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Transfer Function Approach to the Model Matching Problem of Nonlinear Systems ⋆

Miroslav Halás * **Ülle Kotta** ** **Claude H. Moog** ***

* *Institute of Control and Industrial Informatics, Fac. of Electrical Engineering and IT, Slovak University of Technology, Ilkovičova 3, 81219 Bratislava, Slovakia (e-mail: miroslav.halas@stuba.sk)*
** *Institute of Cybernetics, Tallinn University of Technology, Akademia tee 21, 12618 Tallinn, Estonia (e-mail: kotta@cc.ioc.ee)*
*** *IRCCyN, UMR C.N.R.S. 6597, 1 rue de la Noë, BP 92101, 44321 Nantes Cedex 3, France (e-mail: moog@irccyn.ec-nantes.fr)*

Abstract: The mainstream for the analysis and synthesis of nonlinear control systems is the so-called state space approach. The Laplace transform of a nonlinear differential equation is non tracktable and any transfer function approach was not developed until recently. Herein, we show that one may use such mathematical tools to recast and solve the model matching problem. Note that the latter was originaly stated for linear time invariant systems, in terms of equality of the transfer function of both the model and the compensated system.

Keywords: nonlinear systems, model matching problem, transfer functions, transfer equivalence, output feedback

1. INTRODUCTION

The model matching problem was solved within the state space approach by various authors, using problem statements which are slighlty different from one contribution to the other (Benedetto and Isidori (1984); Benedetto (1990); Huijberts (1992); Conte et al. (2007)). In the model matching problem presented in this paper we study the problem of designing a compensator for a nonlinear control system under which the input-output map of the compensated system becomes transfer equivalent to a prespecified model. By transfer equivalence one means that the systems admit the same irreducible input-output differential equation (Conte et al. (2007)). In general, the model is also assumed to be nonlinear.

The model matching problem is a typical design problem in the sense that it plays a role in various other problems like the input-output linearization and the (disturbance) decoupling. In the linear case, one requires the equality of the transfer functions of the model and of the compensated system. However, the transfer function formalism was recently developed also for nonlinear systems, see Zheng and Cao (1995); Halás and Huba (2006); Halás (2008). Such a formalism generalizes well known results valid for linear time invariant systems and was, for instance, already employed in Perdon et al. (2007) to investigate some structural properties of nonlinear systems. Transfer function approach to the model matching problem, of course, represents a very natural tool.

The nonlinear model matching problem has been studied earlier (Benedetto and Isidori (1984); Benedetto (1990); Huijberts (1992); Conte et al. (2007)) in the state-space setting. Our problem statement and solution consider, however, a more general case, since neither the control system itself, nor the model and the compensator are required to be realizable in the state-space form. In particular, this gives us a chance to find realizable compensators for nonlinear systems not having the state-space realization. The paper studies both feedforward and feedback compensators. In case of a feedforward compensator we show that, in contrast to what happens in linear case, a class of nonlinear systems for which the solution exists is quite restricted. In case of a feedback compensator the approach presented here has contact points with that of Rudolph (1994) and of Glad (1990) where the input-output behaviour of the nonlinear system and a model are described by the differential polynomials and differential algebraic tools, particularly Ritt's remainder algorithm have been employed to find the controller equations.

2. TRANSFER FUNCTIONS OF NONLINEAR SYSTEMS

We will use the algebraic formalism of Halás and Huba (2006); Halás (2008) which introduce transfer functions of nonlinear systems.

Consider the SISO nonlinear system defined by an input-output equation of the form

$$y^{(n)} = \varphi(y, \dot{y}, \ldots, y^{(n-1)}, u, \dot{u}, \ldots, u^{(m)}) \tag{1}$$

where φ is assumed to be an element of the field of meromorphic functions $\mathcal{K}$.

Remark 1. Even if one starts with a state-space representation it is always possible to eliminate the state variables to get an input-output equation of the form (1), see Conte et al. (2007).

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The left skew polynomial ring $\mathcal{K}[s]$ of polynomials in s over $\mathcal{K}$ with the usual addition, and the (non-commutative) multiplication given by the commutation rule

$$sa = as + \dot{a} \tag{2}$$

where $a \in \mathcal{K}$, represents the ring of linear ordinary differential operators that act over vector space of one-forms $\mathcal{E} = \text{span}_{\mathcal{K}}\{\text{d}\xi; \xi \in \mathcal{K}\}$ in the following way

$$\left(\sum_{i=0}^{k} a_i s^i\right) v = \sum_{i=0}^{k} a_i v^{(i)}$$

for any $v \in \mathcal{E}$.
The commutation rule (2) actually represents the rule for differentiating.

Lemma 2. (Ore condition). For all non-zero $a, b \in \mathcal{K}[s]$, there exist non-zero $a_1, b_1 \in \mathcal{K}[s]$ such that $a_1 b = b_1 a$.

Thus, the ring $\mathcal{K}[s]$ can be embedded to the non-commutative quotient field $\mathcal{K}\langle s\rangle$ by defining quotients (Halás and Huba (2006); Halás (2008)) as

$$\frac{a}{b} = b^{-1} \cdot a$$

The addition and multiplication in $\mathcal{K}\langle s\rangle$ are defined as

$$\frac{a_1}{b_1} + \frac{a_2}{b_2} = \frac{\beta_2 a_1 + \beta_1 a_2}{\beta_2 b_1}$$

where $\beta_2 b_1 = \beta_1 b_2$ by Ore condition and

$$\frac{a_1}{b_1} \cdot \frac{a_2}{b_2} = \frac{\alpha_1 a_2}{\beta_2 b_1} \tag{3}$$

where $\beta_2 a_1 = \alpha_1 b_2$ again by Ore condition.
Due to the non-commutative multiplication (2) they, of course, differ from the usual rules. In particular, in case of the multiplication (3) we, in general, cannot simply multiply numerators and denominators, nor cancel them in a usual manner. We neither can commute them as the multiplication in $\mathcal{K}\langle s\rangle$ is non-commutative as well.

Example 3. Consider two quotients

$$\frac{1}{s-y}, \frac{1}{s}$$

Then

$$\frac{1}{s-y} + \frac{1}{s} = \frac{2s - y - 2\dot{y}/y}{s^2 - (y + \dot{y}/y)s}$$

and

$$\frac{1}{s-y} \cdot \frac{1}{s} = \frac{1}{s^2 - ys - \dot{y}} \neq \frac{1}{s} \cdot \frac{1}{s-y} = \frac{1}{s^2 - ys}$$

Once the fraction of two skew polynomials is defined we can introduce the transfer function of the nonlinear system (1) as an element $F(s) \in \mathcal{K}\langle s\rangle$ such that $\text{d}y = F(s)\text{d}u$.

After differentiating (1) we get

$$\text{d}y^{(n)} - \sum_{i=1}^{n-1} \frac{\partial\varphi}{\partial y^{(i)}} \text{d}y^{(i)} = \sum_{i=0}^{m} \frac{\partial\varphi}{\partial u^{(i)}} \text{d}u^{(i)}$$

or alternatively

$$a(s)\text{d}y = b(s)\text{d}u$$

where $a(s) = s^n - \sum_{i=1}^{n-1} \frac{\partial\varphi}{\partial y^{(i)}} s^i$, $b(s) = \sum_{i=0}^{m} \frac{\partial\varphi}{\partial u^{(i)}} s^i$ and $a(s), b(s) \in \mathcal{K}[s]$. Then

$$F(s) = \frac{b(s)}{a(s)}$$

Example 4. Consider the system

$$\dot{y} = yu$$

After differentiating

$$\text{d}\dot{y} = u\text{d}y + y\text{d}u$$

$$(s - u)\text{d}y = y\text{d}u$$

and the transfer function is

$$F(s) = \frac{y}{s-u}$$

Remark 5. The transfer function is defined by employing the standard algebraic formalism of differential forms, following the lines in Conte et al. (2007) which introduces the notion of a one-form in a formal and abstract way. Hence, it is not necessary to deal here with the linearization of the system along a trajectory using the Kähler-type differential which leads to a time-varying linear system.

In the linear case to each proper rational function an input-output differential equation of a control system can be associated. However, things are different in the nonlinear case. Though one can always associate to a proper rational function $F(s) = \frac{b(s)}{a(s)}$ a corresponding input-output differential form, $\omega = a(s)\text{d}y - b(s)\text{d}u$, this one-form is not necessarily integrable. If the input-output differential form is integrable, or can be made integrable, then there exists an input-output differential equation of the form (1) such that the transfer function of this input-output equation is $F(s)$. In other words, not every quotient of skew polynomials necessarily represents a control system. Of course, this will play a crucial role in designing compensators.

Transfer functions of nonlinear systems satisfy many properties we expect from transfer functions (Halás (2008)):

- They characterize a nonlinear system uniquely; that is, each nonlinear system has a unique transfer function, no matter what state-space realization one starts with.
- They characterize only accessible and observable subsystem.
- They provide an input-output description.
- They allow us to use transfer function algebra when combining systems in series, parallel or feedback connection.

Some additional structural properties of transfer functions of nonlinear systems are discussed in Perdon et al. (2007). In terms of the model matching problem we remark that

- Two nonlinear systems are locally transfer equivalent (admit the same irreducible input-output differential equation) if and only if they have the same transfer function.

It is now easy to conclude that in the nonlinear model matching problem one, as in linear case, requires the equality of the transfer functions of the model and that of the compensated system.

3. FEEDFORWARD COMPENSATOR

One of the basic tasks in solving the model matching problem is to be interested in finding solution in terms of a feedforward compensator.

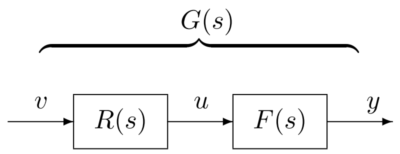

Fig. 1. Compensated system

Problem statement. Consider a nonlinear system F and a model G described by the transfer functions

$$F(s) = \frac{b_F(s)}{a_F(s)}$$
$$G(s) = \frac{b_G(s)}{a_G(s)}$$

respectively. Find a (proper) feedforward compensator R (not necessarily state-space realizable), described by the transfer function

$$R(s) = \frac{b_R(s)}{a_R(s)}$$

such that the transfer function of the compensated system coincides with that of the model G

$$G(s) = F(s) \cdot R(s)$$

The situation is depicted in Fig. 1.

Theorem 6. Given $F(s) \neq 0$ and $G(s)$, there exists a feedforward compensator $R(s)$ which solves the model matching problem if $a_R(s)\mathrm{d}u - b_R(s)\mathrm{d}v$ is integrable, where $\frac{b_R(s)}{a_R(s)} = F^{-1}(s) \cdot G(s)$.

Proof. By the transfer function algebra (Halás (2008)) we get

$$G(s) = F(s) \cdot R(s)$$

Hence, the compensator

$$R(s) = F^{-1}(s) \cdot G(s) \tag{4}$$

Clearly, the existence of such a compensator is determined by the integrability of the compensator's equation $a_R(s)\mathrm{d}u = b_R(s)\mathrm{d}v$. □

If one is interested in finding a solution in a class of proper compensators, it is necessary to restrict the relative degree of the model.

Proposition 7. $R(s)$ is proper (causal) if and only if

$$\operatorname{rel\,deg} G(s) \geq \operatorname{rel\,deg} F(s)$$

Proof. *Sufficiency:* Assume $\operatorname{rel\,deg} G(s) \geq \operatorname{rel\,deg} F(s)$. Since

$$\operatorname{rel\,deg} G(s) = \deg a_G(s) - \deg b_G(s)$$
$$\operatorname{rel\,deg} F(s) = \deg a_F(s) - \deg b_F(s)$$

we have

$$\deg a_G(s) - \deg b_G(s) \geq \deg a_F(s) - \deg b_F(s)$$

Now, as $\deg a_G(s) = \deg a_F(s) + \deg a_R(s)$ and $\deg b_G(s) = \deg b_F(s) + \deg b_R(s)$ we get

$$\deg a_R(s) \geq \deg b_R(s)$$

which means a proper $R(s)$.

Necessity: Assume a proper $R(s)$, $\deg a_R(s) \geq \deg b_R(s)$. Since all previous steps can be done in reverse order, $\operatorname{rel\,deg} G(s) \geq \operatorname{rel\,deg} F(s)$. □

Example 8. Given the system F

$$\dot{y} = \dot{u} + u^2$$

with the transfer function

$$F(s) = \frac{s + 2u}{s}$$

Consider the following three models

$$G(s) = \frac{1}{s}$$
$$G'(s) = \frac{1}{s+1}$$
$$G''(s) = \frac{1}{s+2y}$$

By (4) and (3) we get the following transfer functions of the compensators

$$R(s) = \frac{s}{s+2u} \cdot \frac{1}{s} = \frac{1}{s+2u}$$
$$R'(s) = \frac{s}{s+2u} \cdot \frac{1}{s+1} = \frac{s}{(s+1)(s+2u)}$$
$$R''(s) = \frac{s}{s+2u} \cdot \frac{1}{s+2y} = \frac{s - \dot{y}/y}{(s + 2y - \dot{y}/y)(s+2u)}$$

While $R(s)$ and $R'(s)$ result in the integrable compensators

$$\begin{aligned} R &: & \dot{u} + u^2 &= v \\ R' &: & \ddot{u} + 2u\dot{u} + \dot{u} + u^2 &= \dot{v} \end{aligned}$$

$R''(s)$ does not. We can also easily check, see Conte et al. (2007), that both R and R' are state-space realizable. Finally, note that in multiplying transfer functions one always has to follow the rule (3) which, in general, yields a different result from the usual multiplication, as can be seen for instance in the case of $R''(s)$.

Example 9. Consider the system from Example 4

$$F(s) = \frac{y}{s-u}$$

Let the desired dynamics be given by the transfer function

$$G(s) = \frac{1}{s^2 + 2s + 1}$$

The compensator reads

$$R(s) = F^{-1}(s) \cdot G(s) = \frac{s-u}{y} \cdot \frac{1}{s^2 + 2s + 1}$$

which after rearrangement yields a non-integrable compensator's equation.

So, in contrast to what happens in the linear time-invariant or time-varying systems (Marinescu and Bourlès (2003)), a class of nonlinear systems for which the solution in terms of a feedforward compensator exists is, due to the integrability condition, quite restricted. Hence, it is natural to be interested in finding a solution in a (more general) class of feedback compensators.

4. FEEDBACK COMPENSATOR

Problem statement. Consider a nonlinear system F and a model G described by the transfer functions

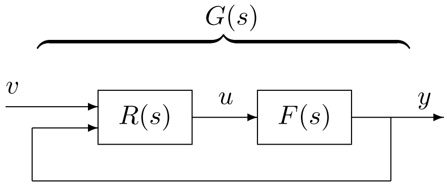

Fig. 2. Compensated system

$$F(s) = \frac{b_F(s)}{a_F(s)} \tag{5}$$

$$G(s) = \frac{b_G(s)}{a_G(s)} \tag{6}$$

respectively. Find a (proper) feedback compensator R (not necessarily state-space realizable), described by the transfer functions

$$R_v(s) = \frac{b_{Rv}(s)}{a_R(s)} \tag{7}$$

$$R_y(s) = \frac{b_{Ry}(s)}{a_R(s)} \tag{8}$$

with

$$\mathrm{d}u = R_v(s)\mathrm{d}v + R_y(s)\mathrm{d}y$$

such that the transfer function of the compensated system coincides with that of the model G

$$G(s) = \frac{F(s)R_v(s)}{1 - F(s)R_y(s)}$$

The situation is depicted in Fig. 2.

Theorem 10. Given $F(s) \neq 0$ and $G(s)$, there exists a feedback compensator $R(s)$ which solves the model matching problem.

Proof. By the transfer function algebra (Halás (2008)) we get

$$G(s) = \frac{F(s)R_v(s)}{1 - F(s)R_y(s)}$$

When considering (5), (7) and (8)

$$G(s) = \frac{\frac{b_F(s)}{a_F(s)} \cdot \frac{b_{Rv}(s)}{a_R(s)}}{1 - \frac{b_F(s)}{a_F(s)} \cdot \frac{b_{Ry}(s)}{a_R(s)}} = \frac{b_{Rv}(s)}{a_R(s) \cdot \frac{a_F(s)}{b_F(s)} - b_{Ry}(s)}$$

After matching the latter to (6) we can set

$$b_G(s) = b_{Rv}(s)$$
$$a_G(s) = a_R(s) \cdot \frac{a_F(s)}{b_F(s)} - b_{Ry}(s)$$

If $a_R(s)$ is chosen to be $q(s)b_F(s)$ then

$$a_G(s) = q(s)a_F(s) - b_{Ry}(s)$$

Assume, without loss of generality, that $\deg a_G(s) \geq \deg a_F(s)$. Then one can think of $q(s)$ as a right skew polynomial quotient and of $-b_{Ry}(s)$ as a skew polynomial remainder of skew polynomials $a_G(s)$ and $a_F(s)$.

So, from given $a_G(s)$ and $a_F(s)$ one can, by Euclidean division algorithm, determine the compensator

$$\mathrm{d}u = R_v(s)\mathrm{d}v + R_y(s)\mathrm{d}y$$
$$a_R(s)\mathrm{d}u = b_{Rv}(s)\mathrm{d}v + b_{Ry}(s)\mathrm{d}y \tag{9}$$

with

$$a_G(s) = q(s)a_F(s) - b_{Ry}(s)$$
$$a_R(s) = q(s)b_F(s)$$
$$b_{Rv}(s) = b_G(s)$$

In comparison to what happens in case of a feedforward model matching problem, now such a compensator is always integrable. Equation (9) can be restated as

$$q(s)b_F(s)\mathrm{d}u = (q(s)a_F(s) - a_G(s))\mathrm{d}y + b_G(s)\mathrm{d}v$$
$$q(s)(b_F(s)\mathrm{d}u - a_F(s)\mathrm{d}y) = b_G(s)\mathrm{d}v - a_G(s)\mathrm{d}y$$

One-forms $b_F(s)\mathrm{d}u - a_F(s)\mathrm{d}y$, $b_G(s)\mathrm{d}v - a_G(s)\mathrm{d}y$ are clearly exact and applying $q(s)$ to an exact one-form results in an exact one-form as well. □

Remark 11. Assumption $\deg a_G(s) \geq \deg a_F(s)$ in the proof is clearly necessary to get a reasonable solution to Euclidean division algorithm of $a_G(s)$ and $a_F(s)$. However, it is not restrictive, for if we have a model

$$G(s) = \frac{b_G(s)}{a_G(s)}$$

with $\deg a_G(s) < \deg a_F(s)$ then we can, without loss of generality, use the model

$$G'(s) = \frac{s^k b_G(s)}{s^k a_G(s)}$$

such that $\deg s^k a_G(s) \geq \deg a_F(s)$. Clearly, the model $G'(s)$ is transfer equivalent to the model $G(s)$.

Roughly speaking, in the sense of transfer equivalence there always exists a feedback compensator which solves the model matching problem.

In case one is interested in finding a solution in a class of proper compensators, the situation is the same as in case of a feedforward compensator.

Proposition 12. $R(s)$ is proper (causal) if and only if

$$\mathrm{rel\,deg}\, G(s) \geq \mathrm{rel\,deg}\, F(s)$$

Proof. *Sufficiency:* Assume $\mathrm{rel\,deg}\, G(s) \geq \mathrm{rel\,deg}\, F(s)$. Since

$$\mathrm{rel\,deg}\, G(s) = \deg a_G(s) - \deg b_G(s)$$
$$\mathrm{rel\,deg}\, F(s) = \deg a_F(s) - \deg b_F(s)$$

we have

$$\deg a_G(s) - \deg b_G(s) \geq \deg a_F(s) - \deg b_F(s)$$

Now, as $a_G(s) = q(s)a_F(s) - b_{Ry}(s)$ we get

$$\deg q(s) + \deg a_F(s) - \deg b_G(s) \geq \deg a_F(s) - \deg b_F(s)$$
$$\deg q(s) + \deg b_F(s) \geq \deg b_G(s)$$

Or, by taking into account that $a_R(s) = q(s)b_F(s)$ and $b_G(s) = b_{Rv}(s)$

$$\deg a_R(s) \geq \deg b_{Rv}(s)$$

which means a proper $R(s)$.

Necessity: Assume a proper $R(s)$, $\deg a_R(s) \geq \deg b_{Rv}(s)$. Since all previous steps can be done in reverse order, $\mathrm{rel\,deg}\, G(s) \geq \mathrm{rel\,deg}\, F(s)$. □

Example 13. Consider the system and the model from Example 9 where we were not able to find a solution in terms of a feedforward compensator.

$$F(s) = \frac{y}{s - u}$$

$$G(s) = \frac{1}{s^2 + 2s + 1}$$

Now, we have $b_F(s) = y$, $a_F(s) = s - u$, $b_G(s) = 1$, $a_G(s) = s^2 + 2s + 1$. From Euclidean division algorithm we get

$$q(s) = s + 2 + u$$
$$b_{Ry}(s) = -1 - \dot{u} - 2u - u^2$$

such that $a_G(s) = q(s)a_F(s) - b_{Ry}(s)$. The compensator is determined by

$$b_{Rv}(s) = b_G(s) = 1$$
$$b_{Ry}(s) = -1 - \dot{u} - 2u - u^2$$
$$a_R(s) = q(s)b_F(s) = ys + \dot{y} + 2y + uy$$

that is

$$a_R(s)\mathrm{d}u = b_{Rv}(s)\mathrm{d}v + b_{Ry}(s)\mathrm{d}y$$
$$y\mathrm{d}\dot{u} + \dot{y}\mathrm{d}u + 2y\mathrm{d}u + uy\mathrm{d}u = \mathrm{d}v - \mathrm{d}y - \dot{u}\mathrm{d}y - 2u\mathrm{d}y - u^2\mathrm{d}y$$

Note that $\dot{y}$ is not independent, $\dot{y} = yu$, and after substituting, the last equation represents the differential of

$$y\dot{u} + 2yu + y + u^2y = v$$

The compensator has the following state-space realization

$$\dot{\xi} = -2\xi - 1 - \xi^2 + \frac{v}{y}$$
$$u = \xi$$

Example 14. Consider the system and the models from Example 8

$$F(s) = \frac{s + 2u}{s}$$

$$G(s) = \frac{1}{s}$$
$$G'(s) = \frac{1}{s+1}$$
$$G''(s) = \frac{1}{s+2y}$$

We get now the following compensators

$$R : \quad \dot{u} + u^2 = v$$
$$R' : \quad \dot{u} + u^2 = v - y$$
$$R'' : \quad \dot{u} + u^2 = v - y^2$$

all of them state-space realizable. Note also that while R does not differ from its feedforward counterpart R' does.

5. MODEL MATCHING PROBLEM FOR NONREALIZABLE SYSTEMS

The input-output approach to the model matching problem, as presented here, has one strong point. It is, in fact, more general in that sense that it is applicable to nonlinear systems not having the state-space realization. We do not require this from the original system equations neither from compensator equations. So there is a chance to find realizable compensators for nonrealizable systems in both feedforward and feedback cases, as demonstrated by the following example.

Example 15. Consider the system

$$\ddot{y} = -y + \dot{u}^2 + u$$

which has, according to Conte et al. (2007), no state-space realization. The transfer function is

$$F(s) = \frac{2\dot{u}s + 1}{s^2 + 1}$$

Let the desired dynamics be given by the transfer function

$$G(s) = \frac{1}{s^2}$$

To find a feedforward compensator we compute

$$R(s) = F^{-1}(s) \cdot G(s) = \frac{s^2 + 1}{2\dot{u}s + 1} \cdot \frac{1}{s^2} = \frac{s^2 + 1}{s^2(2\dot{u}s + 1)}$$

The compensator's equation is integrable

$$2\ddot{u}^2 + 2\dot{u}u^{(3)} + \ddot{u} = \ddot{v} + v$$

and has the following state-space realization

$$\dot{\xi}_1 = \sqrt{\xi_2 - \xi_1 + v}$$
$$\dot{\xi}_2 = \xi_3$$
$$\dot{\xi}_3 = v$$
$$u = \xi_1$$

In case we are interested in finding a feedback compensator we get

$$q(s) = 1$$
$$b_{Ry}(s) = 1$$
$$b_{Rv}(s) = b_G(s) = 1$$
$$a_R(s) = q(s)b_F(s) = 2\dot{u}s + 1$$

and finally

$$(2\dot{u}s + 1)\mathrm{d}u = \mathrm{d}v + \mathrm{d}y$$
$$\dot{u}^2 + u = v + y$$

This time, the compensator has the state-space realization

$$\dot{\xi} = \sqrt{v + y - \xi}$$
$$u = \xi$$

6. CONCLUSIONS

In this paper the transfer function formalism was employed to recast and solve the model matching problem of single-input single-output nonlinear control systems. This resulted in designing compensators, both feedforward and feedback, under which the input-output map of the compensated system becomes transfer equivalent to a prespecified model. It was shown that the existence of a feedforward compensator requires a restrictive integrability condition. A feedback compensator exists whenever the system is nontrivial, that is $F(s) \neq 0$. Obviously, the properness of the compensator requires the standard inequality on the relative degrees of the system and that of the model. It is argued that the transfer function approach is the most natural one, in comparison with the state space approaches. In addition, it may be applied also to nonrealizable nonlinear systems where it is possible to find realizable compensators.

Results of this paper may be extended from a several points of view. Obviously, when following Halás and Kotta (2007) this approach carries over quite easily to the nonlinear discrete-time systems. Another natural extension consists in applying ideas of this work to the case of nonlinear time-delay systems. For the corresponding transfer function formalism, see Halás (2007). One of the future tasks is to apply the transfer function approach to solve the model matching problem for square multi-input multi-output system using inverses of transfer function matrices. Another topic is the model matching problem with stability. Clearly, to get a stable solution to the model matching problem the assumption that the set of the unstable zeros of the system is included in the set of the unstable zeros of the model, in both feedforward and feedback case, has to be met. In addition, in case of a feedforward compensator, one has to assume that the system is stable itself, for the solution is, in fact, based on the zero-pole cancellation.

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