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Control of Formations of UAVs for Surveillance and Reconnaissance Missions

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Abstract: This paper presents a method for control of formations of Unmanned Aerial Vehicles (UAVs) in urban environments with several obstacles. Therefore the trajectories for each UAV are planned using mixed integer quadratic programming (MIQP) to describe a minimization problem. The result of this minimization problem then characterizes a collision free trajectory for each UAV using the commanded formations to fulfil the missions. The description of the UAVs, the inter UAV collision avoidance, the collision avoidance with obstacles as well as the description of formations will be shown in detail together with some simulation results in this paper. In addition the introduction explains the fields of interest in such formations of UAVs and what kind of advantage they can bring in comparison to today's solutions. The novelty in the approach in this paper is the description of formations of UAVs used in combination with MIQP to change formations, to add additional UAVs into an existing formation and to split formations, simply by changing some parameters in the description of the formation.

Keywords: Autonomous Systems, Description of Formation Flights, Hybrid Systems, Optimal Control, Mixed Integer Quadratic Programming

1. INTRODUCTION

Until now Unmanned Aerial Vehicles (UAVs) are mostly used as HALE (High Altitude Long Endurance) or MALE (Medium Altitude Long Endurance) in directly military missions or for observer missions. For civil missions it is rather difficult to get an allowance for UAVs in the civil airspace as different national laws in several countries are not prepared for fully autonomous systems in the sky. Due to facts like these in many cases where an UAV would be able to do the job there is simply no interest in an UAV as there would be several problems to obtain a flight permission.

For small UAVs this is often not the case. They can easily obtain flight permissions for low level flights below the air traffic routes of commercial aircrafts. The problem for small UAVs (mini UAVs) is that in many cases their payload is really limited and flight endurance is often less than one hour. Nevertheless for several specific cases this will be enough time to fulfil beneficial missions that could not be accomplished today.

Missions where small UAVs could be a helpful extension to actual concepts are in rescue missions when it is often difficult for the mission command on the ground to get a good situational awareness and to identify the positions where people need help. In these cases a group of small UAVs could easily fulfil a reconnaissance mission to support the rescue troops on the ground with pictures of the scenario, detailed scans for life and heat signatures. These sensors like a vision camera together with a bolometer, a carbon dioxide measuring device and CBRN sensors, are often too heavy to be able to integrate all necessary sensors in a single small UAV. In such cases the sensors could be distributed between a group of mini UAVs that then scan the area of interest in a formation. This will allow to use the small UAVs close to the ground and to investigate also the inside of buildings. Also the small UAVs can be easily carried by fire fighters or policemen on the ground in their vehicles without the need of special start platforms so that they can be easily transported and used.

Another interesting field for small UAVs are surveillance missions along borders of industrial complexes, harbours or camps in dangerous regions. Especially when there is much traffic along the borders it is often difficult to guarantee total control only by fixedly based cameras and guards on fixed routes. For events like football games and demonstrations formations of small UAVs could help to get a better view of the situation and to identify possible problems early. In such missions the UAVs would patrol in formation autonomously and if a point of interest is

identified the operator on the ground can select a specific UAV and use it with remote control to get a good look at things in detail. During this time the other UAVs will then automatically change their formation to guarantee an as good as possible view over the complete ground sector and change their formation back to original when the temporary teleoperated UAV comes back into the formation.
Therefore this paper will show a method how formations of UAVs can be coordinated and their flight routes can be created. This will be done by using mixed integer programming (MIP) for the complete generation of the trajectories for all UAVs in opposition to other publications like Schouwenaars [2006] or Richards et al. [2005] where normally mixed integer programming is used for receding horizon control where only parts of the trajectories are planned directly. Based on the work in Kopfstedt et al. [2007] the algorithms here are designed for formation flights of many UAVs with respect to the work presented by Xia et al. [2007].
Therefore in the second section we will present the UAV model, the method to avoid collisions between UAVs and with obstacles. The third section then introduces the description of formations of UAVs and how changes in the formations can be realized. Based on the information from sections two and three in section four the optimization problem itself will be described and in the following section simulation results for changes of formations due to additional UAVs, the splitting of a formation and encounter situations of two formations of UAVs are shown.

2. BASIC MIXED INTEGER MODELS

2.1 UAV Model

At first a dynamic model of the system is needed. As dynamic models for small UAVs with dynamics like mini helicopters and quadrotor systems have been already investigated and verified models are existing as described in Gavrilets et al. [2001]. The dynamic UAV model

$$\begin{bmatrix} \dot{x}_j(t) \\ \dot{y}_j(t) \\ \dot{z}_j(t) \\ \ddot{x}_j(t) \\ \ddot{y}_j(t) \\ \ddot{z}_j(t) \end{bmatrix} = \begin{bmatrix} 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 0&0&0&-\frac{1}{\tau_l}&0&0 \\ 0&0&0&0&-\frac{1}{\tau_l}&0 \\ 0&0&0&0&0&-\frac{1}{\tau_v} \end{bmatrix} \cdot \begin{bmatrix} x_j(t) \\ y_j(t) \\ z_j(t) \\ \dot{x}_j(t) \\ \dot{y}_j(t) \\ \dot{z}_j(t) \end{bmatrix} + \begin{bmatrix} 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ \frac{k_l}{\tau_l}&0&0 \\ 0&\frac{k_l}{\tau_l}&0 \\ 0&0&\frac{k_v}{\tau_v} \end{bmatrix} \cdot \begin{bmatrix} f_{x,j}(t) \\ f_{y,j}(t) \\ f_{z,j}(t) \end{bmatrix} \quad (1)$$

of vehicle j is directly based on the model described in Schouwenaars [2006] and can be used by parameterisation for various mini UAVs. The model is a simple second order dynamic model for an UAV that is originally a nonlinear system. To describe the system with (1) it is necessary to combine the effects in the x and the y directions. Due to this there is a time constraint τ_l and a gain constraint k_l for the planar motion along the xy-plane and decoupled constraints τ_v and k_v for the vertical flight direction. The forces $f_{x,j}(t)$, $f_{y,j}(t)$ and $f_{z,j}(t)$ represent the components of the engine force that controls the UAV. All three components are linked together so that large forces into one direction reduce the maximum possible forces into the other directions.

2.2 UAV Collision Avoidance Model

If the UAVs are not flying in a predefined formation with given distances between them it could happen that two UAVs collide with each other. To avoid this it is necessary to create at least virtual boxes around the position of the UAVs and to test if another UAV is inside of the same box as it has been already done by Richards et al. [2001] and Tanaka et al. [2006]. If a second UAV appears inside a box around a UAV this would result in a collision of two UAVs. To avoid this the following equation

$$\begin{aligned} s_{t,k} - s_{t,j} - S \cdot \xi_{t,p,1} &\leq s_{min} \\ -s_{t,k} - s_{t,j} - S \cdot \xi_{t,p,2} &\leq -s_{min} \end{aligned} \quad (2)$$

has to be included into the MIP minimization problem together with

$$\xi_{t,p,1} + \xi_{t,p,2} \leq L \quad (3)$$

taken from Kopfstedt et al. [2006] to ensure that the UAV j described by $s_{t,j}$ and k described by $s_{t,k}$ do not come closer than s_{min} to each other at step t, if $s_{min} = (x_{min}, y_{min}, z_{min})$ applies. With S as a large positive number, $\xi_{t,p,1}$ and $\xi_{t,p,2}$ as Boolean vectors of the lenght three, $L = (1,1,1)^T$ and $s = (x,y,z)^T$. As theoretically collisions between all UAVs could occur these equations are needed for all possible inter-UAV-collisions for all steps of the simulation. This results in $(M-1)!$ possibilities for each step t, when M is the total number of UAVs.

2.3 Model for Obstacles

In addition to collisions with other UAVs also collisions with obstacles like trees, walls and buildings can occur, as mini UAVs are often flying at low altitudes where the environment around the UAVs is full of obstacles, in contrast to UAVs that fly at high altitudes also. For the description of obstacles in 3D various methods for the description are existing starting with simple boxes as used in Kuwata et al. [2004] and described by Richards et al. [2005] up to complex polygons as in Schouwenaars [2006]. The following description of obstacles allows the description of convex polygons and is a 3D extension of the algorithm based on Kopfstedt et al. [2006]. A possible obstacle that can be described by this algorithm is shown in Fig. 1. The description of such obstacles is done by a single vector of the form

$$Obs_i = [x_{st}, y_{st}, z_{st}, \alpha_1, \alpha_2, ..., \alpha_{n-1}, l_1, l_2, ..., l_{n-1}, z_{height}] \quad (4)$$

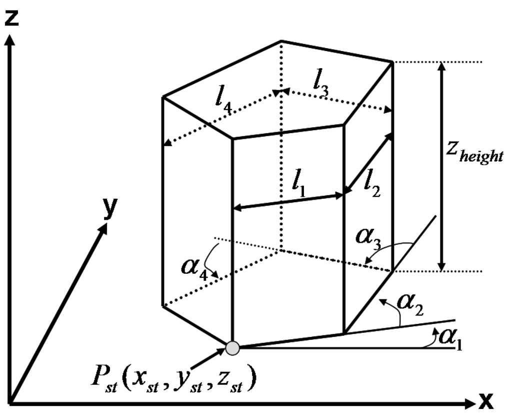

Fig. 1. Visualization of a possible convex 3D obstacle

when the first edge of the obstacle can be described as

$$P_1\left(x_1, y_1, z_1\right) = \begin{bmatrix} x_{st} \\ y_{st} \\ z_{st} \end{bmatrix} \tag{5}$$
$$\alpha_{obs\_1} = 0$$

and all following edges in the xy-plane can be calculated using (4) and (5) as inputs by

$$P_i\left(x_i, y_i, z_i\right) = \begin{bmatrix} x_{i-1} + l_{i-1} \cdot \cos\left(\alpha_{obs\_i-1}\right) \\ y_{i-1} + l_{i-1} \cdot \sin\left(\alpha_{obs\_i-1}\right) \\ z_{st} \end{bmatrix} \tag{6}$$
$$\alpha_{obs\_i} = \alpha_{obs\_i-1} + \alpha_{i-1}$$

where x_{st}, y_{st} and z_{st} define the base point of the obstacle, $\alpha_1, \alpha_2, ..., \alpha_{n-1}$ define the angular difference between two following straight lines of the obstacles in the xy-plane, $l_1, l_2, ..., l_{n-1}$ define the length of the segments of the obstacles along the xy-plane and z_{height} is the height of the obstacle towards the base height z_{st} of the obstacle. Based on these data it is possible, if the angle α_{obs_i} is $\frac{\pi}{2}$, $\frac{3}{2}\pi$ or differs only with less the the machine precision e towards one of these angles, to use the equation

$$-x_{t,j} \cdot sgn\left(y_{i+1} - y_i\right) - \epsilon_i \cdot S \leq -\left(x_i + e\right) \cdot sgn\left(y_{i+1} - y_i\right) \tag{7}$$

to test if a collision of the UAV j in the step t could occur with the obstacle. In all other cases the more complex equation

$$y_{t,j} \cdot f_i - x_{t,j} \cdot f_i \cdot g_i - \epsilon_i \cdot S \leq -f_i \cdot \left(e + h_i\right) \tag{8}$$

is necessary, which needs

$$f_i = sgn\left(\cos\left(\alpha_{obs\_i+1}\right)\right)$$
$$g_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$
$$h_i = g_i \cdot x_i - y_i \ .$$

as help equations to become solvable. While S represents a large positive number and ϵ_i is a Boolean variable. With the equations from (7) and (8) it is tested if the x and y positions of the UAV could collide with the obstacle. In addition the two equations

$$z_{t,j} - \epsilon_i \cdot S \leq z_{st} + e \tag{9}$$
$$-z_{t,j} - \epsilon_i \cdot S \leq -(z_{st} + z_{height}) + e$$

are necessary to test if the z position of the UAV is also inside of the obstacle. If

$$\sum_{i=1}^{n+2} \epsilon_i \leq n + 1 \tag{10}$$

is fulfilled the UAV is outside of the obstacle. If (10) cannot be fulfilled there is a collision of UAV j at step t with this obstacle. The result is that for each possible collision between an UAV and one obstacle for each step t of the UAV $n+3$ equations are needed if n is the number of edges of the obstacles in the xy-plane.

3. FORMATIONS OF UAVS

3.1 Description of a single Formation

With the equations from the previous section it is possible to describe each single UAV and to avoid possible collisions between UAV j and UAV k as well as collisions with obstacles in the environment. To allow descriptions of different designs of formations a flexible algorithm is necessary as the one presented in Kopfstedt et al. [2006] that always describes only the distance between two UAVs. Due to this it is possible to describe various types of formations simply by defining different connections between UAVs in the formation and variations of these relative distances. The connection between two UAVs is visualised in Fig. 2 where $s_{dist,j} = \left[x_{dist,j}, y_{dist,j}, z_{dist,j}\right]^T$ are the nominal distance between UAV j and UAV k. If a little variation inside of the formation is allowed the vector $s_{dist\_max,j} = \left[x_{dist\_max,j}, y_{dist\_max,j}, z_{dist\_max,j}\right]^T$ can be used to define the maximum allowed distance between two UAVs inside of the formation and the vector $s_{dist\_min,j} = \left[x_{dist\_min,j}, y_{dist\_min,j}, z_{dist\_min,j}\right]^T$ can be used to define the minimum allowed distance between the UAVs j and k. If formations with many UAVs need to be described it cannot be ensured that for the description of the connection between the UAVs j and k the position of UAV k towards UAV j in all three axes is larger or equal to zero. Due to this fact the vector orientated description

$$\lambda \cdot s_{t,k} - \lambda \cdot s_{t,j} \leq \lambda \cdot s_{dist\_max,j}$$
$$-\lambda \cdot s_{t,k} + \lambda \cdot s_{t,j} \leq -\lambda \cdot s_{dist\_min,j} \tag{11}$$
$$\lambda \cdot s_{t,k} - \lambda \cdot s_{t,j} - \lambda \cdot n_{t,j} = \lambda \cdot s_{dist,j}$$

for the description of formation is needed to describe the connection between UAV j and k. The vector $n_{t,j}$ defines the distance between the UAVs j and k in relation to the optimal distance that is defined by $s_{dist,j}$. The 3x3 Matrix

$$\lambda = \begin{bmatrix} \lambda_x & 0 & 0 \\ 0 & \lambda_y & 0 \\ 0 & 0 & \lambda_z \end{bmatrix} \tag{12}$$

is used to determine if the UAV k has to be above UAV j ($\lambda_z = 1$) or below UAV j ($\lambda_z = -1$), if the relative position of UAV k towards UAV j is left ($\lambda_x = -1$), right ($\lambda_x = 1$), before ($\lambda_y = 1$) or behind ($\lambda_y = -1$).
If the values *dist_max* and/or *dist_min* differ from *dist* also

$$\min_n \sum_{t=1}^{T} \sum_{j=1}^{M-1} n_{t,j}^2 \tag{13}$$

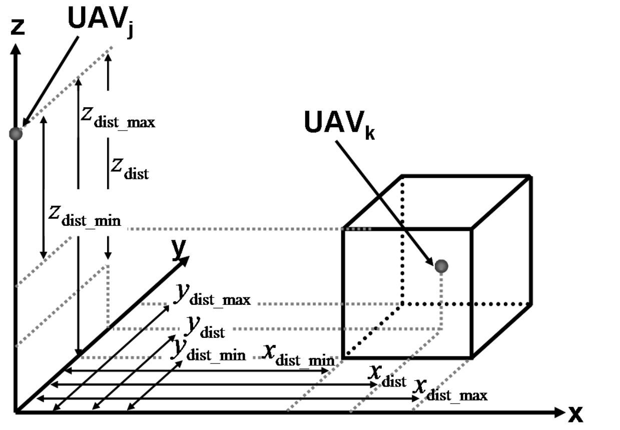

Fig. 2. Method for the description of formation

will be needed to ensure that in normal situations the UAVs j and k fly in distance $s_{dist,j}$ to each other. Only in cases when there are environment effects like obstacles or other UAVs the distance inside of the formation can change in order to pass the obstacle or UAV. The minimization criteria (13) has to be added to the overall minimization criteria of the complete MIQP problem.

3.2 Description of Changes between Formations

With the equation in the previous subsection it is possible to describe all possible formations as only the links between two connected UAVs are described. For the creation of a large formation out of two smaller ones or the splitting of a large formation into several smaller ones this is not enough. For these cases at least in some simulation steps two different excluding connections between UAVs must be possible, so that, depending how fast the change between the two formations is possible, the UAVs can stay in the old formation a bit longer or reach the new type of formation earlier than necessary. The realization of these both formations is described separately by the equations from the previous subsection; in addition for both formations and if the UAV is in no formation a value for the minimization criteria must be fixed. These values are described by β_r when R is the number of existing described formations plus one value for the case the UAV is in no formation. The variable $\epsilon_{t,r}$ is a Boolean variable and describes if the UAVs are at time t in the formation r or not. The resulting minimization criteria for the formation switching condition is

$$\min \sum_{t=1}^{T} \sum_{r=1}^{R} \beta_r \epsilon_{t,r} \tag{14}$$

and has to be added to the total minimization criteria for the MIP whenever switching between formations is described.

4. FORMULATION OF THE OPTIMIZATION CRITERIA

For the optimization it is important to describe the goal condition. In our cases the goal of each optimization is to reach the goal position in the desired formation. Therefore each UAV has to fulfil in its last step the equation $s_{T,j} = s_{goal,j}$. As the focus of this paper is on the formation flights and the changing between formations the paths of the single UAVs underlie only

$$\min \sum_{t=1}^{T-1} \sum_{j=1}^{M} \Delta s_{t,j}^2 \tag{15}$$

which means that the UAVs will try to fly with constant velocity values over all steps and to take the shortest possible path, when $\Delta s_{t,j}$ represents the vector $[\Delta x_{t,j}, \Delta y_{t,j}, \Delta z_{t,j}]^T$, with $\Delta x_{t,j} = x_{t,j} - x_{t-1,j}$ and similar for the y and z components of the vector for the UAV j at the step t. Other more complex optimization criteria for the paths of the individual UAVs like fuel reduction, minimum flight time, ideal acceleration and deceleration could also be added, but will not be presented here. By combination of (13), (14) and (15) the following optimal control problem can be formulated

$$\min \left(\begin{array}{l} \displaystyle\sum_{t=1}^{T-1} \sum_{j=1}^{M} \varphi_1 \Delta s_{t,j}^2 + \sum_{t=1}^{T} \sum_{j=1}^{M-1} \varphi_2 n_{t,j}^2 \\ \displaystyle + \sum_{t=1}^{T} \sum_{r=1}^{R} \varphi_3 \beta_r \epsilon_{t,r} \end{array} \right) \tag{16}$$

$$\text{subject to} \left[\begin{array}{c} (1) \\ (7) \text{ and/or } (8) \text{ and } (9) \text{ and } (10) \\ \text{description of formation one to } R-1 \\ (2) \text{ and } (3) \\ s_{T,j} = s_{goal,j} \end{array} \right]$$

which delivers the position of each UAV for all steps T by using a MIQP solver like CPLEX. With the parameters φ_1, φ_2 and φ_1 the influence of the different parts of the minimization criterion can be controlled. This allows to focus more on the length of the trajectories of the UAVs by using a high value for φ_1 or more on the accuracy of the formation with a high value for φ_2. Alternatively it is also possible to concentrate on specific formations with the parameter φ_3. For good results in the most cases it has been shown that the highest relative value should be set on the parameter φ_1 to control the length of the trajectories while for the parameters φ_2 and φ_3 small values should be used.

As the planning of complete missions before mission start is often impossible the algorithms presented in this paper are used to plan mission parts between waypoints given by the operator in detail. With this planning horizion on the one hand the scenarios for the optimization are big enough to define specific flight routes and on the other hand the scenarios become not too complexe as this would result in the need of big amounts of time for the complete optimization. If the flight plans for formations of UAVs are planned with this algorithms always only a few minutes into the future the optimization itself is realtime capable and also the human operator is flexible in the mission planning and can react on not predicted events.

5. SIMULATION

5.1 Splitting of a Formation

As explained in the introduction in several cases it can become necessary to split a formation of UAVs into two or more parts or to separate a single UAV from the formation.

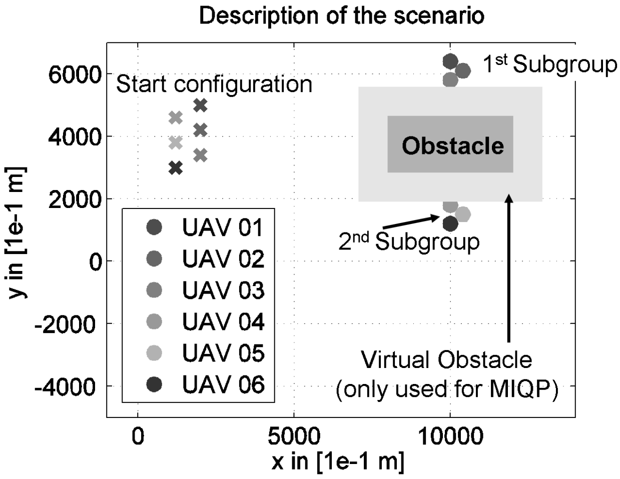

Fig. 3. Mission task: Splitting of a formation

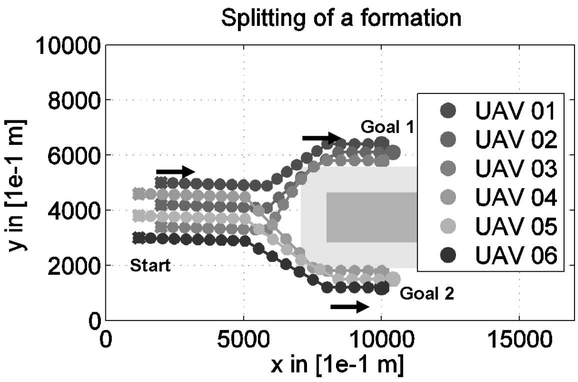

Fig. 4. Trajectories for the UAVs to fulfil the mission

In the example described in Fig. 3 the formation of UAVs is split into two parts. For visualization effects the formations are all created as 2D formations and for the optimization itself it is only possible to control the UAVs in their x and y position components. The z value is set for this simulation example to a fixed value.

The resulting trajectory for each UAV to fulfil the change from the start formation into one as defined in Fig. 3 is presented in Fig. 4. As it is shown there the formation of the UAVs can be split into two smaller formations without any collision to the obstacle or between the UAVs.

5.2 Fusion of two Formations

The second simulation example describes the fusion of two UAV formations into one new formation where all UAVs take part. This mission task is visualised in 5 and as in the example above the UAVs are only allowed to change positions in x and y while the height z is fixed to allow a clear visualization in the following figures. The solution of the mission described in Fig. 5 is presented in Fig. 6 and demonstrates effectively that the creation of a larger formation out of two smaller ones even in environments with the presence of obstacles is possible without any collisions. To show the capabilities of the algorithm the

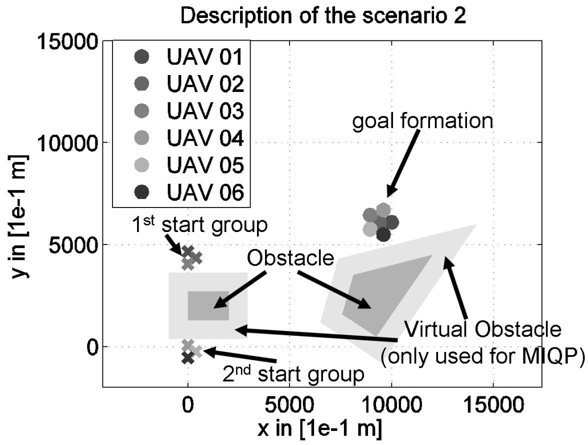

Fig. 5. Mission task: creation of a formation

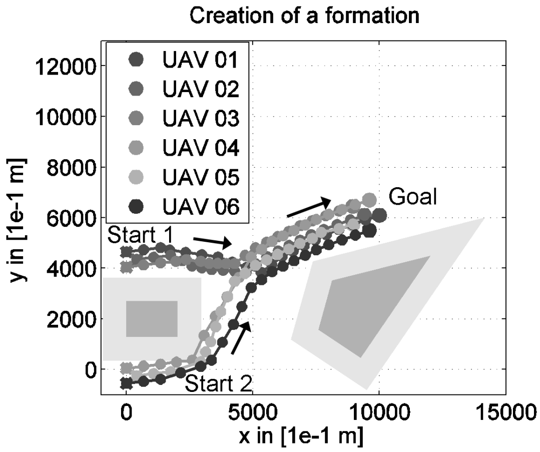

Fig. 6. Trajectories for the UAVs to fulfil the mission

UAVs of the formations do not simply get merged into one bigger formation. As the numerical order in Fig. 5 shows the UAV have to reach specific positions inside of the formation that are placed in a way that the UAVs have to avoid possible collisions with several other UAVs to reach the positions in the formation. In addition the obstacle is placed in this scenario in a manner that it reduces the space for the UAVs so that they have in addition also to avoid collisions with the obstacle during the fusion of the two formations into a singular new one, to demonstrate that even in complex situations the MIQP minimization problem can be used and produces with an MIQP solver a good result even for such situations.

5.3 Collision Avoidance between two Formations

In difference to the two examples above the following simulation has been done without any specific limitation along the z plane so that the UAVs are able to move in this simulation into all directions, to show the full capabilities of the equations and algorithm explained in this paper. With the use of CPLEX on a Intel Pentium 4 with 1,6 GHz the used time for optimization was several tens of seconds due to the high complexity of the optimization problem. The task in this simulation is that two formations of UAVs cross each other on optimal routes and this is visualised in Fig. 7.

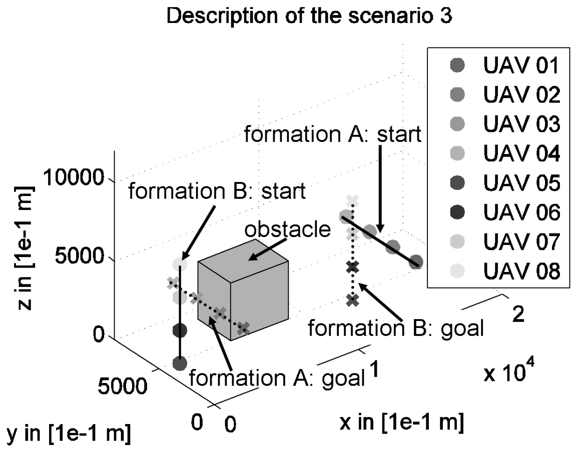

Fig. 7. Planned Start and Goal Situations

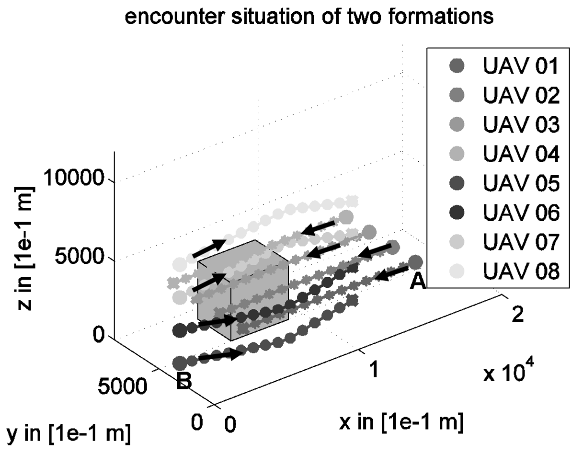

Fig. 8. Trajectories for the UAVs to fulfil the mission

As result Fig. 8 shows that collisions between all UAVs and between UAVs and the obstacle have been totally avoided. Also after the encounter situation of the UAVs formation B is recreated and can continue its flight. The formation B has left the formation flight during the encounter situation as the description of the formations was in a way that the formation which is marked in Fig. 7 with B can be left during an encounter situation while the formation which is marked with A is not allowed to do this. Due to this difference between the two formations the formation A keeps totally in formation also during the encounter situation while the formation B is temporarily not existing but is directly recreated after the encounter situation. If UAVs have to fly in a fixed formation where a separation of single UAVs out of the fixed relative positions inside of the formation would result in a mission failure the formation type A has to be used for the description of formations. In all other cases the UAVs in one formation should be allowed to temporarily leave the formation if obstacles or other formations have to be passed as this allows shorter paths in total and sometimes only when the fixed positions in a formation are left by the UAVs it is possible to pass narrow passages while formations which have to fly exactly in formation have to surround such areas.

6. CONCLUSION

The algorithms presented in this paper have been tested in simulation and, based on the optimization criteria, are able to create the optimal paths of the UAVs. If a solution can be found also the formation conditions are fulfilled. The novelty of this approach is that complex formations in combination with formation switching, path planning and collision avoidance can be done in total for a complete mission by a single optimization task completely described as an MIQP problem for UAVs including a simplified second order model for each UAV. As future work these algorithms have to be validated also in complex experimental tests to ensure that the models of the UAVs and the resulting trajectories can be flown in stable UAV formations also in rain, wind and other environmental conditions which have not been implemented in the environment of the simulation.

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