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Design of Robust Quadratic-Optimal Controllers for Linear Multivariable Output Feedback PID Uncertain Control Systems

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Abstract: By complementarily fusing the robust regional eigenvalue-assignability condition, the orthogonal-functions approach (OFA) and the hybrid Taguchi-genetic algorithm (HTGA), an integrative method is proposed in this paper to design the robust optimal eigenvalue-assignable output feedback PID (proportional-integral-derivative) controller such that (i) the eigenvalues of a linear multivariable uncertain closed-loop system can be retained inside the same specified region as the nominal closed-loop system does, and (ii) a quadratic finite-hoziron integral performance index for the linear nominal multivariable control system can be minimized. A design example of the robust optimal eigenvalue-assignable output feedback PID controller for an uncertain stirred tank system is given to demonstrate the applicability of the proposed integrative approach.

1. INTRODUCTION

The PID (proportional-integral-derivative) controller is the most common form of feedback in use today, and is successfully used for a wide range of application: process control, motor drives, magnetic and optic memories, automotive, flight control, instrumentation and so on (Tan et al., 1999; Isaksson and Hagglund, 2002). But the problem for the performance design of linear multivariable PID control systems is still a real challenge to control system engineers (Saeki, 2006). Besides, to ensure both stability robustness and certain performance robustness, it is important to guarantee that the eigenvalues of a linear time-invariant multivariable system under parameter uncertainties remain in a specified region. Thus, recently, Chen et al. (2006) have discussed the robustness analysis problem of eigenvalue-clustering in a specified region for the linear multivariable PID control systems with parameter uncertainties. However, to the authors' best knowledge, there are no literatures to study the issue of designing the robust optimal eigenvalue-assignable output feedback PID controller such that (i) the eigenvalues of a multivariable uncertain closed-loop system can be retained inside the same specified region as the nominal closed-loop system does, and (ii) a quadratic integral performance criterion for the linear nominal multivariable control system can be minimized. On the other hand, very recently, Ho and Chou (2007) have proposed a computational optimization method, which integrates the orthogonal-functions approach (OFA) and the hybrid Taguchi-genetic algorithm (HTGA), to design the optimal fuzzy controllers. Since the method proposed by Ho and Chou (2007) only involves the algebraic computation and is straightforward and well-adapted to computer implementation, the design procedures of the optimal fuzzy controllers may be either greatly reduced or much simplified accordingly. Summing up the above statements and reasons, the purpose of this paper is to propose an integrative optimization method to design the robust optimal eigenvalue-assignable output feedback PID controllers for the linear multivariable uncertain systems. The proposed integrative method complementarily fuses the OFA, the HTGA and the robust regional eigenvalue-assignability condition, where the robust regional eigenvalue-assignability condition is derived in this paper for ensuring that the eigenvalues of a linear multivariable uncertain closed-loop system can be retained inside the same specified region as nominal closed-loop system does.

2. PROBLEM STATEMENT

Consider the linear uncertain system described by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

and

$$y(t) = Cx(t) \tag{2}$$

with the PID controller of the form

$$u(t) = K_P y(t) + K_I \int_0^t y(\theta)d\theta + K_D \dot{y}(t), \tag{3}$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^q$ is the output vector, and $u(t) \in R^r$ is the input vector;

$$A = A_0 + \sum_{i=1}^{\overline{m}} k_i A_i, \quad B = B_0 + \sum_{i=1}^{\overline{m}} k_i B_i, \text{ and } C = C_0 + \sum_{i=1}^{\overline{m}} k_i C_i \tag{4}$$

are the system matrix, the input matrix and the output matrix, respectively, in which k_i $(i = 1, 2, \ldots, \overline{m})$ are the elemental uncertainties; A_i, B_i and C_i $(i = 1, 2, \ldots, \overline{m})$ are, respectively, the given $n \times n$, $n \times r$ and $q \times n$ constant matrices which are prescribed prior to denote the linearly dependent information on elemental uncertainties k_i's; $\overline{m}$ is the number of independent uncertain parameters. The matrices K_P, K_I, $K_D \in R^{r\times q}$, respectively, are the proportional feedback gain matrix, the integral feedback gain matrix and the derivative feedback gain matrix of the output feedback PID controller.

Let a new state variable be $\bar{x}(t)=\left[x^T(t), \int_0^t x^T(\theta)d\theta, \dot{x}^T(t)\right]^T$ and the new output be $\bar{y}(t)=\left[y^T(t), \int_0^t y^T(\theta)d\theta, \dot{y}^T(t)\right]^T$ (Zheng et al., 2002), then the system in (1) and (2) with the PID controller in (3) can be expressed as the following uncertain closed-loop generalized state-space system:

$$\begin{aligned}\bar{E}\dot{\bar{x}}(t) &= \bar{A}\bar{x}(t)+\sum_{i=1}^{\bar{m}} k_i\bar{A}_i\bar{x}(t)+\sum_{i=1}^{\bar{m}}\sum_{j=1}^{\bar{m}} k_ik_j\bar{A}_{ij}\bar{x}(t)\\ &= \bar{A}\bar{x}(t)+\Delta\bar{A}\bar{x}(t),\end{aligned} \tag{5}$$

where

$$\bar{E}=\begin{bmatrix} I_n & 0 & 0\\ 0 & I_n & 0\\ 0 & 0 & 0\end{bmatrix},\ \bar{A}=\begin{bmatrix} 0 & 0 & I_n\\ I_n & 0 & 0\\ A+BK_PC & BK_IC & BK_DC-I_n\end{bmatrix},$$

$$\Delta\bar{A}=\sum_{i=1}^{\bar{m}} k_i\bar{A}_i+\sum_{i=1}^{\bar{m}}\sum_{j=1}^{\bar{m}} k_ik_j\bar{A}_{ij},$$

$$\bar{A}_i=\begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ A_i+B_0K_PC_i+B_iK_PC_0 & B_0K_IC_i+B_iK_IC_0 & B_0K_DC_i+B_iK_DC_0\end{bmatrix},$$

$$\bar{A}_{ij}=\begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ B_iK_PC_j+B_jK_PC_i & B_iK_IC_j+B_jK_IC_i & B_iK_DC_j+B_jK_DC_i\end{bmatrix},$$

and I_n denote the $n\times n$ identity matrix.

For the linear uncertain singular system $(\bar{E},\bar{A}+\Delta\bar{A})$, we assume that a set of PID feedback gain matrix $K=[K_P,K_I,K_D]$ has been specified in advance to make the nominal system $(\bar{E},\bar{A})$ be regular and impulse-free, and to have all its finite eigenvalues located within a specified region *D*, then we can see that $(s\bar{E}-\bar{A})^{-1}$ is a proper rational matrix. Since $(s\bar{E}-\bar{A})^{-1}$ is a proper rational matrix, it can be uniquely decomposed as (Fang, 1997):

$$(sE-\bar{A})^{-1}=G_{sp}(s)+\bar{J}, \tag{6}$$

where $G_{sp}(s)$ is a strictly proper matrix part of $(s\bar{E}-\bar{A})^{-1}$, and $\bar{J}$ is a constant matrix part. In what follows, we present a robust eigenvalue-assignability criterion to analyze whether the linear uncertain singular system $(\bar{E},\bar{A}+\Delta\bar{A})$ remains regular and impulse-free, and has all its finite eigenvalues retained inside the same specified region as the nominal system $(\bar{E},\bar{A})$ does. That is, we propose a robust eigenvalue-assignability criterion to analyze whether the linear multivariable output feedback PID uncertain control system has all its eigenvalues kept within the same specified region as the linear multivariable output feedback PID nominal control system does, where the PID feedback gain matrices have been specified in advance.

Theorem:

Assume that a set of PID feedback gain matrix $K=[K_P,K_I,K_D]$ has been specified in advance to make the nominal system $(\bar{E},\bar{A})$ be regular and impulse-free, and to have all its finite eigenvalues located inside a specified region *D*. The linear uncertain singular system $(\bar{E},\bar{A}+\Delta\bar{A})$ is still regular and impulse-free, and has all its finite eigenvalues retained within the same specified region as the nominal system $(\bar{E},\bar{A})$ does, if the following both inequalities are simultaneously satisfied:

$$\sum_{i=1}^{\bar{m}} k_i\varphi_i+\sum_{i=1}^{\bar{m}}\sum_{j=1}^{\bar{m}} k_ik_j\varphi_{ij}<1 \tag{7a}$$

and

$$\sum_{i=1}^{\bar{m}} k_i\phi_i+\sum_{i=1}^{\bar{m}}\sum_{j=1}^{\bar{m}} k_ik_j\phi_{ij}<1, \tag{7b}$$

where

$$\varphi_i=\begin{cases}\mu(\bar{J}\bar{A}_i), & \text{for } k_i\ge 0;\\ -\mu(-\bar{J}\bar{A}_i), & \text{for } k_i<0;\end{cases} \tag{8a}$$

$$\varphi_{ij}=\begin{cases}\mu(\bar{J}\bar{A}_{ij}), & \text{for } k_ik_j\ge 0;\\ -\mu(-\bar{J}\bar{A}_{ij}), & \text{for } k_ik_j<0;\end{cases} \tag{8b}$$

$$\phi_i=\begin{cases}\sup_{\tilde{q}}\mu\left((\tilde{q}\bar{E}-\bar{A})^{-1}\bar{A}_i\right), & \text{for } k_i\ge 0;\\ -\sup_{\tilde{q}}\mu\left(-(\tilde{q}\bar{E}-\bar{A})^{-1}\bar{A}_i\right), & \text{for } k_i<0;\end{cases} \tag{8c}$$

$$\phi_{ij}=\begin{cases}\sup_{\tilde{q}}\mu\left((\tilde{q}\bar{E}-\bar{A})^{-1}\bar{A}_{ij}\right), & \text{for } k_ik_j\ge 0;\\ -\sup_{\tilde{q}}\mu\left(-(\tilde{q}\bar{E}-\bar{A})^{-1}\bar{A}_{ij}\right), & \text{for } k_ik_j<0;\end{cases} \tag{8d}$$

in which $\mu(V)$ denotes the matrix measure of the matrix $V\in C^{n\times n}$ (Desoer and Vidyasagar, 1975); $\bar{J}$ is given in (6); $\tilde{q}\in Q$ and *Q* denotes the boundary of the specified region *D*.

Proof: Following the same proof procedures given in the work presented by the authors of this paper (Chen et al., 2006), we can obtain that, if both inequalities (7a) and (7b) are satisfied, the linear uncertain singular system $(\bar{E},\bar{A}+\Delta\bar{A})$ is still regular and impulse-free, and has all its finite eigenvalues retained inside the specified region *D*.

Q.E.D.

The problem considered in this paper is how to specify the PID feedback gain matrices K_P K_I and K_D in (3) such that (i) the constraint of robust eigenvalue-assignability criterion in (7) for the linear closed-loop uncertain singular system in (5) can be satisfied, and (ii) such that the optimal control performance for the linear nominal multivariable system

$$\dot{x}(t)=A_0x(t)+B_0u(t),\ \ y(t)=C_0x(t) \tag{9}$$

can be achieved by minimizing the following quadratic finite-horizon integral performance index:

$$J = \int_0^{\bar{q}t_f} \left[x^T(t)Q_{xx}x(t) + u^T(t)R_{uu}u(t)\right]dt$$
$$= \sum_{k=0}^{\bar{q}-1} \int_{kt_f}^{(k+1)t_f} \left[x^T(t)Q_{xx}x(t) + u^T(t)R_{uu}u(t)\right]dt, \quad (10)$$

that is, such that the optimal control performance for the linear closed-loop nominal singular system

$$\bar{E}\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) \quad (11)$$

can be achieved by minimizing the following quadratic integral performance index:

$$J = \sum_{k=0}^{\bar{q}-1} \int_{kt_f}^{(k+1)t_f} \bar{x}^T(t)\left[\bar{Q} + \bar{C}^T K^T R_{uu} K \bar{C}\right]\bar{x}(t)dt, \quad (12)$$

where t_f denotes a small time interval which is chosen for the independent variable t, $\bar{q}$ is a positive integer specified by designer, Q_{xx} is a symmetric positive-semidefinite matrix, R_{uu} is a symmetric positive-definite matrix,

$$\bar{Q} = \begin{bmatrix} Q_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \bar{C} = \begin{bmatrix} C_0 & 0 & 0 \\ 0 & C_0 & 0 \\ 0 & 0 & C_0 \end{bmatrix}, \text{ and } K = [K_P, K_I, K_D].$$

Here the time interval of interest is designated as being from $t = 0$ to $t = \bar{q}t_f$, where $t = 0$ is the initial time and $t = \bar{q}t_f$ is the final time of the control period. The problem to be studied in this paper can be named the robust optimal eigenvalue-assignable output feedback PID controller design problem of a linear uncertain multivariable system, and the design procedures for the PID controller can be described as following:

Step 1: Check the constraint of robust regional eigenvalue-assignability criterion in (7).

Step 2: Minimize the quadratic finite-horizon integral performance index in (12) for the linear closed-loop nominal singular system in (11).

That is, the design problem of the robust optimal eigenvalue-assignable output feedback PID controller for a linear uncertain multivariable system is a constrained optimization problem. In the next section, we will integrate the OFA, the HTGA and the presented robust eigenvalue-assignability criterion to solve this PID controller design problem.

3. ROBUST OPTIMAL EIGENVALUE-ASSIGNABLE PID CONTROLLER DESIGN

Here, consider the time interval $kt_f \le t \le (k+1)t_f$, where t_f is chosen for the independent variable t, and let us define

$$t = kt_f + \eta, \quad (13)$$

and

$$\bar{x}_k = \bar{x}(kt_f), \quad (14)$$

in which $k = 0, 1, 2, \ldots, \bar{q}-1$, and $0 \le \eta \le t_f$.

The state vector $\bar{x}(t)$, within $kt_f \le t \le (k+1)t_f$, can be represented by the truncated orthogonal functions (OF) as

$$\bar{x}(t) = \sum_{s=0}^{m-1} x_s^{(k)} T_s(t) = \tilde{x}^{(k)} T(t), \quad (15)$$

where m is the number of terms required for the orthogonal functions, $T(t) = [T_0(t), T_1(t), \ldots, T_{m-1}(t)]^T$ denotes the $m \times 1$ OF basis vector, $T_s(t)$ $(s = 0, 1, \ldots, m-1)$ denote the orthogonal functions, $x_s^{(k)}$ $(s = 0, 1, \ldots, m-1)$ are the $n \times 1$ coefficient vector, and $\tilde{x}^{(k)} = \left[x_0^{(k)}, x_1^{(k)}, \ldots, x_{m-1}^{(k)}\right]$ is the $n \times m$ coefficient matrix.

Substituting the truncated OF representation of $\bar{x}(t)$ in (15) into the quadratic integral performance index in (12), the quadratic integral performance index J becomes the following algebraic form:

$$J = \sum_{k=0}^{\bar{q}-1} \text{trace}\left[\tilde{W}\left(\tilde{x}^{(k)}\right)^T \left(\bar{Q} + \bar{C}^T K^T R_{uu} K \bar{C}\right)\left(\tilde{x}^{(k)}\right)\right], \quad (16)$$

where the constant matrix $\tilde{W}$ is the product-integration-matrix of two OF basis vectors (Ho and Chou, 2007).

Integrating (11) from $t = kt_f$ to $t = t$ within $kt_f \le t \le (k+1)t_f$, we obtain

$$\bar{E}\bar{x}(t) - \bar{E}\bar{x}(kt_f) = \bar{A}\int_{kt_f}^{t} \bar{x}(t)dt. \quad (17)$$

Using the following integral property of the OF:

$$\int_{kt_f}^{t} T(t)dt = HT(t), \quad (18)$$

applying (14) and (15), and making use of the Kronecker product, the explicit form for the coefficient matrix $\tilde{x}^{(k)}$ can be obtained from (17) as

$$\hat{x}^{(k)} = \left[I_m \otimes \bar{E} - (H^T \otimes \bar{A})\right]^{-1} \hat{Q}^{(k)}, \quad (19)$$

where I_m denotes the $m \times m$ identity matrix, $\hat{x}^{(k)} = \left[x_0^{(k)T}, x_1^{(k)T}, \ldots, x_{m-1}^{(k)\,T}\right]^T$, $\hat{Q}^{(k)} = \left[(\bar{E}\bar{x}_k)^T, 0^T, 0^T, \ldots, 0^T\right]^T$, $\otimes$ denotes the Kronecker product (Barnet, 1979), and H is the operational matrix of integration for the OF (Ho and Chou, 2007). This implies that $\tilde{x}^{(k)}$ can be obtained from (19).

Now, if one set of PID feedback gain matrices $\{K_P, K_I, K_D\}$ is given, then $\tilde{x}^{(k)}$ $(k = 0, 1, 2, \ldots, \bar{q}-1)$ can be calculated from the following algorithm only involving algebraic computation.

Detailed Steps: Algebraic Algorithm

Step 1: Give a small time interval t_f, the specified positive integer $\bar{q}$, and the initial state vector $\bar{x}(0)$, and set $k = 0$.

Step 2: Calculate $\hat{x}^{(k)}$ from (19).

Step 3: Compute $\bar{x}_{k+1}$ by using $\bar{x}_{k+1} = \bar{x}((k+1)t_f) = \tilde{x}^{(k)}T((k+1)t_f)$.

Step 4: Set $k = k+1$. If $k > \bar{q}-1$, then stop; otherwise go to Step 2.

From the above algorithm, it is obvious that if one set of PID feedback gain matrices $\{K_P, K_I, K_D\}$ is specified, then $\tilde{x}^{(k)}$ $(k = 0, 1, 2, \ldots, \bar{q}-1)$ can be determined, and thus the value of the performance index in (16) corresponding to this set of

$\{K_P, K_I, K_D\}$ can be calculated. Given another set of PID feedback gain matrices $\{K_P, K_I, K_D\}$, there obtains another value of the performance index in (16). That is, the value of the performance index of algebraic form in (16) is actually dependent on the set of PID feedback gain matrices $\{K_P, K_I, K_D\}$, which means

$$J = F(K_{11}, K_{12}, \ldots, K_{r\alpha}), \tag{20}$$

where K_{ij} $(i = 1, 2, \ldots, r,\quad j = 1, 2, \ldots, \alpha,$ and $\alpha = 3 \times q)$, respectively, denote the elements of the PID gain matrices K_P, K_I and K_D. Hence, the design problem of the robust optimal eigenvalue-assignable output feedback PID controller for the linear uncertain multivariable system is to search for the optimal K_{ij} such that (i) the robust eigenvalue-assignability criterion in (7) is satisfied, and (ii) the performance index of algebraic form in (16) for the linear nominal singular system in (11) is minimized. This is equivalent to the static parameter constrained-optimization problem

$$\text{minimize } J = F(K_{11}, K_{12}, \ldots, K_{r\alpha}) \tag{21a}$$

$$\text{subject to } \begin{cases} |K_{ij}| \le D_{ij}, \\ \sum_{i=1}^{\overline{m}} k_i \varphi_i + \sum_{i=1}^{\overline{m}} \sum_{j=1}^{\overline{m}} k_i k_j \varphi_{ij} < 1, \\ \sum_{i=1}^{\overline{m}} k_i \phi_i + \sum_{i=1}^{\overline{m}} \sum_{j=1}^{\overline{m}} k_i k_j \phi_{ij} < 1, \end{cases} \tag{21b}$$

where D_{ij} $(i = 1, 2, \ldots, r,\quad j = 1, 2, \ldots, \alpha,$ and $\alpha = 3 \times q)$ are the positive real numbers given from the practical consideration, respectively. This means that, by using the OFA and the robust eigenvalue-assignability criterion, the robust optimal eigenvalue-assignable output feedback PID control problem for the linear uncertain multivariable system can be replaced by a static constrained-optimization problem represented by the algebraic equations with constraints; thus greatly simplifying the robust optimal eigenvalue-assignable PID control problem. Then, the HTGA can be employed to search for the optimal solution of the static constrained-optimization problem in (21), where (21a) is a nonlinear function with the continuous variables. The detailed steps of the HTGA are described in the work proposed by Tsai et al. (2004).

4. DESIGN EXAMPLE

In this section, a design example for a stirred tank is given for illustrating the application of the proposed integrative approach. The state-space equation of a stirred tank (Kwakernaak and Sivan, 1972) is considered and given as

$$\dot{x}(t) = \begin{bmatrix} -\dfrac{F_0}{2V_0} & 0 \\ 0 & -\dfrac{F_0}{V_0} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ \dfrac{c_1 - c_0}{V_0} & \dfrac{c_2 - c_0}{V_0} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \tag{22a}$$

and

$$y(t) = \begin{bmatrix} \dfrac{F_0}{2V_0} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \tag{22b}$$

where $x_1(t)$ is the incremental volume of the fluid in the tank, $x_2(t)$ is the incremental concentration in the tank, and $u_1(t)$ and $u_2(t)$ are the flow rates. The following data are used in this example: $V_0 = 1\,(\text{m}^3)$, $0.019 \le F_0 \le 0.0212\,(\text{m}^3/\text{s})$, $c_0 = 1.25\,(\text{kmol/m}^3)$, $0.0094 \le c_1 \le 1.0006\,(\text{kmol/m}^3)$ and $1.9992 \le c_2 \le 2.0009\,(\text{kmol/m}^3)$. Hence, the dynamic equation in (22) can be described as (1) and (2) with

$$A = \begin{bmatrix} -0.01 & 0 \\ 0 & -0.02 \end{bmatrix} + k_1 \begin{bmatrix} -0.01 & 0 \\ 0 & -0.02 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 1 \\ -0.25 & 0.75 \end{bmatrix} + k_2 \begin{bmatrix} 0 & 0 \\ 0.01 & 0 \end{bmatrix} + k_3 \begin{bmatrix} 0 & 0 \\ 0 & 0.01 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix} + k_1 \begin{bmatrix} 0.01 & 0 \\ 0 & 0 \end{bmatrix},\quad k_1 \in [-0.05\quad 0.06],$$

$k_2 \in [-0.06\quad 0.06]$, and $k_3 \in [-0.08\quad 0.09]$.

The quadratic finite-horizon integral performance index is

$$\begin{aligned} J &= \int_0^{\overline{q}t_f} \left[x^T(t) Q_{xx} x(t) + u^T(t) R_{uu} u(t)\right] dt \\ &= \sum_{k=0}^{\overline{q}-1} \int_{kt_f}^{(k+1)t_f} \left[x^T(t) Q_{xx} x(t) + u^T(t) R_{uu} u(t)\right] dt \\ &= \sum_{k=0}^{\overline{q}-1} \int_{kt_f}^{(k+1)t_f} \overline{x}^T(t) \left[\overline{Q} + \overline{C}^T K^T R_{uu} K \overline{C}\right] \overline{x}(t) dt, \end{aligned} \tag{23}$$

in which $\overline{q} = 3000$, $t_f = 0.01$, $Q_{xx} = I_2$ and $R_{uu} = I_2$. The initial state vector for this system is $x(0) = [0.01\,(\text{m}^3), 0.1\,(\text{kmol/m}^3)]^T$. In this example, we want to assign all the finite eigenvalues of the resulting closed-loop uncertain singular system $(\overline{E}, \overline{A} + \Delta\overline{A})$ to be retained inside a specified rectangular region $D = \{(\tilde{x} + \tilde{y}\tilde{j}) \mid -3 \le \tilde{x} \le 0, -1.5 \le \tilde{y} \le 1.5\}$, where $\tilde{j} = \sqrt{-1}$.

In the following, we will apply the proposed approach, which integrates the OFA, the HTGA and the robust eigenvalue-assignability criterion, to design the robust optimal eigenvalue-assignable output feedback PID controller. In the OFA, the type of OF considered in this example is the shifted-Chebyshev functions. The evolutionary environments of the HTGA used in this paper are: the population size is 30, the crossover rate is 0.9, the mutation rate is 0.5, and the generation number is 30.

After using the proposed integrative approach with $m = 4$ and $|K_{ij}| \le 30$ in which K_{ij} $(i = 1, 2$ and $j = 1, 2, \ldots, 6)$ are the elements of the PID gain matrices K_P, K_I and K_D, we can obtain that the quadratic performance index is $J = 0.00067791$, and the robust optimal eigenvalue-assignable PID gain matrices are

$$K_P = \begin{bmatrix} -6.74356 & -2.6056 \\ 5.5500 & -5.93757 \end{bmatrix},$$

$$K_I = \begin{bmatrix} -4.64159 & -5.14905 \\ -4.97923 & -7.94637 \end{bmatrix},$$

and

$$K_D = \begin{bmatrix} -1.45965 & -2.20348 \\ -0.6831 & -0.806 \end{bmatrix},$$

and we have

(i) $\sum_{i=1}^{3} k_i \varphi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \varphi_{ij} \le 0.0235 < 1,$

for $k_1 \in [0 \;\; 0.06]$, $k_2 \in [0 \;\; 0.06]$, and $k_3 \in [0 \;\; 0.09]$; (24a)

(ii) $\sum_{i=1}^{3} k_i \varphi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \varphi_{ij} \le 0.0247 < 1,$

for $k_1 \in [0 \;\; 0.06]$, $k_2 \in [-0.06 \;\; 0]$, and $k_3 \in [0 \;\; 0.09]$; (24b)

(iii) $\sum_{i=1}^{3} k_i \varphi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \varphi_{ij} \le 0.0239 < 1,$

for $k_1 \in [0 \;\; 0.06]$, $k_2 \in [-0.06 \;\; 0]$, and $k_3 \in [-0.08 \;\; 0]$; (24c)

(iv) $\sum_{i=1}^{3} k_i \varphi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \varphi_{ij} \le 0.0227 < 1,$

for $k_1 \in [0 \;\; 0.06]$, $k_2 \in [0 \;\; 0.06]$, and $k_3 \in [-0.08 \;\; 0]$; (24d)

(v) $\sum_{i=1}^{3} k_i \varphi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \varphi_{ij} \le 0.02336 < 1,$

for $k_1 \in [-0.05 \;\; 0]$, $k_2 \in [0 \;\; 0.06]$, and $k_3 \in [0 \;\; 0.09]$; (24e)

(vi) $\sum_{i=1}^{3} k_i \varphi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \varphi_{ij} \le 0.0246 < 1,$

for $k_1 \in [-0.05 \;\; 0]$, $k_2 \in [-0.06 \;\; 0]$, and $k_3 \in [0 \;\; 0.09]$; (24f)

(vii) $\sum_{i=1}^{3} k_i \varphi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \varphi_{ij} \le 0.0135 < 1,$

for $k_1 \in [-0.05 \;\; 0]$, $k_2 \in [-0.06 \;\; 0]$, and $k_3 \in [-0.08 \;\; 0]$; (24g)

(viii) $\sum_{i=1}^{3} k_i \varphi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \varphi_{ij} \le 0.0225 < 1,$

for $k_1 \in [-0.05 \;\; 0]$, $k_2 \in [0 \;\; 0.06]$, and $k_3 \in [-0.08 \;\; 0]$; (24h)

(ix) $\sum_{i=1}^{3} k_i \phi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \phi_{ij} \le 0.9655 < 1,$

for $k_1 \in [0 \;\; 0.06]$, $k_2 \in [0 \;\; 0.06]$, and $k_3 \in [0 \;\; 0.09]$; (24i)

(x) $\sum_{i=1}^{3} k_i \phi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \phi_{ij} \le 0.9645 < 1,$

for $k_1 \in [0 \;\; 0.06]$, $k_2 \in [-0.06 \;\; 0]$, and $k_3 \in [0 \;\; 0.09]$; (24j)

(xi) $\sum_{i=1}^{3} k_i \phi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \phi_{ij} \le 0.9061 < 1,$

for $k_1 \in [0 \;\; 0.06]$, $k_2 \in [-0.06 \;\; 0]$, and $k_3 \in [-0.08 \;\; 0]$; (24k)

(xii) $\sum_{i=1}^{3} k_i \phi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \phi_{ij} \le 0.9071 < 1,$

for $k_1 \in [0 \;\; 0.06]$, $k_2 \in [0 \;\; 0.06]$, and $k_3 \in [-0.08 \;\; 0]$; (24l)

(xiii) $\sum_{i=1}^{3} k_i \phi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \phi_{ij} \le 0.9865 < 1,$

for $k_1 \in [-0.05 \;\; 0]$, $k_2 \in [0 \;\; 0.06]$, and $k_3 \in [0 \;\; 0.09]$; (24m)

(xiv) $\sum_{i=1}^{3} k_i \phi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \phi_{ij} \le 0.9858 < 1,$

for $k_1 \in [-0.05 \;\; 0]$, $k_2 \in [-0.06 \;\; 0]$, and $k_3 \in [0 \;\; 0.09]$; (24n)

(xv) $\sum_{i=1}^{3} k_i \phi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \phi_{ij} \le 0.5117 < 1,$

for $k_1 \in [-0.05 \;\; 0]$, $k_2 \in [-0.06 \;\; 0]$, and $k_3 \in [-0.08 \;\; 0]$; (24o)

(xvi) $\sum_{i=1}^{3} k_i \phi_i + \sum_{i=1}^{3}\sum_{j=1}^{3} k_i k_j \phi_{ij} \le 0.9285 < 1,$

for $k_1 \in [-0.05 \;\; 0]$, $k_2 \in [0 \;\; 0.06]$, and $k_3 \in [-0.08 \;\; 0]$. (24p)

From the results in (24), we can conclude that the linear closed-loop uncertain singular system $(\overline{E}, \overline{A} + \Delta\overline{A})$ is regular and impulse-free, and has all its finite eigenvalues retained within the specified region *D*. That is, the designed linear multivariable optimal output feedback PID uncertain control system has all its eigenvalues retained inside the same specified region *D* as the linear multivariable optimal output feedback PID nominal control system does. The state responses for the uncertain stirred tank system with the designed robust optimal eigenvalue-assignable PID controller are, respectively, shown in Fig. 1. From Fig. 1, it can be seen that the proposed approach, which integrates the OFA, the HTGA and the robust eigenvalue-assignability criterion, may provide an effective way for designing the robust optimal eigenvalue-assignable output feedback PID controller of the linear multivariable uncertain system.

5. CONCLUSIONS

The robust optimal eigenvalue-assignable output feedback PID control problem of a linear multivariable uncertain control system is transformed into the robust optimal eigenvalue-assignable static output feedback control problem of a linear uncertain singular system. By using the OFA, an algebraic algorithm is presented in this paper to solve the linear nominal singular feedback dynamic equation. Then, the presented algebraic algorithm is complementarily fused with the HTGA to design the robust optimal eigenvalue-assignable static output feedback controller of the linear uncertain singular system such that the control objective of directly minimizing a quadratic integral performance index subject to the constraint of robust regional eigenvalue-assignability criterion can be achieved. The illustrative example regarding a control problem of an uncertain stirred tank system has shown that the proposed integrative approach is effective for designing the robust optimal eigenvalue-assignable output feedback PID controller of the linear multivariable uncertain system.

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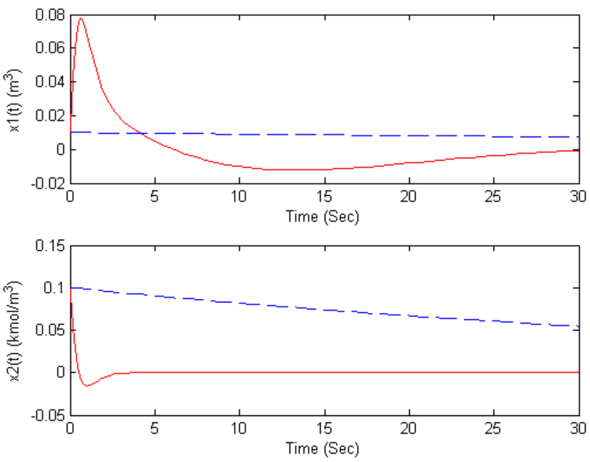

Fig. 1. State responses (incremental volume and incremental concentration) of the uncertain stirred tank system with/without the designed robust optimal eigenvalue-assignable output feedback PID controller (solid line: controlled results; dash line: uncontrolled results).