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Decentralized and Robust Target Tracking with Sensor Networks

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Abstract: In this paper we address the problem of decentralized and robust linear filtering for target tracking using networks of (radar) sensors taking nonlinear range and bearing measurements. The algorithm introduced in this paper permits efficient data fusion from multiple sensors through a summation style fusion architecture. Moreover, we prove that the state estimation error for the linear filtering algorithm is bounded.

1. INTRODUCTION

The problem of decentralized target tracking using a (radar) sensor network and robust linear filtering is examined in this paper. Radar based tracking with range and bearing measurements typically involves linear dynamic models in the Cartesian coordinates and nonlinear measurement models; see Li and Jilkov, 2001). Thus, nonlinear filters such as the extended Kalman filter (EKF) are often used; see Li and Jilkov (2001). Alternatively, measurement conversion methods have been explored for tracking problems; see Lerro and Bar-Shalom (1993); Li and Jilkov (2001); Schlosser (2004); Zhao et al. (2004).

The idea of the measurement conversion methods is to transform nonlinear measurements into a linear combination of the Cartesian coordinates, estimate the bias and covariance of the converted measurement noise, and then use the standard Kalman filter; see Zhao et al. (2004). This technique has been shown to outperform the EKF in general; e.g. see Lerro and Bar-Shalom (1993); Schlosser (2004); Li and Jilkov (2001); Zhao et al. (2004). One shortcoming of the EKF and the measurement conversion methods is the lack of a rigorous proof on the boundedness of the estimation errors. It is also well-known that the estimate produced by the EKF may diverge from the true state in practice; see e.g. Petersen and Savkin (1999).

The contributions of this paper include the development of a decentralized and robust linear estimator for target tracking using nonlinear radar measurements. This filter is designed using ideas and methods from modern robust state estimation theory; see e.g. Petersen and Savkin (1999); Savkin and Petersen (1998, 1996); Savkin and Evans (2002); Matveev and Savkin (2008); Bishop et al. (2007b). The robust algorithm designed here permits efficient data fusion through a simple summation fusion structure; see Rao and Durrant-Whyte (1993b,a); Spanos et al. (2005). The algorithm complexity is scalable with the number of sensors.

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In Section 2 we discuss the typical linear target dynamic model and in Section 3 we introduce the notation related to the radar sensor network. In Section 4 we derive a decentralized robust linear filter for target tracking with converted radar sensor measurements. Furthermore, in Section 4 we discuss the robust filter architecture and the data fusion protocol. In Section 5 we give some illustrative examples and in Section 6 we give our concluding remarks.

2. TARGET DYNAMIC MODELS

The target state is represented by the vector $\mathbf{x} \in \mathbb{R}^n$. Traditionally, the dynamics of a moving target are usually modeled in Cartesian coordinates. Typically, point targets are considered and the models are linear; see Li and Jilkov. For a maneuverable target, the state of a target in 2D is given by $\mathbf{x} = [x_1\ x_2\ x_3\ x_4]' \in \mathbb{R}^4$ (or $\mathbf{x} = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6]' \in \mathbb{R}^6$ depending on the particular dynamic model), where x_1 and x_2 denote the x and y directions respectively. That is, in the external global coordinate system, x_1 is the x-component of the target's position and x_2 is the y-component of the target's position. In addition, $x_3 = \dot{x}_1$, $x_4 = \dot{x}_2$, $x_5 = \ddot{x}_1$ and $x_6 = \ddot{x}_2$. In 3D, the target's state vector takes a similar form, $\mathbf{x} = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9]' \in \mathbb{R}^9$, where x_1, x_2 and x_3 denote the x, y and z directions respectively. Also, $x_4 = \dot{x}_1$, $x_5 = \dot{x}_2$, $x_6 = \dot{x}_3$, $x_7 = \ddot{x}_1$, $x_8 = \ddot{x}_2$ and $x_9 = \ddot{x}_3$. The state-space model in discrete time is given by

$$\mathbf{x}(k) = \mathbf{A}\mathbf{x}(k-1) + \mathbf{B}\mathbf{w}(k) \tag{1}$$

where the matrices $\mathbf{A}$ and $\mathbf{B}$ depend on the specific dynamic model employed. The noise input $\mathbf{w}(k)$ can represent the uncertain acceleration input of the target. The structure of $\mathbf{w}(k)$ and consequently $\mathbf{B}$, should be defined specifically for a given problem, depending on the likely nature of the target maneuvers and system model uncertainties. The input $\mathbf{w}(k)$ is usually assumed to be a white Gaussian distributed random process. However, in this paper and for the filter developed in this paper, the noise input $\mathbf{w}(k)$ can be represented by any bounded function of time. In fact, $\mathbf{w}(k)$ can be probabilistically bounded and Gaussian processes can be considered special cases.

3. RADAR NETWORK CONVENTIONS

Consider a network of N stationary sensors (e.g. radar sensors) positioned in 2D or 3D and communicating over a complete graph. That is, a network of sensors with an undirected communication topology $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where $\mathcal{V} = \{1, \ldots, N\}$ represents the graph vertices, i.e. the sensors, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of inter-sensor communication links. Each sensor i communicates with a set of neighbors $\mathcal{N}_i \subseteq \mathcal{V}$ and $j \in \mathcal{N}_i \Leftrightarrow i \in \mathcal{N}_j$, $\forall i, j \in \mathcal{V}$. For notational simplicity it is always assumed that $i \in \mathcal{N}_i$. If $\mathcal{N}_i = \mathcal{V}$ then the communication graph is complete.

Now consider the tracking problem in 2D, then the position of the i^{th} sensor is given by $\mathbf{s}_i = [s_{i1}\ s_{i2}]'$ where s_{i1} and s_{i2} denote the traditionally denoted x and y positions of sensor i. This means, in the external global coordinate system, s_{i1} is the x-component of the i^{th} sensor's position and s_{i2} is the y-component of the i^{th} sensor's position. Consider the tracking problem in 3D, then the position of the i^{th} sensor is given by $\mathbf{s}_i = [s_{i1}\ s_{i2}\ s_{i3}]'$ where s_{i1}, s_{i2} and s_{i3} denote the traditionally denoted x, y and z positions of i^{th} sensor. Each sensor i knows its own position $\mathbf{s}_i$ in the global coordinate system.

4. ROBUST AND DECENTRALIZED TRACKING

4.1 Tracking in 2D

A typical radar sensor can measure the range r and azimuth angle ϕ to a target in 2D. At the i^{th} sensor, these parameters are related to the target's Cartesian position coordinates via the following nonlinear equations,

$$\widehat{r}_i = r_i + v_{i1} = \sqrt{(x_1 - s_{i1})^2 + (x_2 - s_{i2})^2} + v_{i1} \quad (2)$$

$$\widehat{\phi}_i = \phi_i + v_{i2} = \arctan\left(\frac{x_2 - s_{i2}}{x_1 - s_{i1}}\right) + v_{i2} \quad (3)$$

where v_{ij} is an additive error term to be described subsequently. Assume that the target motion is described by (1) where the matrix $\mathbf{A}$ is non-singular. Define the following analytical transformations

$$\widehat{x}_{i1}(k) = \widehat{r}_i(k)\cos(\widehat{\phi}_i(k)) \quad (4)$$

$$\widehat{x}_{i2}(k) = \widehat{r}_i(k)\sin(\widehat{\phi}_i(k)) \quad (5)$$

where $\widehat{x}_{i1}$ and $\widehat{x}_{i2}$ are the so-called converted measurements for x_1 and x_2 respectively and measured by the i^{th} sensor. Note that $\widehat{x}_{i1}$ and $\widehat{x}_{i2}$ are relative measurements and essentially *place* sensor i at the origin of the global coordinate system. Therefore, it is possible to define the following stacked and relative measurement vector

$$\mathbf{y}_i(k) = [\widehat{x}_{i1}(k)\ \widehat{x}_{i2}(k)]' \quad (6)$$

along with the following absolute measurement vector

$$\mathbf{m}_i(k) = \mathbf{y}_i(k) + \mathbf{s}_i \quad (7)$$

where again $\mathbf{s}_i$ is the position of the i^{th} sensor which is known only at the i^{th} sensor. Let $0 < p_0 \leq 1$ be a given constant. It is presumed that the system initial condition $\mathbf{x}(0)$, noise $\mathbf{w}(k)$ and the measurement noises $v_{i1}(k)$ and $v_{i2}(k)$ satisfy the following assumption.

Assumption 1. The following inequalities with probability p_0 simultaneously hold:

$$|v_{i1}(k)| \leq \alpha_{i1} r_i(k) \quad \text{and} \quad |v_{i2}(k)| \leq \alpha_{i2} \quad (8)$$

$$(\mathbf{x}(0) - \mathbf{x}_0)'\mathbf{N}(\mathbf{x}(0) - \mathbf{x}_0) + \sum_{k=0}^{T-1} \mathbf{w}(k)'\mathbf{Q}(k)\mathbf{w}(k) \leq d \quad (9)$$

Here $0 \leq \alpha_{i1} < 1$ and $0 \leq \alpha_{i2} < \frac{\pi}{2}$ are given constants, $\mathbf{x}_0$ is an initial state estimate, $\mathbf{N} = \mathbf{N}' > 0$ and $\mathbf{Q} = \mathbf{Q}' > 0$ are given weighting matrices, $d > 0$ is a given constant associated with the system and $T > 0$ is a given time. The matrices $\mathbf{A}$ and $\mathbf{B}$ and the initial estimate $\mathbf{x}_0$, $\mathbf{N}$ and $\mathbf{Q}$ are known at every sensor $i \in \{1, \ldots, N\}$ and are consistent.

Notice that the inequality (9) is a sum-quadratic constraint that is common in modern robust filtering theory; see e.g. Petersen and Savkin (1999).

Introduce the following Riccati difference equations

$$\mathbf{F}_i(k+1) = \hat{\mathbf{B}}\left[\hat{\mathbf{B}}'\mathbf{S}_i(k)\hat{\mathbf{B}} + \mathbf{Q}(k)\right]^{-1}\hat{\mathbf{B}}'\mathbf{S}_i(k)\hat{\mathbf{A}} \quad (10)$$

$$\mathbf{S}_i(k+1) = \hat{\mathbf{A}}'\mathbf{S}_i(k)\left[\hat{\mathbf{A}} - \mathbf{F}_i(k+1)\right] + \sum_{i\in\mathcal{N}_i} \mathbf{C}_i'\mathbf{C}_i - \sum_{i\in\mathcal{N}_i} \mathbf{K}_i'\mathbf{K}_i \quad (11)$$

$$\mathbf{S}_i(0) = \mathbf{N}$$

where $\hat{\mathbf{A}} \triangleq \mathbf{A}^{-1}$ and $\hat{\mathbf{B}}(k) \triangleq \mathbf{A}^{-1}\mathbf{B}$ and

$$\mathbf{C}_i \triangleq \begin{bmatrix} \beta_{i1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_{i1} & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{K}_i \triangleq \begin{bmatrix} \bar{\alpha_{i1}} & \bar{\alpha_{i2}} & 0 & 0 & 0 & 0 \\ \bar{\alpha_{i2}} & \bar{\alpha_{i1}} & 0 & 0 & 0 & 0 \end{bmatrix} \quad (12)$$

Then define the following parameters

$$\beta_{i1} \triangleq \frac{1}{2}\left(1 + \alpha_{i1} + (1 - \alpha_{i1})\cos(\alpha_{i2})\right) \quad (13)$$

$$\bar{\alpha_{i1}} \triangleq \frac{1}{2}\left((1 - \alpha_{i1})\cos(\alpha_{i2}) - (1 + \alpha_{i1})\right) \quad (14)$$

$$\bar{\alpha_{i2}} \triangleq -(1 + \alpha_{i1})\sin(\alpha_{i2}) \quad (15)$$

Now introduce the following set of state equations

$$\boldsymbol{\eta}_i(k+1) = \left[\hat{\mathbf{A}} - \mathbf{F}_i(k+1)\right]'\boldsymbol{\eta}_i(k) + \sum_{i\in\mathcal{N}_i} \mathbf{C}_i\mathbf{y}_i(k+1) + \sum_{i\in\mathcal{N}_i} [\mathbf{s}_i'\ 0\ \ldots\ 0]' \quad (16)$$

$$\boldsymbol{\eta}(0) = \mathbf{N}\mathbf{x}_0$$

where $\mathbf{m}_i(k)$ is the absolute measurement vector compiled at (and by) the i^{th} sensor and

$$g_i(k+1) = g_i(k) + \sum_{i\in\mathcal{N}_i} \mathbf{m}_i(k+1)'\mathbf{m}_i(k+1) - \boldsymbol{\eta}_i(k)'\hat{\mathbf{B}}\left[\hat{\mathbf{B}}'\mathbf{S}_i(k)\hat{\mathbf{B}} + \mathbf{Q}\right]^{-1}\hat{\mathbf{B}}'\boldsymbol{\eta}_i(k) \quad (17)$$

$$g_i(0) = \mathbf{x}_0'\mathbf{N}\mathbf{x}_0$$

The introduced state equations (16) and Riccati equations (10) lead to a robust implementation of a Kalman filter-like state estimator which is designed for uncertainties obeying Assumption 1; e.g. see Petersen and Savkin (1999).

Theorem 1. Let $0 < p_0 \leq 1$ be given, and suppose that Assumption 1 holds. Then the state $\mathbf{x}(T)$ of the system (1) with probability p_0 belongs to the ellipsoid

$$\mathcal{X}_T^i \triangleq \left\{ \begin{array}{l} \mathbf{x}_T \in \mathbb{R}^n : \|(\mathbf{S}_i(T)^{\frac{1}{2}}\mathbf{x}_T - \mathbf{S}_i(T)^{-\frac{1}{2}}\boldsymbol{\eta}_i(T))\|^2 \\ \leq \rho + d \end{array} \right\} \tag{18}$$

where $\rho_i \triangleq \boldsymbol{\eta}_i(T)'\mathbf{S}_i(T)^{-1}\boldsymbol{\eta}_i(T) - g_i(T)$ and $\boldsymbol{\eta}_i(T)$ and $g_i(T)$ are defined by (16) and (17).

Proof. It follows from (6) and (8) that

$$\widehat{x}_{i1}(k) = \beta_{i1}x_1(k) + n_{i1}(k) \tag{19}$$

$$\widehat{x}_{i2}(k) = \beta_{i2}x_2(k) + n_{i2}(k) \tag{20}$$

where $x_i(k)$ is the i^{th} component of the state vector $\mathbf{x}(k)$ of the system (1) and the inequalities

$$|n_{i1}(k)| \leq \bar{\alpha_{i1}}|x_1(k)| + \bar{\alpha_{i2}}|x_2(k)| \tag{21}$$

$$|n_{i2}(k)| \leq \bar{\alpha_{i1}}|x_2(k)| + \bar{\alpha_{i2}}|x_1(k)| \tag{22}$$

hold together with (9) with probability p_0. Therefore, this immediately implies that

$$\mathbf{m}_i(k) = \mathbf{C}_i\mathbf{x}(k) + \mathbf{s}_i + \mathbf{n}_i(k) = \mathbf{y}_i(k) + \mathbf{s}_i \tag{23}$$

where $\mathbf{y}_i(k) = \mathbf{C}_i\mathbf{x}(k) + \mathbf{n}_i$ and $\mathbf{n}_i(k) = [n_{i1}(k)\ n_{i2}(k)]'$ and the condition

$$\|\mathbf{n}_i(k)\|^2 \leq \|\mathbf{K}_i\mathbf{x}(k)\|^2 \tag{24}$$

holds together with (9) with probability p_0. Note that $\mathbf{z}_i(k) = \mathbf{K}_i\mathbf{x}(k)$ is the so-called uncertainty output considering only sensor i while $\mathbf{z}(k) = \mathbf{K}\mathbf{x}(k)$ is the entire decentralized system's uncertainty output. The measurement noise $\mathbf{n}_i(k)$ depends dynamically on the system uncertainty output $\mathbf{z}_i(k)$. Thus, the measurement noise exhibits parametric-like uncertainties which are not accounted for by the traditional measurement conversion based tracking algorithms. From (9) and (24) it follows that the subsequent sum quadratic constraint is satisfied

$$\begin{aligned} &(\mathbf{x}(0) - \mathbf{x}_0)'\mathbf{N}(\mathbf{x}(0) - \mathbf{x}_0) + \\ &\sum_0^{T-1} \left(\mathbf{w}(k)'\mathbf{Q}(k)\mathbf{w}(k) + \sum_{i \in \mathcal{N}_i} \|\mathbf{n}_i(k+1)\|^2 \right) \\ &\leq d + \sum_0^{T-1} \sum_{i \in \mathcal{N}_i} \|\mathbf{K}_i\mathbf{x}(k+1)\|^2 \end{aligned} \tag{25}$$

with probability p_0. Now it follows from Theorem 5.3.1 of Petersen and Savkin (1999), see also Savkin and Petersen (1998), that the state $\mathbf{x}(T)$ of the system (1), (23) belongs to the ellipsoid (18) with probability p_0. □

The noise input $\mathbf{w}(k)$ might also depend dynamically on $\mathbf{z}(k)$ if the system is assumed to exhibit parametric-like uncertainties.

Corollary 1. A so-called point value state estimate can be obtained from the bounded ellipsoidal set's center and is given by $\widehat{\mathbf{q}}_i(k) = \mathbf{S}_i(k)^{-1}\boldsymbol{\eta}_i(k)$.

The set-valued estimate $\mathcal{X}_T^i$ for any T and $\forall i$, illustrates the boundedness of the estimation error. It leads to a measure of state estimate uncertainty. For online tracking applications, the point-valued estimate given by $\widehat{\mathbf{q}}_i(T) = \mathbf{S}_i(T)^{-1}\boldsymbol{\eta}_i(T)$, $\forall i \in \mathcal{V}$ is useful in decision-making.

4.2 Tracking in 3D

A typical radar sensor can measure the range r, azimuth bearing ϕ and target elevation θ in 3D. At the i^{th} sensor, these parameters are related to the target's Cartesian position coordinates via the following transformations,

$$\widehat{r}_i = r_i + v_{i1} = \|[x_1\ x_2\ x_3]' - \mathbf{s}_i\| + v_{i1} \tag{26}$$

$$\widehat{\phi}_i = \phi_i + v_{i2} = \arctan\left(\frac{x_2 - s_{i2}}{x_1 - s_{i1}}\right) + v_{i2} \tag{27}$$

$$\widehat{\theta}_i = \theta_i + v_{i3} = \operatorname{asin}\left(\frac{x_3 - s_{i3}}{\|[x_1\ x_2\ x_3]' - \mathbf{s}_i\|}\right) + v_{i3} \tag{28}$$

where v_{ij} is an additive error term to be described. Assume that the target motion is described by (1) where the matrix A is non-singular. Now define the following analytical transformations

$$\widehat{x}_{i1}(k) = \widehat{r}_i(k)\cos(\widehat{\phi}_i(k))\cos(\widehat{\theta}_i(k)) \tag{29}$$

$$\widehat{x}_{i2}(k) = \widehat{r}_i(k)\sin(\widehat{\phi}_i(k))\cos(\widehat{\theta}_i(k)) \tag{30}$$

$$\widehat{x}_{i3}(k) = \widehat{r}_i(k)\sin(\widehat{\theta}_i(k)) \tag{31}$$

where $\widehat{x}_{i1}$, $\widehat{x}_{i2}$ and $\widehat{x}_{i3}(k)$ are the converted measurements for x_1, x_2 and x_3 respectively and measured by the i^{th} sensor. Again, it is important to note that $\widehat{x}_{i1}$, $\widehat{x}_{i2}$ and $\widehat{x}_{i3}(k)$ are relative measurements. Define the following stacked and relative measurement vector

$$\mathbf{y}_i(k) = [\widehat{x}_{i1}(k)\ \widehat{x}_{i2}(k)\ \widehat{x}_{i3}(k)]' \tag{32}$$

along with the following absolute measurement vector

$$\mathbf{m}_i(k) = \mathbf{y}_i(k) + \mathbf{s}_i \tag{33}$$

where here $\mathbf{s}_i$ is the position of the i^{th} sensor in 3D and $\mathbf{s}_i$ is known only at the i^{th} sensor. Let $0 < p_0 \leq 1$ be a given constant and suppose the following assumption holds.

Assumption 2. The following inequalities with probability p_0 simultaneously hold:

$$|v_{i1}(k)| \leq \alpha_{i1}r_i(k) \quad |v_{i2}(k)| \leq \alpha_{i2} \quad |v_{i3}(k)| \leq \alpha_{i3} \tag{34}$$

$$(\mathbf{x}(0) - \mathbf{x}_0)'\mathbf{N}(\mathbf{x}(0) - \mathbf{x}_0) + \sum_{k=0}^{T-1} \mathbf{w}(k)'\mathbf{Q}(k)\mathbf{w}(k) \leq d \tag{35}$$

Here $0 \leq \alpha_{i1} < 1$ and $0 \leq \alpha_{i2}, \alpha_{i3} < \frac{\pi}{2}$ are given constants, $\mathbf{x}_0$ is an initial state estimate, $\mathbf{N} = \mathbf{N}' > 0$ and $\mathbf{Q} = \mathbf{Q}' > 0$ are given weighting matrices, $d > 0$ is a given constant associated with the system and $T > 0$ is a given time. The matrices $\mathbf{A}$ and $\mathbf{B}$ and the estimate $\mathbf{x}_0$, $\mathbf{N}$ and $\mathbf{Q}$ are known at every sensor $i \in \mathcal{V}$ and are consistent.

Once again, introduce the the following Riccati difference equations

$$\mathbf{F}_i(k+1) = \hat{\mathbf{B}}\left[\hat{\mathbf{B}}'\mathbf{S}_i(k)\hat{\mathbf{B}} + \mathbf{Q}(k)\right]^{-1}\hat{\mathbf{B}}'\mathbf{S}_i(k)\hat{\mathbf{A}} \tag{36}$$

$$\begin{aligned} \mathbf{S}_i(k+1) &= \hat{\mathbf{A}}'\mathbf{S}_i(k)\left[\hat{\mathbf{A}} - \mathbf{F}_i(k+1)\right] + \\ &\sum_{i \in \mathcal{N}_i} \mathbf{C}_i'\mathbf{C}_i - \sum_{i \in \mathcal{N}_i} \mathbf{K}_i'\mathbf{K}_i \end{aligned} \tag{37}$$

$$\mathbf{S}_i(0) = \mathbf{N}$$

where $\hat{\mathbf{A}} \triangleq \mathbf{A}^{-1}$ and $\hat{\mathbf{B}}(k) \triangleq \mathbf{A}^{-1}\mathbf{B}$ and,

$$\mathbf{C}_i \triangleq \begin{bmatrix} \beta_{i1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_{i1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_{i2} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (38)$$

$$\mathbf{K}_i \triangleq \begin{bmatrix} \bar{\alpha}_{i1} & \bar{\alpha}_{i2} & \bar{\alpha}_{i3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{\alpha}_{i2} & \bar{\alpha}_{i1} & \bar{\alpha}_{i3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{\alpha}_{i2} & \bar{\alpha}_{i2} & \bar{\alpha}_{i4} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (39)$$

Then define the following parameters

$$\beta_{i1} \triangleq \frac{1}{2}\left(1 + \alpha_{i1} + (1 - \alpha_{i1})\cos(\alpha_{i2})\cos(\alpha_{i3})\right) \quad (40)$$

$$\beta_{i2} \triangleq \frac{1}{2}\left(1 + \alpha_{i1} + (1 - \alpha_{i1})\cos(\alpha_{i3})\right) \quad (41)$$

and

$$\bar{\alpha}_{i1} \triangleq \frac{1}{2}\left((1 - \alpha_{i1})\cos(\alpha_{i2})\cos(\alpha_{i3}) - (1 + \alpha_{i1})\right) \quad (42)$$

$$\bar{\alpha}_{i2} \triangleq -(1 + \alpha_{i1})\sin(\alpha_{i2}) \quad (43)$$

$$\bar{\alpha}_{i3} \triangleq -(1 + \alpha_{i1})\sin(\alpha_{i3}) \quad (44)$$

$$\bar{\alpha}_{i4} \triangleq \frac{1}{2}\left((1 - \alpha_{i1})\cos(\alpha_{i3}) - (1 + \alpha_{i1})\right) \quad (45)$$

Again, introduce the following set of state equations

$$\begin{aligned} \boldsymbol{\eta}_i(k+1) &= \left[\hat{\mathbf{A}} - \mathbf{F}_i(k+1)\right]' \boldsymbol{\eta}_i(k) + \sum_{i \in \mathcal{N}_i} \mathbf{C}_i \mathbf{y}_i(k+1) \\ &\quad + \sum_{i \in \mathcal{N}_i} [\mathbf{s}_i' \; 0 \; \ldots \; 0]' \qquad (46) \\ \boldsymbol{\eta}(0) &= \mathbf{N}\mathbf{x}_0 \\ g_i(k+1) &= g_i(k) + \sum_{i \in \mathcal{N}_i} \mathbf{m}_i(k+1)'\mathbf{m}_i(k+1) - \\ &\quad \boldsymbol{\eta}_i(k)'\hat{\mathbf{B}}\left[\hat{\mathbf{B}}'\mathbf{S}_i(k)\hat{\mathbf{B}} + \mathbf{Q}\right]^{-1}\hat{\mathbf{B}}'\boldsymbol{\eta}_i(k) \qquad (47) \\ g_i(0) &= \mathbf{x}_0'\mathbf{N}\mathbf{x}_0 \end{aligned}$$

The following is the main result of this section.

Theorem 2. Let $0 < p_0 \leq 1$ be given, and suppose that Assumption 2 holds. Then the state $\mathbf{x}(T)$ of the system (1) with probability p_0 belongs to the ellipsoid

$$\mathcal{X}_T^i \triangleq \left\{ \begin{array}{l} \mathbf{x}_T \in \mathbb{R}^n : \|(\mathbf{S}_i(T)^{\frac{1}{2}}\mathbf{x}_T - \mathbf{S}_i(T)^{-\frac{1}{2}}\boldsymbol{\eta}_i(T))\|^2 \\ \leq \rho + d \end{array} \right\} \quad (48)$$

where $\rho_i \triangleq \boldsymbol{\eta}_i(T)'\mathbf{S}_i(T)^{-1}\boldsymbol{\eta}_i(T) - g_i(T)$ and $\boldsymbol{\eta}_i(T)$ and $g_i(T)$ are defined (46) and (47).

Proof. It follows from (32) and (34) that

$$\widehat{x}_{i1}(k) = \beta_{i1}x_1(k) + n_{i1}(k) \quad (49)$$

$$\widehat{x}_{i2}(k) = \beta_{i1}x_2(k) + n_{i2}(k) \quad (50)$$

$$\widehat{x}_{i3}(k) = \beta_{i2}x_3(k) + n_{i3}(k) \quad (51)$$

where $x_i(k)$ is the i^{th} component of the state vector $x(k)$ of the system (1) and the inequalities

$$|n_{i1}(k)| \leq \bar{\alpha}_{i1}|x_1(k)| + \bar{\alpha}_{i2}|x_2(k)| + \bar{\alpha}_{i3}|x_3(k)| \quad (52)$$

$$|n_{i2}(k)| \leq \bar{\alpha}_{i2}|x_1(k)| + \bar{\alpha}_{i1}|x_2(k)| + \bar{\alpha}_{i3}|x_3(k)| \quad (53)$$

$$|n_{i3}(k)| \leq \bar{\alpha}_{i2}|x_1(k)| + \bar{\alpha}_{i2}|x_2(k)| + \bar{\alpha}_{i4}|x_3(k)| \quad (54)$$

hold together with (35) with probability p_0. Therefore, this implies that

$$\mathbf{m}_i(k) = \mathbf{C}_i\mathbf{x}(k) + \mathbf{s}_i + \mathbf{n}_i(k) = \mathbf{y}_i(k) + \mathbf{s}_i \quad (55)$$

where $\mathbf{n}(k) \triangleq [n_{i1}(k) \; n_{i2}(k) \; n_{i3}(k)]'$ and $\mathbf{y}_i(k) = \mathbf{C}_i\mathbf{x}(k) + \mathbf{n}_i(k)$ and the condition

$$\|\mathbf{n}_i(k)\|^2 \leq \|\mathbf{K}_i\mathbf{x}(k)\|^2 \quad (56)$$

holds together with (35) with probability p_0. From (35) and (56) it follows that the subsequent sum quadratic constraint is satisfied

$$\begin{aligned} &(\mathbf{x}(0) - \mathbf{x}_0)'\mathbf{N}(\mathbf{x}(0) - \mathbf{x}_0) + \\ &\sum_{k=0}^{T-1}\left(\mathbf{w}(k)'\mathbf{Q}(k)\mathbf{w}(k) + \sum_{i \in \mathcal{N}_i}\|\mathbf{n}_i(k+1)\|^2\right) \\ &\leq d + \sum_{k=0}^{T-1}\sum_{i \in \mathcal{N}_i}\|\mathbf{K}_i\mathbf{x}(k+1)\|^2 \qquad (57) \end{aligned}$$

with probability p_0. It now follows from Theorem 5.3.1 of Petersen and Savkin (1999); see also Savkin and Petersen (1998), that the state $\mathbf{x}(T)$ of the system (1), (55) belongs to the ellipsoid (48) with probability p_0. □

4.3 Comments on the Robust Tracking Algorithm

The uncertain system described by (1) with ((7) or (32)) and ((9) or (35)) is represented by the diagram in Fig. 1.

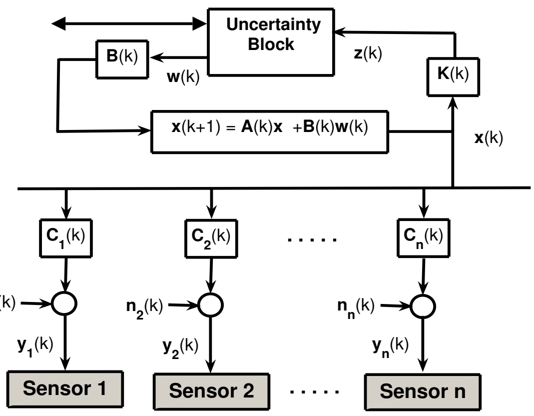


Fig. 1. *Block diagram representation of a multi-sensor uncertain system.*

The algorithm derived in this paper is general and parametric-like system uncertainties satisfying Assumption 1 can be accommodated. Thus, unmodeled target dynamics are likely to cause less problems in the algorithm derived in this section than with traditional approaches, e.g. EKF based algorithms, which inherently require accurate knowledge of the system dynamics. The target dynamics are not required to be modeled using Gaussian random inputs. However, the following remark is given.

Remark 1. Gaussian noise is bounded by the first standard deviation with probability $p_0 \approx 0.68$ and within two standard deviations with $p_0 \approx 0.95$ etc. Thus, no generality is lost by assuming uncertainties satisfying Assumption 1. Further, there are systems in place in practical situations to remove large Gaussian outliers (e.g. gating etc.).

4.4 Comments on Decentralized Data Fusion

The goal of decentralized target tracking is to efficiently exploit the measured radar data from multiple sensors. Specifically, decentralized target tracking is concerned with the problem of efficiently estimating the set $\mathcal{X}_T\left[\mathbf{x}_0, \mathbf{y}(k), d\right]$ of all possible states $\mathbf{x}(T)$ at time T and at each of the sensors $i \in \{1, \ldots, N\}$. Recall that $\mathbf{x}(k) \in \mathbb{R}^n$.

The structure of the given state equations and Riccati equations permit efficient data fusion. Each sensor need only have, and maintain, specific knowledge of it's own measurement model. At every time step, each sensor i receives $|\mathcal{N}_i|$ copies of an $n \times n$ matrix $\mathbf{C}_j'\mathbf{C}_j$, $\forall j \in \mathcal{N}_i$ and $|\mathcal{N}_i|$ copies of a $n \times n$ matrix $\mathbf{K}_j'\mathbf{K}_j$, $\forall j \in \mathcal{N}_i$. Note that if each sensor is identical, then $\mathbf{C}_j'\mathbf{C}_j$, $\forall j \in \mathcal{N}_i$ and $\mathbf{K}_j'\mathbf{K}_j$, $\forall j \in \mathcal{N}_i$ are known at sensor i since sensor i has knowledge of $\mathbf{C}_i$ and $\mathbf{K}_i$. Thus the, the communication required can be decreased significantly for networks of identical sensors. Each sensor i receives $|\mathcal{N}_i|$ copies of a $n \times 1$ vector $\mathbf{C}_j'\mathbf{y}_j(k+1) + [\mathbf{s}_i' \ 0 \ \ldots \ 0]'$, $\forall j \in \mathcal{N}_i$ and $|\mathcal{N}_i|$ copies of a scalar $\mathbf{m}_j(k+1)'\mathbf{m}_j(k+1)$, $\forall j \in \mathcal{N}_i$. Clearly, each sensor must only maintain knowledge of its own measurement model and does not require knowledge of any other sensor's position.

The received data at sensor i is added to the appropriate recursive equations. No overhead processing or maintenance needs to be performed by the sensor. The summation based architecture for data fusion is scalable with the number of sensors. The network topology can be time-dependent. If, for example, sensor i has no neighbors for a period, then the algorithm reduces to a single sensor tracking filter. Neither the standard EKF or the measurement conversion based algorithms permit such efficient multi-sensor data fusion; see Lerro and Bar-Shalom (1993); Schlosser (2004); Li and Jilkov (2001); Zhao et al. (2004).

The communication and data structure maintained by each sensor i is illustrated in Fig. 2 for a sensor network communicating over a complete network topology.

Note that $\mathcal{X}_T^i = \mathcal{X}_T^j$, for any T and $\forall i, j \in \{1, \ldots, N\}$ when we assume a complete graph $\mathcal{G}$ topology.

5. NUMERICAL EXAMPLES

The simulation results presented in this section assume the target maneuvers in 2D. More extensive simulations are given in Bishop et al. (2007a) where the algorithm is compared against the best linear unbiased estimator (BLUE) given in Zhao et al. (2004).

The radar network consists of N sensors, denoted by the set $\mathcal{V}$, with communication topology $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. For simplicity, the sensors are assumed to be identical and the communication topology is assumed to be complete with $\mathcal{E} = \mathcal{V} \times \mathcal{V}$ and $|\mathcal{E}| = N^2$. As such, the performance of the tracking filter is identical at each sensor. That is, $\mathcal{X}_T^i = \mathcal{X}_T^j$ and $\widetilde{\mathbf{q}}_i(T) = \widetilde{\mathbf{q}}_j(T)$ for any T and $\forall i, j \in \mathcal{V}$. Thus, the performance of a single sensor (chosen randomly during each simulation run) is examined.

For each example given, the results of 10000 individual simulation runs are analyzed. The root-mean-squared (RMS) position and velocity error is calculated and shown.

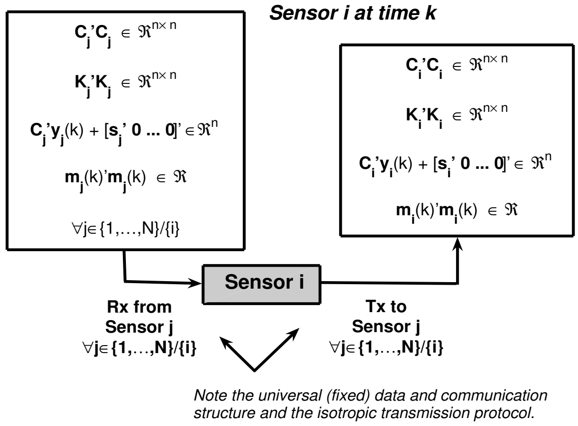


Fig. 2. *Block diagram representation of the communication and data structure of sensor i at time k. The filter protocol is decentralized and permits efficient data fusion with fixed communication requirements. This figure assumes a complete graph and non-identical sensors. If the communication graph is not complete than each sensor i only transmits to, and receives data from, the set of neighbor sensors $\mathcal{N}_i$. In general, $\mathcal{X}_T^i \neq \mathcal{X}_T^j$ when the topology is not complete. If the sensors are identical, both $\mathbf{C}_j'\mathbf{C}_j$ and $\mathbf{K}_j'\mathbf{K}_j$ are known at sensor i and need not be transmitted.*

The specific discussions and examples on 2D tracking in this section, assume the state of the target obeys $\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4]' \in \mathbb{R}^4$. The system model is given by (1) with the following system matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & k_s & 0 \\ 0 & 1 & 0 & k_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} \frac{k_s^2}{2} & 0 \\ 0 & \frac{k_s^2}{2} \\ k_s & 0 \\ 0 & k_s \end{bmatrix} \tag{58}$$

where k_s is the measurement and system sampling interval. The noise input $\mathbf{w}(k)$ represents the uncertain acceleration input of the target. The noise input $\mathbf{w}(k)$ is usually assumed to be a white Gaussian distributed random process. However, in this section and for the filter developed in this paper, the noise input $\mathbf{w}(k)$ can be represented by any bounded function of time. In fact, $\mathbf{w}(k)$ need only be bounded in a probabilistic sense, such that Gaussian processes can be considered as special cases.

The measurement noise in all cases is uniformly distributed such that the error in the range measurement is characterized by $v_{i1}(k) \sim \mathcal{U}(-0.1r_i(k), 0.1r_i(k))$ and the error in the bearing measurement is given by $v_{i2}(k) \sim \mathcal{U}(-5^o, 5^o)$ where $r_i(k)$ is the true range to the target at time k. Knowledge of the sensor's accuracy is assumed such that the noise parameters in the robust linear filter are given accurately by $\alpha_1 = 0.1$ and $\alpha_2 = 5^o$.

Remark 2. It is reasonable to assume (at least partial) knowledge of the sensor's error statistics due to the routine sensor testing and calibration operations performed. However, it is dangerous to assume knowledge of the target's uncertainty statistics characterizing it's maneuvers.

The measurement sampling time is $k_s = 1$ and the entire tracking interval is 600 units. The true initial state of the target is given by $\mathbf{x}(0) = [500\ 5000\ 13\ 11]'$ in all simulation examples involving multiple sensors. The matrix $\mathbf{N}$ is set by $\mathbf{N} = \mathbf{I}$ where $\mathbf{I}$ is the identity matrix. This is a robust (and practical) choice for $\mathbf{N}$ in the absence of any knowledge of the error in $\mathbf{x}_0$. The robust algorithm can withstand a large initial estimate error since no Tayor-series approximation is used and the problem is solved in the linear domain.

5.1 Simulation Example 1 - Five Sensors

In this example case, 5 sensors randomly distributed in a 10000×10000unit region of interest during each simulation run. The process noise $\mathbf{w}(k)$ is used to represent the unknown acceleration of the target. In this example, the process noise is a bounded (partially deterministic) random function of time given in by

$$\mathbf{w}(k) = \begin{bmatrix} 2\sin(0.5k) + \kappa_1(k) \\ \cos(2k) - 5\sin(0.0001k) + \kappa_2(k) \end{bmatrix} \quad (59)$$

where $\kappa_1(k) \sim \mathcal{U}(-0.5, 0.5)$ and $\kappa_2(k) \sim \mathcal{U}(-1, 1)$ are uniformly distributed random variables. In all cases the process noise weighting matrix for the robust filter $\mathbf{Q}(k)$ was chosen such that $\mathbf{Q}(k) = 10^5\mathbf{I}$ for all k. The scenario is depicted in Figure 3 during a specific simulation run.

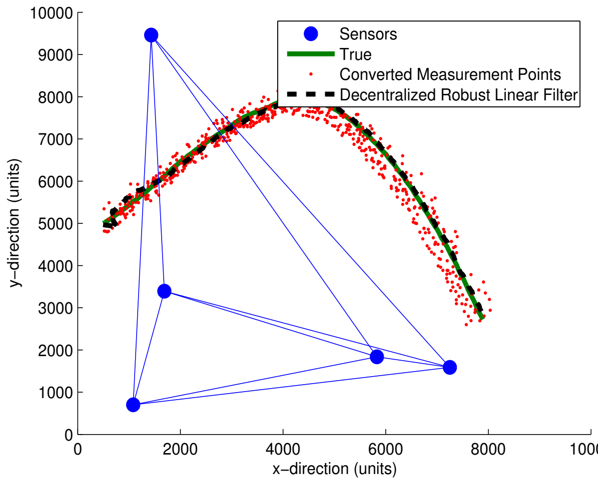
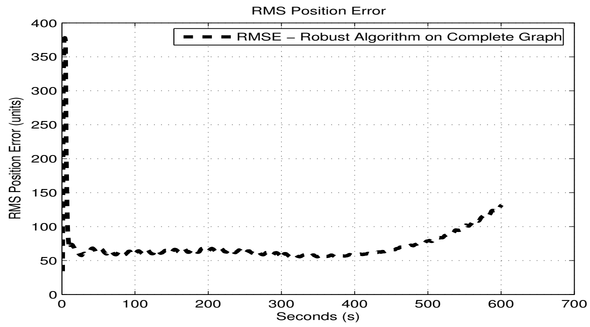


Fig. 3. *The tracking scenario with* 5 *sensors for simulation example 1 during a particular simulation run.*

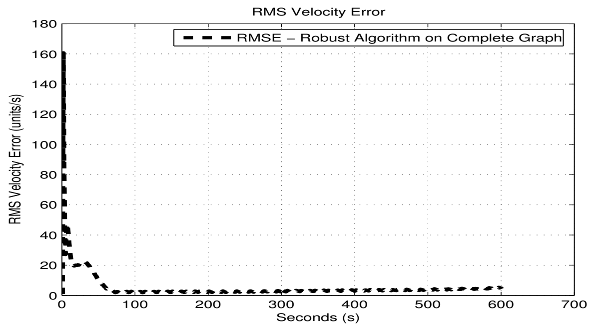
Figure 3 shows the true target path during the particular simulation run along with the robust (decentralized) linear filtering estimate at a particular, randomly chosen, sensor i (recall that each sensor's estimate is identical for a complete network topology). Figure 3 also shows the original converted measurement points for the same specific sensor. The converted measurement points are generated from that the converted measurement vector

$$\mathbf{m}_i(k) = [\widehat{x}_{i1}(k)\ \ \widehat{x}_{i2}(k)]' + \mathbf{s}_i$$

The RMS position and velocity error over 10000 simulation runs is given in Figure 4.



(a).



(b).

Fig. 4. *The RMS position (a) and velocity (b) error graphs generated using 10000 simulation runs for the simulation example case 1.*

Note from Figure 4(a) that the position estimate accuracy begins slowly to degrade as the target moves away from the region within which the sensor's are generally distributed. In this example, only a relatively small number of sensors were considered and it is expected that the performance of the tracking estimate will improve as N increases.

In Bishop et al. (2007a), the BLUE filter and the robust linear filter derived in this paper are compared extensively for simulations involving the standard Gaussian measurement and process noise assumptions.

5.2 Simulation Example 2 - 25 Sensors

In this example case, 25 sensors randomly distributed in the 10000×10000unit region of interest during each simulation run. The simulation parameters here are identical to simulation example case 2. The scenario is depicted in Figure 5 during a specific simulation run.

Figure 5 shows the true target path during a particular simulation run, along with the robust (decentralized) linear filtering estimate at a particular, randomly chosen, sensor i (remember that each sensor's estimate is identical). Figure 5 also shows the original converted measurement points for the same randomly chosen sensor i during the specific simulation run (i.e. similarly to simulation case 2). The RMS position error over 1000 simulation runs for simulation case 3 is given in Figure 6.

Note from Figure 6 that the position estimate accuracy is improved with the additional sensors. Also, the summation fusion structure means that the algorithm is scalable.

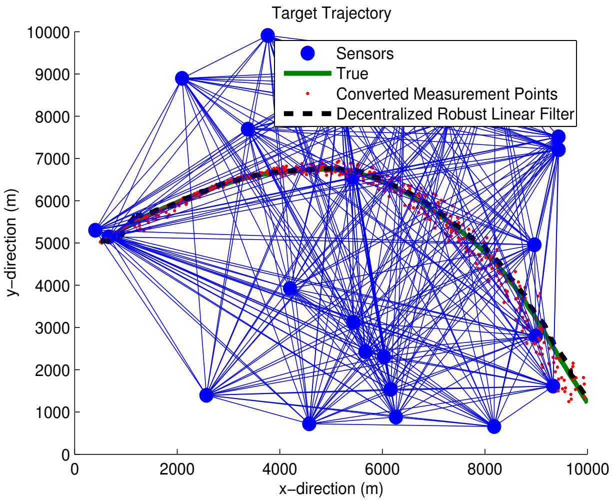
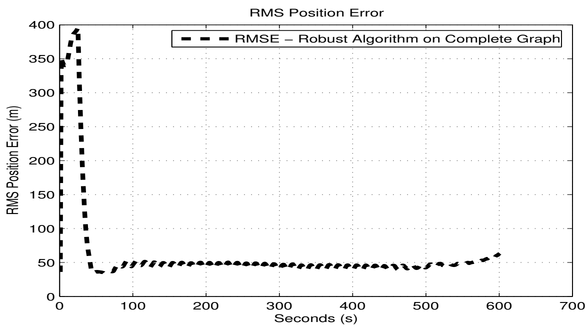
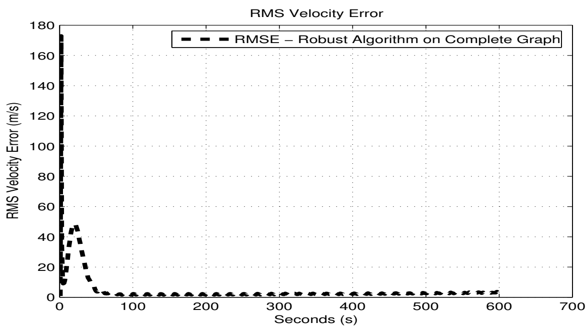


Fig. 5. *The tracking scenario during a particular simulation run for simulation example 2 with 25 sensors.*



(a).



(b).

Fig. 6. *The RMS position (a) and velocity (b) error graphs generated using 10000 simulation runs for the simulation example case 2.*

6. CONCLUSION

In this paper, a novel and robust linear filter was derived for target tracking using converted radar sensor measurements. The converted radar measurements can be modeled using parametric-type uncertainty models; e.g. see Zhao et al. (2004); Bishop et al. (2007a). Therefore, methods and concepts from robust estimation are well-suited to solving the newly-converted filtering problem. The problem of decentralized filtering was also examined, and the derived filter provided a novel and efficient/scalable solution to the sensor network based tracking problem. A mathematically rigorous proof of the boundedness of the estimation error was given. In fact, the given filter belongs to a class of set-valued state estimators and the true target state was shown to belong to a given ellipsoidal set estimate with an arbitrarily high probability. The derived technique does not require Taylor-series based approximations, and builds upon a solid foundation of robust estimation ideas.

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