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Nonlinear ANC using a third-order Volterra filter with an LDL^T-FAP algorithm

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Abstract: Active noise control (ANC) systems employing the conventional fast affine projection (FAP) algorithms may lead to low ANC performance when the non-unity step size is chosen. To solve the problem, an LDL^T- fast affine projection algorithm was proposed recently for the *linear* ANC. In this paper, the LDL^T-FAP algorithm is further utilized, along with a third-order Volterra filter, for *nonlinear* ANC. Simulation results show that the proposed approach yields the good nonlinear ANC performance even in wide range of step sizes.

1. INTRODUCTION

Active noise control (ANC) is one of effective methods to cancel or suppress acoustic noises, being utilized to many applications in communication fields, signal processing areas, etc. For decades, many adaptive filter techniques, yielding better performance and/or less computational complexity, have been suggested for ANC systems (Kuo and Morgan, 1999). In particular, the filtered-x least mean square (FX-LMS) algorithm has been widely used for linear ANC due to its stability and efficiency. More specifically, the LMS algorithm is relatively simple and stable. However, it shows slow convergence. On the other hand, the recursive least-squares (RLS) algorithm leads to much faster convergence than the LMS algorithm, since its utilizes all the information contained in the input data from the start of the adaptation up to the present. However, it requires much more computational complexity than the LMS algorithm. Recently, affine projection (AP) algorithms and their computationally efficient versions, e.g., fast affine projection (FAP) algorithms, are widely used due to their good trade-off between the convergence speed and computational complexity (Douglas, 1995). Furthermore, while Gauss-Seidel FAP (GS-FAP) algorithms provide best performance and convergence speed (Albu, *et al*., 2002; Bouchard and Albu, 2005), the GS-FAP has several limitations in choosing the step size (i.e., the step size is equal to one), which can be a major cause of low convergence. To solve the problem, the LDL^T factorization FAP algorithm has been proposed for the linear ANC system (Bouchard and Ding, 2007), which yields a good performance even in various step sizes. However, in the real systems, nonlinear distortions can be generated from nonlinear (primary and/or secondary) paths of the ANC system, and, accordingly, those nonlinear distortions should be compensated for by employing some nonlinear approaches. In particular, Volterra filtering has been used for the nonlinear filtering and system identification in many applications (Tan and Jing, 2001; Carini and Sicuranza, 2004). In this paper, a nonlinear ANC algorithm is proposed by utilizing third-order Volterra filtering with a LDL^T -FAP adaptation scheme.

This paper is organized as follows: In Section 2, Volterra system modeling for nonlinear ANC is discussed, and Volterra filtering with an LDL^T -FAP adaptive algorithm is introduced in Section 3 for nonlinear ANC system, and some simulation results are provided in Section 4. Finally, conclusion is drawn in Section 5.

2. VOLTERRA SYSTEM MODELLING FOR NONLINEAR ANC

In this section, a third-order Volterra series modelling is discussed for a nonlinear ANC system. Volterra series with a linear adaptive scheme has been applied to various nonlinear communication and control systems (Ahn , *et al*., 2005; Mathews and Sicuranza, 2000). In particular, a Volterra FX-LMS (VFX-LMS) algorithm was proposed for nonlinear ANC (Tan and Jing, 2001), where each channel output in the VFX-LMS scheme consists of nonlinear (i.e., linear, quadratic, and cubic) combinations of the input signal. The output of a nonlinear Volterra system is a function of its input, which can be expressed in a multidimensional convolution form. Accordingly, the linear filter theory can be utilized for Volterra system analysis. More specifically, the input-output relation of a discrete-time third-order Volterra system can be expressed as follows:

$$\begin{aligned} y[n] &= \sum_{m_1=0}^{N-1} w_1[m_1]x[n-m_1] \\ &+ \sum_{m_1=0}^{N-1}\sum_{m_2=m_1}^{N-1} w_2[m_1,m_2]x[n-m_1]x[n-m_2] \\ &+ \sum_{m_1=0}^{N-1}\sum_{m_2=m_1}^{N-1}\sum_{m_3=m_2}^{N-1} w_3[m_1,m_2,m_3]x[n-m_1]x[n-m_2]\,x[n-m_3] \end{aligned} \tag{1}$$

In (1), $x[n]$ is the Volterra system input, $y[n]$ is the Volterra system output, and $w_1[m_1]$ $w_2[m_1,m_2]$ and $w_3[m_1,m_2,m_3]$ correspond to linear, quadratic, and cubic Volterra kernels. Also, N is the system memory size. Furthermore, the Volterra input and kernels can be written in the following vector form by considering the symmetric properties of nonlinear Volterra kernels:

$$\begin{aligned}\mathbf{x}_v[n] = [\,&x[n],\ldots,x[n-N+1],\\ &x^2[n],x[n]x[n-1],\ldots,x[n]x[n-N+1],\ x^2[n-1],\ldots,x[n-1]x[n-N+1],\\ &x^3[n]\ ,\ldots,\ x^2[n]x[n-N+1],\ldots,x^3[n-N+1]\,]^T\end{aligned} \quad (2)$$

$$\begin{aligned}\mathbf{w}_v[n] = [\ &w_1[0],\ldots,w_1[N-1],\\ &w_2[0,0],\ldots,w_2[0,N-1],\ w_2[1,1],\ldots,w_2[1,N-1],\\ &w_3[0,0,0],\ldots,w_3[0,0,N-1],\ldots,w_3[N-1,N-1,N-1]\,]^T\end{aligned} \quad (3)$$

Also, (1) can be expressed in the following vector form by using (2) and (3):

$$y[n] = \mathbf{w}_v^T[n]\,\mathbf{x}_v[n] \quad (4)$$

The general structure of a FX-ANC system is shown in Fig. 1, where $P[z]$ denotes a nonlinear primary path and $S[z]$ a nonlinear secondary path. A nonlinear ANC system using a LMS-based Volterra filtering was reported (Tan and Jing, 2001; Kim, *et al.*, 2006), where the input-output representation of a Volterra system $\widetilde{\mathbf{W}}$ as in Fig. 2 can be described by

$$\widetilde{y}[n] = \widetilde{\mathbf{W}}(x[n]) \quad (5)$$

where $\widetilde{\mathbf{W}}$ is composed of a secondary path $\hat{\mathbf{s}}$, a Volterra kernel vector $\mathbf{w}_v = [\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3]$, and the inverse of the secondary path (i.e., $\hat{\mathbf{s}}^{-1}$). Also, $\widetilde{y}[n]$ is the output of Volterra adaptive filter, and $x[n]$ is an input signal. Furthermore, $u[n]$ is the filtered signal of the input vector $\mathbf{x}[n]$ to the secondary path $\hat{\mathbf{s}}$: That is,

$$u[n] = \hat{\mathbf{s}}\mathbf{x}[n] \quad (6)$$

$$\mathbf{w}_v[n+1] = \mathbf{w}_v[n] - \mu\mathbf{u}_v[n]e[n] \quad (7)$$

$$e[n] = d[n] - \hat{\mathbf{s}}^T\widetilde{\mathbf{y}}[n] \quad (8)$$

In (7), $\mathbf{u}_v[n]$ is an expanded form of $u[n]$ by using the Volterra input vector, and the Volterra kernel vector $\mathbf{w}_v = [\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3]$ can be updated by $\mathbf{u}_v[n]$ and the error signal $e[n]$ as in (7) (Kim, *et al.*, 2006).

3. VOLTERRA FILTERING WITH A FILTERED-X LDL^T-FAP ALGORITHM for NONLINEAR ANC

The LDL^T -FAP algorithms (Bouchard and Ding, 2007; Ding, 2007) were proposed for the *linear* ANC, yielding good converging performance even in case of wide range of step-sizes chosen. Also, the GS-FAP algorithms have been recently proposed, leading to a better ANC performance than conventional FAP methods. However, the GS-FAP shows poor performance in case of a non-unity step size. In this section, the LDL^T-FAP algorithm is further utilized for nonlinear ANC by employing a third-order Volterra filter.

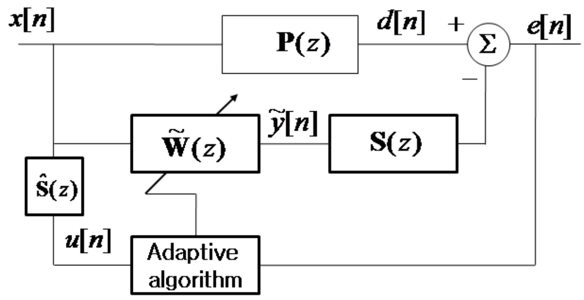

Fig. 1 The structure of a filtered-x ANC

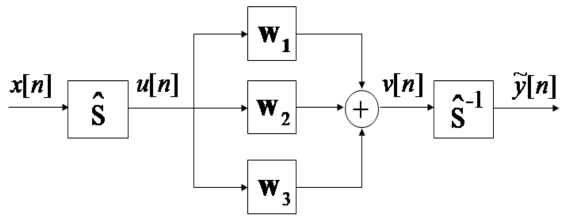

Fig. 2 The adaptive Volterra filter $\widetilde{\mathbf{W}}(z)$.

The Volterra filtering with the LDL^T-FAP algorithm can be given by

$$v[n] = \mathbf{w}_v^T[n]\mathbf{u}_v[n] - \mathbf{r}_v^T[n]\overline{\boldsymbol{\eta}}[n-1] \quad (9)$$

$$\mathbf{R}_v[n] = \mathbf{U}_v[n]\,\mathbf{U}_v^T[n] + \delta\mathbf{I} \quad (10)$$

$$\mathbf{U}_v[n] = [\mathbf{u}_v[n],\mathbf{u}_v[n-1],\ldots,\mathbf{u}_v[n-M+1]] \quad (11)$$

where $v[n]$ is a Volterra filter output, $\mathbf{w}_v$ is a Volterra kernel vector, and, $\mathbf{u}_v[n]$ is a Volterra input vector. Also, $\mathbf{r}_v^T[n]\overline{\boldsymbol{\eta}}[n-1]$ is a compensation term (Bouchard and Ding, 2007). In (9), $\mathbf{r}_v^T$ is the first column (M-1 elements) of an autocorrelation matrix $\mathbf{R}_v[n]$, and, $\overline{\boldsymbol{\eta}}[n-1]$ is a vector consisting of M-1 upper most elements of $\boldsymbol{\eta}$ [n-1]. In addition, the autocorrelation matrix $\mathbf{R}_v[n]$ is calculated from (10), where δ is a regularization factor to avoid the autocorrelation matrix being ill-conditioned, and, $\mathbf{I}$ is a identity matrix of size $M \times M$. $\mathbf{U}_v[n]$ from (11) is a matrix consisting of past M vectors made from $\mathbf{u}_v[n]$ which is an expanded version of the filtered input vector. Finally,

$$\mathbf{R}_v[n]\mathbf{p}[n] = \overline{\mathbf{e}}[n] \quad (12)$$

$$\overline{\mathbf{e}}[n] = [\ e[n]\ \ (1-\mu)\overline{\mathbf{e}}_{M-1}[n-1]]^T \quad (13)$$

where $\overline{\mathbf{e}}[n]$ is a vector of size M (here, M is the affine projection order), $\overline{\mathbf{e}}_{M-1}[n-1]$ is a vector consisting of upper most M-1 elements of $\bar{\mathbf{e}}[n-1]$, and, μ is a step size $(0 \le \mu < 1)$. For the update of the adaptive filter coefficients, the nonlinear primary path can be estimated by using the LDL$^{\mathbf{T}}$ factorization with forward and backward substitutions. Utilization of the LDL$^{\mathbf{T}}$ factorization for the solution to (12) and its implementation procedure are is summarized in Table 1.

Table 1. Update equation for (12) by using the LDL$^{\mathbf{T}}$ – factorization.

$\mathbf{R}_v[n] = LDL^T$ $\mathbf{p}^{(1)}, \mathbf{p}^{(2)}, \mathbf{p}$ is a vector of size M
- Forward substitution to solve $L\mathbf{p}^{(1)} = \bar{\mathbf{e}}$, find $\mathbf{p}^{(1)}$ vector
$p_1^{(1)} = \frac{\bar{e}_1}{L_{11}}$ $p_i^{(1)} = \frac{1}{L_{ii}}\left[\bar{e}_i - \sum_{j=1}^{i-1} L_{ij} p_j^{(1)}\right]$
- Scaling to solve $D\mathbf{p}^{(2)} = \mathbf{p}^{(1)}$, find $\mathbf{p}^{(2)}$ vector
- Backward substitution to solve $L^T\mathbf{p} = \mathbf{p}^{(2)}$, find $\mathbf{p}$ vector
$p_M = \frac{p_M^{(2)}}{L^T_{MM}}$ $p_i = \frac{1}{L_{ii}^T}\left[p_i^{(2)} - \sum_{j=i+1}^{M} L_{ij}^T p_j\right]$

$$\boldsymbol{\eta}[n] = \mu\,\mathbf{p}[n] + [\mathbf{0}\ \bar{\boldsymbol{\eta}}[n-1]]^T \tag{14}$$

$$\mathbf{w}_v[n+1] = \mathbf{w}_v[n] - \mathbf{u}_v[n-M+1]\,\eta_{M-1}[n] \tag{15}$$

where $\boldsymbol{\eta}[n]$ is a vector consisting of a sum of the projected error vector $\mathbf{p}[n]$, and, $\eta_{M-1}[n]$ is a scalar value of the last element of the vector $\boldsymbol{\eta}[n]$. Furthermore, (15) is an update for the Volterra kernel vector. In the LDL$^{\mathbf{T}}$-FAP method, no assumption is made just as in the GS-FAP algorithm ($\bar{\mathbf{e}} \approx [e[n]\ 0 \ldots 0]^T$).

4. SIMULATION RESULTS

For the nonlinear ANC, the nonlinear Primary path (Tan and Jing, 2001) is modelled as the following third-order Volterra system:

$$d[n] = t[n] + 0.08 \cdot t^2[n] - 0.04 \cdot t^3[n] \tag{16}$$

$$t[n] = \mathbf{a}^T \mathbf{x}[n] \tag{17}$$

where $\mathbf{a}$ is the impulse response vector of the linear primary path described by 10 coefficients, and the secondary path is represented by 5 coefficients. Also, the noise cancellation performance of the proposed approach is compared with that of the conventional GS-FAP based on the following normalized mean square error (NMSE):

$$\text{NMSE} = 10\log_{10}\frac{E\{e^2[n]\}}{\sigma_d^2} \tag{18}$$

Fig. 3 and Fig. 4 show the NMSE performances of the proposed and conventional nonlinear ANC algorithms, respectively, in case of three different step sizes ($\mu = [0.01,\ 0.1,\ 1]$). In case of the unit step size, both approaches result in similar converging performance. When the step size is changed to 0.1, the convergence speed in case of the GS-FAP approach is slower than that of the proposed approach. Finally, when the step size is changed to 0.01, it can be seen that the convergence speed of the proposed LDL$^{\mathbf{T}}$-FAP-based approach are much faster than that of the conventional GS-FAP-based approach.

5. CONCLUSTION

In this paper, a nonlinear ANC method is proposed where the Volterra filter with the LDL$^{\mathbf{T}}$-FAP algorithm is utilized. Simulation results show that the proposed algorithm yields better convergence performance for nonlinear ANC than the conventional GS-FAP algorithm even in case of non-unity step sizes.

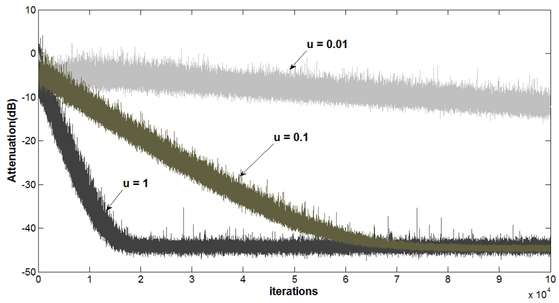

Fig. 3 Convergence curves by the conventional GS-FAP algorithm for nonlinear ANC.

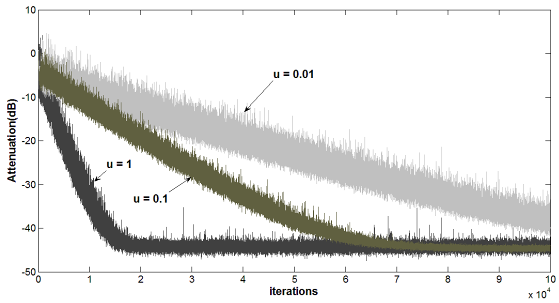

Fig. 4 Convergence curves by the proposed approach for the nonlinear ANC.

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