Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Adaptive Trajectory Tracking and Stabilization for Omnidirectional Mobile Robot with Dynamic Effect and Uncertainties

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Abstract: This paper presents an adaptive backstepping control method for trajectory tracking and stabilization of an omnidirectional wheeled mobile robot with parameter variations and uncertainties caused by friction and slip. The dynamic model of the robot with three independent driving omnidirectional wheels equally spaced at 120 degrees from one another is briefly introduced. With the dynamic model, the adaptive controller to achieve both trajectory tracking and stabilization is synthesized via adaptive backstepping approach. Experimental results are conducted to show the merit of the proposed control method.

Keywords: Backstepping, dynamic model, omnidirectional mobile robot, trajectory tracking.

1. INTRODUCTION

Recently, omnidirectional mobile robots have attracted much attention in the robotics and control systems societies. Compared to the more common car-like robot (Samson, 1995; Jiang, *et al*., 1999; Dixon, *et al*., 2000, and Lee, *et al*., 2001), the omnidirectional mobile mechanism has the superior agile capability to move towards any position and to simultaneously attain any desired orientation. Modeling and control of omnidirectional mobile robots incorporating with dynamic effect has been investigated by several researchers. Pin *et al.* (1994) presented the concepts for a family of holonomic wheeled platforms. Jung *et al.* (2000) constructed a kind of omnidirectional base, derived its kinematical and dynamic models. Bétourné and Campion (1996) introduced the dynamic modeling of a class of omnidirectional mobile robots and proposed the output feedback linearization law to achieve a good trajectory tracking. Kalmár-Nagy *et al.* (2004) in Cornell University offered the dynamic model and the time-optimal control for an omnidirectional robot. William II *et al.* (2002) presented a dynamic model for omnidirectional wheeled mobile robots, considering the occurrence of slip between the wheels and motion surface. Driessen (2006) presented a continuous adaptive controller for the robot adaptive tracking problem with unknown robot parameter, unknown actuator parameter, and unavailable joint acceleration measurement. With the dynamic model from (Kalmár-Nagy *et al.,* 2004*)*, Tsai and Wang (2005) proposed the stabilization and trajectory tracking method via backstepping. Overall, the aforementioned methods did not consider the issues of the parameter variations and the uncertainties from friction and slip.

The backstepping approach has been widely used to solve for the simultaneous trajectory tracking and stabilization problem for the dynamic models of nonholonomic mobile robots. Fierro and Lewis (1998) presented a control structure that makes possible the integration of a kinematic controller. Based on the results presented by Fierro and Lewis, Fukao *et al.* (2000) used an adaptive backstepping approach to find the adaptive tracking controller for nonholonomic mobile robots incorporating with dynamic effect and unknown parameters. Dong and Kuhnert (2005) revisited the path following problem of a nonholonomic wheeled mobile robot with both parameter and nonparameter uncertainties. Do *et al.* (2004) further used the backstepping approach to propose a time-varying global adaptive controller at the torque level. However, the simultaneous tracking and stabilization (regulation) problem of omni-directional mobile robots using the adaptive backstepping approach still remains open. From the viewpoint of controller design, omnidirectional robots are much easier than two-wheeled nonholonomic mobile robots. Nevertheless, this kind of tracking and stabilization control problem with dynamic effect and uncertainties still deserves further study not only for the annual RoboCup competition, but also for possible applications to several kinds of wheeled service robots that require maneuvering capability or omnidirectional mobility.

The goal of this paper is to apply the well-known adaptive backstepping approach to construct a unified control framework to achieve stabilization, trajectory tracking for the omni-directional robot incorporating with the dynamic effect and uncertainties. The proposed controller will be proven with the property of globally asymptotical stability via the Lyapunov stability theory. This type of controller is of practical interest because the voltage control is easier to achieve than torque control or current control.

The rest of this paper is organized as follows. In Section II, the dynamic model of the omni-directional wheeled mobile robot with slip is briefly presented. To achieve both trajectory tracking and stabilization, Section III synthesizes an adaptive controller via the adaptive backstepping approach. Section IV conducts several experiments to show the efficacy of the proposed control method. Section V concludes the paper.

2. BRIEF DESCRIPTION OF THE DYNAMIC MODEL WITH SLIP

This section is devoted to briefly describing the dynamic model of an omnidirectional mobile robot with three independent driving wheels equally spaced at 120 degrees from one another. Fig.1 depicts its structure and geometry that is used to find the dynamic model of the robot, where θ denotes the vehicle orientation. Due to structural symmetry, the vehicle has the property that the center of geometry coincides with the center of mass.

Before introducing the dynamic model, let us recall the following robot's kinematic equation described by Kalmár-Nagy *et al.*(2004)

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} r\omega_1 \\ r\omega_2 \\ r\omega_3 \end{bmatrix} = P(\theta)\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \tag{1}$$

where

$$P(\theta) = \begin{bmatrix} -\sin\theta & \cos\theta & L \\ -\sin(\frac{\pi}{3}-\theta) & -\cos(\frac{\pi}{3}-\theta) & L \\ \sin(\frac{\pi}{3}+\theta) & -\cos(\frac{\pi}{3}+\theta) & L \end{bmatrix}$$

r denotes the radius of the driving wheels; L represents the distance from the wheel's center to the center of mass of the robot; V_i and ω_i, i=1,2,3, respectively denote the linear and angular velocities of each wheel. Moreover, the matrix $P(\theta)$ is always nonsingular for any θ, i.e.,

$$P^{-1}(\theta) = \begin{bmatrix} -\frac{2}{3}\sin\theta & -\frac{2}{3}\sin(\frac{\pi}{3}-\theta) & \frac{2}{3}\sin(\frac{\pi}{3}+\theta) \\ \frac{2}{3}\cos\theta & -\frac{2}{3}\cos(\frac{\pi}{3}-\theta) & -\frac{2}{3}\cos(\frac{\pi}{3}+\theta) \\ \frac{1}{3L} & \frac{1}{3L} & \frac{1}{3L} \end{bmatrix}$$

In order to derive the robot's dynamic model, one assumes that the robot has two unknown but constant parameters, the total mass m and the moment of inertia J of the robot, and three uncertain but bounded forces exerted on the driving wheels, and neglects the servomotors' dynamics. The assumption of unknown but constant parameters, m and J, is relevant not only at the modeling process , but also during operation for mission execution in case of payload changes. The uncertain but bounded friction forces may come from several factors, such as the static friction between the wheel and the surface, and the slip phenomena where the force may vary with the surface made by the used materials. Note that the friction force exerted on wheel i is divided into two components: the first friction component F_{Wi} in the wheel rolling direction and the second friction component F_{Ti} in the transverse direction (normal to the first).

The total force resulting from the force F_i from the ith DC servomotor motor and the friction force F_{Wi} exerted in the rotation direction of wheel i is given by

$$F_i = \alpha U_i - \beta V_i - F_{Wi},\ i = 1,2,3. \tag{2}$$

where U_i is the applied voltage of each motor; F_{Wi} satisfies the inequality $-\frac{mg}{3}\mu_{W\max} \le F_{Wi} \le \frac{mg}{3}\mu_{W\max}$ where $\mu_{W\max}$ is the maximum static friction coefficient in the direction of wheel rotation and g is the acceleration of gravity. The two parameters α and β in (2) can be determined by $\alpha = \frac{k_t}{rR_a}$, $\beta = r_g\frac{k_e k_t}{rR_a}$ where R_a is the armature resistance of the servomotors, r is the radius of the omnidirectional wheel, r_g is the gear ratio, k_e denotes the back-emf coefficient, and k_t represents the torque coefficient. Note that these two parameters α and β in (2) can be obtained from the motors' experiments.

With the force equation (2) and the friction force F_{Ti} exerted in the transverse direction of wheel i, one obtains the following equations from Newton's second law for translation and rotation

$$m\ddot{P}_0 = m\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \sum_{i=1}^{3} F_i R(\theta) D_i - \begin{bmatrix} F_{T1}\cos\theta + F_{T2}\cos(\theta+\frac{2\pi}{3}) + F_{T3}\cos(\theta+\frac{4\pi}{3}) \\ F_{T1}\sin\theta + F_{T2}\sin(\theta+\frac{2\pi}{3}) + F_{T3}\sin(\theta+\frac{4\pi}{3}) \end{bmatrix},\quad J\ddot{\theta} = L\sum_{i=1}^{3} F_i \tag{3a}$$

where $D_1 = [0\ \ 1]^T$, $D_2 = -\frac{1}{2}\left[\sqrt{3}\ \ 1\right]^T$, $D_3 = \frac{1}{2}\left[\sqrt{3}\ \ -1\right]^T$; the rotation matrix $R(\theta)$ is given by $R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$; F_{Ti} satisfies the inequality $-\frac{mg}{3}\mu_{T\max} \le F_{Ti} \le \frac{mg}{3}\mu_{T\max}$, where $\mu_{T\max}$ is the maximum static friction coefficient for the transverse wheel direction. Substituting (2) into (3a) yields a second-order dynamic model of the omnidirectional mobile robot.

$$\begin{bmatrix} m\ddot{x} \\ m\ddot{y} \\ J\ddot{\theta} \end{bmatrix} = \alpha \begin{bmatrix} -\sin\theta & -\sin(\frac{\pi}{3}-\theta) & \sin(\frac{\pi}{3}+\theta) \\ \cos\theta & -\cos(\frac{\pi}{3}-\theta) & -\cos(\frac{\pi}{3}+\theta) \\ L & L & L \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} - \frac{3}{2}\beta \begin{bmatrix} \dot{x} \\ \dot{y} \\ 2L^2\dot{\theta} \end{bmatrix} - \begin{bmatrix} \bar{f}_1 \\ \bar{f}_2 \\ \bar{f}_3 \end{bmatrix} \tag{3b}$$

$$= \alpha P^T(\theta)\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} - \frac{3}{2}\beta\begin{bmatrix} \dot{x} \\ \dot{y} \\ 2L^2\dot{\theta} \end{bmatrix} - \bar{f}$$

where

$$\bar{f} = \left[\bar{f}_1\ \ \bar{f}_2\ \ \bar{f}_3\right]^T$$

$$\bar{f}_1 = F_{W1}\sin\theta + F_{W2}\sin(\frac{\pi}{3}-\theta) - F_{W3}\sin(\frac{\pi}{3}+\theta) + F_{T1}\cos\theta - F_{T2}\cos(\frac{\pi}{3}-\theta) - F_{T3}\cos(\frac{\pi}{3}+\theta)$$

$$\bar{f}_2 = -F_{W1}\cos\theta + F_{W2}\cos(\frac{\pi}{3}-\theta) + F_{W3}\cos(\frac{\pi}{3}+\theta) + F_{T1}\sin\theta + F_{T2}\sin(\frac{\pi}{3}-\theta) - F_{T3}\sin(\frac{\pi}{3}+\theta),$$

$\bar{f}_3 = L\sum_{i=1}^{3} F_{Wi}$, and the uncertain friction force vector satisfies the inequality, i.e., $\|\bar{f}\|_\infty \le k_{\max}$, where $\|\bar{f}\|_\infty$ denotes the infinity-norm of the vector $\bar{f}$ and $k_{\max}$ is the least upper bound of $\|\bar{f}\|_\infty$.

By defining the following two vectors $Z_1 = [x\ \ y\ \ \theta]^T$

and $Z_2 = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$, the second-order dynamic model (3b) can be rewritten in the standard state space from as follows;

$$\dot{Z}_1 = Z_2 \tag{4a}$$

$$M\dot{Z}_2 = \alpha P^T(\theta)U - BZ_2 - \overline{f} \tag{4b}$$

where

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J \end{bmatrix},\ U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix},\ B = \begin{bmatrix} 1.5\beta & 0 & 0 \\ 0 & 1.5\beta & 0 \\ 0 & 0 & 3L^2\beta \end{bmatrix}$$

3. ADAPTIVE CONTROLLER DESIGN

This section synthesizes an adaptive controller for the robot's dynamic model (4) with two unknown but constant parameters, m and J, and three uncertain but bounded friction forces exerted on the driving wheels. This controller aims at steering the robot to reach the destination pose or exactly follow desired trajectory with maneuvering capability. In order to solve for the adaptive control problem, the desired and differentiable trajectory is described by $Z_r(t) = [x_r(t) \quad y_r(t) \quad \theta_r(t)]^T$, Note that if the desired trajectory Z_r is independent of time, that is, a fixed destination pose, then the control problem is referred to the so-called regulation problem or stabilization problem; otherwise, the control problem is the so-called trajectory tracking. Unlike conventional two-wheeled or four-wheeled (car-like) mobile robots, the desired trajectory can not be generated from the robot's kinematics, but can be any smooth but differentiable time function. In the following we elucidate how to use the adaptive backstepping approach to synthesize the adaptive controller. Throughout the paper, $\|x\|_1$, $\|x\|_2$ and $\|x\|_\infty$ respectively denote the one-norm, the two-norm and the infinity-norm for the vector x, and the vehicle weight is assumed equally distributed on each wheel.

To proceed with the controller design for the dynamic model (4), one defines the tracking error vector by $Z_e = Z_1 - Z_r$. Differentiating Z_e and Z_r with respect to time yields

$$\dot{Z}_e = \dot{Z}_1 - \dot{Z}_r = Z_2 - \dot{Z}_r \tag{5}$$

Considering Z_2 as a virtual control which is designed as $Z_2 = \phi(Z_e) = -K_P Z_e + \dot{Z}_r$ where the matrix K_P is diagonal, one obtains $\dot{Z}_e = -K_P Z_e + \dot{Z}_r - \dot{Z}_r = -K_P Z_e$. The asymptotic stability of the Z_e dynamics can be shown via selecting the quadratic Lyapunov function $V_1 = \frac{1}{2} Z_e^T M K_P^2 Z_e$, thus yielding $\dot{V}_1 = Z_e^T M K_P^2 \dot{Z}_e = Z_e^T M K_P^2(-K_P Z_e) = -Z_e^T M K_P^3 Z_e < 0$. To achieve the controller deign, we define the backstepping error vector

$$\eta = Z_2 - \phi(Z_e) = Z_2 - (-K_P Z_e + \dot{Z}_r) = Z_2 + K_P Z_e - \dot{Z}_r \tag{6}$$

With the defined error vector (6), it follows that

$$\dot{Z}_e = Z_2 - \dot{Z}_r = (Z_2 + K_P Z_e - \dot{Z}_r) - K_P Z_e = \eta - K_P Z_e \tag{7}$$

For the control goal to design the control law for U such that $Z_e \to 0$ and $\eta \to 0$ as $t \to \infty$, the following adaptive control law is proposed by

$$U = \left(P^T(\theta)\right)^{-1} \alpha^{-1} \left(BZ_2 + \hat{M}\ddot{Z}_r - (K + \hat{M}K_P)\eta - \hat{k}\,\mathrm{sgn}(\eta)\right) \tag{8}$$

where $\hat{M} = diag\{\hat{m}, \hat{m}, \hat{J}\}$; the matrix K_P is diagonal, i.e., $K_P = diag\{k_{P1}, k_{P2}, k_{P3}\}$; the matrix K is symmetric and positive-definite and the control gain $\hat{k}$ is a real and positive number. The parameter adjustment rules for $\hat{m}$, $\hat{J}$ and $\hat{k}$ are to be determined in the following. Substituting (8) into (4b) gives

$$\begin{aligned} M\dot{\eta} &= \alpha P^T(\theta)\left(P^T(\theta)\right)^{-1} \alpha^{-1}\left(BZ_2 + \hat{M}\ddot{Z}_r - (K + \hat{M}K_P)\eta - \hat{k}\,\mathrm{sgn}(\eta)\right) \\ &\quad - BZ_2 + MK_P\eta - MK_P^2 Z_e - M\ddot{Z}_r - \overline{f} \\ &= -(M - \hat{M})\ddot{Z}_r - K\eta + (M - \hat{M})K_P\eta - MK_P^2 Z_e - \hat{k}\,\mathrm{sgn}(\eta) - \overline{f} \\ &= -\tilde{M}\ddot{Z}_r + \tilde{M}K_P\eta - K\eta - MK_P^2 Z_e - \hat{k}\,\mathrm{sgn}(\eta) - \overline{f} \end{aligned} \tag{9}$$

where $\tilde{M} = diag\{m - \hat{m}, m - \hat{m}, J - \hat{J}\}$. The closed-loop stability of the feedback error system and the parameter adjustment rules for $\hat{m}$, $\hat{J}$ and $\hat{k}$ can be simultaneously accomplished by the Lyapunov stability theory. For the goal, we choose the same radial and unbounded Lyapunov function

$$V_2 = \frac{1}{2} Z_e^T M K_P^2 Z_e + \frac{1}{2}\eta^T M \eta + \frac{1}{2\lambda_m}\tilde{m}^2 + \frac{1}{2\lambda_J}\tilde{J}^2 + \frac{1}{2\lambda_k}\tilde{k}^2,\ \lambda_m > 0, \lambda_J > 0, \lambda_k > 0 \tag{10}$$

where $\tilde{k} = k_{\max} - \hat{k}$, $\tilde{J} = J - \hat{J}$ and $\tilde{m} = m - \hat{m}$. Taking the time derivative of V_2 along with the trajectories of (6) and (9) and using the inequality $\eta^T \overline{f} \le \|\eta\|_1 \|\overline{f}\|_\infty \le \|\eta\|_1 k_{\max}$ yield

$$\begin{aligned} \dot{V}_2 &= Z_e^T(MK_P^2)\dot{Z}_e + \eta^T M\dot{\eta} + \frac{1}{\lambda_m}(-\tilde{m}\dot{\hat{m}}) + \frac{1}{\lambda_J}(-\tilde{J}\dot{\hat{J}}) + \frac{1}{\lambda_k}(-\tilde{k}\dot{\hat{k}}) \\ &= -Z_e^T MK_P^3 Z_e - \eta^T K\eta + \tilde{m}\left(-\frac{1}{\lambda_m}\dot{\hat{m}} - (\eta_1\ddot{x}_r + \eta_2\ddot{y}_r) + (k_{P1}\eta_1^2 + k_{P2}\eta_2^2)\right) \\ &\quad + \tilde{J}\left(-\frac{1}{\lambda_J}\dot{\hat{J}} - \eta_3\ddot{\theta}_r + k_{P3}\eta_3^2\right) - \hat{k}\eta^T\mathrm{sgn}(\eta) - \eta^T\overline{f} + \frac{1}{\lambda_k}(-\tilde{k}\dot{\hat{k}}) \\ &\le -Z_e^T MK_P^3 Z_e - \eta^T K\eta + \tilde{m}\left(-\frac{1}{\lambda_m}\dot{\hat{m}} - (\eta_1\ddot{x}_r + \eta_2\ddot{y}_r) + (k_{P1}\eta_1^2 + k_{P2}\eta_2^2)\right) \\ &\quad + \tilde{J}\left(-\frac{1}{\lambda_J}\dot{\hat{J}} - \eta_3\ddot{\theta}_r + k_{P3}\eta_3^2\right) - \hat{k}\|\eta\|_1 + k_{\max}\|\eta\|_1 + \frac{1}{\lambda_k}(-\tilde{k}\dot{\hat{k}}) \\ &= -Z_e^T MK_P^3 Z_e - \eta^T K\eta + \tilde{m}\left(-\frac{1}{\lambda_m}\dot{\hat{m}} - (\eta_1\ddot{x}_r + \eta_2\ddot{y}_r) + (k_{P1}\eta_1^2 + k_{P2}\eta_2^2)\right) \\ &\quad + \tilde{J}\left(-\frac{1}{\lambda_J}\dot{\hat{J}} - \eta_3\ddot{\theta}_r + k_{P3}\eta_3^2\right) + \tilde{k}\left(\|\eta\|_1 - \frac{1}{\lambda_k}\dot{\hat{k}}\right) \end{aligned} \tag{11}$$

If the following parameter update laws (12), (13) and (14) for $\hat{m}$, $\hat{J}$ and $\hat{k}$ are chosen

$$\dot{\hat{m}} = -\lambda_m(\eta_1\ddot{x}_r + \eta_2\ddot{y}_r) + \lambda_m(k_{P1}\eta_1^2 + k_{P2}\eta_2^2) \tag{12}$$

$$\dot{\hat{J}} = \lambda_J(-\eta_3\ddot{\theta}_r + k_{P3}\eta_3^2) \tag{13}$$

$$\dot{\hat{k}} = \lambda_k \|\eta\|_1 \tag{14}$$

then one obtains

$$\dot{V}_2 \le -Z_e^T MK_P^3 Z_e - \eta^T K\eta \le 0 \tag{15}$$

which shows that $\dot{V}_2$ is negative semidefinite. Similarly, the use of Barbalat's lemma indicates hat $Z_e \to 0$ and $\eta \to 0$ as time tends to infinity and the estimates $\hat{m}$, $\hat{J}$ and $\hat{k}$ are globally uniformly bounded. Hence, the globally asymptotical stability of the closed-loop error system is ensured. Before closing this step, the following theorem is stated.

Theorem 1. Consider the robot's dynamic model (4) with the desired differentiable trajectory $Z_r(t)=[x_r(t) \quad y_r(t) \quad \theta_r(t)]^T \in C^2$, two unknown but constant parameters m and J, and three uncertain but bounded forces exerted on the driving wheels. If the control (8) along with the parameter adjustment rules (12-14) are applied, then the robot can be steered to achieve trajectory tracking and stabilization in the sense of globally asymptotical stability, i.e., $Z_1 \to Z_r$ and $Z_2 \to \dot{Z}_r$ as $t \to \infty$.

Remark 1. Given the positive time constant τ, if the parameter adjustment rules (12), (13) and (14) for $\hat{m}$, $\hat{J}$ and $\hat{k}$ are modified based on the e-modification as below

$$\dot{\hat{m}} = -\lambda_m(\eta_1\ddot{x}_r + \eta_2\ddot{y}_r) + \lambda_m(k_{P1}\eta_1^2 + k_{P2}\eta_2^2) - \lambda_m\tau\|\eta\|_2\hat{m} \quad (16)$$

$$\dot{\hat{J}} = \lambda_J(-\eta_3\ddot{\theta}_r + k_{P3}\eta_3^2) - \lambda_J\tau\|\eta\|_2\hat{J} \quad (17)$$

$$\dot{\hat{k}} = \lambda_k\|\eta\|_1 - \lambda_k\tau\|\eta\|_2\hat{k} \quad (18)$$

then $\dot{V}_2$ becomes

$$\dot{V}_2 \le -Z_e^T MK_P^3 Z_e - \eta^T K\eta + \tau\|\eta\|_2(\hat{m}\tilde{m} + \tilde{J}\hat{J} + \tilde{k}\hat{k}^2)$$

$$\le -Z_e^T MK_P^3 Z_e - \bar{k}_{\min}\|\eta\|_2^2 + \tau\|\eta\|_2((m-\tilde{m})\tilde{m} + (J-\tilde{J})\tilde{J} + (k_{\max}-\tilde{k})\tilde{k})$$

where $\bar{k}_{\min}$ denotes the minimum positive eigenvalue of the diagonal matrix K. Hence, $\dot{V}_2$ is negative semidefinite outside the compact set $\{\eta \mid \|\eta\|_2 < \tau(m^2 + J^2 + k_{\max}^2)/4\bar{k}_{\min}\}$; this reveals that the tracking errors Z_e and η are uniformly ultimate bounded (UUB) and the estimates $\hat{m}$, $\hat{J}$ and $\hat{k}$ are also uniformly ultimate bounded (UUB). These results indicate that the proposed adaptive control (8) with the parameter adaptation rules (16-18) is capable of steering the robot with the dynamic model (4) to reach any destination pose or follow any differentiable and time-varying trajectory in the sense of uniformly ultimately bounded (UUB) stability

4. EXPERIMENTAL RESULTS AND DISCUSSION

The aims of the experiments are to examine the effectiveness and performance of the proposed adaptive control law (8) to an omnidirectional mobile robot incorporated with the dynamic effect and friction. These experimental result were conducted with the following parameters: L=0.23m, $r = 0.058$ m, m=35Kg, $J = 10.0$ Kg-m^2, $\alpha = 0.757$, $\beta = 25.74$, $K = diag\{250,250,250\}$ and $K_P = diag\{5,5,5\}$. All the friction forces, F_{Wi} and F_{Ti}, i=1,2,.3, are assumed to be 5 Newtons.

4.1 Brief Description of the Experimental Omnidirectional Mobile Robot

As shown in Fig.2, the experimental omnidirectional mobile robot is equipped with the following components: i) one 7" LCD monitor; ii) one personal computer (PC); iii) two encoders mounted on the driving motors; iv) two 12V serial batteries; v) three DC24V brushless servomotors with their drivers from Oriental Motor Co. ; vi) three omnidirectional wheels from Kornylak Corporation. The PC is composed of one parallel digital input and output circuit card with three 20-bit counters HCTL2020, and one digital-to-analog card (PIO-DA9). Three driving omnidirectional wheels are driven by three DC24V brushless servomotors with three mounted encoders of 300 pulses per revolution. The three encoders were employed to provide the velocities of the three DC brushless motors to achieve the dead-reckoning of the robot. The adaptive controller (8) for stabilization and trajectory tracking was implemented using C++ codes and standard programming techniques.

4.2 Adaptive Stabilization

The first experiment was conducted to investigate the stabilization performance of the proposed adaptive control law (8). The initial pose of the omnidirectional mobile robot was assumed at the origin, i.e., $[X_0 \quad Y_0 \quad \theta_0]^T = [0\text{cm} \quad 0\text{cm} \quad 0\text{rad}]^T$, and the desired final eight goal poses were located on a circle with the radius of 100cm and given by $(X_d, Y_d, \theta_d)^T = (100\times\cos(\theta), 100\times\sin(\theta), \theta)^T$, where θ=0, $\pi/4$, $\pi/2$, $3\pi/4$, π, $5\pi/4$, $3\pi/2$, $7\pi/4$, respectively. Fig. 3 depicts all the experimental trajectories of the omnidirectional mobile base from the origin to the eight goal poses. Fig. 4 shows the heading behavior of the proposed adaptive control law for the robot moving towards the desired orientation $\pi/2$. These results in Fig.3 indicate that these trajectories have almost minimum distances, namely that the robot moved along with straight lines towards the goal poses. Through experiment results, the mobile robot with the proposed adaptive controller has been shown capable of reaching the desired postures with satisfactory performance.

4.3 Adaptive Line Trajectory Tracking

The second experiment was used to study straight-line trajectory tracking performance of the control law (8). The initial pose of the robot was given by $(X_o, Y_o, \theta_o)^T = (30\text{cm}, 0\text{cm}, 0\text{rad})^T$ and the desired straight-line trajectory was described by

$$\left(X_d(t) \quad Y_d(t) \quad \theta_d(t)\right)^T = \left(X_{do} + V_x t \quad Y_{do} + V_y t \quad \frac{\pi}{2}\right)^T$$

where $X_{do} = Y_{do} = 0$cm, $V_x = V_y = 3$ (cm/s). Fig.5 shows the behavior of the experimental straight-line trajectory of the omnidirectional mobile robot. The result indicated that the adaptive control law (8) was capable of steering the robot to follow the desired line path.

4.4 Adaptive Elliptical Trajectory Tracking

The last experiment was employed to explore how the adaptive controller (8) steers the mobile robot to follow an elliptic trajectory described by

$$\left(X_d \quad Y_d \quad \theta_d\right)^T = \left(X_{do} + r_1\times\cos(\omega_o + \omega_r t) \quad Y_{do} + r_2\times\sin(\omega_o + \omega_r t) \quad \frac{\pi}{2}\right)^T$$

The experiment assumed that the robot got started at the origin, i.e., $(X_o, Y_o, \theta_o)^T = (0\text{cm}, 0\text{cm}, 0\text{rad})$. The parameters in the elliptic trajectory tracking experiment were taken as follows: $\omega_o = 0$ (rad/sec), $\omega_r = 0.3$ (rad/sec), $r_1 = 20$ (cm), $r_2 = 30$ (cm), $Y_{do} = 0$ (cm) and $X_{do} = 0$ (cm). Fig.6 depicts the experimental elliptic trajectory of the robot. The result showed that the adaptive controller was capable of steering the omnidirectional mobile robot to exactly track the elliptic trajectory.

5. CONCLUSIONS

This paper has presented an adaptive backstepping control method for both trajectory tracking and stabilization of an omnidirectional mobile robot with three independent driving wheels equally spaced at 120 degrees from one another. With the simplified characteristics of the used DC servomotors, the second-order nonlinear dynamic model with two unknown parameters and bounded uncertainties has been constructed via the Newtonian second motion law. The adaptive controller has been designed using adaptive backstepping to achieve trajectory tracking and regulation with the matched uncertainties. Experimental results have shown that the proposed controller is capable of accomplishing the basic navigation problems: stabilization about a desired posture and tracking a reference trajectory.

ACKNOWLEDGEMENT

The authors gratefully acknowledge financial support from the National Science Council, Taiwan, R.O.C., under the grant NSC 95-2213-E-005-001.

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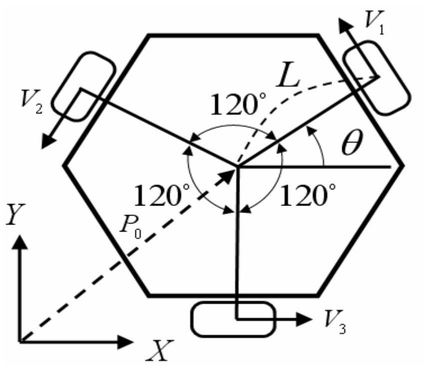

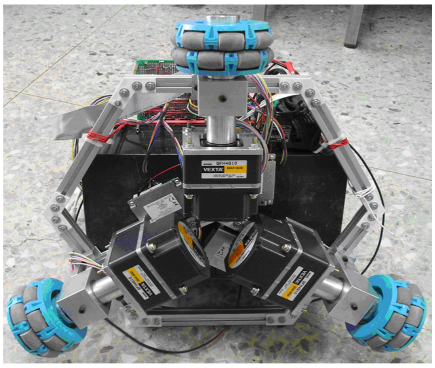

Fig.1. Structure and geometry of the omnidirectional mobile robot.

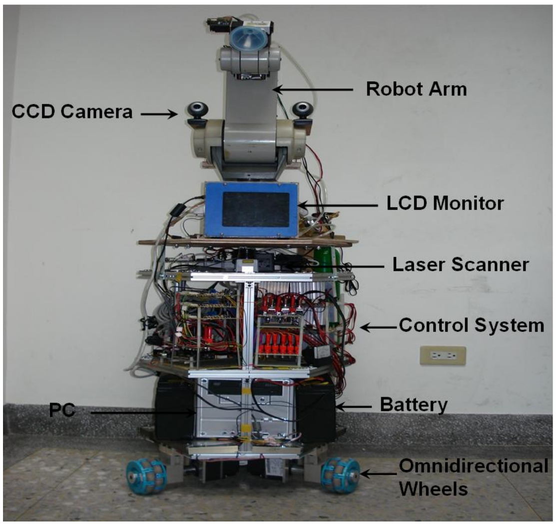

Fig.2. Picture of the experimental omnidirectional mobile robot.

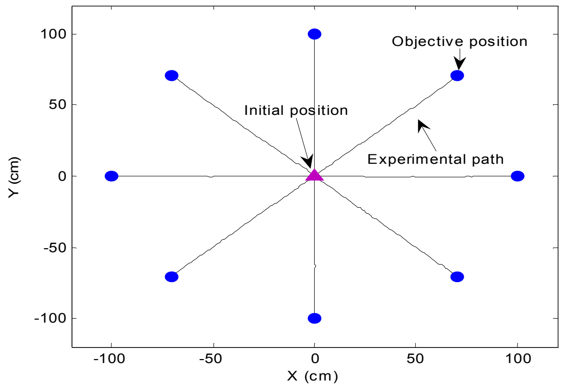

Fig.3. Experimental results of the adaptive controller for stabilization.

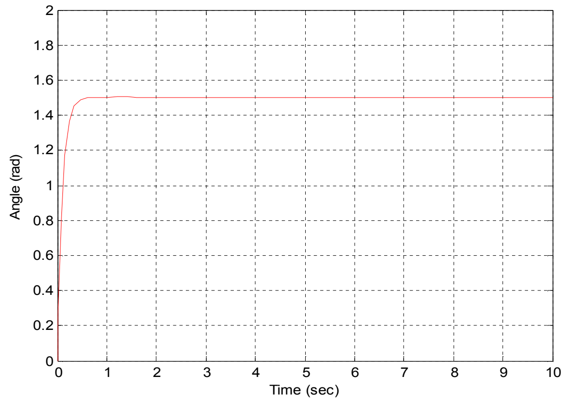

Fig.4. Illustration of the heading behavior of the adaptive control law for the robot moving towards the desired orientation of $\pi/2$.

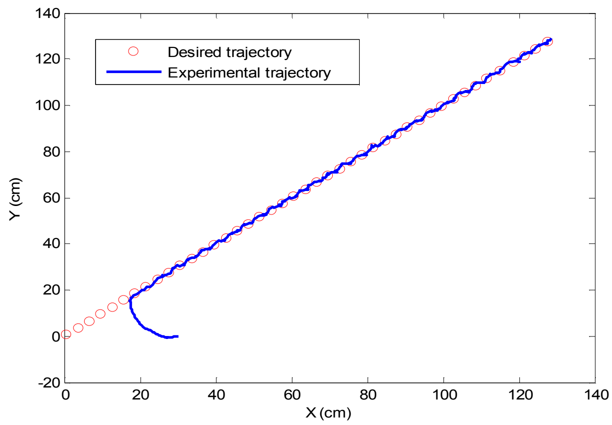

Fig.5. Experiment straight-line trajectory tracking result.

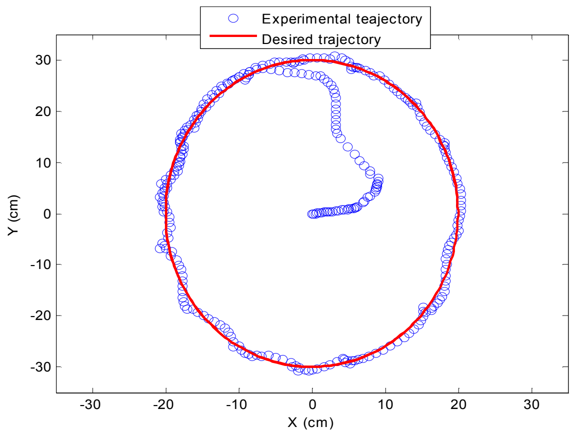

Fig.6. Experimental result of the elliptic trajectory tracking.