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A Note on Unknown Input Fault Detection Filter Design

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Abstract: This note addresses the problems related to the design of fault detection filters which are perfectly decoupled from unknown inputs. The major focuses are on the study of existence conditions, the design of reduced order and minimum order fault detection filters. For the design purpose, different algorithms are provided. The achieved results are finally illustrated by an example.

Keywords: Fault detection filter, residual generator, perfect unknown input decoupling, geometric approach, inverted pendulum model

1. INTRODUCTION AND PROBLEM FORMULATION

Consider a linear time invariant (LTI) system given by

$$\dot{x} = Ax + Bu + E_d d + E_f f \quad (1)$$

$$y = Cx + Du + F_d d + F_f f \quad (2)$$

where $x \in \Re^n$, $y \in \Re^m$ and $u \in \Re^{k_u}$ are the system state vector, input and output vectors respectively. $d \in \Re^{k_d}$ represents the unknown input vector and $f \in \Re^{k_f}$ is the vector of the faults to be detected. It is well known that the first step to a successful model based fault detection (FD) is the so-called residual generation. To this end, the fault detection filter (FDF) scheme is widely used (Frank [1997], Chen and Patton [1999], Kinnaert [2003]). Given system (1)-(2), an FDF is a dynamic system of the form

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x} - Du) \quad (3)$$

$$r = V(y - C\hat{x} - Du) \quad (4)$$

where r is the so-called residual vector and L, V are the observer gain and the post-filter which should be selected such that r is independent of u and $x(0)$ for $t \to \infty$. The dynamics of FDF (3)-(4) is governed by

$$\dot{e} = (A - LC)\, e + (E_d - LF_d)d + (E_f - LF_f)f \quad (5)$$

$$r = V\,(Ce + F_d d + F_f f)\,, e = x - \hat{x}. \quad (6)$$

For a reliable FD, it is desirable that the residual vector would only be influenced by the fault. For this purpose, L and V should be so designed that the FDF is stable and

$$G_{rd}(s) = V(C(sI - A + LC)^{-1}(E_d - LF_d) + F_d) = 0 \quad (7)$$

$$G_{rf}(s) = V\left(C(sI - A + LC)^{-1}(E_f - LF_f) + F_f\right) \neq 0. \quad (8)$$

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An FDF satisfying (7) is known as unknown input FDF (UIFDF) and the design issue as *P*erfect *U*nknown *I*nput *D*ecoupling *P*roblem (PUIDP), which has been intensively studied in the past two decades, see for instance Chen and Patton [1999], Gertler [1998] and Blanke [2003]. The major objective of this note is to provide a practical UIFDF solution to the PUIDP.

In the past two decades, a number of significant UIFDF schemes have been reported. In the pioneer work by Massoumnia [1986], geometric approach was used for the design of UIFDF. In Ding and Frank [1990], factorization approach aided UIFDF design was proposed. Wuenenburg [1990] and Patton and Hou [1998] reported successful UIFDF design using different forms of the matrix pencil method. In addition, Chen and Patton [1999] have applied the well-established unknown input observer (UIO) technique for the UIFDF design.

Recently, during the attempt to integrate the UIFDF design into a MATLAB based FDI toolbox developed in our institute (Ding et al. [2006]), we notice that (a) by different design schemes the existence conditions are given in different ways and in terms of different system structural properties (b) these conditions may differ from (in fact stronger than) the well recognized conditions

$$rank\,(G_{rd}(s)) < m \quad (9)$$

for FDF satisfying (7) as well as

$$rank\,(G_{rd}(s)) < rank\,[\,G_{rd}(s)\;\; G_{rf}(s)\,] \quad (10)$$

for FDF satisfying (7) and (8), as given in Ding and Frank [1990] or recently in Frisk [2001] and Varga [2002] (c) the implementation of those algorithms may be involved and requires in particular special knowledge of linear control theory like, for instance, the geometric approach or matrix pencil method. This observation motivated our study reported in this note. Using some results well known in linear system theory, we shall present an existence condition for the UIFDF and then give a modified form of UIFDF whose existence condition is identical with (10).

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This modified UIFDF is a reduced order one. Based on this result, we shall also give an algorithm for designing minimum order UIFDF which is of considerable practical interests.

Throughout this note, the standard notation, for instance, the one adopted in Kailath [1980], will be used. This note is organized as follows. In Section 2, UIFDF design is addressed. Section 3 is devoted to the design of a reduced order UIFDF whose existence condition is given by (10). In Section 4, an algorithm is provided for the design of minimum order UIFDF. Section 5 is dedicated to the extension of the results. And finally in Section 6, an example is given to illustrate the achieved results.

2. ON THE UIFDF DESIGN

For the sake of simplicity, let us first assume that $F_d = 0$. Later on, in Section 5, we will generalize our results for $F_d \neq 0$.

For our purpose, we first introduce a well known result on the design of L such that for given

$$\dot{x} = (A - LC)x + E_d d, y = Cx \tag{11}$$

the pair $((A-LC), E_d)$ becomes maximally uncontrollable by d, which is equivalently to the fact that the pair $((A-LC)^T, E_d^T)$ is maximally unobservable. The terminologies *maximally uncontrollable* and *maximally unobservable* are used to express the uncontrollable and unobservable subspaces with the maximal dimensions (Kailath [1980], Wonham [1979]).

Lemma 1 *Kailath [1980]* : Suppose *L makes $(A-LC, E_d)$ maximally uncontrollable by d, i.e. $((A-LC)^T, E_d^T)$ is maximally unobservable. Then by a suitable choice of output and state bases, $\bar{V}$ and T, the resulting realization can be described by*

$$T(A - LC)T^{-1} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ 0 & \bar{A}_{22} \end{bmatrix}, TE_d = \begin{bmatrix} \bar{E}_{d1} \\ 0 \end{bmatrix} \tag{12}$$

$$\bar{C} = \bar{V}CT^{-1} = \begin{bmatrix} \bar{C}_1 & 0 \\ 0 & \bar{C}_2 \end{bmatrix}$$

where the realization $(\bar{A}_{11}, \bar{E}_{d1}, \bar{C}_1)$ is perfectly controllable.

Remark: A system (A, B, C) is called perfectly controllable if

$$\forall\lambda, \begin{bmatrix} A - \lambda I & B \\ C & 0 \end{bmatrix} \text{ has full row rank.}$$

Now, we apply Lemma 1 for our UIFDF study.

Let $L_{\max}$ be the observer gain that makes $(A-L_{\max}C, E_d)$ maximally uncontrollable by d. When $\bar{C}_2 \neq 0$, we construct, according to Lemma 1, the following FDF

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} - L_{11}\bar{C}_1 & \bar{A}_{12} - L_{12}\bar{C}_2 \\ 0 & \bar{A}_{22} - L_{22}\bar{C}_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + TBu + T\left(L_{\max} + L_0\bar{V}\right)y \tag{13}$$

$$r = [\,0 \;\; v_2\,] \left(\bar{V}y - \begin{bmatrix} \bar{C}_1 & 0 \\ 0 & \bar{C}_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}\right), v_2 \neq 0 \tag{14}$$

with

$$TL_0 = \begin{bmatrix} L_{11} & L_{12} \\ 0 & L_{22} \end{bmatrix} \tag{15}$$

and L_{11}, L_{22} ensuring the stability of $\bar{A}_{11} - L_{11}\bar{C}_1$ and $\bar{A}_{22} - L_{22}\bar{C}_2$. Introducing

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, e = Tx - z = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

gives

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} - L_{11}\bar{C}_1 & \bar{A}_{12} - L_{12}\bar{C}_2 \\ 0 & \bar{A}_{22} - L_{22}\bar{C}_2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} \bar{E}_{d1} \\ 0 \end{bmatrix} d \tag{16}$$

$$r = [\,0 \;\; v_2\,] \begin{bmatrix} \bar{C}_1 & 0 \\ 0 & \bar{C}_2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = v_2\bar{C}_2e_2. \tag{17}$$

It is evident that residual signal r is perfectly decoupled from d. Thus, (13)-(14) is an UIFDF. For our purpose, we now study the existence condition for UIFDF (13)-(14).

Lemma 2 *Kailath [1980]: Under the same conditions as given in Lemma 1, we have*

- *$\bar{C}_2 \neq 0$ if and only if $rank\,(C) > rank\,(E_d)$*
- *$(\bar{A}_{22}, \bar{C}_2)$ is equivalent to*

$$(\bar{A}_{22}, \bar{C}_2) \sim \left(\begin{bmatrix} \bar{A}_{221} & 0 \\ \bar{A}_{222} & \bar{A}_{223} \end{bmatrix}, [\,\bar{C}_{21} \;\; 0\,]\right)$$

where $(\bar{A}_{221}, \bar{C}_{21})$ is perfectly observable, the eigenvalues of matrix $\bar{A}_{223}$ are the invariant zeros of (A, E_d, C) and they are unobservable.

Following Lemma 2, we know that $\bar{A}_{22} - L_{22}\bar{C}_2$ becomes stable only if (A, E_d, C) has no unstable invariant zero. Moreover, the PUIDP is solvable if and only if

$$rank\,(C) > rank\,(E_d)$$

which leads to

$$rank \begin{bmatrix} A - sI & E_d \\ C & 0 \end{bmatrix} < n + m. \tag{18}$$

As a result, the following theorem and corollary are proven.

Theorem 1: *Given system (5)-(6) with $F_d = 0$, then there exists an FDF that solves the PUIDP if and only if*

$$rank \begin{bmatrix} A - sI & E_d \\ C & 0 \end{bmatrix} < n + m$$

and (A, E_d, C) has no unstable invariant zero.

Corollary 1: *Given system (5)-(6) with $F_d = 0$, then there exists an FDF that satisfies (7)-(8) if*

$$rank \begin{bmatrix} A - sI & E_d \\ C & 0 \end{bmatrix} < rank \begin{bmatrix} A - sI & E_f & E_d \\ C & F_f & 0 \end{bmatrix} \leq n + m \tag{19}$$

and the invariant zeros of (A, E_d, C) are stable.

Comparing the existence conditions given in Theorem 1 and Corollary 1 with the ones given in (9) and (10), we can clearly see that the existence conditions (18) as well as (19) are stronger than (9) and (10) due to the additional requirement on the invariant zeros of (A, E_d, C). This motivates our further study described in the next section.

3. DESIGN OF REDUCED ORDER UIFDF

The basic idea behind our effort is to construct the residual generator only using those state variables which are decoupled from d and stable. For this purpose, we consider FDF in (13). Instead of constructing a full order observer, we now define the sub-system related to z_2, i.e.

$$\dot{z}_2 = \left(\bar{A}_{22} - L_{22}\bar{C}_2\right) z_2 + T_2 B u + T_2 \left(L_{\max} + L_0 \bar{V}\right) y \tag{20}$$

$$r = v_2 \left(\bar{V}_2 y - \bar{C}_2 z_2\right) \tag{21}$$

with

$$T := \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}, \bar{V} := \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}.$$

It is evident that (20) is a reduced order UIFDF which is decoupled from d. In order to solve the problem with unstable invariant zeros, suppose, without loss of generality, that $(\bar{A}_{22}, \bar{C}_2)$ is of the form

$$\bar{A}_{22} = \begin{bmatrix} \bar{A}_{221} & 0 \\ \bar{A}_{222} & \bar{A}_{223} \end{bmatrix}, \bar{C}_2 = \begin{bmatrix} \bar{C}_{21} & 0 \end{bmatrix} \tag{22}$$

as described in Lemma 2, i.e.

$$T(A - LC)T^{-1} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{121} & \bar{A}_{122} \\ 0 & \bar{A}_{221} & 0 \\ 0 & \bar{A}_{222} & \bar{A}_{223} \end{bmatrix} \tag{23}$$

$$\bar{C} = \bar{V}CT^{-1} = \begin{bmatrix} \bar{C}_1 & 0 & 0 \\ 0 & \bar{C}_{21} & 0 \end{bmatrix}, TE_d = \begin{bmatrix} \bar{E}_{d1} \\ 0 \\ 0 \end{bmatrix}. \tag{24}$$

Corresponding to the decomposition given in (24), we now further split z_2, L_{22} and T_2 into

$$z_2 = \begin{bmatrix} z_{21} \\ z_{22} \end{bmatrix}, L_{22} = \begin{bmatrix} L_{221} \\ L_{222} \end{bmatrix}, T_2 = \begin{bmatrix} T_{21} \\ T_{22} \end{bmatrix}$$

and construct the following residual generator

$$\dot{z}_{21} = \left(\bar{A}_{221} - L_{221}\bar{C}_{21}\right) z_{21} + T_{21} B u + T_{21} \left(L_{\max} + L_0 \bar{V}\right) y \tag{25}$$

$$r = v_2 \left(\bar{V}_2 y - \bar{C}_{21} z_{21}\right).$$

It is straightforward to prove that for $e_{21} = T_{21}x - z_{21}$

$$\dot{e}_{21} = \left(\bar{A}_{221} - L_{221}\bar{C}_{21}\right) e_{21}, r = v_2 \bar{C}_{21} e_{21}.$$

That means residual generator (25) is stable and perfectly decoupled from d.

Theorem 2: *Given system (5)-(6) with $F_d = 0$, and suppose that*

$$rank \begin{bmatrix} A - sI & E_d \\ C & 0 \end{bmatrix} < n + m. \tag{26}$$

Then residual generator (25) delivers a residual signal decoupled from d.

In this way, we have removed the requirement on the invariable zeros of (A, E_d, C). As a result, the following corollary is evident.

Corollary 2: *Given system (5)-(6) with $F_d = 0$, then there exists a reduced order FDF that satisfies (7)-(8) if*

$$rank \begin{bmatrix} A - sI & E_d \\ C & 0 \end{bmatrix} < rank \begin{bmatrix} A - sI & E_f & E_d \\ C & 0 & 0 \end{bmatrix} \le n + m \tag{27}$$

Remark: *Note that residual generator (25) is in fact a so-called fault diagnosis observer Chen and Patton [1999]. To be consistent with the term FDF used in the last section and considering the basic idea behind the design scheme, we call it reduced order FDF.*

Below, we summarize the major results for the reduced order UIFDF design into two algorithms, which can be easily programed.

Algorithm 1: *Computation of observer gain L_{max} for generating maximally uncontrollable subspace*

Step 0: Setting initial condition: find a maximal solution of

$$E_d^T V_0 = 0$$

for V_0

Step 1: find a maximal solution of

$$W_i \begin{bmatrix} C^T & V_{i-1} \end{bmatrix} = 0,$$

for W_i

Step 2: find a maximal solution of

$$\begin{bmatrix} E_d^T \\ W_i A^T \end{bmatrix} V_i = 0,$$

for V_i

Step 3: check

$$rank\,(V_i) = rank\,(V_{i-1})$$

If no, increase $i = i + 1$ and go to Step 1, otherwise set $\tilde{V} = V_i$,

Step 4: find a solution of

$$A^T \tilde{V} = \begin{bmatrix} C^T & \tilde{V} \end{bmatrix} \begin{bmatrix} K \\ P \end{bmatrix}$$

for (K, P)

Step 5: solve

$$K = L_{max}^T \tilde{V}$$

for the observer gain L_{max}.

Remark: *The term maximal solution is used to denote the maximally dimensional solution X of an equation $MX = 0$ (or $XM = 0$ for a given M). Step 0 to Step 3 are the algebraic version of the algorithm proposed by Bhattacharyya [1978], Wonham [1979] for the computation of the supremal(A^T, C^T)-invariant subspace contained in the null-space of E_d^T. As a result, the dual representation of system (11) becomes maximally unobservable.*

Algorithm 2: *Computation of reduced order UIFDF*

Step 1: Determine $L_{\max}$ that makes $(A - L_{\max}C, E_d, C)$ maximally uncontrollable by using Algorithm 1

Step 2: Transform $(A - L_{\max}C, E_d, C)$ into (12) by a state and an output transformation (controllability and observability decomposition)

Step 3: Transform $(\bar{A}_{22}, \bar{C}_2)$ into (22) by a state transformation (observability decomposition)

Step 4 Select L_{221} ensuring the stability of $\bar{A}_{221} - L_{221}\bar{C}_{21}$

Step 5: Construct residual generator (25)

4. DESIGN OF MINIMUM ORDER UIFDF

Construction of minimum order residual generators is of strong practical interest for the real time application. In this section, we shall propose a design procedure for constructing minimum order UIFDF based on the results achieved in the last two sections.

Assume that the existence condition (26) for a PUIDP is satisfied. Then, applying Algorithm 2 leads to an observable pair $\left(\bar{C}_{21}, \bar{A}_{221}\right)$, see (22). In Ding et al. [1998, 1999] and Ding [2007], it has been proven that for a given observable system (C, A) the minimum order of any LTI residual generator is the minimum observability index.

Moreover, an algorithm has been proposed for the construction of the minimum order residual generator, which is summarized in the following algorithm.

Algorithm 3: Computation of minimum order LTI residual generator

Given a system

$$\dot{x} = Ax + Bu, \quad y = Cx$$

where $C \in R^{m\times n}, A \in R^{n\times n}, x \in R^n$ and $y \in R^m$ *denote the minimum observability index by* $\sigma_{\min}$

Step 1: Solve

$$v_s \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{\sigma_{\min}} \end{bmatrix} = 0$$

for $v_s \in R^{1\times m\sigma_{\min}}, v_s = \begin{bmatrix} v_{\sigma_{\min},0} & v_{\sigma_{\min},1} & \cdots & v_{\sigma_{\min},\sigma_{\min}} \end{bmatrix}$
Step 2: Form

$$G = \begin{bmatrix} G_o & g \end{bmatrix}, G_o = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \in \mathcal{R}^{\sigma_{\min}\times(\sigma_{\min}-1)}$$

$$g = \begin{bmatrix} g_1 \\ \vdots \\ g_{\sigma_{\min}} \end{bmatrix} \in \mathcal{R}^{\sigma_{\min}},$$

$$L = -\begin{bmatrix} v_{\sigma_{min},0} \\ v_{\sigma_{min},1} \\ \vdots \\ v_{\sigma_{min},\sigma_{min}-1} \end{bmatrix} - gv_{\sigma_{\min},\sigma_{\min}}$$

$$H = \begin{bmatrix} v_{\sigma_{\min},1} & v_{\sigma_{\min},2} & \cdots & v_{\sigma_{\min},\sigma_{\min}} \\ v_{\sigma_{\min},2} & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \vdots \\ v_{\sigma_{\min},\sigma_{\min}} & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{\sigma_{\min}-1} \end{bmatrix} B$$

$$w = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} \in \mathcal{R}^{\sigma_{\min}}, v = v_{\sigma_{\min},\sigma_{\min}}$$

where g is so selected that G is stable
Step 3: Construct residual generator

$$\dot{z} = Gz + Hu + Ly, r = vy - wz. \tag{28}$$

Remark: *The dynamics of residual generator (28) is governed by Ding [2007]*

$$\dot{\zeta} = G\zeta, r = w\zeta.$$

Now, applying the above result and Algorithm 3 to the observable pair $\left(\bar{C}_{21}, \bar{A}_{221}\right)$ delivers a residual generator of the order $\sigma_{2,\min}$ with $\sigma_{2,\min}$ denoting the minimum observability index of the pair $\left(\bar{C}_{21}, \bar{A}_{221}\right)$. To show that $\sigma_{2,\min}$ is also the minimum order of (reduced order) UIFDF, we call the reader's attention to the following facts: given system model (11)

- any pair (L, V) that solves the PUIDP leads to

$$(A - LC, E_d, VC) \sim$$
$$\left(\begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ 0 & \tilde{A}_{22} \end{bmatrix}, \begin{bmatrix} \tilde{E}_{d1} \\ 0 \end{bmatrix}, \begin{bmatrix} \tilde{C}_{11} & \tilde{C}_{12} \\ 0 & \tilde{C}_{22} \end{bmatrix}\right)$$

- the subspace spanned by $\left(\tilde{A}_{11}, \tilde{E}_{d1}, \tilde{C}_{11}\right)$ includes the perfect controllable subspace $(\bar{A}_{11}, \bar{E}_{d1}, \bar{C}_1)$ given in Lemma 1
- by a suitable selection of a pair $\left(\tilde{L}_1, \tilde{V}_1\right)$,

$$\left(\tilde{A}_{11} - \tilde{L}_1\tilde{C}_{11}, \tilde{E}_{d1}, \tilde{V}_1\tilde{C}_{11}\right) \sim$$
$$\left(\begin{bmatrix} \tilde{A}_{11,11} & \tilde{A}_{11,12} \\ 0 & \tilde{A}_{11,22} \end{bmatrix}, \begin{bmatrix} \tilde{E}_{d1,1} \\ 0 \end{bmatrix}, \begin{bmatrix} \tilde{C}_{11,11} & \tilde{C}_{11,12} \\ 0 & \tilde{C}_{11,22} \end{bmatrix}\right)$$

  where $\left(\tilde{A}_{11,11}, \tilde{E}_{d1,1}, \tilde{C}_{11,11}\right)$ is perfect controllable.
- Due to the special form of

$$\left(\begin{bmatrix} \tilde{A}_{11,22} & X \\ 0 & \tilde{A}_{22} \end{bmatrix}, \begin{bmatrix} \tilde{C}_{11,22} & X \\ 0 & \tilde{C}_{22} \end{bmatrix}\right) \tag{29}$$

  with X denoting some block of no interest, it is evident that the minimum order of the residual generator for the pair (29) is not larger than the minimum order of the residual generator for the pair $\left(\tilde{A}_{22}, \tilde{C}_{22}\right)$
- the pair (29) is equivalent to the pair $(\bar{A}_{22}, \bar{C}_2)$ given in Lemma 2.

Based on these facts, the following theorem becomes clear.

Theorem 3: *Given system (5)-(6) with* $F_d = 0$. *Using Algorithms 1 - 3,* L, V *can be found that delivers a minimum order UIFDF.*

5. AN EXTENSION

Remember that the results achieved in the last three sections are on the assumption $F_d = 0$. In this section, we extend these results to the case when $F_d \neq 0$. To this end, we rewrite system model (1)-(2) into

$$\begin{bmatrix} \dot{x} \\ \dot{d} \\ \dot{f} \end{bmatrix} = \begin{bmatrix} A & E_d & E_f \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ d \\ f \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} \dot{d} + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \dot{f} \tag{30}$$

$$y = \begin{bmatrix} C & F_d & F_f \end{bmatrix} \begin{bmatrix} x \\ d \\ f \end{bmatrix} + Du. \tag{31}$$

Noting that

$$rank \begin{bmatrix} A - sI & E_d & E_f & 0 & 0 \\ 0 & -sI & 0 & I & 0 \\ 0 & 0 & -sI & 0 & I \\ C & F_d & F_f & 0 & 0 \end{bmatrix} =$$
$$rank \begin{bmatrix} A - sI & E_d & E_f \\ C & F_d & F_f \end{bmatrix} + k_d + k_f$$

$$rank \begin{bmatrix} A - sI & E_d & 0 \\ 0 & -sI & I \\ C & F_d & 0 \end{bmatrix} = rank \begin{bmatrix} A - sI & E_d \\ C & F_d \end{bmatrix} + k_d$$

it holds

$$rank \begin{bmatrix} A - sI & E_d & E_f & 0 & 0 \\ 0 & -sI & 0 & I & 0 \\ 0 & 0 & -sI & 0 & I \\ C & F_d & F_f & 0 & 0 \end{bmatrix} \leq n + m + k_d + k_f$$

$$\Longleftrightarrow rank \begin{bmatrix} A - sI & E_d & E_f \\ C & F_d & F_f \end{bmatrix} \leq n + m$$

$$rank \begin{bmatrix} A - sI & E_d & 0 \\ 0 & -sI & I \\ C & F_d & 0 \end{bmatrix} < n + k_d + m \Longleftrightarrow$$

$$rank \begin{bmatrix} A - sI & E_d \\ C & F_d \end{bmatrix} < n + m.$$

Recalling the definition of invariant zeros, the results given in Theorem 1 and Corollary 1 can be extended to the following corollary.

Corollary 3: *Given system (1)-(2), then the PUIDP is solvable if and only if*

$$rank \begin{bmatrix} A - sI & E_d \\ C & F_d \end{bmatrix} < n + m.$$

Moreover, there exists a (reduced order) UIFDF that satisfies (7)-(8) if

$$rank \begin{bmatrix} A - sI & E_d \\ C & F_d \end{bmatrix} < rank \begin{bmatrix} A - sI & E_d & E_f \\ C & F_d & F_f \end{bmatrix} \leq n + m.$$

Using system model (30)-(31), we can also apply Algorithms 1-3 to design UIFDF for system *(1)-(2)*.

6. DESIGN EXAMPLE

In this section we apply the proposed Algorithms 1 - 3 to design full order, reduced order as well as minimum order UIFDF for inverted pendulum model. We consider the linear model of the system which is linearized at $\theta = 0$. $\theta = 0$ is the upright position of the pendulum. There are three sensors measuring the cart position r, cart velocity $\dot{r}$ and the pendulum position θ and the corresponding faults are f_r, f_{rd} and f_θ. Similarly perfect measurements are assumed, that means no measurements noises are considered. The state variable vector is $(r, \theta, \dot{r}, \dot{\theta})$. The system matrices given blow are taken from Ding [2006].

$$A = \begin{bmatrix} 0 & 0 & -1.95 & 0 \\ 0 & 0 & 0 & 1.0 \\ 0 & -0.12864 & -1.9148 & -0.0082 \\ 0 & 21.4745 & 26.31 & -0.1362 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ -6.1343 \\ 84.303 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = 0_{3\times1}, \quad E_d = B \quad F_d = D$$

$$E_f = [B \;\; 0_{4\times3}], \quad F_f = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It is also noticed that the system

$$\begin{bmatrix} A - sI & E_d \\ C & F_d \end{bmatrix}$$

has no transmission zeros. The existence conditions for the solution of PUIDP given in Corollary 1 are satisfied. The parameters computed for full, reduced and minimum order UIFDF are given as follows.

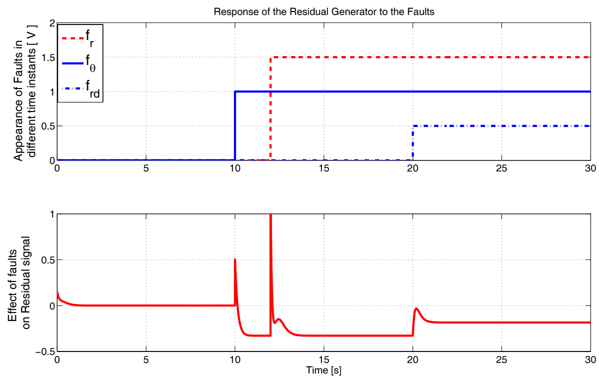

Fig. 1. Response of Full order Residual Generator to simultaneous faults

Full order UIFDF: The parameters for Full order UIFDF (13) are computed as follows

$$L_{\max} = \begin{bmatrix} 0 & 0.0000 & -1.9500 \\ 0 & 0.0000 & -13.7429 \\ 0 & 0.7131 & 0.2470 \\ 0 & 0.0519 & 0.0180 \end{bmatrix}$$

Similarly the state and output transformations are

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -0.1630 & 0 \\ 0.0055 & 0.9473 & 0.3193 & 0.0232 \\ 0.0676 & 0.3191 & -0.9428 & -0.0686 \\ 0.9977 & -0.0269 & 0.0621 & 0.0045 \end{bmatrix}$$

$$\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

where

$$T_1 \in \Re^{1\times4}, T_2 \in \Re^{3\times4}$$
$$\bar{V}_1 \in \Re^{1\times3}, \bar{V}_2 \in \Re^{2\times3}$$

$$TL_0 = \begin{bmatrix} -1.0000 & 0.5000 & 0.5000 \\ 0 & 11.0500 & 11.8438 \\ 0 & 0.8796 & 5.1779 \\ 0 & -0.1223 & 7.0497 \end{bmatrix}$$

$$V = [\,0 \;\; v_2\,] = [\,0 \;\; 0.500 \;\; 1.000\,]$$

Reduced order UIFDF: The parameters of Reduced order UIFDF (20) are given as follows

$$\bar{C}_2 = \begin{bmatrix} 0.9473 & 0.3191 & -0.0269 \\ 0.0055 & 0.0676 & 0.9977 \end{bmatrix}$$

$$L_{11} = -1.000, \quad L_{22} = \begin{bmatrix} 11.0500 & 11.8438 \\ 0.8796 & 5.1779 \\ -0.1223 & 7.0497 \end{bmatrix}$$

and $v_2, \bar{V}_2, T_2$ in equation (20) is given above.

Minimum order UIFDF: The parameters of minimum order UIFDF (28) are obtained as follows

$$L = [0 \;\; 0 \;\; 1.000] \quad G = -1, \quad H = 0$$
$$w = 1, \quad v = [0 \;\; 0 \;\; 1];$$

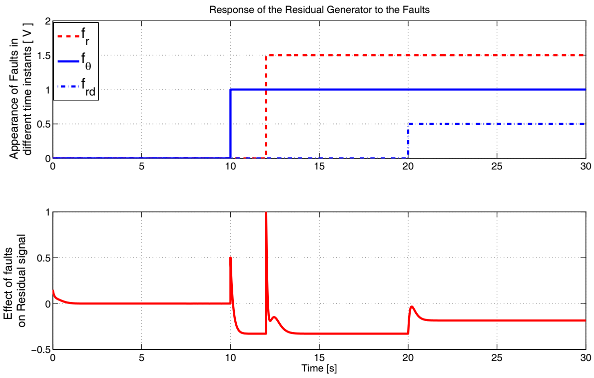

Fig. 2. Response of Reduced order residual generator to simultaneous faults

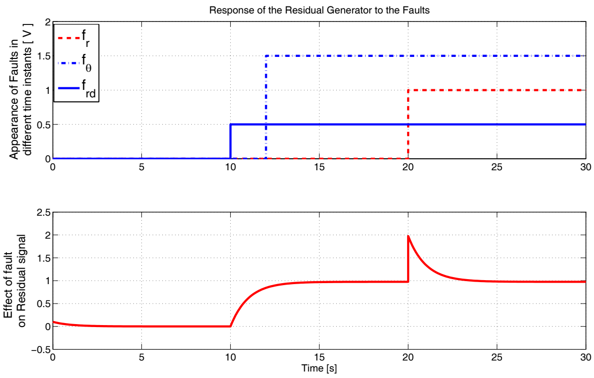

Fig. 3. Response of minimum order residual generator to simultaneous faults

The simulation results for all three types of FDF are shown in Fig. (1) - (3). The faults are simulated using step functions activated at different time instants. The results for reduced order UIFDF are similar to full order UIFDF, which means that reduced order UIFDF can be efficiently used instead of Full order UIFDF. Perfect decoupling of disturbances and sensitivity to faults in the simulation results, also help us that minimum order UIFDF can be used for this purpose as well. The price we paid for this decoupling is the actuator fault is not detected. However this is not the case in general. It is because of the system structure as the disturbance and the actuator input are forming the same subspace. So by decoupling the residual from the disturbance indeed we make it insensitive to the the actuator fault.

7. CONCLUSION

In this work we have presented the solutions of problems related to design of fault detection filters which are perfectly decoupled from unknown inputs. The major focus has been on the study of existence conditions for the solution of perfect unknown input decoupling problem using reduced order UIFDF. The algorithms for reduced order unknown input fault detection filter and minimum order UIFDF are provided. The simulation results show the effectiveness of the proposed approaches.

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