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Robust stabilization of nonlinear sandwich plants containing generalized hysteresis nonlinearities

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Abstract: In this paper an approach for the stabilization problem of sandwich nonlinear systems containing a general nonsmooth nonlinearity is presented. The proposed solution is based on variable structure control theory and ensures the robust ultimate boundedness of the system trajectories in a neighborhood of the origin. Theoretical results have been validated by simulation on a mechanical system representing a robot-like system with one link, preceded by an hysteretic block and a first order actuator dynamics representing a DC motor and the simulation results have been confirmed the effectiveness of the proposed solution.

1. INTRODUCTION

The presence of nonsmooth nonlinear characteristics such as dead-zone, backlash, hysteresis and piecewise linearity is common in actuators and sensors.

There are systems in which these nonsmooth nonlinearities are present at the input or the output of a linear or a nonlinear block. Moreover an increasing number of contributions are currently being devoted to the control of systems, named *sandwich systems*, where nonsmooth nonlinearities are placed between two dynamic blocks.

The research on the control of sandwich system was proposed by Taware et al. [1999] and Taware et al. [2002].

Several approaches are proposed to control the systems with such nonlinearities. In order to compensate for the effect of hysteresis, one of the often applied methods is to construct the inverse model of hysteresis (Taware et al. [1999] and Taware et al. [2002]). Unfortunately, such inverse compensation techniques can have serious drawbacks when they are applied in practice, as confirmed by experimental findings (Lewis et al. [1997]).

Other approaches proposed are (Tao et al. [2001]) where the authors proposed a different control strategy based on time-optimal control techniques when the backlash gap is crossed. In the recent paper (Zhao et al. [2006]) the authors design a neural network based inverse model to compensate for the effect of the first dynamic block of the sandwich system.

This paper addresses the stabilization problem of a sandwich nonlinear system with a general piecewise-linear nonsmooth nonlinearity between two dynamical blocks. The proposed solution is based on an output feedback sliding mode controller (V. I. Utkin [1992]) designed as to ensure the convergence of the system trajectories in a neighborhood of the origin.

The remainder of the paper is organized as follows. Section 2 describes system model and problem statement, while preliminaries and notations are reported in section 3. The main results are discussed in section 4. Simulation results are presented in section 5 and, finally, conclusions are briefly summarized in section 6.

2. SYSTEM MODEL AND PROBLEM STATEMENT

Consider a SISO uncertain dynamical system described by:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{\Delta f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u + \mathbf{d}(\mathbf{x}) \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the system state vector at time t, $u(t) \in \mathbb{R}$ is the system input, $\mathbf{g}(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}^n$ is the smooth state-input map, $\mathbf{f}(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}^n$ is a smooth function describing the known plant dynamics, and finally $\mathbf{\Delta f}(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}^n$ and $\mathbf{d}(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}^n$ account for parameter variations and exogenous disturbance respectively. The nonlinear system is assumed to be preceded by an actuating device (see figure 1) whose dynamics are described by:

$$\dot{\mathbf{x}}_{\mathbf{A}} = \mathbf{f}_{\mathbf{A}}(\mathbf{x}_{\mathbf{A}}) + \mathbf{\Delta f}_{\mathbf{A}}(\mathbf{x}_{\mathbf{A}}) + \mathbf{g}_{\mathbf{A}}(\mathbf{x}_{\mathbf{A}})v + \mathbf{d}_{\mathbf{A}}(\mathbf{x}_{\mathbf{A}}) \tag{2}$$

$$\mathbf{w} = \mathbf{h}_{\mathbf{A}}(\mathbf{x}_{\mathbf{A}}) \tag{3}$$

$$u = \mathcal{H}(\mathbf{w}) \tag{4}$$

where $\mathbf{x}_{\mathbf{A}}(t) \in \mathbb{R}^{n_A}$ is the actuator state vector at time t, $v(t) \in \mathbb{R}$ is the input signal of the actuator, $\mathbf{g}_{\mathbf{A}}(\mathbf{x}_{\mathbf{A}}) : \mathbb{R}^{n_A} \to \mathbb{R}^{n_A}$ is the smooth actuator state-input map, $\mathbf{f}_{\mathbf{A}}(\mathbf{x}_{\mathbf{A}}) : \mathbb{R}^{n_A} \to \mathbb{R}^{n_A}$ is a smooth function describing the known plant dynamics, and finally $\mathbf{\Delta f}_{\mathbf{A}}(\mathbf{x}_{\mathbf{A}})$ and $\mathbf{d}_{\mathbf{A}}(\mathbf{x}_{\mathbf{A}})$ describe eventual uncertain terms in the actuator.

The output of the actuator dynamics $\mathbf{w}(t) \in \mathbb{R}$ is connected to the system input $u(t)$ by a nonsmooth block (see figure 2) modeled by the function $\mathcal{H}$. Following the idea introduced in Tao et al. [1996], a compact analytical description of such nonlinearity has the following structure:

$$u(t) = \mathcal{H}(\mathbf{w}) = \begin{cases} F_i(\mathbf{w}(t), \mathbf{w}(t_k), u(t_k), t_k) \\ \qquad \text{if } \mathbf{w}(\cdot) \text{ is increasing over } [t_k, t_{k+1}] \\ F_d(\mathbf{w}(t), \mathbf{w}(t_k), u(t_k), t_k) \\ \qquad \text{if } \mathbf{w}(\cdot) \text{ is decreasing over } [t_k, t_{k+1}] \end{cases} \tag{5}$$

where $F_i(\cdot)$ and $F_d(\cdot)$ are two different piecewise linear functions describing the memory effect associated to the hysteretic behavior of the interconnection between actuator and system. Indeed, they describe the relation between the output of the actuator dynamics $\mathbf{w}$ and system input u for (respectively) positive or negative increments starting from the point $(\mathbf{w}(t_k), u(t_k))$ at time t_k. More details on the hysteresis model are also present in Corradini et al. [2004].

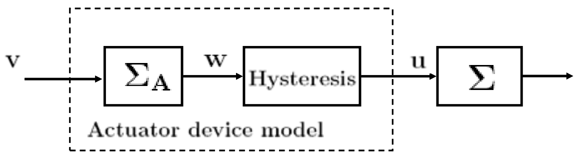

Fig. 1. Block scheme of the system model.

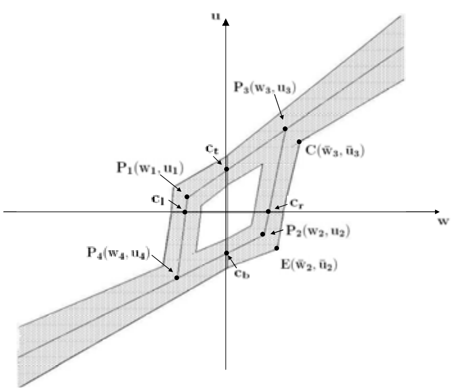

Fig. 2. Sketch of the nonsmooth nonlinearity.

With reference to the hysteresis model, the following assumption is assumed:

Assumption 2.1. Coefficients describing the nonsmooth nonlinearity are uncertain with bounded uncertainties. Hysteresis loop slopes of the two external half-lines are assumed strictly positive. This latter assumption is added in order to exclude saturation as a special case of such nonlinearity, but it is easy to show, in Corradini et al. [2004], that backlash, dead-zone and the model of hysteresis adopted in Tao et al. [1996] can be obtained as special cases of (5) for a suitable choice of parameters.

Further assumptions on the system and actuator dynamics need to be introduced.

Assumption 2.2. There exists a smooth function

$$s(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R} \tag{6}$$

such that:

- system dynamics on the surface $s(\mathbf{x}) = 0$ are asymptotically stable.
- function $s(\mathbf{x})$ is such that the following relation holds:

$$\frac{\partial s(\mathbf{x})}{\partial \mathbf{x}} \mathbf{g}(\mathbf{x}) = \nabla s \cdot \mathbf{g}(\mathbf{x}) \neq 0 \quad \forall \mathbf{x} \in \mathbb{R}^n$$

Assumption 2.3. The actuator dynamics satisfy

$$\nabla \mathbf{h_A}(\mathbf{x_A}) \mathbf{g_A}(\mathbf{x_A}) \neq 0 \quad \forall \mathbf{x_A} \in \mathbb{R}^{n_A}$$

and, without any loss of generality, it is possible to assume

$$\nabla \mathbf{h_A}(\mathbf{x_A}) \mathbf{g_A}(\mathbf{x_A}) > 0$$

Assumption 2.4. There exist smooth functions $(\rho_f(\mathbf{x}))_i : \mathbb{R}^n \to \mathbb{R}$, $(\rho_d(\mathbf{x}))_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, n$ such that:

$$|[\Delta \mathbf{f}(\mathbf{x})]_i| \leq (\rho_f(\mathbf{x}))_i \quad \forall \mathbf{x} \in \mathbb{R}^n \quad i = 1, \ldots, n$$
$$|[\mathbf{d}(\mathbf{x})]_i| \leq (\rho_d(\mathbf{x}))_i \quad \forall \mathbf{x} \in \mathbb{R}^n \quad i = 1, \ldots, n$$

So it follows that

$$|\nabla s[\Delta \mathbf{f}(\mathbf{x}) + \mathbf{d}(\mathbf{x})]| \leq \rho(\mathbf{x}) \quad \forall \mathbf{x} \in \mathbb{R}^n$$

for a known positive scalar $\rho(\mathbf{x})$.

Assumption 2.5. In the present problem, it is assumed that signals available for measurements are system Σ state variables $\mathbf{x}$, system input u, actuator input v. With respect to $\mathbf{w}$ a twofold solution is proposed, considering signal $\mathbf{w}$ known at each time instant or estimating it. The actuator state vector $\mathbf{x_A}$ is assumed unknown, but an initial estimate of the state vector $\widehat{\mathbf{x}}_\mathbf{A}(0)$ is available such that the estimation error is bounded by a known constant $\|\mathbf{e}_{\xi_\mathbf{A}}(0)\| < \chi_{\xi_A}$. Moreover it is assumed (see Marino et al. [1993] and Khalil [2002]) that actuator dynamics (2), (3) can be transformed using a global state-space diffeomorphism $\xi_\mathbf{A} = T(\mathbf{x_A})$ into

$$\dot{\xi}_\mathbf{A} = \mathbf{A_c} \xi_\mathbf{A} + \mathbf{B_c} v + \mathbf{g}_{\xi_\mathbf{A}}(\mathbf{w}, v) \tag{7}$$
$$\mathbf{w} = \mathbf{C_c} \xi_\mathbf{A} \tag{8}$$

where $\mathbf{g}_{\xi_\mathbf{A}}$ represents a norm bounded perturbation term, i.e. $\forall\ (\mathbf{w}, v) \in \mathbb{R}^2\ , \exists\ \mu_\xi \in \mathbb{R} : \|\mathbf{g}_{\xi_\mathbf{A}}(\mathbf{w}, v)\| < \mu_\xi$. In this representation the relation in assumption 2.3 becomes $[\mathbf{C_c}\mathbf{B_c}] \neq 0$ and, without loss of generality, we assume that $\mathbf{C_c}\mathbf{B_c} > 0$. Using the above linear dominant representation it is possible to design a Luenberger-like observer choosing L such that $(\mathbf{A_c} - L\mathbf{C_c})$ is asymptotically stable

$$\dot{\widehat{\xi}}_\mathbf{A} = \mathbf{A_c} \widehat{\xi}_\mathbf{A} + \mathbf{B_c} v + L(\mathbf{w} - \mathbf{C_c} \widehat{\xi}_\mathbf{A}) \tag{9}$$

whose estimation error follows the dynamics

$$\dot{\mathbf{e}}_{\xi_\mathbf{A}} = (\mathbf{A_c} - L\mathbf{C_c}) \mathbf{e}_{\xi_\mathbf{A}} + \mathbf{g}_{\xi_\mathbf{A}}(\mathbf{w}, v) \tag{10}$$

and, being $\mathbf{e}_{\xi_\mathbf{A}}(0)$ and $\mathbf{g}_{\xi_\mathbf{A}}$ bounded respectively by $\chi_{\xi_\mathbf{A}}$ and μ_ξ, the estimation error $\mathbf{e}_{\xi_\mathbf{A}}(t)$ is bounded by a computable function $\bar{\mu}_\xi(t)$. Whenever signal $\mathbf{w}$ is not directly available for measurement, it can be estimated by the system input u so estimator updating rule needs a slight correction, as explained in the next section.

Problem 2.1. The addressed problem, provided that Assumptions 2.1 to 2.5 are satisfied, is to find a feedback controller built on the available signals $\mathbf{x}$, u, v (and eventually $\mathbf{w}$) guaranteeing the practical robust stabilization of system (1) in the presence of the actuating device described by (7), (8) and (5).

3. PRELIMINARIES AND NOTATIONS

In order to concisely state the main results, some definitions are given in the following formalizing some relationships between the sliding surface and system dynamics. Define:

$\omega(\mathbf{x}) \triangleq \nabla s \mathbf{f}(\mathbf{x})$; $r(\mathbf{x}) \triangleq \nabla s \mathbf{g}(\mathbf{x})$; $\delta(\mathbf{x}) \triangleq \nabla s[\Delta \mathbf{f}(\mathbf{x}) + \mathbf{d}(\mathbf{x})]$

As in Corradini et al. [2005], the following function will be used to approximate the *sign* function:

$$\psi_\varepsilon(\zeta) \triangleq \int_{-\infty}^{\zeta} \phi_\varepsilon(\xi) d\xi - 1 \tag{11}$$

where ζ is a real scalar variable, $\varepsilon > 0$ and ϕ_ε is the so called "bell" function, which belong to the C^∞ class over the whole real axis but is nonzero in the interval $(-\varepsilon, \varepsilon)$. Function ψ_ε is by construction a C^∞ function which coincides with the sign function outside the interval $[-\varepsilon, \varepsilon]$. For details see Corradini et al. [2005].
With reference to fig. 3, some symbols relative to the nonsmooth function are described below.

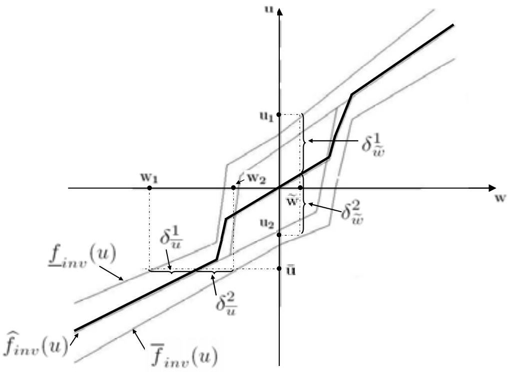

Fig. 3. Nominal hysteresis characteristic and uncertainties.

For any input $\widetilde{\mathbf{w}}$ of the uncertain nonsmooth block, the corresponding output $\widetilde{u} = F(\widetilde{\mathbf{w}})$ is not unique but, in view of assumption 2.1, it surely belongs to a suitable interval $[u_1, u_2]$. The extremal values u_1 and u_2 can be easily computed considering the worst cases of the uncertain parameters describing such nonlinearity. In other words, two "worst-case" functions, $\underline{f}(\cdot)$, $\overline{f}(\cdot)$ can be determined (see Corradini et al. [2004]). They are limiting functions containing that "true" nonlinear function, whichever the uncertainty is. Moreover it is useful to define an average behavior of the uncertain nonsmooth function $\widehat{f}(\cdot)$ such that $\widehat{f}(\mathbf{w}) = \frac{1}{2}(\underline{f}(\mathbf{w}) + \overline{f}(\mathbf{w}))$. It follows that, for any $\mathbf{w}$, the output of the nonsmooth block satisfies:

$$F(\mathbf{w}) \in [\widehat{f}(\mathbf{w}) - \delta^2_{\widetilde{\mathbf{w}}}, \widehat{f}(\mathbf{w}) + \delta^1_{\widetilde{\mathbf{w}}}]$$

Following an analogous approach, the inverse relation between $\mathbf{w}$ and u can be conversely performed using the same monotone functions $\underline{f}(\cdot)$, $\overline{f}(\cdot)$ and $\widehat{f}(\cdot)$. Accordingly, for any desired value of $\overline{u}$ the input must be chosen within the interval $[\underline{f}_{inv}(\overline{u}), \overline{f}_{inv}(\overline{u})]$ which can equivalently be written $[\widehat{f}_{inv}(\overline{u}) - \delta^1_{\overline{u}}, \widehat{f}_{inv}(\overline{u}) + \delta^2_{\overline{u}}]$. It is worth noticing that, if signal $\mathbf{w}$ is not measurable, it can be estimated by u with $\widehat{\mathbf{w}} = \widehat{f}_{inv}(u)$, this estimation differing from $\mathbf{w}$ by an unknown but bounded value, i.e. $\mathbf{w} \in [\widehat{\mathbf{w}} - \delta^1_{\overline{u}}, \widehat{\mathbf{w}} + \delta^2_{\overline{u}}]$. Denote $\Delta_u = \max(\delta^1_{\overline{u}}, \delta^2_{\overline{u}})$ for each u.
Whenever signal $\mathbf{w}$ is not directly available for measurement, estimator updating rule (9) can be chosen as

$$\dot{\widehat{\xi}}_{\mathbf{A}} = \mathbf{A_c}\widehat{\xi}_{\mathbf{A}} + \mathbf{B_c}v + L(\widehat{\mathbf{w}} - \mathbf{C_c}\widehat{\xi}_{\mathbf{A}}) \tag{12}$$

$$\widehat{\mathbf{w}} = \widehat{f}_{inv}(u) \tag{13}$$

$$\widehat{\xi}_{\mathbf{A}}(0) = T(\widehat{\mathbf{x}}_{\mathbf{A}}(0))$$

ensuring an estimation error $\mathbf{e}_{\xi_\mathbf{A}}(t)$ which evolves following

$$\dot{\mathbf{e}}_{\xi_\mathbf{A}} = (\mathbf{A_c} - L\mathbf{C_c})\mathbf{e}_{\xi_\mathbf{A}} + \mathbf{g}_{\xi_\mathbf{A}}(\mathbf{w}, v) + \zeta_\mathbf{w}, \quad \zeta_\mathbf{w} = \mathbf{w} - \widehat{\mathbf{w}}$$

being $(\mathbf{A_c} - L\mathbf{C_c})$ asymptotically stable, $\mathbf{e}_{\xi_\mathbf{A}}(0)$, $\mathbf{g}_{\xi_\mathbf{A}}$ and $\zeta_\mathbf{w}$ bounded by, respectively, $\chi_{\xi_\mathbf{0}}$, μ_ξ, and Δ_u, $\mathbf{e}_{\xi_\mathbf{A}}(t)$ is bounded for each time instant by a computable constant $\overline{\mu}_\xi(t)$.

4. MAIN RESULT

The basic idea of the following theorem is to find a sliding surface $\sigma(\mathbf{w}, \mathbf{x})$ such that, if actuator-system trajectories evolve on it, then system state variables assume values in an ε-neighborhood of the surface $s(\mathbf{x}) = 0$, this guaranteeing ultimate boundedness of system trajectories.
Define the following sliding surface

$$\begin{aligned}\sigma(\mathbf{w}, \mathbf{x}) = \mathbf{w} - \widehat{f}_{inv}(-[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + \\ + \overline{k}(\mathbf{x})\psi_\varepsilon(s(\mathbf{x}))\}) + \Delta_u\psi_\varepsilon(s(\mathbf{x}))\end{aligned} \tag{14}$$

where $\overline{k}(\mathbf{x}) = \rho(\mathbf{x}) + \eta$, with η and ε positive constants. In the following result it is assumed that the time evolution of signal $\mathbf{w}$ is available, this assumption is removed in the further remark 4.1.

Theorem 4.1. Given the system (1) containing uncertain actuator nonlinearities described by (7), (8) and (5), under assumptions 2.1-2.5, there exist computable functions $v_e(t)$, $v_n(t)$ such that the control law

$$v = v_e + v_n \mathrm{sgn}(\sigma(\mathbf{w}, \mathbf{x})) \tag{15}$$

$\sigma(\mathbf{w}, \mathbf{x})$ defined in equation (14), ensures the convergence of system (1) trajectories to an ε-neighborhood of the surface $s(\mathbf{x})$ in finite time, ε a positive design constant, this condition guaranteeing system Σ trajectories boundedness.

Proof: From assumption 2.2, the achievement of a sliding motion on the surface $s(\mathbf{x}) = 0$ guarantees plant asymptotic stabilization. This condition can be achieved if the sliding mode existence condition $s(\mathbf{x})\dot{s}(\mathbf{x}) < -\eta|s(\mathbf{x})|$ is satisfied at each time instant, ensuring a finite reaching time equal to $T_0 = |s(\mathbf{x}(0))|/\eta$:

$$\begin{aligned}s(\mathbf{x})\dot{s}(\mathbf{x}) = s(\mathbf{x})[\nabla s(\mathbf{f}(\mathbf{x}) + \Delta\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u + \mathbf{d}(\mathbf{x}))] = \\ = s(\mathbf{x})[\omega(\mathbf{x}) + r(\mathbf{x})u + \delta(\mathbf{x})] < -\eta|s(\mathbf{x})|.\end{aligned}$$

In order to fulfill the previous relation, if at a time instant $s(\mathbf{x}) > 0$ then input u must be chosen in order to satisfy:

$$u < -[r(\mathbf{x})]^{-1}(\omega(\mathbf{x}) + \overline{k}(\mathbf{x})) \tag{16}$$

on the other hand, if $s(\mathbf{x}) < 0$, then input u must be chosen in order to satisfy:

$$u > -[r(\mathbf{x})]^{-1}(\omega(\mathbf{x}) - \overline{k}(\mathbf{x})) \tag{17}$$

or, equivalently,

$$u = -[r(\mathbf{x})]^{-1}(\omega(\mathbf{x}) + k(\mathbf{x})\mathrm{sgn}(s(\mathbf{x}))) \tag{18}$$

$k(\mathbf{x}) = \Theta\overline{k}(\mathbf{x})$, $\Theta > 1$. This choice would imply that the internal variable u should be discontinuous, so the continuous approximation of the *sign* function with the function (11) still ensures the confinement of system trajectories (Slotine et al. [1983]) within an ε-neighborhood of $s(\mathbf{x}) = 0$ this condition implying practical plant stabilization:

$$u = -[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + k(\mathbf{x})\psi_\varepsilon(s(\mathbf{x}))\} \tag{19}$$

Being $u(t)$ the output of the nonsmooth block, previous condition can be indirectly obtained by imposing:

$$\mathbf{w} = \widehat{f}_{inv}\Big(-[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + k(\mathbf{x})\psi_\varepsilon(s(\mathbf{x}))\}\Big) + \\ -\Delta_u\psi_\varepsilon(s(\mathbf{x})) \quad (20)$$

which can be equivalently written:

$$\mathbf{w} - \widehat{f}_{inv}\Big(-[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + \\ + k(\mathbf{x})\psi_\varepsilon(s(\mathbf{x}))\}\Big) + \Delta_u\psi_\varepsilon(s(\mathbf{x})) = 0 \quad (21)$$

which can be thought as constraining the trajectories of the cascaded "actuator-plant" system on the surface $\sigma(\mathbf{w},\mathbf{x}) = 0$. Whenever this new sliding condition $\sigma(\mathbf{w},\mathbf{x}) = 0$ holds, then $\mathbf{w}$ satisfies (20), and this, in turn, ensures plant Σ practical stabilization.

Now, the next step is to design a control law v allowing the achievement of a sliding mode on (14) in a finite time. The computation of $d\sigma(\mathbf{w},\mathbf{x})/dt$ gives:

$$\frac{d\sigma(\mathbf{w},\mathbf{x})}{dt} = \frac{d\mathbf{w}}{dt} - \widehat{f}_{inv}\Big(-[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + \\ + k(\mathbf{x})\psi_\varepsilon(s(\mathbf{x}))\}\Big)\frac{d}{dt}\Big\{ -[r(\mathbf{x})]^{-1}\omega(\mathbf{x})\Big\} + \\ - \widehat{f}_{inv}\Big(-[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + k(\mathbf{x})\psi_\varepsilon(s(\mathbf{x}))\}\Big) \\ \frac{d}{dt}\Big\{ -[r(\mathbf{x})]^{-1}k(\mathbf{x})\psi_\varepsilon(s(\mathbf{x}))\Big\} + \\ + \varepsilon^{-1}\Delta_u\phi_\varepsilon(s(\mathbf{x}))\nabla s\dot{\mathbf{x}}$$

Passing through successive derivations, we arrive at:

$$\frac{d\sigma(\mathbf{w},\mathbf{x})}{dt} = \frac{d\mathbf{w}}{dt} + \gamma(\mathbf{x})\dot{\mathbf{x}}$$

where

$$\gamma(\mathbf{x}) = -\widehat{f}_{inv}\Big(-[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + \\ + k(\mathbf{x})\psi_\varepsilon(s(\mathbf{x}))\}\Big)\Big\{[r(\mathbf{x})]^{-2}\omega(\mathbf{x})\frac{\partial r(\mathbf{x})}{\partial \mathbf{x}} + \\ - [r(\mathbf{x})]^{-1}\frac{\partial \omega(\mathbf{x})}{\partial \mathbf{x}} + \psi_\varepsilon(s(\mathbf{x}))\Big([r(\mathbf{x})]^{-2}\rho(\mathbf{x})\frac{\partial r(\mathbf{x})}{\partial \mathbf{x}} + \\ - [r(\mathbf{x})]^{-1}\frac{\partial \rho(\mathbf{x})}{\partial \mathbf{x}}\Big) - \varepsilon^{-1}[r(\mathbf{x})]^{-1}k(\mathbf{x})\phi_\varepsilon(s(\mathbf{x}))\nabla s\Big\} + \\ + \varepsilon^{-1}\Delta_u\phi_\varepsilon(s(\mathbf{x}))\nabla s$$

Now, considering equation (8) it is possible to write

$$\mathbf{w} = \mathbf{C_c}\xi_\mathbf{A} = \mathbf{C_c}(\widehat{\xi}_\mathbf{A} + \mathbf{e}_{\xi_\mathbf{A}})$$

and so

$$\frac{d\mathbf{w}}{dt} = \mathbf{C_c}\dot{\widehat{\xi}}_\mathbf{A} + \mathbf{C_c}\dot{\mathbf{e}}_{\xi_\mathbf{A}}$$

with $\dot{\mathbf{e}}_{\xi_\mathbf{A}}$ given by equation (10) obtaining

$$\frac{d\sigma(\mathbf{w},\mathbf{x})}{dt} = \mathbf{C_c}\dot{\widehat{\xi}}_\mathbf{A} + \mathbf{C_c}\dot{\mathbf{e}}_{\xi_\mathbf{A}} + \gamma(\mathbf{x})\dot{\mathbf{x}}$$

Replacing system and actuator dynamics (1), (9) and (10) in the latter expression, it becomes:

$$\frac{d\sigma(\mathbf{w},\mathbf{x})}{dt} = \mathbf{C_c}\Big[\mathbf{A_c}\widehat{\xi}_\mathbf{A} + \mathbf{B_c}v + L\big(\mathbf{w} - \mathbf{C_c}\widehat{\xi}_\mathbf{A}\big)\Big] + \\ + \mathbf{C_c}\Big[\big(\mathbf{A_c} - L\mathbf{C_c}\big)\mathbf{e}_{\xi_\mathbf{A}} + \mathbf{g}_{\xi_\mathbf{A}}\Big] + \gamma(\mathbf{x})\Big[\mathbf{f}(\mathbf{x}) + \\ + \mathbf{\Delta f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u + \mathbf{d}(\mathbf{x})\Big]$$

The final step is to determine the input v imposing the condition

$$\sigma(\mathbf{w},\mathbf{x})\dot{\sigma}(\mathbf{w},\mathbf{x}) < -\nu|\sigma(\mathbf{w},\mathbf{x})| \quad (22)$$

In order to obtain v, assume $\sigma(\mathbf{w},\mathbf{x}) > 0$. In this case (22) corresponds to $\dot{\sigma}(\mathbf{w},\mathbf{x}) < -\nu$. It can be easily verified that, setting:

$$v_e = -\big[\mathbf{C_c}\mathbf{B_c}\big]^{-1}\Big(\mathbf{C_c}\mathbf{A_c}\widehat{\xi}_\mathbf{A} + \mathbf{C_c}L(\mathbf{w} - \mathbf{C_c}\widehat{\xi}_\mathbf{A}) + \\ + \gamma(\mathbf{x})\mathbf{f}(\mathbf{x}) + \gamma(\mathbf{x})\mathbf{g}(\mathbf{x})m_{inv}\mathbf{w}\Big) \quad (23)$$

and

$$v_n = -\big[\mathbf{C_c}\mathbf{B_c}\big]^{-1}\Big(\nu + \alpha(\mathbf{x}) + |\gamma(\mathbf{x})\mathbf{g}(\mathbf{x})|\Delta_u + \\ + \|\mathbf{C_c}(\mathbf{A_c} - L\mathbf{C_c})\|_2\bar{\mu}_\xi + \|\mathbf{C_c}\|_2\mu_\xi\Big) \quad (24)$$

with

$$\alpha(\mathbf{x}) = \sum_{i=1}^{n}\gamma_i(\mathbf{x})[(\rho_f(\mathbf{x}))_i + (\rho_d(\mathbf{x}))_i]$$

then (22) is fulfilled. The same logic can be followed in the case of $\sigma(\mathbf{w},\mathbf{x}) < 0$. In this case the sliding mode condition becomes $\dot{\sigma}(\mathbf{w},\mathbf{x}) > \nu$, and it is possible to write:

$$v = v_e + v_n\mathrm{sgn}(\sigma(\mathbf{w},\mathbf{x}))$$

where v_e and v_n computed in (23) and (24). ■

Remark 4.1. Whenever the interconnection variable $\mathbf{w}$ is not directly available, the result claimed theorem 4.1 still holds using as sliding surface (14) $\sigma(\widehat{\mathbf{w}},\mathbf{x})$ instead of $\sigma(\mathbf{w},\mathbf{x})$, $\widehat{\mathbf{w}}$ built on the estimation of $\mathbf{w}$, (observe (12), (13)). As a matter of fact, when sliding equation (20) becomes:

$$\widehat{f}_{inv}\Big(-[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + k(\mathbf{x})\psi_\varepsilon(s(\mathbf{x}))\}\Big) + \\ - \Delta_u\psi_\varepsilon(s(\mathbf{x})) = \widehat{\mathbf{w}} = \widehat{f}_{inv}(u) \quad (25)$$

being $\widehat{f}_{inv}$ strictly monotone by construction and hence bijective, this ensures that system input and system state variables are connected by

$$u = \widehat{f}_{inv}^{-1}\Bigg(\widehat{f}_{inv}\Big(-[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + k(\mathbf{x})\psi_\varepsilon(s(\mathbf{x}))\}\Big) + \\ - \Delta_u\psi_\varepsilon(s(\mathbf{x}))\Bigg).$$

Consider now $s(\mathbf{x}) > \varepsilon$, function $\widehat{f}_{inv}^{-1}$ is a monotone strictly increasing function because $\widehat{f}_{inv}$ is by construction, so previous relation becomes $u = \widehat{f}_{inv}^{-1}\Big(\widehat{f}_{inv}\big(-[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + k(\mathbf{x}))\}\big) - \Delta_u\Big) < \widehat{f}_{inv}^{-1}\Big(\widehat{f}_{inv}\big(-[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + k(\mathbf{x}))\}\big)\Big) = -[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + k(\mathbf{x})\} < -[r(\mathbf{x})]^{-1}\{\omega(\mathbf{x}) + \bar{k}(\mathbf{x})\}$ and so condition (16) is satisfied. Analogously, if $s(\mathbf{x}) < -\varepsilon$, equation (25) implies condition (17), these two conditions ensuring that attractiveness of system trajectories into an ε neighborhood of the surface $s(\mathbf{x}) = 0$.

5. SIMULATION RESULTS

In order to validate previous theoretical results, the proposed control approach has been applied by simulation on the mechanical system, proposed in Lewis et al. [1997], representing a robot-like system with one link. The system is described by the following model:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\alpha_1 x_2 + \alpha_2 x_2^2 \cos(x_1) - \alpha_3 \sin(x_1) + u \end{cases} \quad (26)$$

where $\alpha_i = \hat{\alpha}_i + \Delta\alpha_i$, $i = 1, \ldots, 3$ are uncertain parameters whose nominal values are given by $\hat{\alpha}_1 = (1/T)$, $\hat{\alpha}_2 = \bar{m}a$, $\hat{\alpha}_3 = \bar{m}ga$, being $\bar{m}$ the load mass, T the motor time constant, a the length and g the gravitational constant. As in Lewis et al. [1997] the following nominal values has been used: $T = 1$ s, $\bar{m} = 1$ kg, $a = 3.5$ m.

The considered plant is preceded by an actuator device representing a DC motor, proposed in T.-L. Chern et al. [1995], whose linearized behavior is described by:

$$\begin{cases} \dot{\xi}_{\mathbf{A}} = -\beta_1 \xi_{\mathbf{A}} + \beta_2 v \\ \mathbf{w} = \xi_{\mathbf{A}} \end{cases} \quad (27)$$

where $\beta_1 = \hat{\beta}_1 + \Delta\beta_1$ is uncertain parameter whose nominal value is given by $\hat{\beta}_1 = (k_b k_t / J_m R_a)$, and $\beta_2 = (k_t / J_m R_a)$, being k_b the BEMF constant, k_t the torque constant, J_m the rotor inertia, and R_a the armature resistance. As in T.-L. Chern et al. [1995] the following values has been used: $k_b = 0.43$ V s/rad, $k_t = 0.43$ N m/A, $J_m = 0.0014$ kg m^2 and $R_a = 2.0\ \Omega$.

A nonsmooth nonlinearity is considered present between the system input $u(t)$ and the actuator output $\mathbf{w}(t)$. According to Corradini et al. [2004], hysteresis nominal parameters have been chosen as follows: $\hat{m}_r = 5$, $\hat{m}_l = 8$, $\hat{m}_t = \hat{m}_s = 0.7$, $\hat{m}_b = \hat{m}_i = 0.5$, $\hat{c}_r = \hat{c}_t = \hat{c}_s = 1$, $\hat{c}_l = \hat{c}_b = \hat{c}_i = -1$. A 25% variation has been applied to system and actuator dynamical parameters α_i and β_1, while hysteresis parameters have been varied of 35% and 15% for c_j and m_j respectively, $j = \{t, l, r, b, s, i\}$.

The following standard sliding surface (6) for the system (1) as been chosen, satisfying the conditions of assumption 2.2:

$$s(\mathbf{x}) = \lambda x_1 + x_2.$$

Two simulation results have been included to show the effectiveness of the proposed controller, addressing both the stabilization and the regulation problem of the mechanical system. Note that no modifications have been found necessary for applying the main result to the regulation case, apart that of considering the error system instead of the plant itself.

In both simulations initial conditions have been set $\mathbf{x}_0 = [1\ 1]^T$, $\widehat{\mathbf{x}}_{\mathbf{A}}(0) = 0.5$. λ parameter has been set $\lambda = 10$. In the regulation problem we chose the desired final position equal to $x_1 = 5$ degrees.

It can be easily observed that control goals stated in problem 2.1 have been achieved. Figures 4 and 5 (a) show the time evolution of state variables x_1 and x_2 of the plant approaching to zero or the desired position ($x_1 = 0.0873$ rad). Panels 4 and 5 (b) show the time evolution of the control law v, in figures 4 and 5 (c) is reported time evolution of sliding surfaces s which behave as expected. Finally panels 4 and 5 (d) display the points (marked) of the hysteresis characteristic used by the controller.

6. CONCLUSIONS

In this work the problem of stabilizing a system with sandwiched nonsmooth nonlinearity with uncertain parameters between two nonlinear uncertain SISO system has been addressed.

This note has proposed an output feedback sliding mode controller ensuring the convergence of the system trajectories in a neighborhood of the origin for a sandwich system with hysteresis.

Simulation results confirm the effectiveness of the presented solution.

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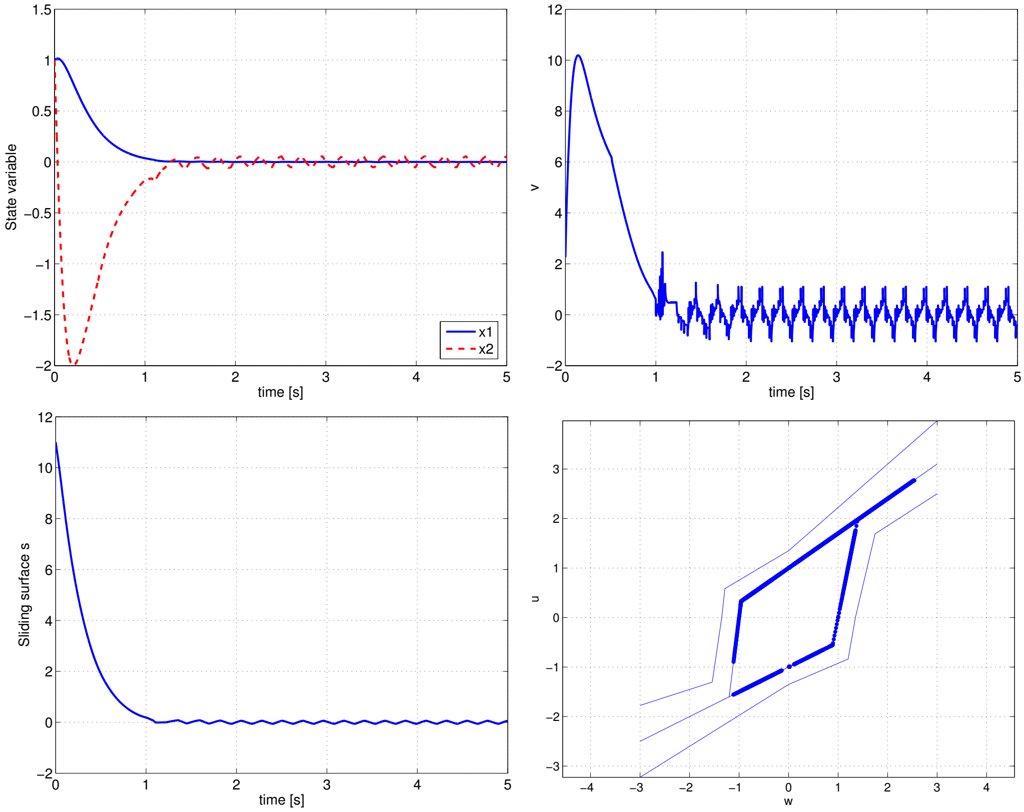

Fig. 4. Simulation results 1: (a) system state variables, (b) control law v, (c) sliding surface s, (d) hysteresis characteristic between actuator output and system input.

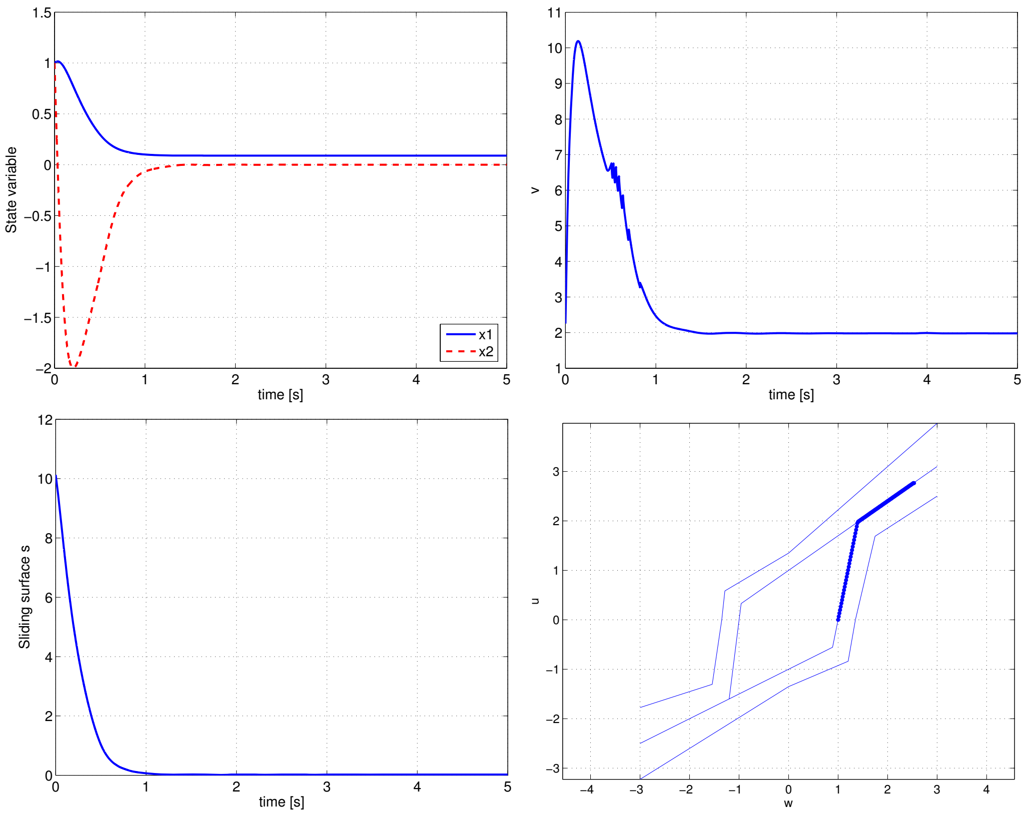

Fig. 5. Simulation results 2: (a) system state variables, (b) control law v, (c) sliding surface s, (d) hysteresis characteristic between actuator output and system input.