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Stabilization of Nonlinear Switched Continuous-Time Complex Systems

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Abstract: The objective of this paper is to propose an approach to decentralized robust stabilization with state-dependent supervisor for a class of nonlinear switched symmetric composite systems. The proposed methodology employs the structural properties of the system to construct a low order control design model as well as the multiple Lyapunov functions technique. Static output feedback gain matrices robustly stabilizing this model are designed by using bilinear matrix inequalities (BMIs). These inequalities can be used as linear matrix inequalities (LMIs) when selecting appropriate parameters in advance. The switching process is decentralized into independent switching rules operating only on local subsystems states. It is shown that if the set of gain matrices of this switching controller is implemented as an identical set into each local switching controller of the global decentralized controller, then the overall closed-loop system is globally asymptotically stable with robust stability degree α.

Keywords: Large-scale complex systems, decentralization, methodologies and tools for analysis of complexity, dynamics and control of large-scale structures

1. INTRODUCTION

There are real world highly complex systems which are not stabilizable by means of any individual continuous feedback controller. Multi-controller switched schemes provide an effective and powerful mechanism to cope with such systems or systems with large uncertainties. Multi-controller switching among smooth controllers provides a good conceptual framework to solve the problem in this case. Switched linear systems provide an attractive approach which bridges the gap between linear systems and the highly complex or uncertain systems. A switching system is a dynamic system consisting of a finite number of continuous-time subsystems and a logical rule that orchestrates switching between them. Switched systems naturally belong to the hybrid dynamic systems framework. The continuous state variables include the state variables of all the continuous-time subsystems, while the discrete variable is the subsystem index. Even if each subsystem is stable, then the overall switched system is not necessarily stable. On the other hand, switching among individually unstable subsystems does not imply necessarily the stability of the switched system. Switched linear systems are relatively easy to handle as many powerful tools from linear analysis are applicable to cope with these systems. The motivation for studying switched dynamic systems arises in very different applications such as for instance in control and sensing of mechanical systems, the automotive industry, aircraft and traffic control, or process control. The importance of multi-controller switched schemes is underlined in large scale complex systems when implementing low-order decentralized controllers. It motivates the development of new control design methods which include the solution using multi-controller switched schemes mainly for large scale complex systems.

1.1 Prior work

One of important problems in uncertain switched systems is the design of switching rules which guarantee quadratic stability and performance. Such switching rules must be independent of uncertainties. A state-dependent switching rule satisfying this requirement which is called the min-projection strategy presents Ji and Wang [2005], Ji and Wang [2005] deals with robust H_∞ control for switched state feedback and static output switched feedback when considering a common Lyapunov function. Bakule [2007] extends these results into a H_∞ decentralized setting including decentralized state-dependent switching rule for continuous-time symmetric composite system, while the

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discrete-time case presents Bakule [2006]. Ji and Wang [2006] derived sufficient conditions for the robust stability and stabilization of nonlinear switched systems, where only the quadratic bound on a nonlinear term must be satisfied. The multiple Lyapunov functions approach analyze in detail Branicky [1998]. Static output feedback stabilization for switched systems by using the multiple Lyapunov functions presents Ji and Wang [2006]. Geromel and Colaneri [2006] deals with the stabilization of switched continuous-time systems by using dynamic output switching control. They use the so-called Lyapunov-Metzler inequalities when applying the multiple Lyapunov functions approach.

Robust Stabilization schemes for nonlinear interconnected systems with nonlinearity satisfying the quadratic bounds were developed in Stanković et al. [2007]. Ji and Wang [2006] proposed sufficient conditions for the synthesis problem of switched nonlinear systems.

The recent results for this class of systems has benefited many real world systems such as for instance control network control Roberts and Stilwell [2006], spatially-distributed systems Steward et al. [2003], or the problem of formations of vehicles in cyclic pursuit which was solved by using circulant matrices in Marshall et al. [2004]. Relevant references on applications surveys are given in Hovd and Skogestad [1994], while theoretic results for this class of systems were presented in Bakule [2005, 2007] and Lam and Huang [2007] including the references therein.

This paper presents the switching stabilizing controller design with state-dependent switching rule for nonlinear continuous-time symmetric composite systems.

To the author's best knowledge, the problem of low-order non-fragile control design for symmetric nonlinear switched symmetric composite systems has not been solved up to now.

1.2 Outline of the paper

This paper presents a novel sufficient condition for the design of decentralized static output switched controller for stabilization of nonlinear switched composite systems. This controller requires the construction of a low order design model as well as the selection of the gain matrix for this model. The gain switched matrices together with switching signal guarantees the global asymptotic asymptotically stable with robust stability degree α of the global closed-loop system when implemented into the global system. A multiple Lyapunov functions approach is used.

2. PROBLEM FORMULATION

Consider a nonlinear switched symmetric composite system consisting of N subsystems, where the ith subsystem is described as follows

$$\begin{aligned} \dot{x}_i(t) &= A_{r_i(t)}x_i(t) + B_{r_i(t)}u_i(t) + s_{z_i}(t) + h_{r_i(t)}(t,x) \\ y_i(t) &= C_{r_i(t)}x_i(t) \qquad i = 1,\dots,N \qquad N \geq 2 \end{aligned} \tag{1}$$

where x_i, u_i, s_{zi}, and y_i are n-, m-, p_s-, and p_y-dimensional vectors of the subsystem states, control inputs, interconnection inputs, and measured outputs, respectively.

Suppose known linear interconnections are described in the form

$$s_{z_i}(t) = \sum_{j=1}^{N} y_{z_j}(t) \tag{2}$$

where $y_{z_j j}$ is the p_z-dimensional vector of the interconnection output from the subsystem j to the subsystem i which is related to the state vector in the form

$$y_{z_j}(t) = L_{r_{ij}} C_{zr_i(t)} x_j(t) \tag{3}$$

where $L_{r_{ii}} = 0$ and $L_{r_{ij}} = L_q \;\; i \neq j$.

$A_{r_i(t)}$, $B_{r_i(t)}$, $C_{r_i(t)}$, $C_{zr_i(t)}$ are time-varying matrices and $r_i(t) \in \{1, \dots \kappa\}$. They take only values in given sets $A_{r_i} \in \{A_1, ..., A_\kappa\}$, $B_{r_i} \in \{B_1, ..., B_\kappa\}$, $C_{r_i} \in \{C_1, ..., C_\kappa\}$, and $C_{zr_i} \in \{C_{z1}, ..., C_{z\kappa}\}$, for all i. A_k, B_k, C_k, C_{zk}, and L_q are constant matrices, $k \in \Lambda$, $\Lambda = \{1, \dots \kappa\}$.

Suppose the structure of unknown nonlinear interconnections as

$$h_{ki}(t,x) = e_{di}(t,x_i)H_d x_i + \sum_{l=1,l\neq i}^{N} e_{1l}(t,x_l)Hx_j \tag{4}$$

where $e_{di}(t,x_i) \in [-1, \; 1]$ and $e_{1l}(t,x_l) \in [-1, \; 1]$ for all i, l.

The nonlinearities $h_{ki}(t,x)$ are uncertain arbitrarily time-varying piecewise-continuous functions for all k, i. They all belong to a class of piecewise-continuous functions $\mathbf{H}_\alpha$ as follows

$$\begin{aligned} \mathbf{H}_\alpha \overset{\text{def}}{=} \{ & h_{ki}(t,x) : \Re^+ \times \Re^n \to \Re^n | h_{ki}(t,x)^T h_{ki}(t,x) \\ & \leq \alpha^2 x^T H_i^T H_i x \subset \mathbf{D}_{di}\} \qquad h_{ki}(t,0) = 0 \end{aligned} \tag{5}$$

over the domains of continuity $\mathbf{D}_{pi}$ for all i, where H_d, H are given constant matrices and $\alpha > 0$ is a given scalar.

Consider a piecewise-continuous function $r_i = r_i(x_i(t)) : \Re^+ \times \Re^n \to \{1, ..., \kappa\}$ to be designed as a switching rule for the ith structural subsystem. $r_i(t, x_i) = k$ means that the kth switching subsystem is activated for the ith subsystem of the global system. We suppose that each r_i has finite number of discontinuities called switching times on every bounded time interval. It takes a constant value on every interval between consecutive switching times. Suppose for concreteness that r_i is continuous from the right everywhere: $r_i(t, x_i) = \lim_{\tau \to t^+} r_i(\tau, x_i)$ for each $\tau \geq 0$.

The goal is to find a decentralized static output feedback switching controller and a decentralized switching rule $r_i(t) = r_i(x_i(t))$ globally asymptotically stabilizing the system (1)–(5). The controller is composed of N local identical controllers of the form

$$u_i(t) = K_{r_i(t)} y_i(t) \qquad i = 1, \ldots, N \tag{6}$$

where y_i is the n-dimensional controller input from the subsystem i. $K_{r_i(t)}$ are time-varying matrices such that $K_{r_i(t)} \in \Omega_K = \{K_1, ..., K_\kappa\}$ for all i, where K_k are constant matrices.

Denote the global description of the system (1)–(5) as follows

$$\begin{aligned} \dot{x}(t) &= \overline{A}_{r(t)} x(t) + \overline{B}_{r(t)} u(t) + \overline{h}_{r(t)}(t, x) \\ y(t) &= \overline{C}_{r(t)} x(t) \end{aligned} \tag{7}$$

where $x = (x_1^T, \; ..., \; x_N^T)^T$, $u = (u_1^T, \; ..., \; u_N^T)^T$ are nN-, mN-dimensional vectors of the system states and control inputs, respectively. The terms used in (7) are defined as follows

$$\begin{aligned} \overline{A}_{r(t)} &= (\overline{A}_{r_{ij}(t)}) \quad \overline{A}_{r_{ii}(t)} = A_{r(t)} \quad \overline{A}_{r_{ij}} = L_q C_{zr_{ij}(t)} \\ \overline{B}_{r(t)} &= \text{diag}(B_{r_1(t)}, \; ..., \; B_{r_N(t)}) \\ \overline{C}_{r(t)} &= \text{diag}(C_{r_1(t)}, \; ..., \; C_{r_N(t)}) \\ \overline{h}_{r(t)}(t, x) &= \text{diag}(h_{r_1(t)}(t, x), \; ..., \; h_{r_N(t)}(t, x)) \end{aligned} \tag{8}$$

The admissible nonlinearities $\overline{h}_k(t, x)$ in (7) are uncertain piecewise-continuous functions satisfying the relations (4), (5).

The bounding matrices $\bar{H}$ are defined as follows

$$\begin{aligned} \overline{H} &= \text{diag}(H_1, \; ..., \; H_N) \\ H_i &= (H ... H \quad H_d \quad H ... H) \end{aligned} \tag{9}$$

with H_d located at the ith position in H_i.

Consider a stabilizing controller for the system (7)–(9) in the form

$$u(t) = \overline{K}_{r(t)} y(t) = \text{diag}(K_{r_1(t)}, \; ..., \; K_{r_N(t)}) y(t) \tag{10}$$

With the controller (10) we associate a vector switching function as follows

$$\overline{r}(x(t)) = (r_1(x_1(t)), \; ... \; r_N(x_N(t))) \tag{11}$$

Denote the resulting closed-loop system as

$$\dot{x}(t) = (\overline{A}_{r(t)} + \overline{B}_{r(t)} \overline{K}_{r(t)} \overline{C}_{r(t)}) x(t) + \overline{h}_{r(t)}(t, x) \tag{12}$$

It is evident that there are κ^N switching signals within the global system, where κ is the maximum number of distinct elements in each set. Each subsystem has assigned only one local switching signal operating independently from other ones.

2.1 The problem

The goal is to derive a complexity-reduced procedure for designing a decentralized switching rule $\overline{r}(x(t))$ (11) and an associated static output feedback $u(t) = \overline{K}_{r(t)} y(t)$ such that the closed-loop switching system (12) is globally asymptotically stable with robust stability degree α for all $\overline{h}_i \in \overline{H}$. Solve the problem by using a multiple Lyapunov function approach.

3. SOLUTION

The solution of the problem requires to find the switching rule as well as the gain matrix of the controller. Let us divide the solution into three parts. The first part consists of the construction of a switching control design model which serves as a low order model for the design of the centralized switching rule and the gain matrix. The second part presents the proper design method for this reduced-order system. The third part shows that when such switching rule and gain matrix are implemented into the global system then the overall closed-loop system is asymptotically stabilized with robust stability degree α

3.1 Reduced-order control design system

Define the matrix $L_q C_z^+$ and $L_q C_z^-$ its each elements is given as an elementwise maximum and minimum from all matrices $L_q C_{zr(t)}$, respectively. It means for that an ijth element of $L_q C_z^+$ or $L_q C_z^-$ is the maximum or the minimum from all ijth elements of $L_q C_{zr(t)}$, respectively.

Introduce the matrices

$$\begin{aligned} L_q C_z &= \frac{L_q C_z^+ + L_q C_z^-}{2} \\ H_q &= \frac{| \, L_q C_z^+ \, | - | \, L_q C_z^- \, |}{2} \end{aligned} \tag{13}$$

where H_q is a constant matrix.

Now, a low-order control design system is derived. Construct the n-dimensional system as follows

$$\begin{aligned} \dot{x}_m(t) &= A_{mr(t)} x_m(t) + B_{r(t)} u_m(t) + h_{mr(t)}(t, x_m) \\ y_m(t) &= C_{r(t)} x_m(t) \end{aligned} \tag{14}$$

where x_m, u_m, and y_m are n-, m-, and p_y-dimensional vectors of the subsystem states, control inputs, and measured outputs, respectively.

The matrix $A_{mr(t)}$ is defined by the expression

$$A_{mr(t)} = A_{r(t)} + (\frac{N}{2} - 1) L_q C_z \tag{15}$$

while $B_{r(t)} \in \{B_1, ..., B_\kappa\}$ and $C_{r(t)} \in \{C_1, ..., C_\kappa\}$.

The nonlinearities in (14) are piecewise-continuous functions defined as

$$\begin{aligned} h_{mr(t)}(t, x_m) &= e_{dm}(t, x_m) H_d x_m(t) \\ &+ \frac{(e_{1m}(t, x_m) + 1)}{2} N H x_m(t) \\ &+ \frac{(e_{2m}(t, x_m) + 1)}{2} N H_q x_m(t) \\ &+ e_{3m}(t, x_m) \frac{N}{2} L_q C_z x_m(t) \end{aligned} \tag{16}$$

where $e_{3m}(t, x_m) \in [-1, \, 1]$.

The quadratic bounds on the nonlinearities $h_{mk}(t, x_m)$ ($k \in \Lambda$) in (14) satisfy the inequalities

$$\mathbf{H}_\alpha \stackrel{\text{def}}{=} \{h_{km}(t,x_m) : \\ \Re^+ \times \Re^n \to \Re^n | h_{km}(t,x_m)^T h_{km}(t,x_m) \\ \leq \alpha^2 x_m^T H_m^T H_m x_m \subset \mathbf{D}_m\} \tag{17}$$

over the domain of continuity $\mathbf{D}_m$. H_m is a constant matrix and $\alpha > 0$ is a scalar given in (4).

The switching signal $r_m(x_m(t)) : \Re^+ \times \Re^n \to \{1, ..., \kappa\}$ is a piecewise-continuous function to be designed for the system (14).

3.2 Control design

Definition 1. The system (14) (with $u_m = 0$) is globally asymptotically stable with robust degree α if the equilibrium $x_m = 0$ is globally asymptotically stable for all $h_{km}(t,x_m) \in \mathbf{H}_\alpha$, where $k \in \Lambda$.

The robust stability problem via switching is to design a switching rule $r_m(x_m(t))$ so that the system (14) (with $u_m = 0$) is globally asymptotically stable with robust degree α.

Consider the switching rule $r_m(x_m(t))$ and the multiple Lyapunov function $V(x_m(t))$ candidate for the system (16) as follows

$$r_m(x_m(t)) = \arg \max_{k\in\Lambda} \{x_m^T P_k x_m\} \quad V(x_m(t)) = x_m^T P_{r(t)} x_m \tag{18}$$

where $P_{r(t)} \in \{P_1 \ldots P_\kappa\}$, $P_k = P_k^T > 0$ are constant matrices for all k and $t \in< 0, +\infty)$.

The following centralized LMI-based formulation of the robust stability problem represents a basis for further elaborations (Ji and Wang [2006]).

The system (14) (with $u_m = 0$) is globally asymptotically stable with robust degree α if there exist matrices $P_k > 0$, constants β_{kl} $(k, j \in \Lambda\ )$ satisfying the inequalities

$$Y_s(A_{mk}) = \begin{pmatrix} S_k & P_k & H_m^T \\ \bullet & -I & 0 \\ \bullet & \bullet & -\gamma I \end{pmatrix} < 0 \qquad \gamma = \frac{1}{\alpha^2} \qquad \forall k, l \in \Lambda \tag{19}$$

where

$$S_k = A_{mk} P_k + P_k A_{mk} + \sum_{k\neq l, l\in\Lambda} \beta_{kl}(P_k - P_l) \tag{20}$$

The Lyapunov function in (18) is not differentiable for all $t \geq 0$. We need to deal with the Dini derivative of $V(x_m(t))$ denoted as $D^+V(x_m(t))$ Garg [1998], (Geromel and Colaneri [2005]). Then, $D^+V(x_m(t)) < 0$ if the switching rule $r_m(x_m(t))$ (18) is applied and $V(x_m(t))$ is the Lyapunov function for this system (Branicky [1998]), (Ji and Wang [2006]).

Consider a stabilizing controller for the system (14) in the form

$$u_m(t) = K_{r(t)} y_m(t) \qquad K_{r(t)} \in \Omega_K \tag{21}$$

The closed-loop system (14), (21) has the form

$$\dot{x}_m(t) = (A_{mr(t)} + B_{r(t)} K_{r(t)} C_{r(t)}) x_m(t) \\ + h_{mr(t)}(t, x_m) \tag{22}$$

Denote Ker(B) and Im(B) the null space and the range space of B, respectively. The matrix $B^\perp$ is introduces as the matrix satisfying the relations Ker($B^\perp$)=Im(B) and $B^\perp B^{\perp T} > 0$ (Ji and Wang [2006]).

The gain matrices $K_k \in \Omega_K$ can be determined by the procedure as follow.

Lemma 1. Consider the system (14). Suppose that all B_k have a full column rank. Then, the system (22) is globally asymptotically stable with robust stability degree α via switched output feedback (21) if there exist $n \times n$ matrices $X_k > 0$, $V_k > 0$, a $n \times m$ matrix N_k, and scalars β_{kl} such that the following matrix inequalities are satisfied

$$Y(A_{mk}) = \begin{pmatrix} T_k & \hat{B}_k^T V_k \hat{B}_k + B_k^{\perp T} X_k B_k^\perp & H_m^T \\ \bullet & -I & 0 \\ \bullet & \bullet & -\gamma I \end{pmatrix} < 0 \\ \gamma = \frac{1}{\alpha^2} \qquad \forall k, l \in \Lambda \tag{23}$$

where

$$T_k = (A_{mk} + \sum_{k\neq l, l\in\Lambda} \beta_{kl} I)^T \hat{B}_k^T V_k \hat{B}_k \\ + B_k^{\perp T} X_k B_k^\perp (A_{mk} + \sum_{k\neq l, l\in\Lambda} \beta_{kl} I) + A_{mk}^T B_k^{\perp T} X_k B_k^\perp \\ + \hat{B}_k^T V_k \hat{B}_k A_{mk} - \sum_{k\neq l, l\in\Lambda} \beta_{kl} (\hat{B}_k^T V_k \hat{B}_k + B_k^{\perp T} X_k B_k^\perp) \\ + B_k N_k C_k + C_k^T N_k^T B_k^T \tag{24}$$

The gain matrices K_k are given as follows

$$K_k = M_k^{-1} N_k \qquad M_k = (B_k^T B_k)^{-1} V_k \tag{25}$$

The switching rule $r_m(x_m(t))$ as well as the multiple Lyapunov function are given by (18) with $P_k = \hat{B}_k^T V_k \hat{B}_k + B_k^{\perp T} X_k B_k^\perp$, where $\hat{B}_k = (B_k^T B_k)^{-1} B_k^T$.

Remark 1. Lemma 1 is an extension of Theorem 2 in (Ji and Wang [2006]) when eliminating equality constraint originally used for static output feedback control design for non-switched systems. The inequalities (23) are bilinear matrix inequalities (BMIs). When selecting parameters β_{kl} in advance, then (23) are linear matrix inequalities (LMIs) which can be solved effectively by using well known tools.

Lemma 1 simplifies for the linear switched system

$$\dot{x}_m(t) = A_{mr(t)} x_m(t) + B_{r(t)} u_m(t) \\ y_m(t) = C_{r(t)} x_m(t) \tag{26}$$

with the linear controller

$$u_m(t) = K_{r(t)} y_m(t) \tag{27}$$

into an important special case given by the following result.

Corollary 1. Consider the system (26). Suppose that all B_k have a full column rank. Then, the system (26), (27) is globally asymptotically stable with robust stability degree α via switched output feedback (27) if there exist $n \times n$ matrices $X_k > 0$, $V_k > 0$, a $n \times m$ matrix N_k, and scalars β_{kl} such that the following matrix inequalities are satisfied

$$\begin{aligned} Y_v(A_{mk}) &= (A_{mk} + \sum_{k\neq l, l\in\Lambda} \beta_{kl} I)^T \hat{B}_k^T V_k \hat{B}_k \\ &+ B_k^{\perp T} X_k B_k^{\perp} (A_{mk} + \sum_{k\neq l, l\in\Lambda} \beta_{kl} I) + A_{mk}^T B_k^{\perp T} X_k B_k^{\perp} \\ &+ \hat{B}_k^T V_k \hat{B}_k A_{mk} - \sum_{k\neq l, l\in\Lambda} \beta_{kl} (\hat{B}_k^T V_k \hat{B}_k + B_k^{\perp T} X_k B_k^{\perp}) \\ &+ B_k N_k C_k + C_k^T N_k^T B_k^T \end{aligned} \tag{28}$$

The gain matrices K_k are given by (25). The switching rule $r_m(x_m(t))$ as well as the multiple Lyapunov function are given in (18).

3.3 Decentralized switching control of the global system

The system (7) has a bounding matrix $\overline{H}$ which has a structure of symmetric composite systems. This structural feature can be exploited by using the transformation of states to get two reduced order models. Consider

$$\tilde{x}(t) = Sx(t) \tag{29}$$

by using the $nN \times nN$ matrix $S = T^{-1}$.

Suppose a real snxsn matrix $T(n, s)$ in the form

$$\begin{aligned} T(n,1) &= I \\ T(n,s) &= \begin{pmatrix} I & 0 & \dots & 0 & I \\ 0 & I & \dots & 0 & I \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I & I \\ -I & -I & \dots & -I & I \end{pmatrix} \qquad s > 1 \end{aligned} \tag{30}$$

where I denotes here nxn identical matrix. Then T is defined as

$$\begin{aligned} T(i) &= \text{diag}[T(n, N-i)I, ..., I] \in \Re^{Nn\times Nn} \\ T &= T(0)\,T(1)\,...\,T(N-1) \qquad i = 0, ..., N-1 \end{aligned} \tag{31}$$

The constructive way how to use this transformation presents Yang and Zhang [1995].

Lemma 2. Consider the matrix $\overline{H}$ by (9) and any given $J = \text{diag}[J_o, ..., J_o]$, where J, J_o are nNxnN, nxn matrices. Then, the following equalities hold

$$\begin{aligned} T^{-1}\overline{H}T &= \text{diag}(H_s, ..., H_s,\ H_c) \\ T^T\overline{H}T &= \text{diag}(2H_s,\ 6H_s, ..., N(N-1)H_s,\ NH_c) \\ T^{-1}J(T^{-1})^T &= \text{diag}(\frac{1}{2}J_o,\ \frac{1}{6}J_o, ..., \frac{1}{N(N-1)}J_o) \\ T^TJT &= \text{diag}(2J_o, ..., N(N-1)J_o,\ NJ_o) \end{aligned} \tag{32}$$

where $H_s = H_p - H$ and $H_c = H_s + NH$.

Applying now the transformation (29) on the system (7) and with respect to (13), we get finally two systems of order n (when, in short, deleting unnecessary dimensionality indices) as

$$\begin{aligned} \dot{x}(t) &= A_{sr(t)}x(t) + B_{r(t)}u(t) + h_{sr}(t,x) \\ y(t) &= C_{r(t)}x(t) \end{aligned} \tag{33}$$

$$\begin{aligned} \dot{x}(t) &= A_{cr(t)}x(t) + B_{r(t)}u(t) + h_{cr}(t,x) \\ y(t) &= C_{r(t)}x(t) \end{aligned} \tag{34}$$

where

$$\begin{aligned} A_{sr(t)} &= A_{r(t)} - L_qC_z \\ A_{cr(t)} &= A_{sr(t)} + NL_qC_z \\ h_{sr(t)}(t,x) &= e_d(t,x)H_dx(t) - e_1(t,x)Hx(t) \\ &\quad - e_2(t,x)H_qx(t) \\ h_{cr(t)}(t,x) &= h_{sr(t)}(t,x) + e_1(t,x)NHx \\ &\quad + e_2(t,x)NH_qx(t) \end{aligned} \tag{35}$$

The structure of (33)–(34) motivates to construct a single model containing both systems by decomposing the difference between the models into the nominal part and uncertain nonlinear part. The term NL_qC_z defining the difference between $A_{cr(t)} - A_{sr(t)}$ in (35) can be considered as composed of a nominal part $\frac{1}{2}NL_qC_z$ and a nonlinear term $e_3(t,x_m)\frac{N}{2}L_qC_z$ with $e_3(t,x_m) \in [-1, 1]$. Analogously, this way of reasoning holds for nonlinear terms. Finally, it leads to the single design model (14)–(17).

Denote the local switching rule for the ith structural subsystem as

$$r_i(x_i(t)) = \arg\max_{k\in\Lambda}\{x_i^T P_{ki} x_i\} \tag{36}$$

where $P_{ki} \in \{P_1 \,...\, P_\kappa\}$. $P_k = P_k^T > 0$ are constant matrices for all k by (18).

The following theorem states the main result.

Theorem 1. Given the symmetric composite nonlinear switched system (7)–(9): (a) Construct the reduced control design system (14)–(17). (b) Select the controller gain matrices K_k by (25) in the controller (21) for the system (14)–(17) satisfying the inequalities (23) for all a priori given β_{kl} and k, l. (c) Determine the set of gain matrices $\Omega_K = \{K_1, \,...\, K_\kappa\}$. (d) Implement the set Ω_K into (10) so that $K_{r_i(t)} \in \Omega_K$ for all i together with the switching rule (11), (36). Then, the overall closed-loop switched system (7)–(11), (36) is globally asymptotically stable with robust stability degree α.

Proof. Consider the multiple Lyapunov function candidate for the system (7)–(9) as

$$\overline{V}(x(t)) = x^T\overline{P}_{r(t)}x = x^T\text{diag}(P_{r_1(t)},\ ...,\ P_{r_N(t)})x \tag{37}$$

where $P_{ri} \in \{P_1 \,...\, P_\kappa\}$ for all i.

The asymptotic stability with robust stability degree α of the global switched closed-loop system (7)–(11) is proved by using the inequality in Lemma 1 being appropriately modified to the global system when directly implementing the gain matrices $\overline{K}_{r(t)} = \text{diag}(K_{r_1(t)},\ ...,\ K_{r_N(t)})$. Note that K_{r_i} are obtained by using Lemma 1 for the closed-loop system (14)–(17). Therefore, there are available also the matrices X_k, V_k, N_k as well as constants β_{kl}. Now, substitute the parameters of the control design system (14) by those of the global system (7). Substitute the controller (21) by the controller (10). Denote simply these changes with the replacements (14)→(7), (21)→(10) and implement them together with $\overline{X}_k = \text{diag}(X_k,\ ...,\ X_k)$,$\overline{V}_k = \text{diag}(V_k,\ ...,\ V_k)$, $\overline{N}_k = \text{diag}(N_k,\ ...,\ N_k)$ into the inequality (23). $\overline{X}_k$, $\overline{V}_k$ and $\overline{N}_k$ are $nN \times nN$, $nN \times nN$, and $nN \times mN$ matrices, respectively. Denote the final matrix $\overline{Y}(\overline{A}_k)$. It has the same structure as $Y(A_{mk})$, but it is reformulated for the global system (7)–(9). Then, it remains to show that the matrix $\overline{Y}(\overline{A}_k) < 0$.

Consider the matrix $\overline{Y}(\overline{A}_k)$. Applying now the transformation of the states S by (29) and Lemma 1, we get the transformed system resulting in the relation

$$P^{-1}T^{-1}\overline{Y}(\overline{A}_k)TP = \text{diag}(Y(A_{sk}), ..., Y(A_{sk}), Y(A_{ck})) \tag{38}$$

with $N-1$ diagonal blocks $Y(A_{sk})$. P is a convenient permutation matrix. P and T are non-singular matrices. If $Y(A_{mk}) < 0$ by Lemma 1, then $Y(A_{sk}) < 0$, $Y(A_{ck}) < 0$ because the system (14)–(17) includes both systems (33), (34) as its special cases.

The switching rule (11) is decentralized as it follows directly from a block diagonal structure of the matrix $\overline{P}_{r(t)}$ in (37) when taking into account the centralized case. Thus, (37) is the Lyapunov function for the system (7)–(9).

Thereby, the closed-loop system (1)–(5), (6) with the gain matrices $K_{r_i(t)} \in \Omega_K$, for all i, is globally asymptotically stable with robust stability degree α. Q.E.D.

4. EXAMPLE

Problem. Consider the system with $N = 4$, the ith subsystem switching rule $r_i = r_i(x_i(t)) : \Re^+ \times \Re^n \to \{1, 2\}$ and the subsystem matrices which are identical for all i as

$$A_1 = \begin{pmatrix} -2 & 0.1 \\ -1 & -1 \end{pmatrix} \quad A_2 = \begin{pmatrix} 1 & 1 \\ -2 & -1.5 \end{pmatrix} \quad B_1 = B_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$C_1 = C_2 = (0.5 \;\; 0) \quad L_q = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix} \quad C_{z1} = \begin{pmatrix} 0 & 0 \\ 0 & 1.5 \end{pmatrix}$$
$$C_{z2} = \begin{pmatrix} 0 & 0 \\ 0 & 0.5 \end{pmatrix} \quad H_d = 0 \quad H = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{39}$$

Results. Applying Lemma 1 on the model (14)–(16) with $\alpha_{12} = \alpha_{21} = 2$ and $\gamma = 0.5$ results in the gain matrices

$$K_1 = -0.7876 \qquad K_2 = 3.899 \tag{40}$$

when using the matrices

$$P_1 = \text{diag}(0.6812,\ 0.0521) \qquad P_2 = \text{diag}(0.0701,\ 1.2051) \tag{41}$$

for the switching rule (18). The global system (12), (39) with the feedback gains (10),(40) and the switching rule (11), (36), (41) is globally asymptotically stable with robust stability degree α.

5. CONCLUSION

The paper contributes by a new complexity-reduced control design method for a class of nonlinear switched symmetric composite systems. Particular structural properties of this class of large scale complex systems are used for the construction of low-order design system. The multiple Lyapunov functions have been selected for the proper design of gain matrices by using the matrix inequalities. The switched controller together with the switching rule designed for the reduced-order control design model is consequently implemented into the original system. The method guarantees that the overall closed-loop switched symmetric composite systems is globally asymptotically stable with robust stability degree α.

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