages more complex detectors, utilizing a model of the system, were developed.

The model-based detectors [Jones, 1973, Basseville and Nikiforov, 1993] can detect faults in the system without any redundant sensors, because more detailed information about system behavior in fault-free and faulty cases is used. The detector usually consists of a residual generator and a decision generator, which are connected in cascade. The residual generator processes input-output data and generates residual signals that are close to zero in fault-free case and deviate from zero in faulty case. The decision generator statistically analyzes these residual signals and outputs a decision on fault.

In all above mentioned cases, the detector $\mathbf{D}$ uses measurements $\mathbf{z}_k$ to generate the decision $\mathbf{d}_k$ in a passive way, as depicted in Fig. 1. The measurements $\mathbf{z}_k$ consist of input-output data, i.e. $\mathbf{z}_k^T = [\mathbf{u}_k^T, \mathbf{y}_k^T]$. If a proper input $\mathbf{u}_k$ is applied to the system $\mathbf{S}$, as depicted in Fig. 2, further improvement of fault detection quality can be achieved. This idea is similar to input design for parameter estimation [Mehra, 1974]. The active detector $\mathbf{AD}$ generates, in addition to the decision $\mathbf{d}_k$, an auxiliary input $\mathbf{u}_k$ that excites the system and improves fault detection. Note that in the fault tolerant control literature, e.g. [Blanke et al., 2003], the terms passive and active are used in a different meaning. The term passive is used for fault tolerant system that is robust to faults and the term active denotes a fault tolerant system that estimates faults and reconfigures the controller.

There are only few works dealing with the active fault detection problem. One of the first attempt to formulate and solve this problem can be found in [Zhang, 1989]. Multiple linear Gaussian models were used for description of the system, the detector was based on the sequential probability ratio test (SPRT) and a clipped harmonic auxiliary input signal was designed to minimize the average sampling number (ASN). This idea was further extended in [Kerestecioğlu, 1993] referring to the fact that minimization of the ASN can increase probability of false detection. A more general formulation of active change detection, based on criterion minimization, was also proposed there, but it was not elaborated in all details. A completely different approach to the active fault detection problem was introduced in [Campbell and Nikoukhah, 2004]. The system was again described using the multiple model approach, but the disturbances were modelled as bounded deterministic signals. Further, it was considered that the behavior of the system does not change during the test period, in which a valid model is determined using the member set approach. Depending on the system and a restriction on the input signal there can be an auxiliary input signal that allows to surely decide on valid model during the test period.

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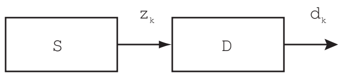

Fig. 1. A passive detector

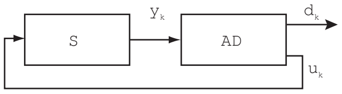

Fig. 2. Active detection

Nevertheless, there are some issues when active fault detection and control should be performed simultaneously. The fundamental question is how the auxiliary input signal should enter the system if all inputs are already used for control purposes. The first possibility is to completely replace the control signal by the auxiliary input signal for a short time period. Another solution is to generate weighted sum of control signal and auxiliary input signal. If these or other heuristic solutions to this issue are used then the resulting closed loop behavior is always questionable. Active fault detection for closed-loop systems is presented in [Niemann, 2006], where a controller is supplemented by a probing signal, which slightly degrades quality of control and manifests itself in residual signal only in faulty case. Another approach that allows to incorporate control objectives in active fault detection was proposed in [Blackmore and Williams, 2006], where control objectives were expressed in terms of equality and inequality linear constraints on input and expected system state trajectories. All above mentioned approaches solve the active detection and control problem in different frameworks and with different levels of generality.

A unified framework that allows to formulate and solve several active fault detection problems was presented in [Šimandl and Punčochář, 2006]. The more detailed derivation of the active fault detector and controller was discussed in [Šimandl and Punčochář, 2007]. The goal of the paper is to elaborate the general approach given in [Šimandl and Punčochář, 2007] for multiple model framework and present a suboptimal solution based on rolling horizon technique.

The paper is organized as follows. Section 2 introduces description of a system, an active detector and controller, and a criterion. The dynamic programming is used for the design of optimal active fault detector and controller in Section 3. Suboptimal solutions of recursive equation and state estimation problem are presented in Section 4. Section 5 deals with a numerical example.

2. PROBLEM FORMULATION

Let the system $\mathbf{S}$ be described at each time step $k \in \mathcal{T} = \{0, 1, \ldots, F\}$ by the jump Markov linear Gaussian model

$$\mathbf{x}_{k+1} = A(\mu_k)\mathbf{x}_k + B(\mu_k)\mathbf{u}_k + G(\mu_k)\mathbf{w}_k, \quad (1)$$

$$\mathbf{y}_k = C(\mu_k)\mathbf{x}_k + H(\mu_k)\mathbf{v}_k, \quad (2)$$

where $\mathbf{y}_k \in \mathcal{R}^{n_y}$ denotes the output and $\mathbf{u}_k \in \mathcal{U}_k \subset \mathcal{R}^{n_u}$ denotes the input. The subset $\mathcal{U}_k$ is a continuous or discrete set that determines a range of admissible values of the input $\mathbf{u}_k$. The immeasurable state $\bar{\mathbf{x}}_k = \left[\mathbf{x}_k^T, \mu_k\right]^T$ of the system consists of variables $\mathbf{x}_k \in \mathcal{R}^{n_x}$ and $\mu_k \in \mathcal{M} = \{1, 2, \ldots, N\}$. The vector $\mathbf{x}_k$ is continuous in values and represents the common state of particular linear Gaussian models. The scalar μ_k is the index of linear Gaussian model that represents the system at time step k. It is assumed that the set of models $\mathcal{M}$ is identical with a set of all possible modes of system (i.e. model set cover fault-free and all faulty behavior patterns of the system) and the number of models N remains constant over time. Practical model selection is described e.g. in [Athans et al., 2006]. The noises $\mathbf{w}_k \in \mathcal{R}^{n_x}$ and $\mathbf{v}_k \in \mathcal{R}^{n_y}$ are mutually independent zero-mean white Gaussian noises with identity covariance matrices, which is written as $\mathcal{N}\{0, I\}$. The initial condition $\mathbf{x}_0$ is described by the Gaussian probability density function $\mathcal{N}\{\hat{\mathbf{x}}_0', P_{x,0}'\}$ and the variable μ_0 is described by the probability function $P(\mu_0)$. The variables $\mathbf{x}_0$ and μ_0 are mutually independent and also independent of the noise processes. The switching between models is governed by the given transition probability $P_{i,j} = P(\mu_{k+1} = j | \mu_k = i)$ for $i, j \in \mathcal{M}$ and the each model μ_k is represented by known matrices $A(\mu_k)$, $B(\mu_k)$, $C(\mu_k)$, $G(\mu_k)$, $H(\mu_k)$ of appropriate dimensions.

The active detector and controller (**ADC**) is considered as one block that processes the output of the system and generates the input and decision about faults. At each time step $k \in \mathcal{T}$, a realizable **ADC** has to be a causal system that utilize all information received up to the current time

$$\begin{bmatrix} d_k \\ \mathbf{u}_k \end{bmatrix} = \boldsymbol{\rho}_k\left(\mathbf{I}_0^k\right), \quad k \in \mathcal{T}, \quad (3)$$

where $d_k \in \mathcal{M}$ denotes decision, $\mathbf{u}_k$ is already defined input and $\mathbf{I}_0^{k\,T} = \left[\mathbf{y}_0^{k\,T}, u_0^{k-1\,T}, d_0^{k-1\,T}\right]$ denotes all available information received up to the actual time step. The notation $\mathbf{y}_0^k = [\mathbf{y}_0{}^T, \mathbf{y}_1{}^T, \ldots, \mathbf{y}_k{}^T]^T$ is used for description of whole history of considered variable or sequence of functions.

The goal is to design the **ADC** that achieves minimum costs connected with wrong decisions and trajectories of the state and input. An appropriate criterion can be chosen as

$$J(\boldsymbol{\rho}_0^F) = \mathrm{E}\left\{\sum_{k=0}^{F} L_k^{\mathrm{d}}(\mu_k, d_k) + \alpha_k L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k)\right\}, \quad (4)$$

where $\mathrm{E}\{\cdot\}$ denotes the expectation operator over all random variables, the cost function $L_k^{\mathrm{d}}(\mu_k, d_k)$ is a real-valued non-negative function representing decision objective, the cost function $L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k)$ is a real-valued non-negative function representing control objective and the coefficient α_k sets a desired compromise between these two objectives.

3. OPTIMAL ACTIVE DETECTOR AND CONTROLLER

3.1 Information processing strategies

In the optimal stochastic control problem, a dynamic optimization can be solved using three basic information processing strategies (IPS's). Open-loop (OL) IPS uses only an a priori information and no future information will be used. Open-loop feedback (OLF) IPS assumes that an additional information will be available in a priori unknown time steps and this information will be used together with the a priori information. Closed-loop (CL) IPS supposes that an additional information will be received and utilized at all future time steps.

These IPS's lead to different values of criterion J. It holds that $J^{\mathrm{OL}} \geq J^{\mathrm{OLF}} \geq J^{\mathrm{CL}}$, where J^{OL} is the value of the criterion when OL IPS is used, J^{OLF} is the value of the criterion when OLF IPS is used and J^{CL} is the value of the criterion when CL IPS is used. The question is if the IPS's are also used in fault detection (FD). The use of the IPS's in FD was firstly elaborated in [Šimandl and Herejt, 2003] with the following conclusions. The OL IPS uses only the prior information and hence it is not used in FD. The OLF IPS corresponds to the standard approaches in FD (e.g. Bayesian approach) and the CL IPS providing the best results is not commonly used in FD. In this paper only the CL IPS will be considered due to its superiority in comparison to OL and OLF IPS's.

3.2 Closed loop information processing strategy

Bellman's principle of optimality [Bertsekas, 1995] is a general approach to solve a dynamic optimization problem. The dynamic optimization problem is solved backward in time from the final time step and the result is an optimal policy that is defined as follows.

An optimal policy has the property that whatever the initial state and the initial decisions/controls are, the remaining decisions/controls must constitute an optimal policy with regard to the state resulting from the first decision/control.

This definition allows to write directly the backward recursive equation for time step $k = F, F-1, \ldots, 0$

$$\begin{aligned} V_k^*\left(\mathbf{I}_0^k\right) = \min_{\substack{d_k \in \mathcal{M} \\ \mathbf{u}_k \in \mathcal{U}_k}} \mathrm{E}\{L_k^{\mathrm{d}}\left(d_k, \mu_k\right) + \alpha_k L_k^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right) \\ + V_{k+1}^*\left(\mathbf{I}_0^{k+1}\right) |\mathbf{I}_0^k, \mathbf{u}_k, d_k\}, \end{aligned} \tag{5}$$

where $\mathrm{E}\{\cdot|\cdot\}$ denotes the conditional expectation operator and $V_k^*(\mathbf{I}_0^k)$ is the so called Bellman function. The Bellman function expresses the minimum of expected costs from current time step k to the final time step F of the detection horizon given all available information $\mathbf{I}_0^k$. The initial condition for this backward recursive equation is $V_{F+1}^* = 0$ and the value of criterion (4) can be expressed as $J^{\mathrm{CL}} = J\left(\boldsymbol{\rho}_0^{F*}\right) = \mathrm{E}\left\{V_0^*\left(\mathbf{y}_0\right)\right\}$.

The pdf's $p\left(\bar{\mathbf{x}}_k|\mathbf{I}_0^k, \mathbf{u}_k, d_k\right)$ and $p\left(\mathbf{y}_{k+1}|\mathbf{I}_0^k, \mathbf{u}_k, d_k\right)$ are needed for evaluation of the conditional expectations in (5). These pdf's can be obtained from a nonlinear filter and they satisfy the following identities

$$p\left(\bar{\mathbf{x}}_k|\mathbf{I}_0^k, \mathbf{u}_k, d_k\right) = p\left(\bar{\mathbf{x}}_k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right), \tag{6}$$

$$p\left(\mathbf{y}_{k+1}|\mathbf{I}_0^k, \mathbf{u}_k, d_k\right) = p\left(\mathbf{y}_{k+1}|\mathbf{y}_0^k, \mathbf{u}_0^k\right). \tag{7}$$

It will be shown that backward recursive equation (5) can be rewritten to a simpler form using identities (6) and (7). The Bellman function at time step $k = F$ is

$$\begin{aligned} V_F^*\left(\mathbf{I}_0^F\right) = \min_{\substack{d_F \in \mathcal{M} \\ \mathbf{u}_F \in \mathcal{U}_F}} \mathrm{E}\big\{L_F^{\mathrm{d}}\left(d_F, \mu_F\right) \\ + \alpha_F L_F^{\mathrm{c}}\left(\mathbf{x}_F, \mathbf{u}_F\right) |\mathbf{I}_0^F, \mathbf{u}_F, d_F\big\}. \end{aligned} \tag{8}$$

Applying (6) the Bellman function $V_F^*\left(\mathbf{I}_0^F\right)$ takes the form

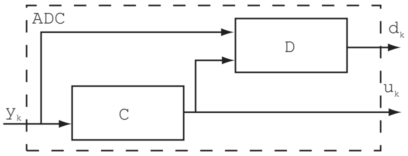

Fig. 3. Optimal active detector and controller

$$\begin{aligned} V_F^*\left(\mathbf{I}_0^F\right) = \min_{\substack{d_F \in \mathcal{M} \\ \mathbf{u}_F \in \mathcal{U}_F}} \Big[\mathrm{E}\big\{L_F^{\mathrm{d}}\left(d_F, \mu_F\right) |\mathbf{y}_0^F, \mathbf{u}_0^{F-1}, d_F\big\} \\ + \alpha_F \mathrm{E}\big\{L_F^{\mathrm{c}}\left(\mathbf{x}_F, \mathbf{u}_F\right) |\mathbf{y}_0^F, \mathbf{u}_0^F\big\}\Big]. \end{aligned} \tag{9}$$

The first term on the right-hand side of (9) is independent of the input $\mathbf{u}_F$ and it can be minimized only over the decision d_F. Moreover, the second term is independent of the decision d_F and it can be minimized only over the input $\mathbf{u}_F$. Therefore, the minimization can be split into two independent minimization problems. The right-hand side of (9) is independent of decisions d_0^F and thus it holds that $V_F^*\left(\mathbf{I}_0^F\right) = V_F^*\left(\mathbf{y}_0^F, \mathbf{u}_0^{F-1}\right)$.

It holds that $V_{k+1}^*\left(\mathbf{I}_0^{k+1}\right) = V_{k+1}^*\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right)$ at a time step $k+1 = F$. At time step k, the Bellman function can be written

$$\begin{aligned} V_k^*\left(\mathbf{I}_0^k\right) = \min_{\substack{d_k \in \mathcal{M} \\ \mathbf{u}_k \in \mathcal{U}_k}} \mathrm{E}\{L_k^{\mathrm{d}}\left(d_k, \mu_k\right) + \alpha_k L_k^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right) \\ + V_{k+1}^*\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right) |\mathbf{I}_0^k, \mathbf{u}_k, d_k\}. \end{aligned} \tag{10}$$

Applying (6) and (7) it can be seen that the Bellman function $V_k^*\left(\mathbf{I}_0^k\right)$ is also independent of decisions d_0^k (i.e. $V_k^*\left(\mathbf{I}_0^k\right) = V_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right)$). The first term on the right-hand side of (10) is again independent of the input $\mathbf{u}_k$ and the remaining two terms are independent of the decision d_k. Thus, the backward recursive equation can be written at each time step $k \in \mathcal{T}$ as

$$\begin{aligned} V_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \min_{d_k \in \mathcal{M}} \mathrm{E}\big\{L_k^{\mathrm{d}}\left(d_k, \mu_k\right) |\mathbf{y}_0^k, \mathbf{u}_0^{k-1}, d_k\big\} \\ + \min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\big\{\alpha_k L_k^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right) \\ + V_{k+1}^*\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right) |\mathbf{y}_0^k, \mathbf{u}_0^k\big\}, \end{aligned} \tag{11}$$

and the optimal decision d_k^* and the optimal input $\mathbf{u}_k^*$ are given as

$$\begin{aligned} d_k^* &= \sigma_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) \\ &= \arg\min_{d_k \in \mathcal{M}} \mathrm{E}\big\{L_k^{\mathrm{d}}\left(d_k, \mu_k\right) |\mathbf{y}_0^k, \mathbf{u}_0^{k-1}, d_k\big\}, \end{aligned} \tag{12}$$

$$\begin{aligned} \mathbf{u}_k^* = \boldsymbol{\gamma}_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \arg\min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\big\{\alpha_k L_k^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right) \\ + V_{k+1}^*\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right) |\mathbf{y}_0^k, \mathbf{u}_0^k\big\}, \end{aligned} \tag{13}$$

where $\sigma_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right)$ is a function describing the optimal passive detector and $\boldsymbol{\gamma}_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right)$ is a function describing the optimal dual controller. The structure of the optimal **ADC** is depicted in Fig. 3.

Finally, some comments concerning the optimal **ADC** are given. The optimal decision d_k^* minimizes the conditional mean value of the cost function $L_k^{\mathrm{d}}\left(d_k, \mu_k\right)$. The block generating optimal decisions can be called a passive detector considering the fact that the decisions do not influence the dual controller as shown in (13). The optimal input $\mathbf{u}_k^*$ is generated by the second block of the optimal **ADC** and represents an optimal tradeoff between exciting and controlling the system. This block can be called an optimal dual controller but the generated input signal differs from the input signal provided by the standard dual controller because of the first term $L_k^{\mathrm{d}}(d_k,\mu_k)$ in the criterion (4).

4. SUBOPTIMAL ACTIVE DETECTOR AND CONTROLLER

This section deals with a practical implementation of the **ADC**, which is based on rolling horizon technique.

4.1 State estimation

This subsection summarizes the solution of the state estimation problem for multiple Gaussian linear models. The aim is to find conditional pdf's $p\left(\mathbf{x}_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right)$, $p\left(\mathbf{y}_{k+1}|\mathbf{y}_0^k,\mathbf{u}_0^k\right)$ and probability $P\left(\mu_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right)$.

The conditional pdf of the variable $\mathbf{x}_k$ has the form of Gaussian sum

$$\begin{aligned} p\left(\mathbf{x}_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right) &= \sum_{\mu_0\in\mathcal{M}}\cdots\sum_{\mu_k\in\mathcal{M}} p\left(\mathbf{x}_k,\mu_0^k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right) \\ &= \sum_{\mu_0\in\mathcal{M}}\cdots\sum_{\mu_k\in\mathcal{M}} p\left(\mathbf{x}_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1},\mu_0^k\right) \\ &\quad P\left(\mu_0^k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right), \end{aligned} \tag{14}$$

where the pdf $p\left(\mathbf{x}_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1},\mu_0^k\right)=\mathcal{N}\{\hat{\mathbf{x}}_k\left(\mu_0^k\right),P_{x,k}\left(\mu_0^k\right)\}$ is Gaussian with the mean value $\hat{\mathbf{x}}_k\left(\mu_0^k\right)$ and the covariance matrix $P_{x,k}\left(\mu_0^k\right)$, and the conditional probability $P\left(\mu_0^k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right)$ can be computed recursively as will be shown later. The mean value $\hat{\mathbf{x}}_k\left(\mu_0^k\right)$ and covariance matrix $P_{x,k}\left(\mu_0^k\right)$ can be obtained using the Kalman filter for t-variant system. The corresponding relations for the filtering step of the Kalman filter are

$$\begin{aligned} \hat{\mathbf{x}}_k\left(\mu_0^k\right) &= \hat{\mathbf{x}}_k'\left(\mu_0^{k-1}\right) \\ &\quad + K\left(\mu_0^k\right)\Big[\mathbf{y}_k - C\left(\mu_k\right)\hat{\mathbf{x}}_k'\left(\mu_0^{k-1}\right)\Big], \end{aligned} \tag{15}$$

$$P_{x,k}\left(\mu_0^k\right)= P_{x,k}'\left(\mu_0^{k-1}\right)-K\left(\mu_0^k\right)C\left(\mu_k\right)P_{x,k}'\left(\mu_0^{k-1}\right), \tag{16}$$

where the Kalman gain $K\left(\mu_0^k\right)$ is given as

$$\begin{aligned} &K\left(\mu_0^k\right) = P_{x,k}{}'\left(\mu_0^{k-1}\right)C\left(\mu_k\right)^T \\ &\Big[C\left(\mu_k\right)P_{x,k}'\left(\mu_0^{k-1}\right)C\left(\mu_k\right)^T + H\left(\mu_k\right)H\left(\mu_k\right)^T\Big]^{-1}, \end{aligned} \tag{17}$$

where $\hat{\mathbf{x}}_k'$ is the predictive mean and $P_{x,k}'$ is the predictive covariance matrix at time k. In the predictive step of the Kalman filter, the mean value $\hat{\mathbf{x}}_{k+1}'\left(\mu_0^k\right)$ and the covariance matrix $P_{x,k+1}'\left(\mu_0^k\right)$, which are parameters of the Gaussian predictive pdf $p\left(\mathbf{x}_{k+1}|\mathbf{y}_0^k,\mathbf{u}_0^k,\mu_0^k\right)=\mathcal{N}\{\hat{\mathbf{x}}_{k+1}'\left(\mu_0^k\right),P_{x,k+1}'\left(\mu_0^k\right)\}$, are computed as

$$\hat{\mathbf{x}}_{k+1}'\left(\mu_0^k\right) = A\left(\mu_k\right)\hat{\mathbf{x}}_k\left(\mu_0^k\right) + B\left(\mu_k\right)\mathbf{u}_k, \tag{18}$$

$$\begin{aligned} P_{x,k+1}'\left(\mu_0^k\right) &= A\left(\mu_k\right)P_{x,k}\left(\mu_0^k\right)A\left(\mu_k\right)^T \\ &\quad + G\left(\mu_k\right)G\left(\mu_k\right)^T. \end{aligned} \tag{19}$$

The conditional probability of the model μ_k is simply given by the following marginalization

$$P\left(\mu_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right) = \sum_{\mu_0}\cdots\sum_{\mu_{k-1}} P\left(\mu_0^k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right). \tag{20}$$

The conditional probability of the model sequence μ_0^k is recursively evaluated according to the relation

$$\begin{aligned} P\left(\mu_0^k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right) &= \frac{p\left(\mathbf{y}_k|\mathbf{y}_0^{k-1},\mathbf{u}_0^{k-1},\mu_0^k\right)}{p\left(\mathbf{y}_k|\mathbf{y}_0^{k-1},\mathbf{u}_0^{k-1}\right)} \\ &\quad P\left(\mu_k|\mu_{k-1}\right)P\left(\mu_0^{k-1}|\mathbf{y}_0^{k-1},\mathbf{u}_0^{k-2}\right), \end{aligned} \tag{21}$$

where the predictive pdf $p\left(\mathbf{y}_k|\mathbf{y}_0^{k-1},\mathbf{u}_0^{k-1}\right)$ is a normalization constant independent of the model sequence μ_0^k and it is computed as

$$\begin{aligned} p\left(\mathbf{y}_k|\mathbf{y}_0^{k-1},\mathbf{u}_0^{k-1}\right) &= \sum_{\mu_0^k} p\left(\mathbf{y}_k|\mathbf{y}_0^{k-1},\mathbf{u}_0^{k-1},\mu_0^k\right) \\ &\quad P\left(\mu_k|\mu_{k-1}\right)P\left(\mu_0^{k-1}|\mathbf{y}_0^{k-1},\mathbf{u}_0^{k-2}\right). \end{aligned} \tag{22}$$

It is obvious that the predictive pdf $p\left(\mathbf{y}_k|\mathbf{y}_0^{k-1},\mathbf{u}_0^{k-1},\mu_0^k\right) = \mathcal{N}\{\hat{\mathbf{y}}_k'\left(\mu_0^k\right),P_{y,k}'\left(\mu_0^k\right)\}$ is also Gaussian with the mean value and the covariance matrix

$$\hat{\mathbf{y}}_k'\left(\mu_0^k\right) = C\left(\mu_k\right)\hat{\mathbf{x}}_k'\left(\mu_0^{k-1}\right), \tag{23}$$

$$\begin{aligned} P_{y,k}'\left(\mu_0^k\right) &= C\left(\mu_k\right)P_{x,k}'\left(\mu_0^{k-1}\right)C\left(\mu_k\right)^T \\ &\quad + H\left(\mu_k\right)H\left(\mu_k\right)^T, \end{aligned} \tag{24}$$

respectively.

The introduced relations provide an exact solution of the state estimation problem for the multiple Gaussian linear models. Unfortunately, at time step k there are N^{k+1} distinct model sequences μ_0^k, for which the state estimation has to be solved. Therefore, it is necessary to reduce exponentially growing number of model sequences at some time steps using sequence pruning, merging, or a combination of these methods [Boers and Driessen, 2005]. The following subsection presents a simple method for approximative state estimation.

4.2 Suboptimal state estimation

The approximation is based on model sequence pruning performed at predefined time steps. During pruning only a sequence that has the highest probability is retained and other sequences that has the same history except last l-steps are pruned.

The conditional probability of the sequence μ_{k-l}^k is

$$P\left(\mu_{k-l}^k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right) = \sum_{\mu_0\in\mathcal{M}}\cdots\sum_{\mu_{k-l-1}\in\mathcal{M}} P\left(\mu_0^k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right) \tag{25}$$

and the filtering pdf $p\left(\mathbf{x}_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1},\mu_{k-l}^k\right)$ has the Gaussian sum form

$$\begin{aligned} p\left(\mathbf{x}_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1},\mu_{k-l}^k\right) &= \sum_{\mu_0\in\mathcal{M}}\cdots\sum_{\mu_{k-l-1}\in\mathcal{M}} \beta\left(\mu_0^k\right) \\ &\quad p\left(\mathbf{x}_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1},\mu_0^k\right), \end{aligned} \tag{26}$$

where

$$\beta\left(\mu_0^k\right) = \frac{P\left(\mu_0^k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right)}{P\left(\mu_{k-l}^k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right)}. \tag{27}$$

Note that dependence of $\beta\left(\mu_0^k\right)$ on $\mathbf{y}_0^k$ and $\mathbf{u}_0^{k-1}$ was simply omitted because of brevity. The Gaussian sum $p\left(\mathbf{x}_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1},\mu_{k-l}^k\right)$ is replaced by one term of the original Gaussian sum, which has highest probability $\beta\left(\mu_0^k\right)$.

Given the terminal sequence μ_{k-l}^{k}, the sequence with the maximum probability is $\mu_0^{k*} = \left[\mu_0^{k-l-1*}, \mu_{k-l}^{k}\right]$, where

$$\mu_0^{k-l-1*} = \arg \max_{\mu_0^{k-l-1}} \beta\left(\mu_0^k\right), \tag{28}$$

and the Gaussian sum (26) is replaced by the single Gaussian pdf $p\left(\mathbf{x}_k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}, \mu_0^{k*}\right)$ with the mean value $\hat{\mathbf{x}}_k\left(\mu_0^{k*}\right)$ and covariance matrix $P_{x,k}\left(\mu_0^{k*}\right)$.

4.3 Application of rolling horizon technique

The closed-form solutions of backward recursive equations, presented in Section 3, can not be derived even in the multi-model framework. It is caused by the fact that the expectations in these backward recursive equations can not be computed analytically. Thus, certain approximation, preserving properties of solution based on the CL IPS, has to be used to obtain a feasible solution. Many approximative solutions were proposed for optimal stochastic control problem, see e.g. [Bertsekas, 1995, Feldbaum, 1960-61], and there are also some feasible design procedures for active detector design [Blackmore and Williams, 2006, Zhang, 1989].

Here, an approximative solution is the following. Discreteness of the set $\mathcal{U}_k$ allows to compute the value of the Bellman function V_k^* recursively forward in time for each input, and choose the optimal one. The conditional expectation of the Bellman function V_k^* is computed numerically using the trapezoidal rule. However, it is a computationally intensive method that is infeasible for long detection horizon F. Therefore, the rolling horizon method [Bertsekas, 1995, Šimandl et al., 2005] is used to overcome this problem. At each time step k, the optimization is performed over an optimization horizon $F_\mathrm{o} \leq F$ instead of the whole detection horizon F. It means that the Bellman function $V_{k+F_\mathrm{o}+1}^*$ is approximated by $\bar{V}_{k+F_\mathrm{o}+1}^* = 0$ and the decision d_k and input $\mathbf{u}_k$ are determined by the design procedure.

5. NUMERICAL EXAMPLE

The proposed **ADC** and a passive detector coupled with a heuristic certainty equivalent controller (**HCEC**)[Wenk and Bar-Shalom, 1980] are compared. A simple system is chosen to make the example clear and understandable .

The detection horizon is $F = 40$ and the system is described by one of two second order stable models

$$\mu_k = 1 : x_{k+1} = \begin{bmatrix} 0.0707 & -0.4826 \\ 0.8579 & 0.4996 \end{bmatrix} x_k + \begin{bmatrix} 0.2145 \\ 0.2224 \end{bmatrix} u_k + \begin{bmatrix} 0.003 & 0 \\ 0 & 0.003 \end{bmatrix} w_k, \tag{29}$$

$$y_k = \left[\, 0 \;\; 2.25 \,\right] x_k + 0.005 v_k, \tag{30}$$

$$\mu_k = 2 : x_{k+1} = \begin{bmatrix} 0.0707 & -0.4826 \\ 0.8579 & 0.4996 \end{bmatrix} x_k + \begin{bmatrix} 0.1973 \\ 0.2104 \end{bmatrix} u_k + \begin{bmatrix} 0.003 & 0 \\ 0 & 0.003 \end{bmatrix} w_k, \tag{31}$$

$$y_k = \left[\, 0 \;\; 2.25 \,\right] x_k + 0.005 v_k. \tag{32}$$

The first model describes fault-free behavior and the second one describes the faulty behavior of the system. The initial state x_0 has mean $x_0' = [0\ 0]^T$ and covariance matrix $P_{x,0}' = 0.1I$. The initial probabilities of models are $P(\mu_0 = 1) = P(\mu_0 = 2) = 0.5$ and the transition probabilities are $P_{1,1} = P_{2,2} = 0.95$ and $P_{1,2} = P_{2,1} = 0.05$. The set of admissible inputs is $\mathcal{U}_k = \{-0.8, -0.7, -0.6, -0.5, -0.4, -0.3, -0.2, -0.15, -0.1, 0, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$. The function penalizing wrong decision is

$$L_k^\mathrm{d}\left(\mu_k, d_k\right) = \begin{cases} 0 \;\; if \;\; \mu_k = d_k \\ 1 \;\; if \;\; \mu_k \neq d_k \end{cases} \tag{33}$$

and the control objective is expressed by the cost function

$$L_k^\mathrm{c}\left(\mathbf{x}_k, \mathbf{u}_k\right) = \left[\mathbf{x}_k^\mathrm{r} - \mathbf{x}_k\right]^T Q_k \left[\mathbf{x}_k^\mathrm{r} - \mathbf{x}_k\right] + \mathbf{u}_k^T R_k \mathbf{u}_k, \tag{34}$$

where $Q_k = I$, $R_k = 0.001$, and $\alpha_k = 8$. The reference state $\mathbf{x}_k^\mathrm{r} = \left[x_{1,k}^\mathrm{r}, x_{2,k}^\mathrm{r}\right]^T$ is defined as follows: $x_{1,k}^\mathrm{r} = 0$ for all $k \in \mathcal{T}$ and $x_{2,k}^\mathrm{r}$ is the rectangular signal with amplitude ± 0.2667 and period 40 steps. The parameters for suboptimal state estimation and optimization were chosen $l = 1$ and $F_o = 1$, respectively.

Table 1. Monte Carlo simulation results

	N_WD	$MSE_{x_k^\mathrm{r} - x_k}$
ADC	25.47	5.8890
HCEC	30.58	5.9047

The results of 300 Monte Carlo simulations are presented in Tab. 1. The difference in the quality of control, measured by the mean square error $MSE_{x_k^\mathrm{r} - x_k}$ of x_k, is not statistically significant, but the average number of wrong decisions N_WD in percents is considerably lesser in the case of **ADC**. Contrary to Tab. 1, Figures 4, 5, and 6 illustrate results for one simulation run. The **HCEC** controls the system in a steady state quite well (see Fig. 5) whatever the correct model is. However, the corresponding passive detector can generate more wrong decisions in such situation as presented in Fig. 6. The same figure shows that the **ADC** automatically generates a probing signal whenever input-output data does not contain information necessary to distinguish between models and it naturally leads to slightly worse quality of control as depicted in Fig. 4.

6. CONCLUSION

The general approach to the optimal active fault detection and dual control was applied to a special case of system description based on multiple model framework. The case was elaborated in detail, especially suboptimal solution of backward recursive equation and state estimation problem. In contrast to the general active fault detection, which provides unified theoretical framework, the multiple model approach can be useful for practical application, because particular models represents considered faults in the system and numerical demands of the corresponding algorithm are acceptable for many processes. The numerical simulation shows that active fault detection and dual control can significantly improves quality of decisions whereas quality of control is usually only slightly decreased.

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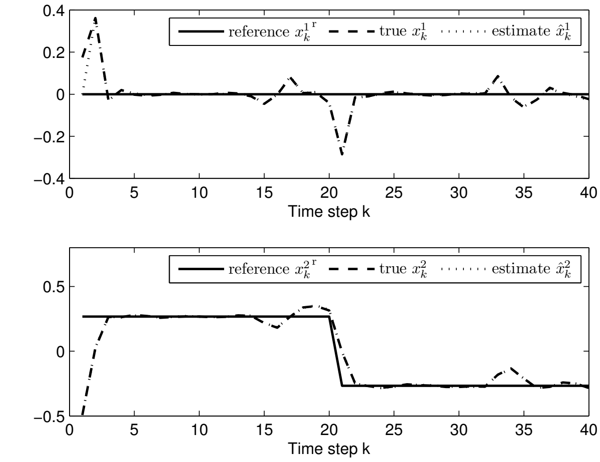

Fig. 4. Reference, true state and its estimate for **ADC**

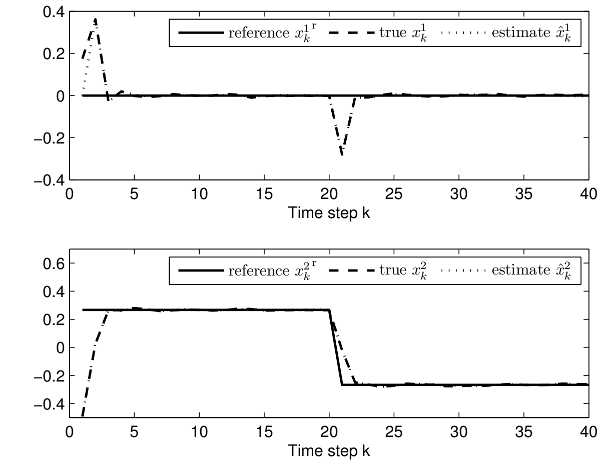

Fig. 5. Reference, true state and its estimate for **HCEC**

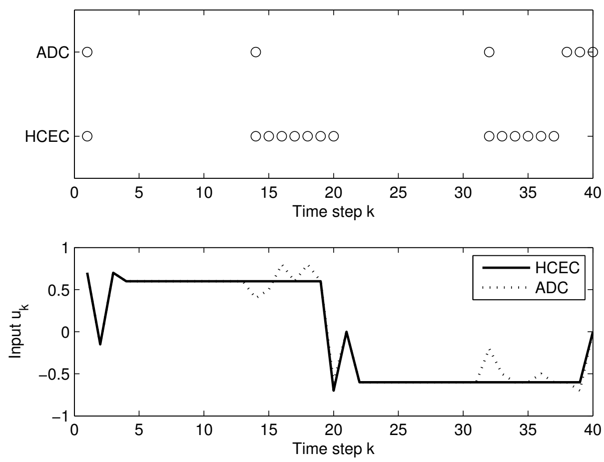

Fig. 6. Upper: Indication of wrong decisions. Bottom: Input trajectories

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