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Multi-model approaches for Integrated Design of Wastewater Treatment Plants with Model Predictive Control

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Abstract: In this work some multi-model and norm based approaches for Integrated Design of processes and constrained Model Predictive Control systems have been proposed. The Integrated Design procedure provides simultaneously the plant dimensions and working point together with the parameters of the control system by solving a multi-objective constrained non-linear optimization problem. Particularly, the cost functions include investment, operating costs, and dynamical indexes based on the weighted sum of the H_∞ and l_1 norms of different closed loop transfer functions matrices of the system, following a novel robust approach. The paper illustrates the application of the proposed methodology for the Integrated Design of the activated sludge process of a wastewater treatment plant (WWTP).

1. INTRODUCTION

Integrated Design (ID) methodology allows for the evaluation of the plant parameters and control system at the same time, making the designed system more controllable [5]. At design stage, controllability indicators are evaluated together with economic considerations, in order to give an optimum plant. Many works apply Integrated Design techniques, particularly to chemical process design. Other works also tackle process and control structure selection by solving a synthesis problem. A comprehensive review of advances in the area is given by [9].

Some good examples of some closed loop ID methodologies applied to the activated sludge process and PI control are given in [2], where a set of Linear Matrix Inequality (LMI) constraints are considered in the problem in order to state stability conditions and some desired closed-loop behaviour.

In more recent works, the same authors use Model Predictive Control (MPC) to improve the control performance within the Integrated Design framework [3]. The reasons were the existence of several successful applications in activated sludge control ([11], [12]) and the easiness of these techniques to deal with constraints and multivariable systems. In those papers, the Integrated Design problem is also stated mathematically as a constrained non-linear multi-objective optimization problem. The solution of the ID problem is obtained using dynamic models and real data records of disturbances together with a set of predefined constraints to evaluate the plant dimensions, the optimal operation points and the control system parameters. During the ID procedure, some objectives to be minimised and some constraints, are expressed by means of dynamical indexes such as ISE (Integral Square Error), AE (turbines Aeration Energy), PE (Pumping Energy), etc. This makes the ID procedure very slow since dynamical simulations must be carried out iteratively to be able to evaluate them.

Taking into account previous considerations the objective of the present work has been to propose norm based ID schemes for activated sludge processes with predictive control systems in which no dynamical simulations are needed, taking as a starting point the single model approach presented in [4].

The main contributions of the paper are related to the cost functions and the constraints selected for ID procedure. Particularly, the new cost functions include investment, operating costs, and dynamical indexes based on the weighted sum of the H_∞ and l_1 norms of different closed loop transfer functions matrices of the system, following a novel multi-model and multi-objective approach. The constraints are selected to ensure that the process variables, some closed loop controllability measures and several closed loop performance criteria lay within specified bounds. Additionally, some robustness conditions are also included as constraints to guarantee that the resulting plant and control system designs are robust in the face of non linearities and disturbances acting on the process.

The proposed methodology for the integrated design is also subdivided in several steps:

1) Input of initial plant and controller information. It includes wastewater and control system characterisation (plant and control type, process models, plant load, etc.)

2) Definition of design objectives and constraints (environmental, economic, operational, control, etc.)
3) Multi-objective optimization procedure.
4) Validation of results.

The paper is organized as follows. First, the activated sludge process is presented and the way to implement an MPC for this process is explained. Secondly, a method for automatic tuning of the MPC is presented and applied to the process. Then, the Integrated Design procedure is stated and solved for the activated sludge process, showing some results to end up with conclusions.

2. Description of the activated sludge process and model predictive controller

2.1. Plant description

The plant layout is represented in Figure 1, consisting of one aerobic tank and one secondary settler. The basis of the process lies in maintaining a microbial population (biomass) into the bioreactor that transforms the biodegradable pollution (substrate) when dissolved oxygen is supplied through aeration turbines. Water coming out of the reactor goes to the settler, where the activated sludge is separated from the clean water and recycled to the bioreactor to maintain there an adequate concentration of microorganisms.

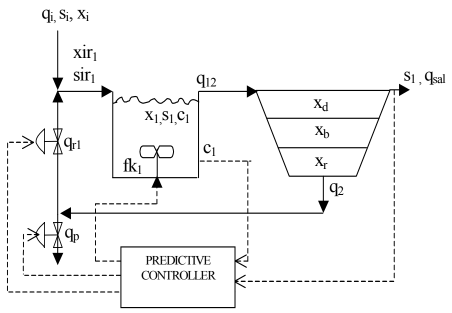

Fig. 1: Plant and controller layout

The set of equations for the nonlinear model (reactor and settler) are the following:

$$
\begin{aligned}
&\frac{dx}{dt} = \mu_{\max} y \frac{sx}{(K_s + s)} - K_d \frac{x^2}{s} - K_c x + \frac{q}{V_1}(xir - x) \\
&\frac{ds}{dt} = -\mu_{\max} \frac{sx}{(K_s + s)} + f_{kd} K_d \frac{x^2}{s} + f_{kd} K_c x + \frac{q}{V_1}(sir - s) \\
&\frac{dc}{dt} = K_{la} fk_1 (c_s - c) - K_{01} \mu_{\max} \frac{x^2}{(K_s + s)} - \frac{q}{V_1} c \qquad (1) \\
&A \cdot l_d \frac{dx_d}{dt} = q_{sal} x_b - q_{sal} x_d - A \cdot vs(x_d) \\
&A \cdot l_b \frac{dx_b}{dt} = q x_1 - q_{sal} x_b - q_2 x_b + A \cdot vs(x_d) - A \cdot vs(x_b) \\
&A \cdot l_r \frac{dx_r}{dt} = q_2 x_b - q_2 x_r + A \cdot vs(x_b)
\end{aligned}
$$

The whole set of variables is presented also in Figure 1. Generically, "x" is used for the biomass concentrations (mg/l), "s" for the organic substrate concentrations (mg/l), "c" for the oxygen concentrations (mg/l) and "q" for flow rates (m^3/h). A detailed description of the mass balances and model parameters is given in [8].

2.2. Control problem

The control of this process aims to keep the substrate at the output (s_1) below a legal value despite the large variations of the flow rate and the substrate concentration of the incoming water (q_i and s_i) (see Fig. 2). This set of disturbances has been determined by COST 624 program and its benchmark [1] Another control objective is to keep dissolved oxygen concentration (c_1) around 2 mg/l, concentration that is necessary for the proper working of activated sludge process.

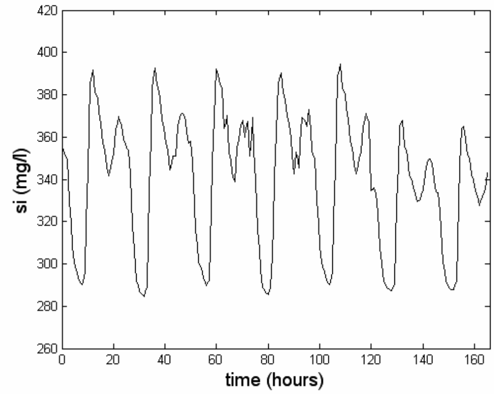

Fig. 2: Substrate disturbances at the influent (s_i)

The general structure of a multivariable controller applied to the activated sludge process can be seen also in Figure 1. Three manipulated variables are considered: recycling flow (qr_1), purge flow (q_p) and aeration factor (fk_1); and also three outputs: biomass (x_1), oxygen (c_1) and substrate (s_1) in the reactor. In our case, although the methodology is general, in order to simplify the problem only substrate control with the recycling flow as manipulated variable is considered.

2.3. MPC applied to the process

A standard linear multivariable MPC has been considered to apply the automatic tuning procedure and the Integrated Design methodology proposed in this paper. It calculates manipulated variables by solving an on-line constrained optimization problem [6].

$$
\begin{aligned}
&\min_{\Delta u} V(k) = \sum_{i=Hw}^{Hp} W_y \cdot (\hat{y}(k+i \mid k) - r(k+i \mid k))^2 + \\
&+ \sum_{i=0}^{Hc-1} W_u \cdot (\Delta \hat{u}(k+i \mid k))^2
\end{aligned} \qquad (2)
$$

subject to constraints on predicted outputs, inputs, and changes in manipulated variables. k denotes the current sampling point, $\hat{y}(k+i|k)$ is the predicted output at time $k+i$, depending of measurements up to time k, $r(k+i|k)$ is the reference trajectory, $\Delta\hat{u}$ are the changes in the manipulated variables, H_p is the upper prediction horizon, H_w is the lower prediction horizon, H_c is the control horizon, W_u is a vector representing the weights of the change of manipulated variables and W_y is a vector representing the weights of the errors of set-points tracking.

The MPC prediction model used in this paper is a linear discrete state space model of the plant obtained by linearizing the model equations (1).

When the MPC controller is linear and unconstrained, it can be represented by the block diagram of figure 3, i.e.

$$u = K_1(r-y) + K_3 d \qquad (3)$$

where K_i are the transfer functions between the control signal and the different inputs (reference r, output y, disturbances d) which depend on the control system tuning parameters (W_u, H_p, H_c). Consequently, the closed loop response can be obtained from

$$y = \frac{GK_1}{1+GK_1} r + \frac{1}{1+GK_1}\tilde{d} \quad \text{where} \quad \tilde{d} = \left(GK_3 + G_d\right) d \qquad (4)$$

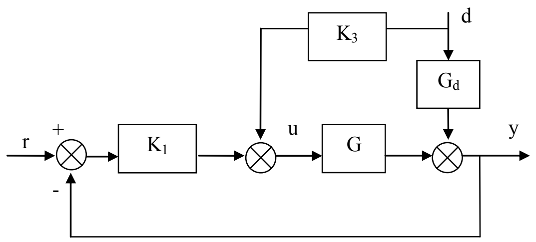

Fig. 3: Equivalent closed loop system

In order to define the automatic tuning problem, we define the sensitivity function S' between the load disturbances (d) and the outputs (y) and M' the Control Sensitivity transfer function defined between the load disturbances (d) and the control signals (u) when the reference is zero. Their calculation is straightforward applying block algebra to diagram of figure 3:

$$S'(s) = \frac{y(s)}{d(s)} = \frac{K_3 G + G_d}{1+GK_1} \quad M'(s) = \frac{u(s)}{d(s)} = \frac{K_3 - K_1 G_d}{1+GK_1} \qquad (5)$$

Note that *u(k)* and *d(k)* vectors are defined for our application in this way: $u(k) = (qr_1)$; $d(k) = (s_i, q_i)$

3. Optimal automatic tuning of MPC

3.1. Mixed sensitivity optimization problem

The problem of finding an optimal MPC is stated as a mixed sensitivity optimization problem ([7]) that takes into account both disturbance rejection and control effort objectives, in the same tuning function. The problem definition is then

$$\min_{K_1,K_3} \|N\|_\infty = \max_w \left|N\left(jw\right)\right| \quad \text{where } N = \begin{pmatrix} W_p S' \\ W_{esf} s \cdot M' \end{pmatrix} \qquad (6)$$

(4)

subject to the set of constraints explained below. K_1 and K_3 are the MPC control compensators (see block diagram of Figure 3) which depend on the tuning parameter vector defined by $c = \left(H_p, H_c, W_u\right)$. W_p and W_{esf} are suitable weights for optimization and w is the frequency. Note that control efforts rather than magnitudes of control are included in the objective function by considering the derivative of the transfer function M'.

3.2. Performance constraints

In order to ensure proper disturbance rejection

$$\left\|W_p \cdot S'\right\|_\infty < 1 \qquad (7)$$

W_p is selected for the specification of load disturbances rejection, what means that its inverse must be smaller in magnitude than the inverse of disturbances spectrums.

3.3. Limits on control and output variables

The maximum value of the control (u_{max}) and the output variable (y_{max}) for the worst case of disturbances can be constrained to be less than certain limits by means of its l_1 norm and the following conditions:

$$\|M'\|_1 < u_{\max} \quad \|S'\|_1 < y_{\max} \qquad (8)$$

The l_1-norm of a stable transfer function such as M', for a SISO system, is defined as follows:

$$\|M'\|_1 = \max_{d(t)} \frac{\|u(t)\|_\infty}{\|d(t)\|_\infty} \qquad (9)$$

3.4. Multi-objective optimization approach

The optimization problems for the automatic tuning within the Integrated Design framework can be defined as a multiobjective optimization problem by defining the following objectives:

$$f_1 = \|N\|_\infty \; ; \; f_2 = \|M'\|_1 \; ; \; f_3 = \|S'\|_1 \tag{10}$$

with the respective goals f_1^*, f_2^*, f_3^*. In order to keep satisfying constraints (8) when the solutions do not get the objectives exactly, goals are chosen in the following way:

$$f_2^* < u_{\max} \; ; \; f_3^* < y_{\max} \tag{11}$$

3.5. Multiple models for robustness

The statement of the problem presented can be modified to include not only the nominal model but also linearized models around a set of working points. For instance, to obtain robust performance in the face of non linearities, the constraint (7) can be rewritten to in the following way:

$$\|W_p \cdot S_i'\|_\infty < 1 \qquad \text{i= 1,…, N} \tag{12}$$

where S_i' are the sensitivity functions obtained with those linearized models, being N the number of multiple models considered.

3.6. Algorithm description and implementation

The main problem when solving this optimization problem is that involves real and integer variables. In this work we propose a two iterative steps algorithm that combines a random search based on the Solis method [10] for tuning the horizons, and the goal attainment multiobjective optimization method for tuning weights W_u.

The controller implementation is based on the MPC Toolbox of MATLAB ® and some modifications of Maciejowski [6]. The real part of the optimization problem is implemented also in MATLAB, specifically in the function *fgoalattain*.

4. Integrated Design of Plant and MPC controller.

The Integrated Design problem consists of determining simultaneously the plant and controller parameters and a steady state working point, while the investment and operating costs are minimized. The algorithm for solving the nonlinear optimization problem generated tackles the problem in an iterative two step approach (see Fig. 4). The first step performs the controller tuning, and the second step the plant design with the previous controller obtained. The loop is finished when a convergence criteria over costs is reached.

The objective functions selected for the plant design step are:

$$f_1 = w_1 \cdot V_{1n} + w_2 \cdot A_n \; ; \; f_2 = \|M'\|_1 \; ; \; f_3 = \frac{\|G\|_1}{\|G_d\|_1} \tag{13}$$

where V_{1n} and A_n are the normalized values for the volume of the reactor and the cross-sectional area of the settler, and w_i=1 (i = 1, 2).

The purpose of objective f_2 is to design plants in which control magnitudes be less than one fixed value for the worst linear case. As for objective f_3, it is related to the input signal needed for perfect control when several disturbances are present. Some nonlinear process and controllability constraints are, for instance:

- Residence time and mass load in the aeration tanks:

$$2.5 \le \frac{V_1}{q_{12}} \le 8 \; ; \; 0.001 \le \frac{q_i s_i + q_{r1} s_1}{V_1 x_1} \le 0.1 \tag{14}$$

- Limits in hydraulic capacity and in the relationship between the input, and recycled flow rates

$$\frac{q_{12}}{A} \le 0.7 \; ; 0.05 \le \frac{q_2}{q_i} \le 0.9 \tag{15}$$

- Constraints on the non-linear differential equations of the plant model to obtain a solution close to a steady state
- Constraints for robustness over a set of controllers and plants defined around the nominal ones.

$$\|W_p \cdot S_i'\|_\infty < 1 \qquad i\text{= 1,…, N+M} \tag{16}$$

where S_i' are the sensitivity functions, being N the number of multiple models of the plant and M the number of multiple controllers.

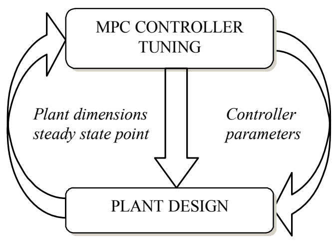

Fig. 4: Iterative loop for Integrated Design

The solution of this optimization problem is also solved with the goal attainment method, and is also subject to lower and upper bounds for optimization variables x=($s_1, x_1, c_1, x_d, x_b, x_r, fk_1, qr_1, q_p, V_1, A$) .

5. Integrated Design results with multiple models

In this point some results are shown when several linear models are considered in order to give some robustness to operational and plant variations. Weights W_p and W_{esf} are kept constant here.

The multiple models have been obtained changing first the working point, because it is the normal operation of the plant, and assuring that the designed plant satisfies the imposed conditions in a region around the nominal point. This case of study of Integrated Design considering multiple models consists of including performance constraints for two new models obtained changing the nominal operation point for the plant influent (s_i±100 mg/l, q_i±220 m^3/h). The optimal plant and MPC obtained considering multiple models rejects disturbances better even for worst case point in the region (s_i+100 mg/l, q_i+220 m^3/h).

TABLE I
RESULTS FOR MULTIPLE MODELS CHANGING S_I,Q_I

	Single model	Multiple models
Operating point	**s_i+100, q_i+220**	**s_i+100, q_i+220**
W_u	0.0044	0.007
H_p	8	7
H_c	2	2
V_1	9476.0	6397.7
A	2084.7	3981.2
S_1	45.32	69.64
max(s1_linear-s1ref)	9.04	4.0485
$\|W_p S'\|_\infty$	37.91	0.71145
Operating point	**s_i+50, q_i+110**	**s_i+50, q_i+110**
Max(qr1)	3500	929.64
Max(s1-s1ref)	21.28	4.24

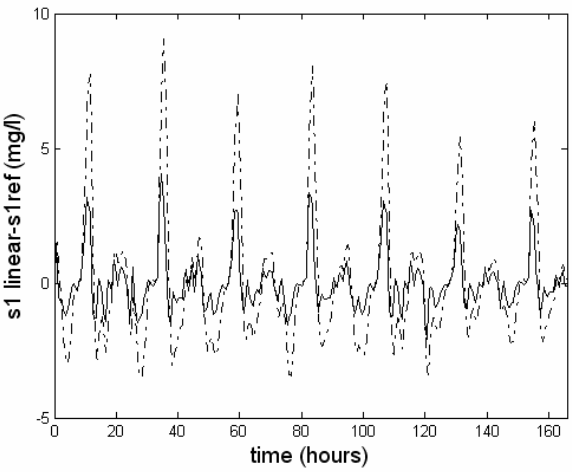

Fig. 5: Linearized model closed loop substrate response for the design with multiple models (solid line) and single model (dashed dotted line).

In order to show that, a comparison between multiple and single model designs simulating the worst case linearized model of the designed plants with the optimal controllers has been performed (see Table I and Fig. 5). The use of the linear model allows for better checking of designs robustness. In Fig. 6 a comparison of the sensitivity functions for both closed loop systems is shown, and it is clear that only the plant designed with multiple models satisfies the constraint imposed by W_p^{-1}. Finally, responses simulating directly the two optimal non linear plants and controllers around the point (s_i+50 mg/l, q_i+110 m^3/h) have been performed (see Fig. 7).

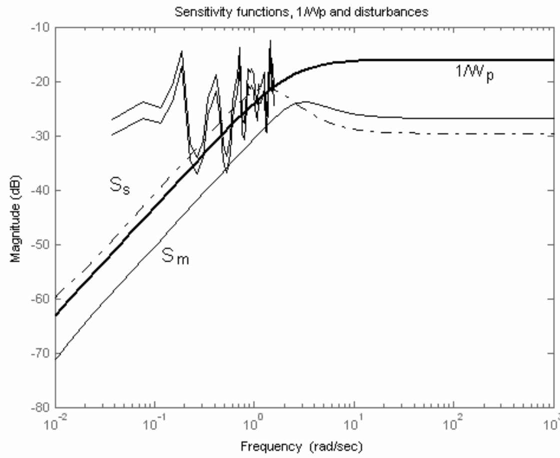

Fig. 6: Sensitivity function for the design with multiple models (solid line, S_m) and single model (dash dotted line, S_s), together with weight W_p^{-1} and the inverse spectrum of influent disturbances

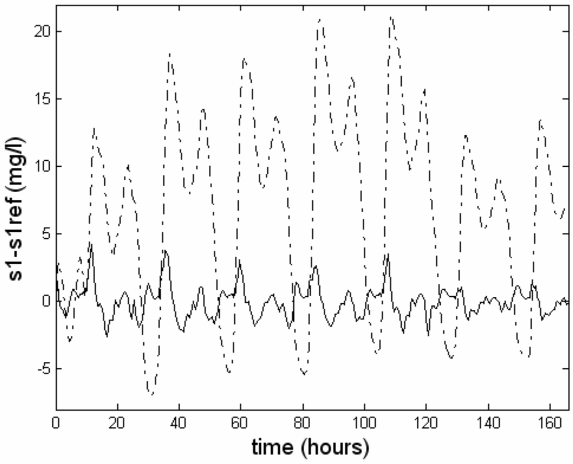

Fig. 7: Non linear plant closed loop substrate response for the design with multiple models (solid line) and single model (dashed dotted line).

The second case of study consists of changing the plant dimensions around nominal values, in order to assure some flexibility to the designed plant in case of future redesigns or to give some error building margin. The optimal plant and

MPC obtained produce better disturbance rejection than those obtained considering only one model, for those plants modified in this way: $V_1 \pm 500\ m^3$, $A \pm 250\ m^2$. Numerical results are shown in Table II for the worst working point.

TABLE II
RESULTS FOR MULTIPLE MODELS CHANGING V_1, A

	Single model	Multiple models
Operating point	**V_1-500, A-250**	**V_1-500, A-250**
W_u	0.0044	0.0018
H_p	8	10
H_c	2	2
V_1	8976.0	4864
A	1834.7	2443
S_1	45.32	64.38
Max(s1_linear-s1ref)	7.0853	3.3016
$\|W_p S'\|_\infty$	1.1994	0.37855

Finally, in the last case of study the multiple models have been obtained changing the plant dimensions ($V_1 \pm 500\ m^3$, $A \pm 250\ m^2$) and the controller weight ($W_u \pm 0.001$), producing similar quality of results that the previous cases. Hence, the controller obtained is robust to plant parameter variations, and the designed plants are robust to controller parameters changes, giving some error margin to further MPC retuning. Numerical results are shown in Table III for the worst case of the plant and the controller.

TABLE III
RESULTS FOR MULTIPLE MODELS CHANGING V_1, A AND CONTROLLER PARAMETER W_U

	Single model	Multiple models
Operating point	**V_1-500, A-250 W_u+0.001**	**V_1-500, A-250 W_u+0.001**
W_u	0.0054	0.0059
H_p	8	10
H_c	2	2
V_1	8976.0	7917.8
A	1834.7	3456.5
S_1	45.32	45.41
Max(qr1_linear)	2125.8	1187.6
Max(s1_linear)	8.23	4.66
$\|W_p S'\|_\infty$	1.4089	0.79344

6. Conclusions

In this paper a multi-model Integrated Design procedure to obtain optimal plants and control systems for activated sludge processes with MPC has been proposed. The approach is based on the optimization of a set of cost functions including investment, operating costs, and dynamical indexes based on the weighted sum of the H_∞ and l_1 norms of different closed loop transfer functions matrices of the system, following a novel multi-objective methodology. Some robustness conditions are also included as constraints to guarantee that the resulting plant and control system designs are robust in the face of non linearities and disturbances acting on the process. A comparison with the single model methodology shows the effectiveness of the new approach. In fact, the optimal plant and MPC obtained produce better disturbance rejection than those obtained considering only one model, even if the working point changes.

Acknowledgments

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