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Performance Limitations arising in the Control of Power Plants

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Abstract: This paper presents results on performance limitations for direct fired coal power plants. A specific feature of this system is the existence of a very large input delay between one of the inputs, namely coal flow, and the two outputs, load and vapour pressure. This problem motivates the main theoretical question addressed in this paper: To examine tracking performance limitations in one process variable when another process variable is constrained. Our main result makes explicit the performance trade-off between the two conflicting objectives, and also links the achievable performance to the delay structure of the plant. These results give insights into the benefits of MIMO control for power plants and into the necessary trade-off between fast tracking of load step changes and the need for minimizing the variations of the vapour pressure around its nominal value. The results provide a benchmark against which practical controller designs for power plants can be assessed.

Keywords: Power Plant Model, Limits of Performance, Multivariable systems, Delays

1. INTRODUCTION

In an increasingly competitive and distributed electricity production market, there is a growing interest in modern control methods for power plants (see Flynn (2003)). However, as recently pointed out by (Poncia (2003)), "the introduction of new concepts and technologies in thermal power plant is difficult". As mentioned by the author, this is partly a cultural obstacle, but it is also due to the fact that, over the years, classical control strategies and procedures have been so well tuned, often using ad-hoc techniques, that operators do not see the additional benefit that modern control technologies can offer.

Recent work from the academic world on this topic has focused on the introduction of multivariable control design for several control loops in power plants (Poncia (2003); Zhao et al. (1999); Poncia and Bittanti (2001); Mortensen et al. (1998)).

In the current contribution, we will focus on the highest control level of a coal-fired power plant, as considered, for example, in Zhao et al. (1999). In this setting, the plant is modelled as a two-input (coal flow and turbine inlet valve position) two-output (vapour pressure and load) system. Rather than focusing on a particular multivariable control design methodology, we will instead study the underlying performance limits associated with such methodologies. In this respect, an important challenge is the presence of a large delay associated with one of the two inputs, namely coal flow. As can be expected, this long delay introduces severe limits on the performance of any (not necessarily multivariable) controller design. Our purpose here is to make these limits explicit. Specifically, we consider a mode of operation of such a plant, similar to *boiler following* mode, where fast reaction to reference changes in load is sought by acting mainly on the turbine inlet valve, while maintaining vapour pressure variations within acceptable limits. In classical power plant operation, such a mode is based on two SISO controllers and is not used in direct fired coal power plants because of the large input delay. We consider here an associated *multivariable* solution and analyze its performance limits.

In order to solve this problem, we will build upon recent results on performance bounds for MIMO systems with arbitrary delay structure (see, e.g., Toker et al. (2002) and Silva and Salgado (2005)). We extend these results to cover the problem of interest here, namely when one has constraints on one output variable. We also refer the reader to Chen et al. (2003) for results that consider control energy penalization.

The remainder of this paper is organised as follows: Section 2 presents the multivariable model that motivates this study. Section 3 presents the main results, showing the performance trade-off between tracking step changes on one output and maintaining the second output around its nominal value. Of special interest here is the specific role of the delay structure in this trade-off. Section 4 presents simulations that illustrates our results. Conclusions are presented in Section 5.

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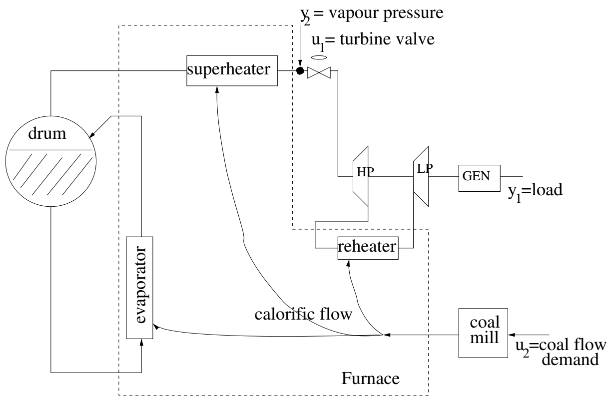

Fig. 1. Simplified scheme of a coal-fired power plant

2. COAL-FIRED POWER PLANTS

2.1 The plant model

This section introduces the model of a coal-fired power plant relevant to our current study. A simplified scheme of the plant is shown on Figure 1. We consider a high-level model of the plant, with coal flow and turbine valve opening as inputs, and load and vapour pressure as outputs. (This assumes that all other local control loops are in place). Similar to the work by Poncia (2003) and Zhao et al. (1999), a linear model has been obtained using black-box identification applied to input-output signals obtained from a nonlinear simulator of a 125 MW coal-fired power plant with natural circulation drum boiler. This model reflects the behaviour of the plant around the following operating point : load = 122 MW, vapour pressure = 136 bar (see Bodeux (2006) for details). The identified continuous time transfer matrix, assuming proper scaling, is given by

$$G_c(s) = \begin{bmatrix} \frac{271.8s-1.72}{3435s^2+244s+1} & \frac{0.2303\, e^{-100s}}{7040s^2+204s+1} \\ \\ \frac{-3.195}{5133s^2+206s+1} & \frac{0.02661\, e^{-100s}}{8052s^2+188s+1} \end{bmatrix}.$$

In this work we will use a discretized version of $G_c(s)$ using a 5 sec sampling period. Assuming a zero order hold at the plant inputs, the corresponding discrete time transfer function is

$$G(z) = \begin{bmatrix} \frac{0.327z-0.3376}{z^2-1.695z+0.7011} & \frac{10^{-4}(3.897z+3.714)}{z^{20}(z^2-1.862z+0.8651)} \\ \\ \frac{-0.007282z-0.006811}{z^2-1.814z+0.8182} & \frac{10^{-4}(3.974z+3.822)}{z^{20}(z^2-1.887z+0.8898)} \end{bmatrix}. \quad (1)$$

The structure of $G(z)$, i.e., its order, number of zeros and poles, delay structure, etc., is similar to that of the model used by Zhao et al. (1999) who dealt with a similar power plant, although of larger size. A special feature of this model is the very large delay on the second input which is typical of coal-fired power plants. This delay is a coarse approximation to the behaviour of the coal mill. (A much more detailed modelling of such coal mills can be found in Rees and Fan (2003)). However, the model used here is adequate to capture the key phenomena of interest in the current context. The value of 100 seconds for the delay is the best fit obtained for one set of experiments, but it is likely to be an underestimate of the actual delay.

Note also that there is a non-minimum phase (NMP) zero in the transfer function from turbine inlet valve to load, i.e., $G_{11}(z)$. [2] This is easily explained from a physical point of view. We note, however, that this zero is *not* a transmission zero of the MIMO transfer function $G(z)$. This is further motivation to consider a MIMO control architecture for the plant, and not just decentralised SISO controllers. Indeed, in the latter case the NMP zero of $G_{11}(z)$ may impose fundamental limitations that are not present in MIMO designs (see, e.g., Goodwin et al. (2005)).

2.2 Control strategy

Here we define the design objective. We want the plant to respond as quickly as possible to load reference variations, while maintaining the vapour pressure variations as small as possible. This is the objective pursued when controlling the power plant in *boiler following* mode. In this mode, the turbine inlet valve controls the load, while the coal flow controls the vapour pressure. This is classically a decentralized control strategy comprising two SISO controllers in parallel. This is commonly used in plants with fast boiler dynamics (e.g., oil or gas fired plants). Direct fired coal power plants, however, usually operate in *turbine following* mode (i.e., the load is controlled by the coal flow and the vapour pressure by the turbine inlet valve).This configuration is favored because the boiler following strategy is unable to maintain the vapour pressure within its bounds due to the large delay on the coal flow. As a result, direct fired coal power plants cannot participate in secondary grid frequency control (which requires the ability to rapidly follow large and frequent load reference variations without delay). This kind of plant may, however, participate in the primary grid frequency control. From a process control point of view, this is achieved through a continuous modulation of the produced load. This is obtained by an additional action on the turbine valve computed by a proportional controller: $\Delta u_1(t) = K\Delta f(t)$ where $\Delta f(t)$ is the instantaneous deviation of the electrical grid frequency from its nominal value and $\Delta u_1(t)$ is added to the value of $u_1(t)$ computed by the pressure controller. The coal flow is also corrected appropriately to limit the impact of this action on the vapour pressure. (Observe that the additional control actions are of the boiler following type). Yet, even in this case, direct fired coal power plants quickly reach their performance limits, because primary control requires an action of the form $\Delta y_1(t) = K\Delta f(t)$ where $\Delta y_1(t)$ is a correction of the produced load. This is equivalent to $\Delta u_1(t) = K\Delta f(t)$ only if the vapour pressure can be perfectly controlled by the coal flow, as there holds $y_1(t) \approx u_1(t)y_2(t)$. It seems clear from the above that the turbine valve can be used to produce more load by exploiting the capacity of the boiler as a steam (and, hence, energy) buffer. However, the new load level won't be maintained unless the vapour pressure fall is arrested by injecting more energy (and, hence, coal) into the system.

The question addressed in this paper is whether a MIMO control strategy could overcome the limitations that arise in decentralized SISO strategies, such as those described above. Formally, we aim at solving the following problem:

[2] for every matrix X, $[X]_{ij} \triangleq X_{ij}$ denotes its (i, j)-th component.

Problem 1. (Main Problem). Consider a MIMO one degree-of-freedom control loop for the two-input two-output model of coal-fired power plant described by $G(z)$ (see (1)). Under these conditions, specify the best achievable performance, as measured by the $\mathcal{L}_2$-norm of the load tracking error for load reference step changes, when there is a bound (say M) on the $\mathcal{L}_2$-norm of the vapour pressure variation.

3. PERFORMANCE BOUNDS

As foreshadowed in Section 2, our aim is to identify the minimal load tracking error norm, whilst satisfying a bound on the vapour pressure deviation norm, associated with a step change in the load reference signal. To give an answer to this question, we will first adopt a more general point of view. We will begin by studying the problem of *simultaneously* minimizing both load tracking error norm and vapour deviation norm. Towards that goal we define

$$J \triangleq \sum_{k=0}^{\infty} e_1(k)^2, \quad R \triangleq \sum_{k=0}^{\infty} e_2(k)^2, \tag{2}$$

where e_i is the i-th component of the tracking error when the reference is given by $r = \epsilon_1 \mu$.[3] J is the corresponding load tracking error norm and R is the corresponding vapour pressure variation norm. With the aid of Parseval's relation and the well known Youla parameterization of controllers for stable plants (see, e.g., Francis (1987)), both J and R can be written as a function of a free parameter $Q(z) \in \mathcal{RH}_\infty$.[4] Namely,

$$J(Q(z)) = \left\| \epsilon_1^T \frac{I - G(z)Q(z)}{z-1} \epsilon_1 \right\|_2^2, \tag{3}$$

$$R(Q(z)) = \left\| \epsilon_2^T \frac{I - G(z)Q(z)}{z-1} \epsilon_1 \right\|_2^2, \tag{4}$$

where $I - G(z)Q(z)$ is the loop sensitivity function (see, e.g., Goodwin et al. (2001)).

The problem of simultaneously minimizing J and R is a special case of so-called multiobjective optimization (see, e.g., Boyd and Vandenberghe (2004)). We define the set of achievable objectives as

$$\mathcal{A} \triangleq \{(\alpha_J, \alpha_R) \in \mathbb{R}^2 : J(Q(z)) \le \alpha_J \text{ and } R(Q(z)) \le \alpha_R, \text{ for some } Q(z) \in \mathcal{RH}_\infty\}. \tag{5}$$

Since both objectives are competing ones, $\mathcal{A}$ does not have a minimal element,[5] i.e., there exist no $Q^*(z)$ such that $J(Q^*(z)) \le J(Q(z))$ and, simultaneously, $R(Q^*(z)) \le R(Q(z))$ for every $Q(z) \in \mathcal{RH}_\infty$. We will thus focus on the points in $\mathcal{A}$ that achieve the best trade-off between both objectives, i.e., we will focus on the Pareto optimal points of $\mathcal{A}$ (see Boyd and Vandenberghe (2004)). The next theorem characterizes these points, as well as those in $\mathcal{A}$.

Theorem 2. (Characterization of achievable specifications). Consider the definition of J and R in (3)-(4) and define

$$L_\lambda(Q(z)) \triangleq J(Q(z)) + \lambda R(Q(z)), \tag{6}$$

$$Q_\lambda(z) \triangleq \arg \min_{Q(z) \in \mathcal{RH}_\infty} L_\lambda(Q(z)). \tag{7}$$

Then:

(1) The set of Pareto optimal points associated with $\mathcal{A}$ is given by

$$\mathcal{P} = \{(\alpha_J, \alpha_R) \in \mathbb{R}^2 : \alpha_J = J(Q_\lambda(z)), \alpha_R = R(Q_\lambda(z)), \text{ for some } \lambda \ge 0\}. \tag{8}$$

(2) The set $\mathcal{A}$ is given by

$$\mathcal{A} = \{(\alpha_J, \alpha_R) \in \mathbb{R}^2 : \alpha_J \ge J(Q_\lambda(z)), \alpha_R \ge R(Q_\lambda(z)), \text{ for some } \lambda \ge 0\}. \tag{9}$$

Proof. The proof follows from well known results and is omitted for the sake of brevity (see, e.g., Section 4.7 in Boyd and Vandenberghe (2004)). ■

Theorem 2 allows one to characterize the set of achievable specifications for load tracking error norm and vapour pressure variation norm, in terms of a set of convex optimization problems. These problems, namely finding $Q_\lambda(z)$ for every $\lambda \ge 0$, can be tackled using standard model matching techniques. To that end, we will denote the i-th row of $G(z)$ by $G_{i*}(z)$ and a left unitary interactor for $G_{i*}(z)$, having unity DC-gain, by $\xi_i(z)$ (see, e.g., Silva and Salgado (2005)). We also define

$$\tilde{G}_{i*}(z) \triangleq \xi_i(z) G_{i*}(z). \tag{10}$$

Lemma 3. (Characterization of $Q_\lambda(z)$). Consider the definition of J and R in (3)-(4) and assume that $G(z)$ is stable and has non singular DC-gain. Then:

(1) If $\lambda > 0$, then

$$Q_\lambda(z)\epsilon_1 = (\xi_\Lambda(z) G_\Lambda(z))^{-1} \Lambda \epsilon_1, \tag{11}$$

where

$$\Lambda \triangleq \operatorname{diag}\left\{1, \sqrt{\lambda}\right\}, \quad G_\Lambda(z) \triangleq \Lambda G(z) \tag{12}$$

and $\xi_\Lambda(z)$ is a left unitary interactor for $G_\Lambda(z)$ having unit DC-gain.

(2) If $\lambda = 0$, then

$$Q_0(z)\epsilon_1 = \left(G(1)^{-1} + \tilde{G}_{1*}(z)^\dagger A_1(z) + \frac{z-1}{z} B_1(z) X_1(z) \right) \epsilon_1. \tag{13}$$

(3) In the limit $\lambda \to \infty$,

$$Q_\infty(z)\epsilon_1 = \left(G(1)^{-1} + \tilde{G}_{2*}(z)^\dagger A_2(z) + \frac{z-1}{z} B_2(z) X_2(z) \right) \epsilon_1. \tag{14}$$

In (13) and (14), and for $i \in \{1, 2\}$, $\tilde{G}_{i*}(z)^\dagger \in \mathcal{RH}_\infty$ is a generalized right inverse of $\tilde{G}_{i*}(z)$,

$$A_i(z) \triangleq \epsilon_i^T - \tilde{G}_{i*}(z) G(1)^{-1}, \tag{15}$$

$$B_i(z) \triangleq I - \tilde{G}_{i*}(z)^\dagger \tilde{G}_{i*}(z), \tag{16}$$

and $X_i(z)$ is any transfer function in $\mathcal{RH}_\infty$.

Proof.

(1) Elementary properties of the 2-norm, and the fact that $\xi_\Lambda(z)$ is unitary and has unit DC-gain, allow one to write

[3] μ denotes a unit step function that changes at $k = 0$ and ϵ_i is the i-th component of the canonical basis in $\mathbb{R}^2$.
[4] $\mathcal{RH}_\infty$ is the set of all stable and proper rational transfer matrices.
[5] except in very special cases.

$$L_\lambda(Q(z)) = \left\| \frac{\Lambda - G_\Lambda(z)Q(z)}{z-1}\epsilon_1 \right\|_2^2 = \left\| \frac{\xi_\Lambda(z) - I}{z-1}\Lambda\epsilon_1 \right\|_2^2 + \left\| \frac{\Lambda - \xi_\Lambda(z)G_\Lambda(z)Q(z)}{z-1}\epsilon_1 \right\|_2^2. \quad (17)$$

Since $G(z)$ is assumed non singular and $\lambda \in (0,\infty)$, then $\xi_\Lambda(z)G_\Lambda(z)$ is non singular. Moreover, the properties of interactors guarantee that $(\xi_\Lambda(z)G_\Lambda(z))^{-1} \in \mathcal{RH}_\infty$. The result is now immediate. We note that if $\lambda \in \{0,\infty\}$, then $G_\Lambda(z)$ is singular and we cannot proceed as above.

(2) If $\lambda = 0$, then standard properties of the 2-norm and the fact that $\xi_1(z)$ is unitary, allow one to write L_λ as follows:

$$L_0(Q(z)) = J(Q(z)) = \left\| \left(\frac{\xi_1(z)\epsilon_1^T - \tilde{G}_{1*}(z)G^{-1}(1)}{z-1} - \frac{\tilde{G}_{1*}(z)\tilde{Q(z)}}{z} \right)\epsilon_1 \right\|_2^2 = \left\| \frac{\xi_1(z)-1}{z-1} \right\|_2^2 + \left\| \left(\frac{A_1(z)}{z-1} - \frac{\tilde{G}_{1*}(z)\tilde{Q}(z)}{z} \right)\epsilon_1 \right\|_2^2, \quad (18)$$

where $\tilde{Q}(z)$ is implicitly defined via [6]

$$Q(z) \triangleq G(1)^{-1} + \frac{z-1}{z}\tilde{Q}(z). \quad (19)$$

We note that $Q(z) \in \mathcal{RH}_\infty \Leftrightarrow \tilde{Q}(z) \in \mathcal{RH}_\infty$ and, by definition of $\xi_1(z)$, $\tilde{G}_{1*}(z)$ is right invertible in $\mathcal{RH}_\infty$. Using a standard generalized inverse result (see, e.g., Chapter 8, Section 6, in Ben-Israel and Greville (2003)), the result follows.

(3) When $\lambda \to \infty$, then minimizing L_λ amounts to minimizing R. Proceeding as above, we conclude that

$$R(Q(z)) = \left\| \left(\frac{A_2(z)}{z-1} - \frac{\tilde{G}_{2*}(z)\tilde{Q}(z)}{z} \right)\epsilon_1 \right\|_2^2, \quad (20)$$

where $\tilde{Q}(z)$ is defined as before. The result is then immediate. ■

Remark 1. Note that (11), (13) and (14) only define the first column of the corresponding Youla parameters. This is because of the problem formulation, where only step references in the first input are considered (see also (3) and (4) where it is becomes clear that only $Q(z)\epsilon_1$, i.e., the first column of $Q(z)$, plays a role in our problem). The same equations, when one drops the ϵ_1 factor, define (not necessarily unique) 2×2 Youla parameters that provide a solution to our problem. ▲

It is also illustrative to study the behaviour of $J(Q_\lambda(z))$. To that end we let m denote the relative degree of $G(z)$, i.e., the number of zeros at infinity, and m_{ij} denote the relative degree of $G_{ij}(z)$ ($(i,j) \in \{1,2\}\times\{1,2\}$).

Lemma 4. (Load tracking error norm as a function of λ). Consider the notation introduced in Lemma 3 and assume that $G(z)$ is stable, minimum phase (MP) and having a non singular DC-gain matrix. If, in addition, $m > m_{21}$, $m_{22} > m_{21}$ and $G_{21}(z)$ is MP, then:

(1) If $\lambda > 0$, then

$$J(Q_\lambda(z)) = \left\| \epsilon_1^T \frac{I - \Lambda^{-1}\xi_\Lambda(z)^{-1}\Lambda}{z-1}\epsilon_1 \right\|_2^2. \quad (21)$$

(2) If $\lambda = 0$, then

$$J(Q_0(z)) = m_{11} \quad (22)$$

(3) When $\lambda \to \infty$, then

$$J(Q_\infty(z)) = m - m_{21} + D(X_2(z)), \quad (23)$$

where $D : \mathcal{RH}_\infty \to \mathbb{R}$ is a function such that

$$\min_{X_2(z)\in\mathcal{RH}_\infty} D(X_2(z)) = 0. \quad (24)$$

Proof.

(1) Immediate from (11), (3) and the definition of $G_\Lambda(z)$.

(2) For $\lambda = 0$ the result follows immediately from the proof of Lemma 3 (see (18)) and Theorem 2 in Silva and Salgado (2005) (restricted to the SISO case).

(3) For $\lambda \to \infty$, we can proceed as follows: Since $m_{22} > m_{21}$ and $G_{21}(z)$ is MP, it follows that a right inverse for $\tilde{G}_{2*}(z)$ is given by

$$\tilde{G}_{2*}(z)^\dagger = \begin{bmatrix} (\xi_2(z)G_{21}(z))^{-1} \\ 0 \end{bmatrix} \in \mathcal{RH}_\infty. \quad (25)$$

From Lemma 3 and Theorem 2 in Silva and Salgado (2005), we have that choosing $\tilde{G}_{2*}(z)^\dagger$ as above yields (after some standard manipulations)

$$J(Q_\infty(z)) = m - m_{21} + D(X_2(z)), \quad (26)$$

where

$$D(X_2(z)) \triangleq \left\| \frac{1 - \frac{z^{m-m_{21}}\det G(z)}{G_{21}(z)}\left(\frac{G_{21}(1)}{\det G(1)} + \frac{z-1}{z}\left[X_2(z)\right]_{21}\right)}{z-1} \right\|_2^2.$$

Since $G(z)$ is MP and stable, so is $\det G(z)$. This, jointly with $m > m_{21}$, implies that the parameter $X_2(z) \in \mathcal{RH}_\infty$ that minimizes D, say $X_2^{opt}(z)$, is such that

$$\left[X_2^{opt}(z)\right]_{21} = \frac{z}{z-1}\left(\frac{G_{21}(z)}{z^{m-m_{21}}\det G(z)} - \frac{G_{21}(1)}{\det G(1)} \right),$$

and, moreover, $D(X_2^{opt}(z)) = 0$ as claimed. ■

Lemma 4 shows that by means of varying λ, one can attain a load tracking error norm that ranges from the relative degree of $G_{11}(z)$ (when $\lambda \to 0$) to the difference between the relative degree of the MIMO model and that of $G_{21}(z)$ (when $\lambda \to \infty$). If, as is the case for the coal fired power plant described in Section 2, the plant has large relative degree (i.e., large delays) concentrated in the second row, then it may be possible to achieve good performance by means of choosing a small λ. Of course, this comes at the expense of large error norm on the second output.

We are now in a position to give a solution to Problem 1. For this purpose, we define the minimal load tracking error norm that is achievable when the vapour pressure variation norm is no greater than M via

$$J_{opt} \triangleq \min_{\substack{Q(z)\in\mathcal{RH}_\infty \\ R(Q(z))\le M}} J(Q(z)), \quad (27)$$

[6] This guarantees that $Q(z)$ defines a controller with integral action, which is needed to be able to have a finite cost J. This constraint was automatically satisfied in Part 1.

and the associated optimal Youla parameter by

$$Q_{opt}(z) \triangleq \arg \min_{\substack{Q(z)\in\mathcal{RH}_\infty \\ R(Q(z))\leq M}} J(Q(z)). \quad (28)$$

Theorem 5. (Solution to Problem 1). Consider the notation and assumptions in Lemmas 3 and 4 and define

$$R_0 \triangleq \min_{X_1(z)\in\mathcal{RH}_\infty} R(Q_0(z)), \quad (29)$$

$$R_\infty \triangleq R(Q_\infty(z)). \quad (30)$$

Then:

(1) If $M \in (R_\infty, R_0)$, then the optimal Youla parameter $Q_{opt}(z)$ is given by

$$Q_{opt}(z) = \left(\xi_{\Lambda_o}(z) G_{\Lambda_o}(z)\right)^{-1} \Lambda_o, \quad (31)$$

and the corresponding minimal cost satisfies

$$J_{opt} = \left\|\epsilon_1^T \frac{I - \Lambda_o^{-1}\xi_{\Lambda_o}(z)^{-1}\Lambda_o}{z-1}\epsilon_1\right\|_2^2, \quad (32)$$

where Λ_o and $\xi_{\Lambda_o}(z)$ are defined as Λ and $\xi_\Lambda(z)$, considering $\lambda = \lambda_o$, and λ_o is the (unique) positive real that satisfies

$$\left\|\epsilon_2^T \frac{I - \Lambda_o^{-1}\xi_{\Lambda_o}(z)^{-1}\Lambda_o}{z-1}\epsilon_1\right\|_2^2 = M. \quad (33)$$

(2) If $M \geq R_0$, then $J_{opt} = m_{11}$ and $Q_{opt}(z) = Q_0(z)$, with $X_1(z) = \arg\min_{X_1(z)\in\mathcal{RH}_\infty} R(Q_0(z))$.

(3) If $M = R_\infty$, then $J_{opt} = m - m_{21}$ and $Q_{opt} = Q_\infty(z)$, with $X_2(z) = \arg\min_{X_2(z)\in\mathcal{RH}_\infty} D(X_2(z))$.

(4) If $M < R_\infty$, then Problem 1 is infeasible.

Proof.

(1) This result is a consequence of the definition of $Q_\lambda(z)$, Pareto optimality and the convexity of the involved functionals.

(2) Immediate from the definition of R_0 and Part 2 of Lemmas 3 and 4.

(3) Immediate from the definition of R_∞ and Part 3 of Lemmas 3 and 4.

(4) Immediate from the definition of R_∞. ■

Theorem 5 characterizes the minimal load tracking error norm as a function of the maximum allowable vapour pressure variation norm, M. Except for special cases, this characterization is made in terms of a single scalar parameter, λ. This parameter can be found by means of a simple line search. Implications of these results on the control of coal-fired power plants are investigated in Section 4.

4. SIMULATION RESULTS

In this section, we shall present simulations illustrating the results of this paper. We consider the model of a power plant as given in Section 2. Fig. 2 shows the set of achievable objectives, $\mathcal{A}$, calculated using Theorem 2 and Lemma 3.

Fig. 2 allows one to better understand the trade-off between vapour pressure variation and load tracking performance. Put differently, if a precise limit on the energy of the regulation error of the vapour pressure can be set, one can obtain from Fig. 2 the optimal tracking performance of the load, when a step reference change occurs in the corresponding reference. For any given M, this optimal performance can be obtained with the controllers parameterized as in Lemma 3 and Theorem 5.

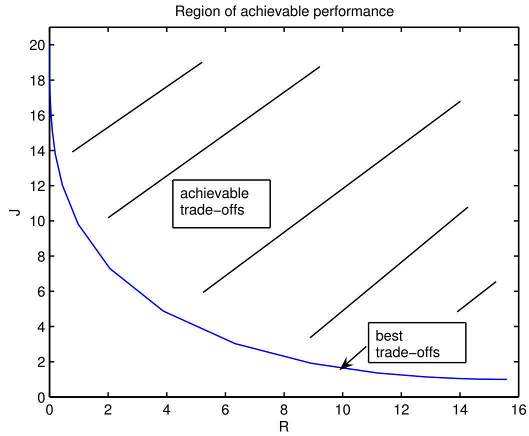

Fig. 2. Power Plant Model : The achievable performance.

As an illustration of the different performance that can be obtained depending on the value of M (and hence of λ in (6)), consider Figure 3. Five different simulations are presented, with $\lambda \in \{10^{-3}, 0.5, 1, 5, 10\}$. (The corresponding values of M are indicated on the figure in the same order.) As shown in Lemma 4, the results for lower values for λ (or larger values for M) achieve faster tracking of the load reference step change at the price of large variations in the vapour pressure. If the latter need to be minimised, then, inevitably, tracking of the load reference step changes can only be achieved by using the second input (coal flow) and here the large model delay comes into play, as predicted by our results.

Note that the controllers obtained by using the results of Lemma 3 are by no means optimised with respect to robustness issues and, as such, are not appropriate for real implementation. They are also constructed based on a discrete time model of the plant and a discrete time formulation of Problem 1. Intersample issues are also of importance in practice and this may be studied by appropriate extensions of recent results on limits of performance for sampled-data systems (see, e.g., Chen et al. (2002)).

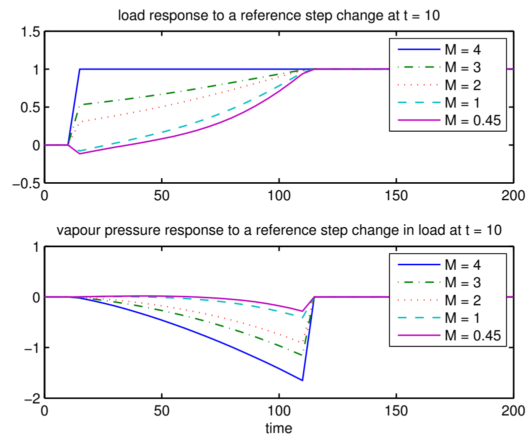

Fig. 3. Power Plant Model : Simulations with various controllers.

(This is the subject of on-going research). Hence, the results presented here are useful as a benchmark with respect to which robust controllers can be assessed.

It is also interesting to study the different terms of the cost function, as a function of the weighting parameter λ (see Figure 4). In that figure, notice, in particular, that the lowest value of the load tracking error norm, $J = 1$, corresponds to λ tending towards zero. The associated cost is equal to the relative degree of $G_{11}(z)$. The maximum value of the cost, $J = 21$, corresponds to λ tending towards ∞, and is equal to $22 - 1$, i.e., the total number of zeros at infinity of the plant minus the number of zeros at infinity of $G_{21}(z)$ (see Lemma 4 and Theorem 5).

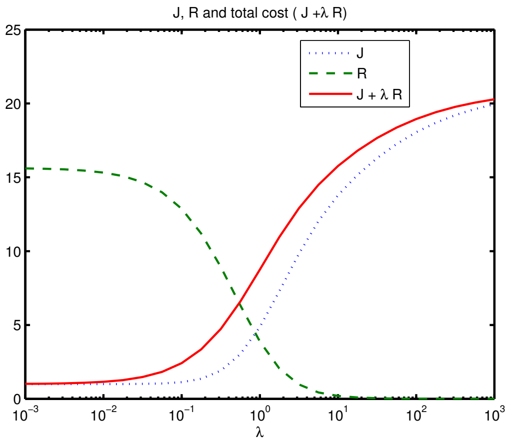

Fig. 4. Power Plant Model : The different cost terms.

5. CONCLUSIONS

This paper has been motivated by a specific application, namely multivariable high-level control of coal fired power plants. We have studied tracking performance limitations arising in one output of two-input two-output stable and minimum phase multivariable discrete time systems, when the second output tracking error norm is constrained. Our results characterise the region of achievable performance, establish limiting values for the optimal tracking cost and propose explicit characterizations of the optimal controllers, given a bound on the norm of the second output variations. A key feature of the results is that, for the class of stable plant models considered here, the values of the tracking cost can be directly related to the delay structure of the plant.

The results presented here form a benchmark against which practical controllers can be assessed. Further work could consider practical designs, robustness issues and intersample behaviour.

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