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Non-regular Feedback Linearization of Switched Nonlinear Systems ⋆

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Abstract: In this paper, the problem of non-regular static state feedback linearization of nonlinear switched systems is considered. Using semi-tensor product, some easily verifiable sufficient conditions for non-regular feedback linearization are obtained. Then an example is presented to illustrate the non-regular linearization process.

1. INTRODUCTION

In recent years the investigation of switched control systems becomes a hot topic in control community Liberzon (1999), Sun (2005). The two main topics considered are stabilization and controllability. Lyapunov theory and its inverse theory for switched systems are the key for stability and stabilization Dayawansa (1999), Liberzon (1999). A particular attention has been paid to the quadratic Lyapunov function for switched linear systems Agrachev (2001), Shorten (2003), Cheng (2003). Some results have also been obtained for stabilization of switched systems Zhao (2001), Sun (2001), Cheng (2004), Cheng (2005). As a generalized Lyapunov approach, LaSalle's invariance principle has also been extended to switched systems Hespanha (2004).

The second major topic is the controllability. For switched linear systems, certain necessary and sufficient conditions for global controllability have been revealed Ge (2001), Xie (2003). When the system is not completely controllable, the controllable submanifolds are investigated in Cheng (2006).

There are also some results about switched nonlinear systems, e.g., Vu (2005), Cheng (2005), Mancilla-Aguilar (2006). But comparing with linear case, there are less systematic control techniques for nonlinear switched systems.

One of the most powerful tools to treat nonlinear systems is the linearization technique. We refer to Isidori (1995) for some classical linearization approaches, and to Cheng (2004), Sun (1997), Ge (2001) and the references therein for non-regular state feedback linearization.

Similar to nonlinear systems, if a switched nonlinear system is state feedback linearizable, both the controllability and the stabilizability problem are solved. This is the motivation for current work.

We first review some preliminaries for later investigation:

For a matrix A, let $\sigma(A) = \lambda = (\lambda_1, \cdots, \lambda_n)$ be its eigenvalues. A is a resonant matrix if there exists $m = (m_1, \cdots, m_n) \in Z_+^n$, and $|m| \geq 2$, i.e., $m_i \geq 0$ and $\sum_{i=1}^{n} m_i \geq 2$, such that for some s, $\lambda_s =< m, \lambda >$. A is non-resonant if it is not resonant.

Theorem 1. (Poincare's Theorem) (Arnold (2001))Consider a C^ω dynamic system

$$\dot{x} = Ax + f_2(x) + f_3(x) + \cdots,\ x \in R^n, \tag{1}$$

where $f_i(x)$, $i \geq 2$ are ith degree homogeneous vector fields. If A is non-resonant, there exists a formal change of coordinates $x = y + h(y)$, where $h(y)$ corresponds to the sum of possibly infinite homogeneous vector polynomials $h_m(y)$, $m \geq 2$, that is $h(y) = h_2(y) + h_3(y) + \cdots$, such that system (1) can be expressed as $\dot{y} = Ay$.

The following proposition provides a sufficient condition for non-resonance.

Proposition 2. (Devanathan (2001)) Let $\lambda = (\lambda_1, \cdots, \lambda_n)$ be the eigenvalues of a given Hurwitz matrix A. A is non-resonant if

$$\max\{|Re(\lambda_i)||\lambda_i \in \sigma(A)\} \leq 2\min\{|Re(\lambda_i)||\lambda_i \in \sigma(A)\}.$$

We give the following assumption:

A.1 A is a diagonal matrix with distinct diagonal elements and is non-resonant.

Definition 3. (Cheng (2004)) Let $A = \text{diag}(\lambda_1, \cdots, \lambda_n)$ be a non-resonant matrix. The Lie-inverse matrices are $n \times n^k$, $k = 2, 3 \cdots$ matrices, its elements are

$$(\Gamma_k^n)_{ij} = \frac{1}{(\sum_{s=1}^{n} \alpha_s^j \lambda_s) - \lambda_i},\ i = 1, \cdots, n; \quad j = 1, \cdots, n^k,$$

where $\alpha_1^j, \cdots, \alpha_n^j$ are respectively the powers of $x_1, \cdots, x_n$ of the jth component of x^k.

For the completeness, we define Γ_1^n as following,

$$(\Gamma_1^n)_{i,j} = \begin{cases} 0, & i = j, \\ \dfrac{1}{\lambda_j - \lambda_i}, & otherwise. \end{cases}$$

Let H_k^n be the set of kth degree homogeneous polynomial vector fields in R^n. Then $ad_{Ax} : H_k^n \to H_k^n$ is a linear mapping.

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Proposition 4. (Cheng (2004))

Suppose $A = diag(\lambda_1, \cdots, \lambda_n)$ is non-resonant. Then $ad_{Ax} : H_k^n \to H_k^n$ is a isomorphism mapping. Moreover, if $V(x) = F_k x^k \in H_k^n$, then

$$ad^{-1}(V(x)) = (\Gamma_k^n \odot F_k)x^k,$$

where $\odot$ is the Hadamard product of matrices.

Here and in the following the matrix product is assumed to be semi-tensor product, which is briefly reviewed in Cheng (2002).

Note that if A is a non-resonant and simple matrix, then there is a non-singular matrix T such that TAT^{-1} satisfies A.1.

Proposition 5. (Zhong (2007)) Let A be non-resonant and TAT^{-1} be a diagonal matrix, then

$$ad_{Ax}^{-1}(V(x)) = T^{-1}(\Gamma_k^n \odot \tilde{F}_k)T(I_n \otimes T)\cdots(I_{n^{k-1}} \otimes T)x^k, \quad (2)$$

where

$$\tilde{F}_k = TF_kT^{-1}(I_n \otimes T^{-1})\cdots(I_{n^{k-1}} \otimes T^{-1}).$$

Using semi-tensor product , we can express system (1) as

$$\dot{x} = Ax + F_2x^2 + F_3x^3 + \cdots, \quad (3)$$

where F_k are $n \times n^k$ constant matrices, $k = 2, 3 \cdots$.

Then we have the following result:

Theorem 6. (Cheng (2004)) Assume A satisfies $A.1$. Then system (3) can be transformed into a linear form

$$\dot{z} = Az$$

by the following coordinate transformation:

$$z = x - \sum_{i=2}^{\infty} E_i x^i,$$

where E_i are determined recursively as

$$E_2 = \Gamma_2 \odot F_2,$$

$$E_s = \Gamma_s \odot \Big(F_s - \sum_{i=2}^{s-1} E_i\Phi_{i-1}(I_{n^{i-1}} \otimes F_{s+1-i})\Big), \ s \geq 3.$$

The paper is organized as follows: In Section 2 some sufficient conditions for non-regular feedback linearization are given. An illustrative example is presented in Section 3. Section 4 is a conclusion.

2. NON-REGULAR STATE FEEDBACK LINEARIZATION

In this section we consider non-regular state feedback linearization of nonlinear switched systems.

Consider the following systems

$$\dot{x} = f^{\sigma(t)}(x) + \sum_{i=1}^{m} g_i{}^{\sigma(t)}(x)u_i, \quad x \in \mathbf{R}^n. \quad (4)$$

where $\sigma(t) : [0, \infty) \to \Lambda = \{1, 2, \cdots, N\}$ is a right continuous piecewise constant mapping called the switching signal.

Definition 7. System (4) is said to be (locally) non-regular static feedback linearizable, if it can be transformed into

$$\dot{z} = A^{\sigma(t)}z + B^{\sigma(t)}v, \quad (5)$$

via state feedbacks

$$u^\lambda(x) = \alpha^\lambda(x) + \beta^\lambda(x)v, \quad \lambda \in \Lambda, \quad (6)$$

with $\alpha^\lambda(0) = 0$, $\beta^\lambda(x) : m \times k$, $k < m$, and a state space diffeomorphism

$$z = \phi(x) \quad (7)$$

When $k = 1$ the linearization is called single-input linearization.

Using Heymann's Lemma (Heymann (1968)), it is easy to prove the following:

Lemma 8. System (4) is said to be non-regular static feedback linearizable, iff it is single-input linearizable, i.e. linearizable by control (6) with $m \times 1$ vectors $\beta^\lambda(x)$, $\lambda \in \Lambda$.

Consider an affine nonlinear system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i, \quad x \in \mathbf{R}^n, \ u \in R^m. \quad (8)$$

Lemma 9. (Sun (1997)) Let $A = \frac{\partial f}{\partial x}|_0$, $B = g(0)$. If the system (8) is linearizable, then (A, B) is completely controllable.

Now consider the linearization of system (4). Denote $A^\lambda = \frac{\partial f^\lambda}{\partial x}|_0$, $B^\lambda = g^\lambda(0)$, and assume (A^λ, B^λ), $\lambda \in \Lambda$ are completely controllable pairs. Then we can find feedbacks K^λ such that $\tilde{A}^\lambda = A^\lambda + B^\lambda K^\lambda$, $\lambda \in \Lambda$ are simple and non-resonant. Hence there exist linear coordinate transformations T^λ such that $T^\lambda A^\lambda T^{-\lambda}$, $\lambda \in \Lambda$ satisfy A.1. For the sake of simplicity, we call the above transformations NR-type transformations.

First, we consider system (3) with A is simple and non-resonant. Then there exists a non-singular matrix T such that $\tilde{A} := TAT^{-1}$ satisfies A.1. Then under the linear coordinate transformation $y = Tx$, system (3) can be expressed as

$$\dot{y} = \tilde{A}y + \tilde{F}_2y^2 + \tilde{F}_3y^3 + \cdots, \quad (9)$$

where

$$\tilde{F}_k = TF_kT^{-1}(I_n \otimes T^{-1})\cdots(I_{n^{k-1}} \otimes T^{-1}), \ k = 2, 3 \cdots.$$

Similar to Theorem 6, we can prove the following :

Proposition 10. System (9) can be transformed into a linear form

$$\dot{z} = \tilde{A}z$$

by the following coordinate transformation:

$$z = y - \sum_{i=2}^{\infty} \tilde{E}_i y^i,$$

where $\tilde{E}_i$ are determined recursively as

$$\tilde{E}_2 = \Gamma_2 \odot \tilde{F}_2,$$

$$\tilde{E}_s = \Gamma_s \odot \Big(\tilde{F}_s - \sum_{i=2}^{s-1} \tilde{E}_i\Phi_{i-1}(I_{n^{i-1}} \otimes \tilde{F}_{s+1-i})\Big), s \geq 3.$$

Now let $\tilde{z} = T^{-1}z$, then we have

$$\dot{\tilde{z}} = T^{-1}\tilde{A}T\tilde{z} = A\tilde{z}.$$

Using Proposition 10, we have :

Proposition 11. System (3) can be transformed into a linear form

$$\dot{\tilde{z}} = A\tilde{z}$$

by the following coordinate transformation:

$$\tilde{z} = x - \sum_{i=2}^{\infty} T^{-1}\tilde{E}_i T(I_n \otimes T)\cdots(I_{n^{i-1}} \otimes T)x^i,$$

where $\tilde{E}_i$ are determined recursively as

$$\tilde{E}_2 = \Gamma_2 \odot \tilde{F}_2,$$

$$\tilde{E}_s = \Gamma_s \odot \left(\tilde{F}_s - \sum_{i=2}^{s-1} \tilde{E}_i \Phi_{i-1}(I_{n^{i-1}} \otimes \tilde{F}_{s+1-i})\right), s \geq 3.$$

In the following, we consider the non-regular linearization of system (4).

Theorem 12. System (4) is non-regular state feedback linearizable, if

(i) (A^λ, B^λ), $\lambda \in \Lambda$ are completely controllable, where $A^\lambda = \dfrac{\partial f^\lambda}{\partial x}|_0$, $B^\lambda = g^\lambda(0)$.

(ii) there exist NR-transformations T^λ, $\lambda \in \Lambda$;

(iii) $T^{-j}\tilde{E}_i^j T^j(I_n \otimes T^j)\cdots(I_{n^{i-1}} \otimes T^j) = T^{-k}\tilde{E}_i^k T^k(I_n \otimes T^k)\cdots(I_{n^{i-1}} \otimes T^k)$, $\forall j, k \in \Lambda$, $j \neq k$, $i = 2, \cdots, \infty$;

(iv) there exist constant vectors b^λ, $\lambda \in \Lambda$ of non-zero component such that

$$b^\lambda \in Span\Big\{(I - \sum_{i=2}^{\infty} \tilde{E}_i^\lambda \Phi_{i-1}(T^\lambda x)^{i-1})g_j^\lambda,\ j = 1, 2, \cdots, m\Big\}.$$

Proof. First, using Taylor series expression on $f^\lambda(x)$, $\lambda \in \Lambda$ with the form of semi-tensor product, we can express system (4) as

$$\dot{x} = A^\lambda x + \sum_{i=1}^{m} g_i^\lambda(x)u_i + F_2^\lambda x^2 + F_3^\lambda x^3 + \cdots, \qquad (10)$$

where $A^\lambda = \dfrac{\partial f^\lambda}{\partial x}|_0$. Let $B^\lambda = g^\lambda(0)$. From the assumption (A^λ, B^λ) are controllable, we can find K^λ such that $\tilde{A}^\lambda := A^\lambda + B^\lambda K^\lambda$, $\lambda \in \Lambda$ are simple and non-resonant. From the condition (ii), there exist linear coordinate transformation T^λ such that $T^\lambda \tilde{A}^\lambda T^{-\lambda}$, $\lambda \in \Lambda$ satisfy A.1. Then under the feedbacks $u^\lambda = \alpha^\lambda(x) + \beta^\lambda(x)v$ and coordinates $y = T^\lambda x$, system (10) can be expressed as

$$\dot{y} = \tilde{\tilde{A}}^\lambda y + \sum_{i=1}^{m} \tilde{g}_i^\lambda(y)v_i + \tilde{F}_2^\lambda y^2 + \tilde{F}_3^\lambda y^3 + \cdots, \qquad (11)$$

where

$$\tilde{\tilde{A}}^\lambda = T^\lambda \tilde{A}^\lambda T^{-\lambda}, \quad \tilde{g}_i^\lambda(y) = T^\lambda g_i^\lambda(T^{-\lambda}y)\beta^\lambda(T^{-\lambda}y),$$

$$\tilde{F}_k^\lambda = T^\lambda F_k^\lambda T^{-\lambda}(I_n \otimes T^{-\lambda})\cdots(I_{n^{k-1}} \otimes T^{-\lambda}),\ k = 2, 3\cdots.$$

Using Proposition 10 and condition (iii), we can transform (11) into

$$\dot{z} = \tilde{\tilde{A}}^\lambda z + \sum_{i=1}^{m} \tilde{\tilde{g}}_i^\lambda(z)v_i, \qquad (12)$$

via the coordinate transformation $z = y - \sum_{i=2}^{\infty} \tilde{E}_i y^i$.

From (iv), we know there exist constant vectors b^λ, $\lambda \in \Lambda$ of non-zero component such that

$$b^\lambda \in Span\Big\{(I - \sum_{i=2}^{\infty} \tilde{E}_i^\lambda \Phi_{i-1}(T^\lambda x)^{i-1})g_j^\lambda,\ j = 1, 2, \cdots, m\Big\}.$$

that is equivalent to

$$b^\lambda \in Span\Big\{\tilde{\tilde{g}}_i^\lambda(z),\ i = 1, \cdots, m\Big\}.$$

Let

$$b^\lambda = \sum_{i=1}^{m} \xi_i^\lambda \tilde{\tilde{g}}_i^\lambda(z), \quad v_i = \xi_i^\lambda \omega,$$

where ω is the input control. Then system (12) can be transformed into

$$\dot{z} = \tilde{\tilde{A}}^\lambda z + b^\lambda \omega.$$

Suppose $\tilde{z} = T^{-\lambda}z$, then we can get

$$\dot{\tilde{z}} = \tilde{A}^\lambda \tilde{z} + T^{-\lambda}b^\lambda \omega.$$

So system (4) is single-input linearizable. □

3. ILLUSTRATIVE EXAMPLE

Example 13. Consider the linearization problem of the following system

$$\dot{x} = f^{\sigma(t)}(x) + \sum_{i=1}^{2} g_i{}^{\sigma(t)}(x)u_i, \quad x \in \mathbf{R}^3, \qquad (13)$$

where $\sigma(t) : [0, \infty) \to \Lambda = \{1, 2\}$ is the switching signal, the two switching modes are respectively as

$$\begin{cases} \dot{x}_1 = -4x_1 - 4x_2e^{-x_3} - 2x_2(e^{x_3} + 2)u_1 \\ \qquad +e^{x_1 - x_2^2}(1 - 2x_2e^{x_3})u_2 \\ \dot{x}_2 = -2x_2 - (e^{x_3} + 2)u_1 - e^{x_1 + x_3 - x_2^2}u_2 \\ \dot{x}_3 = -5x_3 - x_1 + x_2^2 + 4x_2e^{-x_3} + u_1 - e^{x_1 - x_2^2}u_2; \end{cases} \qquad (14)$$

and

$$\begin{cases} \dot{x}_1 = -2x_1 - 4x_2^2 - 6x_2^3e^{x_1} + (e^{x_1} + 12x_2)u_1 + u_2 \\ \dot{x}_2 = -3x_2 + 6u_1 \\ \dot{x}_3 = -4x_3 + 6x_2^3e^{x_1} + (7 - e^{x_1})u_1 - u_2. \end{cases} \qquad (15)$$

We first consider the non-regular state feedback linearization for the first mode. It is easy to get

$$A^1 := \frac{\partial f^1}{\partial x} = \begin{pmatrix} -4 & -4 & 0 \\ 0 & -2 & 0 \\ -1 & 4 & -5 \end{pmatrix},\ b_1^1 := g_1^1(0) = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix},$$

$$b_2^1 := g_2^1(0) = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}.$$

Using state feedback

$$u_1 = v_1,\ u_2 = 4x_2e^{-x_3 - x_1 + x_2^2} + v_2.$$

Therefore (14) can be expressed as

$$\begin{cases} \dot{x}_1 = -4x_1 - 8x_2^2 - 2x_2(e^{x_3} + 2)v_1 \\ \qquad +e^{x_1 - x_2^2}(1 - 2x_2e^{x_3})v_2 \\ \dot{x}_2 = -6x_2 - (e^{x_3} + 2)v_1 - e^{x_1 + x_3 - x_2^2}v_2 \\ \dot{x}_3 = -5x_3 - x_1 + x_2^2 + v_1 - e^{x_1 - x_2^2}v_2; \end{cases} \qquad (16)$$

Hence,

$$\tilde{A}^1 := \frac{\partial \tilde{f}^1}{\partial x} = \begin{pmatrix} -4 & 0 & 0 \\ 0 & -6 & 0 \\ -1 & 0 & -5 \end{pmatrix}.$$

In the following, we will change $\tilde{A}^1$ into a diagonal matrix. Let

$$y = T_1 x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} x$$

Therefore (16) can be expressed as

$$\begin{cases} \dot{y}_1 = -4y_1 - 8y_2^2 - 2y_2(e^{y_3-y_1}+2)v_1 \\ \qquad +e^{y_1-y_2^2}(1-2y_2e^{y_3-y_1})v_2 \\ \dot{y}_2 = -6y_2 - (e^{y_3-y_1}+2)v_1 - e^{y_3-y_2^2}v_2 \\ \dot{y}_3 = -5y_3 - 7y_2^2 + [1-2y_2(e^{y_3-y_1}+2)]v_1 \\ \qquad -2y_2e^{y_3-y_2^2}v_2. \end{cases} \tag{17}$$

Then

$$\lambda_1^1 = -4,\ \lambda_2^1 = -6,\ \lambda_3^1 = -5.$$

Next we search for the coordinate transformation. It is easy to calculate that

$$\tilde{F}_2^1 = \begin{pmatrix} 0&0&0&0&-8&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&-7&0&0&0&0 \end{pmatrix}$$

$$\Gamma_2^3 = \begin{pmatrix} *&*&*&*&-\frac{1}{8}&*&*&*&* \\ *&*&*&*&*&*&*&*&* \\ *&*&*&*&-\frac{1}{7}&*&*&*&* \end{pmatrix}$$

where $*$ does not affect the calculation.

Therefore,

$$\tilde{E}_2^1 = \Gamma_2^3 \odot \tilde{F}_2^1 == \begin{pmatrix} 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \end{pmatrix}$$

Notice that

$$\Phi_1 = I_1 \otimes W_{[3,3]} + I_3 \otimes W_{[1,3]} = \begin{pmatrix} 2&0&0&0&0&0&0&0&0 \\ 0&1&0&1&0&0&0&0&0 \\ 0&0&1&0&0&0&1&0&0 \\ 0&1&0&1&0&0&0&0&0 \\ 0&0&0&0&2&0&0&0&0 \\ 0&0&0&0&0&1&0&1&0 \\ 0&0&1&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0&1&0 \\ 0&0&0&0&0&0&0&0&2 \end{pmatrix}$$

Then we can get

$$\tilde{E}_s^1 = 0,\ \ s \geq 3.$$

Hence the coordinate transformation is

$$z = y - \tilde{E}_2^1 y^2 = \begin{pmatrix} y_1 - y_2^2 \\ y_2 \\ y_3 - y_2^2 \end{pmatrix}.$$

Under coordinates z, (17) can be expressed as

$$\begin{cases} \dot{z}_1 = -4z_1 + e^{z_1}v_2 \\ \dot{z}_2 = -6z_2 - (e^{z_3-z_1}+2)v_1 - e^{z_3}v_2 \\ \dot{z}_3 = -5z_3 + v_1. \end{cases} \tag{18}$$

Setting $\tilde{z} = T_1^{-1}z$, under coordinates $\tilde{z}$, (18) can be expressed as

$$\dot{\tilde{z}} = \begin{pmatrix} -4 & 0 & 0 \\ 0 & -6 & 0 \\ -1 & 0 & -5 \end{pmatrix} \tilde{z} + \begin{pmatrix} 0 \\ -(e^{\tilde{z}_3-2\tilde{z}_1}+2) \\ 1 \end{pmatrix} v_1 + \begin{pmatrix} e^{\tilde{z}_1} \\ -e^{\tilde{z}_3-\tilde{z}_1} \\ -e^{\tilde{z}_1} \end{pmatrix} v_2.$$

Obviously, there is a constant vector b of non-zero component such that

$$b = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -(e^{\tilde{z}_3-2\tilde{z}_1}+2) \\ 1 \end{pmatrix} - e^{-\tilde{z}_1} \begin{pmatrix} e^{\tilde{z}_1} \\ -e^{\tilde{z}_3-\tilde{z}_1} \\ -e^{\tilde{z}_1} \end{pmatrix}.$$

Through feedback

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 4x_2e^{-x_3-x_1+x_2^2} \end{pmatrix} + \begin{pmatrix} 1 \\ -e^{-x_1+x_2^2} \end{pmatrix} w$$

and the coordinate transformation

$$\begin{cases} \tilde{z}_1 = x_1 - x_2^2 \\ \tilde{z}_2 = x_2 \\ \tilde{z}_3 = x_3, \end{cases}$$

(14) is transformed into

$$\dot{\tilde{z}} = \begin{pmatrix} -4 & 0 & 0 \\ 0 & -6 & 0 \\ -1 & 0 & -5 \end{pmatrix} \tilde{z} + \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} w,$$

where w is the control input.

In the following, we consider mode 2.

It is easy to get

$$A^2 := \frac{\partial f^2}{\partial x} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{pmatrix},\ b_2^1 := g_2^1(0) = \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix},$$

$$b_2^2 := g_2^2(0) = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$$

Using feedback

$$u_1 = v_1,\ u_2 = 6x_2^3e^{x_1} + v_2,$$

(15) is transformed into

$$\begin{cases} \dot{x}_1 = -2x_1 - 4x_2^2 + (12x_2 + e^{x_1})v_1 + v_2 \\ \dot{x}_2 = -3x_2 + 6v_1 \\ \dot{x}_3 = -4x_3 + (7 - e^{x_1})v_1 - v_2. \end{cases} \tag{19}$$

Then

$$\lambda_1^2 = -2,\ \lambda_2^2 = -3,\ \lambda_3^2 = -4.$$

It is easy to calculate that

$$\tilde{F}_2^2 = \begin{pmatrix} 0&0&0&0&-4&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$\Gamma_2^3 = \begin{pmatrix} *&*&*&*&-\frac{1}{4}&*&*&*&* \\ *&*&*&*&*&*&*&*&* \\ *&*&*&*&*&*&*&*&* \end{pmatrix}$$

where $*$ does not affect the calculation.

Therefore,

$$\tilde{E}_2^2 = \Gamma_2^3 \odot \tilde{F}_2^2 == \begin{pmatrix} 0\,0\,0\,0\,1\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0 \end{pmatrix}$$

Notice that

$$\Phi_1 = I_1 \otimes W_{[3,3]} + I_3 \otimes W_{[1,3]} = \begin{pmatrix} 2\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,1\,0\,1\,0\,0\,0\,0\,0 \\ 0\,0\,1\,0\,0\,0\,1\,0\,0 \\ 0\,1\,0\,1\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,2\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,1\,0\,1\,0 \\ 0\,0\,1\,0\,0\,0\,1\,0\,0 \\ 0\,0\,0\,0\,0\,1\,0\,1\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,2 \end{pmatrix}$$

Then we can get

$$\tilde{E}_s^2 = 0, \;\; s \geq 3.$$

It is easy to verify that

$$T_1^{-1}\tilde{E}_2^1 T_1(I_3 \otimes T_1) = T_2^{-1}\tilde{E}_2^2 T_2(I_3 \otimes T_2)$$
$$= \begin{pmatrix} 0\,0\,0\,0\,1\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0 \end{pmatrix},$$

where $T_2 = I_3$. So the condition (iii) of Theorem 3.5 is satisfied.

Then under the coordinate transformation

$$\begin{cases} z_1 = x_1 - x_2^2 \\ z_2 = x_2 \\ z_3 = x_3, \end{cases}$$

system (19) can be transformed into

$$\begin{cases} \dot{z}_1 = -2z_1 + e^{z_1+z_2^2}v_1 + v_2 \\ \dot{z}_2 = -3z_2 + 6v_1 \\ \dot{z}_3 = -4z_3 + (7 - e^{z_1+z_2^2})v_1 - v_2. \end{cases} \tag{20}$$

Then it is easy to verify that

$$b = \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix} = \begin{pmatrix} e^{z_1+z_2^2} \\ 6 \\ 7 - e^{z_1+z_2^2} \end{pmatrix} + (1 - e^{z_1+z_2^2}) \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$$

So under the coordinates z and the feedback control

$$u = \alpha(x) + \beta(x)w = \begin{pmatrix} 0 \\ 6x_2^3 e^{x_1} \end{pmatrix} + \begin{pmatrix} 1 \\ 1 - e^{x_1} \end{pmatrix} w,$$

(15) is expressed as

$$\dot{z} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{pmatrix} z + \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix} w, \; z \in R^3,$$

where w is the control input. □

4. CONCLUSIONS

Non-regular static state feedback linearization of switched nonlinear systems was discussed in this paper. Using semi-tensor product some easily verifiable sufficient conditions for non-regular feedback linearization were obtained. An illustrative example was presented to depict the linearization technique.

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