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Realization of Initial Alignment Algorithm for Strapdown Inertial Navigation System using Central Difference Filter

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Abstract: Alignment is the process whereby the orientation of the axes of an inertial navigation system is determined with respect to the reference system. In this paper, the initial alignment error equations of the strapdown inertial navigation system (SINS) with large initial azimuth error have been derived with inclusion of nonlinear characteristics. The second order central difference filter (CDF2) has been used for solution of the alignment problem. Simulations have been carried out to validate and corroborate the stationary alignment case employing a strapdown inertial measurement unit (SIMU). A performance comparison between the extended Kalman filter (EKF), the unscented Kalman filter (UKF) and the CDF demonstrate that the accuracy of attitude error estimation using the CDF2 is better than that of using the EKF or the UKF.

1. INTRODUCTION

The navigation parameters (position, velocity, and attitude) are provided in the navigation frame through the transformation from the body frame to the navigation frame using the attitude transformation matrix. The relation between these two frames is realized by continuously updating this transformation matrix. To limit the errors in the derived navigation parameters, it is very important to determine the initial value of such matrix with high accuracy (Britting, 1971). The process of computing the initial value of transformation matrix is known as the alignment of the SIMU. Alignment is accomplished by two steps, namely coarse alignment (CA), and fine alignment (FA). The purpose of the CA is the determination of approximate values of the attitude angles (roll, pitch, and azimuth) between the body and the navigation frame, the FA then refines the CA estimated attitudes using an iterative optimal estimation technique (Savage, 1997 and Rogers, 2003).

The accelerometer and gyro measurements are referenced to the strapdown inertial measurement unit (SIMU) body frame. In stationary alignment and neglecting sensor errors, these measurements are related to the Earth's gravity and rotation rate vectors. The accelerometer and gyro measurements are averaged over two or three minutes during the CA procedure to determine initial estimates for the pitch, roll and azimuth. Due to the inertial sensor bias errors and measurement noise, the CA cannot provide accurate values for the initial attitude angles that guarantee reliable and precise inertial positioning. Therefore, the FA procedure is utilized to optimally estimate the initial attitude errors as well as the sensor biases and compensate for their effect. This process usually requires about five to ten minutes of static data for a navigation-grade SIMU. The Kalman filter is usually used as an optimal estimation tool during the FA process (Chatfield, 1997).

Estimation in nonlinear systems is extremely important because almost all practical systems involve nonlinearities of one kind or another. Accurately estimating the state of such systems is extremely important for fault detection and control applications, for those suitable extensions to the Kalman Filter have been sought. The most common approach is to use the extended Kalman filter (EKF) (Jason *et al*., 2001) which simply linearizes all nonlinear models so that the traditional linear Kalman filter can be used. Linearization errors in EKF arise from estimation errors due to the dependency of the Jacobian elements on the current state estimate. Such errors lead to suboptimal performance. The EKF is 'only as good as the linearization; if the estimate gets too far off, there comes the EKF divergence. An alternative approach is the unscented Kalman filter (UKF) where the random variable, Gaussian distribution is linearized while the nonlinear model equations are directly used in the calculations (Julier *et al*., 2000, 2001).

The central difference filter (CDF), also called divided difference filter (DDF), is similar to the concept behind the UKF, both of which are considered to be among the class of linear regression Kalman filters, but is based on a multivariable extension of Stirling's interpolation formula. Like the UKF, the CDF generates several points about the mean based on varying the covariance matrix along each dimension. A slight difference from the UKF is in that the CDF evaluates a nonlinear function at two different points for each dimension of the state vector that are divided proportionally by the chosen step size (Nørgaard *et al*., 2000 and Saulson *et al*., 2004).

The CDF performs a better covariance estimate than the UKF by approximating the covariance based on prior covariance as opposed to depending on the covariance of the set of sampled points. Additionally, the CDF provides a faster processing speed than the UKF because it does not need to predict forward every positive and negative sigma point in separate stages from when the measurement prediction, measurement prediction covariance and cross-covariance are solved. Also, the CDF impose less system memory requirement (Nørgaard *et al*., 2000 and Saulson *et al*., 2004).

In this paper, a second order (CDF2) is employed for the initial alignment problem of the SINS. This filter is simple to implement as it needs no derivatives for linearization of the system. The CDF2 implemented here gives the comparable a priori estimate as the UKF described in (Julier *et al*., 2001, 2004), but a better covariance estimate. The initial alignment error equations of the SINS with large initial azimuth error have been derived and nonlinear characteristics are included (Bar-Itzhack, 1985). When the azimuth error is fairly small, nonlinear error equations can be reduced to the linear ones. In the initial alignment problem, it is assumed that latitude and longitude are accurately known. In addition to this, it is assumed that rate of change of these position coordinates is small as compared with the Earth's rate. Under these assumptions, relationships are derived for the Earth's rate, position change rate and the Coriolis acceleration error.

This paper comprises 4 sections; the section 2 presents statement of the problem for the SINS initial alignment using the CDF2; theory and algorithm of the CDF2 is also briefed here; the SINS initial alignment problem for the application of the proposed filter structure is the subject of section 3; simulation results are also depicted in this section, and then in the section 4, some useful conclusions are drawn.

2. STATEMENT OF THE PROBLEM

The nonlinear system's dynamic and observation equations in discrete form are given as (Jazwinski, 1970)

$$x_k = f(x_{k-1}, k-1) + w_{k-1} \tag{1}$$

$$z_k = h_k(x_k) + \vartheta_k \tag{2}$$

where $f_k() \in \mathbb{R}^{n \times n}$ is the process model; $x_k \in \mathbb{R}^n$ is the state vector; $w_k \in \mathbb{R}^n$ is the system noise; $z_k \in \mathbb{R}^p$ is the measurement vector; $h_k() \in \mathbb{R}^{p \times n}$ is the measurement model and $\vartheta_k \in \mathbb{R}^p$ is the measurement noise. It is assumed that noise is uncorrelated Gaussian white noise sequences with mean and covariances as follows:

$$E\{w_i\} = 0; \quad E\{w_i w_j\} = Q\delta_{ij}$$

$$E\{\vartheta_i\} = 0; \quad E\{\vartheta_i \vartheta_j\} = R\delta_{ij}$$

$E\{w_i \vartheta_j\} = 0$, for all i, j

where $E\{\cdot\}$ denotes the expectation, and δ_{ij} is the Kronecker delta function. Q and R are bounded positive definite matrices (*Q>0, R>0*). Initial state x_0 is normally distributed with zero mean and covariance P_0.

The problem here is to use nonlinear filter that is more efficient and estimation accuracy is more than the conventional EKF. The reason for including the CDF in this study is due to the rapidly growing support for the linear regression filters over the EKF in potentially all applications of nonlinear state estimation. The motivation behind including the CDF in this study is due to its improvement over the EKF and the UKF.

2.1 The Central Difference Filter

Historically the first of these approximate nonlinear filters was the EKF (Jazwinski, 1970) which linearizes the system and observation equations about a single sample point with the assumption that the a priori distribution is Gaussian, and uses the Kalman filter to obtain estimates for the state and covariance of these estimates. The single sample point is chosen as the best estimate, that is, the approximation of the conditional mean.

Recently, there have been interesting developments in derivative-free nonlinear state estimation techniques as efficient alternatives to the EKF (Nørgaard *et al*., 2000, Julier *et al*., 1995, 1997 and van der Merwe *et al*., 2004). These include the UKF and the CDF. These are called sigma point filters (SPFs) and belong to the simulation based nonlinear filters (Lee *et al*., 2003). The UKF (Julier *et al*., 2001) works on the principle that a set of discretely sampled sigma points can be used to parameterize the mean and covariance of the Gaussian random variables, and the posterior mean and covariance are propagated through the true nonlinear function without the linearization steps.

The UKF has advantages over the EKF in that 1) it can lead to a more accurate, stable estimate of both the state and covariance, 2) the new filter can estimate with discontinuous functions, 3) no explicit derivation of the Jacobian or Hessian matrix is necessary, and 4) the new filter is suitable for parallel processing.

The CDF adopts an alternative linearization method called a central difference approximation (Nørgaard *et al*., 2000 and Saulson *et al*., 2004) in which derivatives are replaced by functional evaluations, and an easy expansion of the nonlinear functions to higher order terms is possible. This put ups easy and efficient implementation of the filters in nonlinear estimation applications.

2.2 The CDF Algorithm Equations

In this section, the algorithm employed, referred to as the CDF2 (Nørgaard *et al*., 2000) is presented. The CDF2 is described as a SPF in a unified way where the filter linearizes the nonlinear dynamic and measurement functions by using an interpolation formula through systematically chosen sigma points. The linearization is based on polynomial approximations of the nonlinear transformations that are obtained by Stirling's interpolation formula, rather than the derivative-based Taylor series approximation.

Conceptually, the implementation principle resembles that of the UKF, the implementation, however, is significantly simpler because it is not necessary to formulate the Jacobian matrices of the nonlinear dynamic and measurement equations. Thus, the CDF2 can replace the UKF, EKF and its higher-order estimators in practical real-time applications that require accurate estimation, but less computational cost (Christopher *et al*., 2006).

The CDF2 filter makes use of first and second order central differences (CDs) to approximate nonlinear transformation of the state and covariance.

Initialization: It is same as that of the EKF except the square-root decompositions of the covariance, process noise, and measurement noise matrices which are defined as

$$Q = D_w D_w^T,\quad R = D_\vartheta D_\vartheta^T,\quad P^- = D_x^- D_x^{-T},\quad \hat{P} = \hat{D}_x \hat{D}_x^T$$

Also, the jth column of D_x^- shall be referred to as $d_{x,j}^-$ and likewise for the other matrices. The factorization of the noise covariance matrices can usually be made in advance. D_x^- and $\hat{D}_x$ are updated directly during application of the filter.

Time update: Matrices containing first order CDs are

$$\left.\begin{aligned} D'_{x\hat{x},k} &= [F_p^+ - F_p^-]/(2m), & D'_{xw,k} &= [F_q^+ - F_q^-]/(2m) \\ D'_{z\hat{x},k} &= [H_p^+ - H_p^-]/(2m), & D'_{z\vartheta,k} &= [H_r^+ - H_r^-]/(2m) \end{aligned}\right\} \quad (3)$$

where m is the central difference perturbing parameter, and

$F_p^+ = f(\hat{x}_k + m d_{x,j}^-) + \bar{w}_k$, $F_p^- = f(\hat{x}_k - d_{x,j}^-) + \bar{w}_k$,

$F_q^+ = f(\hat{x}_k) + (\bar{w}_k + m d_{w,j})$, $F_q^- = f(\hat{x}_k) + (\bar{w}_k - m d_{w,j})$,

$H_p^+ = h(\hat{x}_{k+1} + m d_{x,j}^-) + \bar{\vartheta}_{k+1}$, $H_p^- = h(\hat{x}_{k+1} - m d_{x,j}^-) + \bar{\vartheta}_{k+1}$,

$H_r^+ = h(\hat{x}_{k+1}) + (\bar{\vartheta}_{k+1} + m d_{v,j})$, $H_r^- = h(\hat{x}_{k+1}) + (\bar{\vartheta}_{k+1} - m d_{x,j}^-)$

The matrices of second order CDs are defined as

$$\left.\begin{aligned} D''_{x\hat{x},k+1} &= \frac{\sqrt{m^2-1}}{2m^2}[F_p^+ + F_p^+ - F^\circ] \\ D''_{xw,k+1} &= \frac{\sqrt{m^2-1}}{2m^2}[F_q^+ - F_q^- - F^\circ] \\ D''_{z\hat{x},k+1} &= \frac{\sqrt{m^2-1}}{2m^2}[H_p^+ - H_p^- - H^\circ] \\ D''_{z\vartheta,k+1} &= \frac{\sqrt{m^2-1}}{2m^2}[H_r^- - H_r^- - H^\circ] \end{aligned}\right\} \quad (4)$$

where $F^\circ = 2(f(\hat{x}_k) + \bar{w}_k)$, and $H^\circ = 2(h(\hat{x}_{k+1}) + \bar{\vartheta}_{k+1})$

The state, state root covariance, measurement, and measurement root-covariance predictions are given by

$$\hat{x}_{k+1}^- = \frac{m^2-(n_x+n_w)}{m^2} F^\circ + \frac{1}{2m^2}\left(\sum_{j=1}^{n_x}(F_p^+ + F_p^-) + \sum_{j=1}^{n_x}(F_q^+ + F_q^-)\right) \quad (5)$$

$$D_{x,k+1}^- = \mathcal{H}\left([D'_{x\hat{x},k+1} \quad D'_{xw,k+1} \quad D''_{x\hat{x},k+1} \quad D''_{xw,k+1}]\right) \quad (6)$$

$$\hat{z}_{k+1}^- = \frac{m^2-(n_x+n_w)}{m^2} H^\circ + \frac{1}{2m^2}\left(\sum_{j=1}^{n_x}(H_q^+ + H_q^-) + \sum_{j=1}^{n_x}(H_r^+ + H_r^-)\right) \quad (7)$$

$$D_{z,k+1}^- = \mathcal{H}\left([D'_{z\hat{x},k+1} \quad D'_{z\vartheta,k+1} \quad D''_{z\hat{x},k+1} \quad D''_{x\vartheta,k+1}]\right) \quad (8)$$

where n_x and n_w denote the dimensions of the state and process noise vector, respectively; $\mathcal{H}$ represents a Householder transformation of the argument matrix (Nørgaard *et al.*, 2000).

Measurement update: Lastly, the state, Kalman gain and state root-covariance update equations are given by

$$\hat{x}_{k+1} = x_{k+1}^- + K_{k+1}(z_{k+1} - \hat{z}_{k+1}^-) \quad (9)$$

$$K_{k+1} = D_{x,k+1}^-(D'_{z\hat{x},k})^T [D_{z,k+1}^-(D_{z,k+1}^-)^T]^{-1} \quad (10)$$

$$\hat{D}_{x,k+1} = \mathcal{H}([D_{x,k+1}^- - K_{k+1}D'_{z\hat{x},k} \quad K_{k+1}D'_{z\vartheta,k} \quad K_{k+1}D''_{z\hat{x},k} \quad K_{k+1}D''_{z\vartheta,k}]) \quad (11)$$

Here, the same state and noise perturbations, used to calculate the first order CDs, are again used to compute the second order CDs. This point has important implications with regard to the computational costs, suggesting that the CDF2 may not require a great deal more computing time than the CDF1.

The CDF2 implementation algorithm is depicted in Fig. 1.

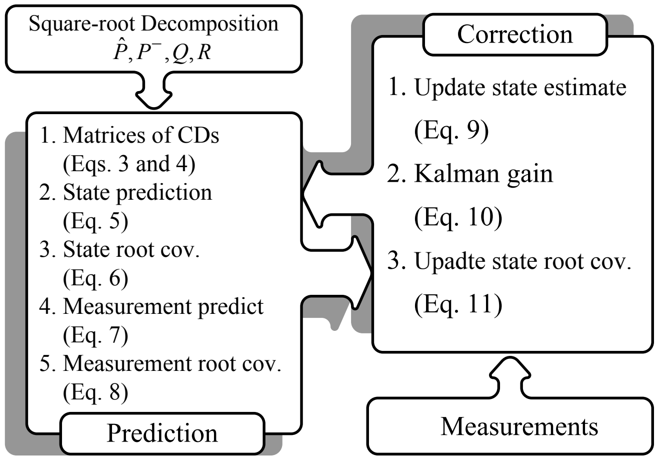

Fig. 1. The CDF2 implementation algorithm

3. ERROR MODEL FOR THE INITIAL ALIGNMENT

3.1 Coordinate Frames

Coordinate frames used in the case study of initial alignment of the SINS are as follows (Britting, 1971):

Inertial frame (i-frame): It has origin at Earth center; z-axis is normal to the equatorial plane; x-axis lies in equatorial plane, its direction can be specified arbitrarily; y-axis complements the right handed system.

Body frame (b-frame): It has origin at center of mass of vehicle; x-axis is along longitudinal axis; z-axis is perpendicular to longitudinal plan of symmetry and y-axis complements the right handed system.

The Earth fixed frame (e-frame): It is the Earth fixed coordinate frame used for position location definition. Its z-axis is coincident with the Earth's polar axis while the other two axes are fixed to the Earth within the equatorial plane.

The navigation frame (n-frame): It is a local geographic coordinate frame; z-axis is parallel to the upward vertical at the local Earth surface referenced position location; x-axis points towards east, and y-axis points towards north.

3.2 Error Analysis

Consider a vector $\bar{v}$ that is known in the geographical frame and can be obtained by processing the sensor outputs. Ideally, their relationship can be written as (Jiang, 1998)

$$\bar{v}^n = C_b^n \bar{v}^b \quad (12)$$

where C_b^n represents the true transformation matrix.

However, it is inevitable that the inertial sensing signals will be contaminated with uncertainties in a practical strapdown system. Therefore, only the computed transformation matrix, $\hat{C}_b^n$, and the estimated vector $\hat{\bar{v}}^b$ are available. As finishing the alignment process, the relationship (12) has to be written as

$$\hat{\dot{v}}^n = \hat{C}_b^n \hat{\dot{v}}^b \tag{13}$$

When the alignment process has completed, $\hat{C}_b^n$ and C_b^n can be related with

$$\hat{C}_b^n = (I + S - \Phi) C_b^n \tag{14}$$

where I is the identity matrix, and S represents the deviation of matrix $\hat{C}_b^n$ from the orthogonal form. The matrix S is symmetric with the form of

$$S = \begin{bmatrix} s_x & e_z & e_y \\ e_z & s_y & e_x \\ e_y & e_x & s_z \end{bmatrix} \tag{15}$$

where the diagonal elements represent the scale errors and off the diagonal elements represent the skew misalignment angles.

When the $\hat{C}_b^n$ matrix is made orthogonal, S is zero. But $\hat{C}_b^n$ will still be different from C_b^n. Then, Φ provides a measure of the difference between the two rotations. The matrix Φ is a skew symmetric with elements denoting axes misalignment angles. It is important to study the case of large uncertainties in the azimuth and low uncertainties in tilt angles. Therefore, for small pitch and roll errors, we have

$$C_n^{n'} = \begin{bmatrix} \cos\phi_z & \sin\phi_z & -\phi_y \\ -\sin\phi_z & \cos\phi_z & \phi_x \\ \phi_{y'} & -\phi_{x'} & 1 \end{bmatrix} \tag{16}$$

Small angle approximations used here are

$\sin\phi_x \approx \phi_x, \sin\phi_y \approx \phi_y, \cos\phi_x \approx 1, \cos\phi_y \approx 1, \phi_x\phi_y \approx 0$

$\phi_{y'} = \phi_y \cos\phi_z + \phi_x \sin\phi_z$ and $\phi_{x'} = -\phi_y \sin\phi_z + \phi_x \cos\phi_z$

3.3 Velocity and Attitude Error Model

The vehicle's acceleration relative to Earth in the navigation frame is (Britting, 1971).

$$\dot{\bar{v}}^n = \bar{f}^n - \Delta\bar{f}^n + \bar{g}^n \tag{17}$$

where $\dot{\bar{v}}^n$ represents acceleration, $\bar{f}^n$ is the specific force, $\Delta\bar{f}^n$ is the Coriolis acceleration and the vehicle's centripetal acceleration around the Earth and $\bar{g}^n = [0 \quad 0 \quad -g]^T$ is the gravitational acceleration vector.

The estimated acceleration of the SINS is

$$\hat{\dot{\bar{v}}}^n = \hat{\bar{f}}^n - \Delta\hat{\bar{f}}^n + \bar{g}^n \tag{18}$$

Now considering the accelerometer bias, estimated specific force is given as

$$\hat{\bar{f}}^n = \bar{f}^{n'} + \bar{\nabla}^{n'} \tag{19}$$

where $\bar{f}^{n'}$ and $\bar{\nabla}^{n'}$ is the specific force and accelerometer bias, respectively in the computed geographic frame. Above equation can also be written as

$$\hat{\bar{f}}^n = C_n^{n'} \bar{f}^n + \bar{\nabla}^{n'} \tag{20}$$

Multiplying both sides of above equation by $C_{n'}^n$, we get

$$C_{n'}^n \hat{\bar{f}}^n = C_{n'}^n C_n^{n'} \bar{f}^n + C_{n'}^n \bar{\nabla}^{n'} \tag{21}$$

Simplification and rearrangement yields

$$\bar{f}^n = C_{n'}^n \hat{\bar{f}}^n - \bar{\nabla}^n \tag{22}$$

Subtracting (17) from (18), and using (22) yields

$$\Delta\dot{\bar{v}}^n = \hat{\dot{\bar{v}}}^n - \dot{\bar{v}}^n = \hat{\bar{f}}^n - C_{g'}^g \hat{\bar{f}}^n + \bar{\nabla}^n - (\Delta\hat{\bar{f}}^n - \Delta\bar{f}) = (I - C_{n'}^n) - \delta\bar{f}^n + \bar{\nabla}^n \tag{23}$$

where I is the identity matrix and $\delta\bar{f}^n = \Delta\hat{\bar{f}}^n - \Delta\bar{f}^n$

For the matrix $C_n^{n'}$, by virtue of the Poisson equation, we have

$$\dot{C}_n^{n'} = -\Omega_n^{n'} C_{n'}^n = -\Omega_i^{n'} C_n^{n'} + C_n^{n'} \Omega_i^n \tag{24}$$

where Ω_i^n is the skew symmetric matrix of the angular velocity of the geographical frame rotation about the inertial frame and it is expressed as

$\bar{\omega}_i^n = [-\dot{\varphi} \quad (\omega_{ie} + \dot{\lambda})\cos\varphi \quad (\omega_{ie} + \dot{\lambda})\sin\varphi]^T$ where $\Omega_i^{n'}$ and $\Omega_n^{n'}$ represent matrices of the angular velocities of the computed geographical frame relative to the inertial frame and of the computed geographical frame relative to the geographical frame.

From (24), for small angle approximation of ϕ_x, ϕ_y, we find

$$\dot{\bar{\phi}}^{n'} = \bar{\omega}_n^{n'} = [\omega_x^n \quad \omega_y^n \quad \omega_z^n]^T \tag{25}$$

Post multiplying both sides of (24) by $C_{n'}^n$ and simplification yields

$$\Omega_n^{n'} = \Omega_i^{n'} - C_n^{n'} \Omega_i^n C_{n'}^n \tag{26}$$

Equation (26) corresponds to the vector equality as

$$\bar{\omega}_n^{n'} = \bar{\omega}_i^{n'} - C_n^{n'} \bar{\omega}_i^n \tag{27}$$

The angular velocity $\bar{\omega}_i^{n'}$ of the computed coordinate system for the SINS orientation algorithm is derived on the information about $\bar{\omega}_i^n$ obtained from SINS as

$$\bar{\omega}_i^{n'} = \bar{\omega}_i^n + \delta\bar{\omega}_i^n + \bar{\varepsilon}^n \tag{28}$$

where $\bar{\varepsilon}^n$ is the gyro drift vector in the computed navigation frame, $\delta\bar{\omega}_i^n$ is the angular rate error in $\bar{\omega}_i^n$ obtained from the SINS.

From (27), we get $\bar{\omega}_i^{n'}$ and equating it with (28) yields

$$\bar{\omega}_n^{n'} = \dot{\bar{\phi}}^n = -(C_n^{n'} - I)\bar{\omega}_i^n + \delta\bar{\omega}_i^n + \bar{\varepsilon}^{n'} = -\Delta C_n^{n'} \bar{\omega}_i^n + \delta\bar{\omega}_i^n + \bar{\varepsilon}^n \tag{29}$$

In the initial alignment problem, it is assumed that latitude and longitude are accurately known. Besides this, it is assumed that rate of change of these position coordinates is small as compared with the Earth's rate. Under these assumptions, relationships for the Earth's rate, position change rate and the Coriolis acceleration error are (Dmitriyev *et al.*, 1997).

$$\bar{\omega}_i^n = [0 \quad \omega_{ie}\cos\varphi \quad \omega_{ie}\sin\varphi]^T \tag{30}$$

$$\delta\bar{\omega}_i^n = [-\Delta v_y / R \quad \Delta v_x / R \quad \Delta v_x \tan\varphi / R]^T \tag{31}$$

$$\delta\bar{f}^n = \begin{bmatrix} 2\omega_{ie}\cos\varphi\Delta v_z - 2\omega_{ie}\sin\varphi\Delta v_y \\ 2\omega_{ie}\cos\varphi\Delta v_x \\ -2\omega_{ie}\cos\varphi\Delta v_x \end{bmatrix} \tag{32}$$

Substituting these relationships given by (30) in (23), we get velocity error differential equations

$$\Delta\dot{\bar{v}}^n = \begin{bmatrix} (1-\cos\phi_z)\hat{f}_x^n + \sin\phi_z \hat{f}_y^n - g\phi_{y'} - 2\omega_{ie}\cos\varphi\Delta v_z + 2\omega_{ie}\sin\varphi\Delta v_y + \nabla_x^n \\ -\sin\phi_z \hat{f}_x^n + (1-\cos\phi_z)\hat{f}_y^n + g\phi_{x'} - 2\omega_{ie}\cos\varphi\Delta v_x + \nabla_y^n \\ \phi_{y'}\hat{f}_x^n - \phi_{x'}\hat{f}_y^n + 2\omega_{ie}\cos\varphi\Delta v_x + \nabla_z^n \end{bmatrix} \quad (33)$$

Substitution of (30) in (29) yields attitude error differential equations as

$$\dot{\bar{\phi}}^n = \begin{bmatrix} -\sin\phi_z\omega_{ie}\cos\varphi + \phi_y\omega_{ie}\sin\varphi - \Delta v_y / R + \varepsilon_x^n \\ (1-\cos\phi_z)\omega_{ie}\cos\varphi - \phi_x\Delta v_x \tan\varphi / R + \varepsilon_y^n \\ \phi_{x'}\omega_{ie}\cos\varphi + \Delta v_x \tan\varphi / R + \varepsilon_z^n \end{bmatrix} \quad (34)$$

3.4 Simulation

To validate and corroborate the alignment problem described in this paper, the primary sensor system used is the SINS that generates position, velocity and attitude information. The goal of the initial alignment scheme is to provide accurate attitude transformation matrix and improved estimates of the SINS error sources. Velocity error from the SINS yields observation to the CDF2. All the simulations have been carried out using a real set of data. Here, the SINS computations are carried out at 20 Hz; while the CDF2 update interval is 1 Hz. Simulations have been carried out for 400 seconds.

In the simulation, the designed values of matrices for the process noise covariance Q and the measurement noise covariance R are as follows:

$Q = diag([Q_{g,i}, Q_{a,j}]), \quad i = x,y,z; \quad j = x,y$

where $Q_{g,i} = (0.01^\circ / h)^2$ for gyros and $Q_{a,j} = (50\mu g)^2$ for accelerometers

$R = diag([(0.1m/s)^2, (0.1m/s)^2])$

The initial state vector x_0 is assumed to be zero and the initial error covariance is defined as

$P_0 = diag([P_{\phi_i}, P_{v_j}, P_{\varepsilon_i}, P_{\nabla_j}]), \quad i = x,y,z; \quad j = x,y$

where $P_{\phi_i} = (1^\circ)^2$, $P_{v_j} = (0.1m/s)^2$, $P_{\varepsilon_i} = (0.02^\circ / h)^2$ and $P_{\nabla_j} = (100\mu g)^2$

Measurement model used in this paper is linear, i.e., horizontal velocity components as estimated by the navigation system are used as observation to the proposed filter implementation. State vector consists of ten system states, i.e., 3 attitude errors, 2 horizontal velocity errors, 3 gyro drifts, and 2 level accelerometer biases.

The discrete filter realization used in this paper is in the direct feedback mode where the estimated attitude errors are fed back to the SINS, thus minimizing the evolution of the observed velocity errors those are to be delivered as an observation to the filter. In this simulation, quaternion is obtained from the corrected attitude matrix and is fed back for attitude error compensation.

3.5 Results and Discussion

Simulation results for the initial alignment problem are shown in Fig. 2. These results illustrate that among the three misalignment angles, leveling misalignment angles can be estimated effectively. The estimation of ϕ_E and ϕ_N converges fast but the convergence of ϕ_U is very slow. Slow convergence rate of ϕ_U is due to the unobservable state of ε_E.

Additionally, the CDF provides a faster processing speed than the UKF because it does not need to predict forward every positive and negative sigma point in separate stages from when the measurement prediction, measurement prediction covariance and cross-covariance are solved. Both the CDF and UKF provide substantial performance increase over the EKF in state estimation problems.

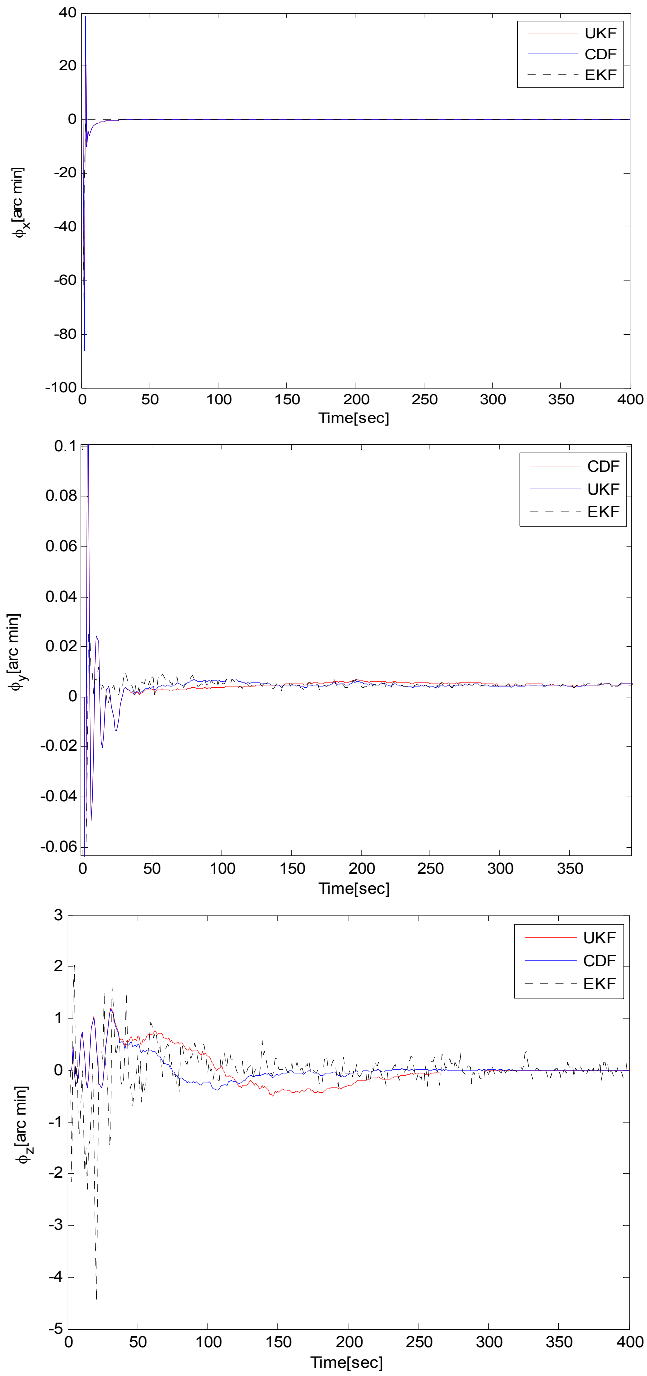

Fig. 2. Axes misalignment angles

Quantitative comparison between the three nonlinear filtering techniques has been depicted in the Table 1. In the table, θ,

ψ and γ stands for vehicle's pitch, heading and roll angles, respectively.

Table 1. Attitude accuracy comparison

Filter	θ["]	ψ[∘]	γ["]
EKF	13.25	2.43	24.32
UKF	7.34	1.92	17.45
CDF	6.57	1.76	16.12

4. CONCLUSIONS

In terms of estimation accuracy, there is insignificant difference between the EKF, UKF, and CDF. However, the EKF may diverge due to poor functional linearization and the UKF may result in large errors and slow convergence due to poorly proportioned and overly large covariance matrices while the CDF is much less sensitive to this problem. In terms of speed, there is an approximately 1.0:2.5:5.0 (EKF:CDF:UKF) ratio for speed, while the ratio for memory is approximately 1.0:1.6:2.0 (EKF:CDF:UKF) for maximum number of elements stored. The EKF requires that the partial derivative at a particular point is available, causing very low reliability in some cases. The UKF and CDF do not always meet the requirement that the state covariance be positive definite, therefore the square-root UKF and CDF should be used instead, which are said to be more efficient and robust.

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