

# Adaptive Friction Compensation: Application to a Robotic Manipulator

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**Abstract:** This paper presents a feed-forward model-based friction compensation technique using the LuGre friction model. An off-line method is given to estimate the model's parameters based on simple ramp-response experiments. In addition, an on-line parameter adaptation procedure for the two most important model parameters is provided. Experimental results obtained with a robotic manipulator are presented to illustrate the merits of the friction compensation technique.

Keywords: friction compensation, adaptive control, nonlinear control, LuGre model, robotics

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## 1. INTRODUCTION

Precise motion control is essential in applications like machining, wafer steppers or robotic systems. Friction is one of the main factors that limit the performance achievable with linear control methods [Canudas de Wit et al., 1995, Canudas de Wit and Lischinsky, 1997]. It affects both static and dynamic performance and it may cause limit cycles, steady-state error or even instability. Failure to compensate the friction effects can substantially degrade the performance of a motion control system, especially in low-velocity tasks.

Because of these limitations, methods have been developed to minimize the effects of friction. The basic idea of these methods is to compensate the friction force. The most well-known approach is model-based friction compensation. This method relies on a friction model, identification of the model parameters and an implementation of a friction compensator in the control scheme. An appropriate friction model is a basic requirement for good compensation results. A dynamic model called LuGre model [Canudas de Wit et al., 1995] has frequently been used, as it offers a good compromise between complexity and accuracy. It covers the main phenomena of friction, i.e., viscous friction, Coulomb friction, stiction and also the dynamic bristle behavior at the contact surface, all captured in a relatively compact formula. This model has also been recommended by many other authors [Waiboer, 2007], [Alpeter, 1999], [Tan and Kanellakopoulos, 1999] and [Panteley et al., 1998]. Variations of environmental factors such as the normal force, temperature, lubricant conditions, etc., affect the friction model parameters. It is therefore necessary to extend the compensation approach to an adaptive version that can cope with environmental changes and model uncertainties.

There are also model-free compensation methods, which do not require a detailed model of friction. A well-known

example is the sliding mode controller. In this approach, friction is regarded as a bounded disturbance signal and the control strategy is designed such that the tracking error converges to zero. This approach is robust against friction model uncertainty. However, the main drawback of sliding-mode control is the chattering of the control signal, which is undesired in practice because it shortens the lifetime of mechanical components. Because of this drawback, we concentrate on the model-based friction compensation.

This paper contributes a detailed description of the LuGre friction model, an off-line estimation method for all the model's parameters based on simple, low-cost ramp-response experiments and an on-line parameter adaptation procedure for the two most important model parameters. Experimental results obtained with a robotic manipulator are presented to illustrate the merits of the friction compensation technique.

## 2. FRICTION MODEL

### 2.1 DC Motor Actuator

Consider a robot's joint actuated by a DC motor. The differential equations describing the motor are:

$$L\dot{i} + Ri = V - k_t\dot{y} \quad (1)$$

$$J\ddot{y} = k_t i - T_f \quad (2)$$

With  $L$  and  $R$  being the armature inductance and resistance, respectively,  $k_t$  the motor torque constant and  $J$  the combined inertia of the rotor and the link attached to it. The input voltage is denoted by  $V$ , the joint angle by  $y$  and the friction torque by  $T_f$ .

### 2.2 Friction Model

To model  $T_f$ , we use the LuGre model introduced by Canudas de Wit et al. [1995]:

$$T_f = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \alpha_2 v \quad (3)$$

$$\frac{dz}{dt} = v - \frac{\sigma_0}{g(v)} z |v| \quad (4)$$

$$g(v) = \alpha_0 + \alpha_1 e^{-(v/v_s)^2} \quad (5)$$

This model has one input variable, the joint velocity,  $v = \dot{y}$ , and one state variable,  $z$ , representing the bristle displacement in the pre-sliding phase. The model has the following six parameters:

$\alpha_0$	Coulomb friction	$v_s$	Stribeck velocity
$\alpha_1$	stiction	$\sigma_0$	bristle stiffness
$\alpha_2$	viscous friction	$\sigma_1$	bristle damping

Methods to estimate these parameters off line and on line are given in the following section.

### 3. PARAMETER ESTIMATION

#### 3.1 Friction Torque Computation

To estimate the model parameters, the friction torque must be known. The friction torque cannot be measured directly, but it can be computed using the DC motor model (1)–(2). Neglecting the armature inductance, from (1) we can express  $i$

$$i = \frac{V - k_t \dot{y}}{R} \quad (6)$$

and substitute it into (2) to obtain:

$$J \ddot{y} = k_t \frac{V - k_t \dot{y}}{R} - T_f \quad (7)$$

At a constant velocity,  $\ddot{y} = 0$ , this yields:

$$T_{ss}^*(v) = k_t \frac{V - k_t v}{R} \quad (8)$$

Clearly, with the knowledge of the motor parameters  $k_t$  and  $R$ , we can compute the steady-state friction torque  $T_{ss}^*$  from the constant-velocity data. These parameters are easy to determine, either from the motor's data sheet or by means of direct static measurements of the applied voltage, the resulting current and torque or force.

#### 3.2 Off-Line Parameter Estimation

The initial values of the LuGre model parameters can be estimated off line by using data from dedicated experiments. We split the parameters into 'static' and 'dynamic' parameters. For the static ones, take the steady-state form of equations (3)–(5), i.e., set  $\frac{dz}{dt} = 0$ , to obtain:

$$T_{ss}(v) = \left( \alpha_0 + \alpha_1 e^{-(v/v_s)^2} \right) \text{sgn}(v) + \alpha_2 v \quad (9)$$

The four parameters in this equation ( $\alpha_0, \alpha_1, \alpha_2, v_s$ ) are called the static parameters. The remaining two parameters ( $\sigma_0$  and  $\sigma_1$ ) are the dynamic parameters.

*Estimation of Static Parameters.* Closed-loop experiments at constant velocities (angle ramp response) were performed. Using (8), these experiments yield data sets consisting of velocity – friction torque pairs. The goal is to find the parameters ( $\alpha_0, \alpha_1, \alpha_2, v_s$ ) such that (9) optimally fits the experimental data (in the least-square sense).

Recall that  $T_{ss}^*$  is the torque computed from the data and  $T_{ss}$  is the friction model output. The following cost function is used to quantify the fit:

$$J_s = \sum_{j=1}^n \left( T_{ss}^*(v_j) - T_{ss}(v_j) \right)^2 \quad (10)$$

where  $j$  is the data sample index and  $n$  is the total number of samples. As (9) is nonlinear with respect to  $v_s$ , we apply linear-least square estimation for an array of pre-selected values of  $v_s$ . We first restate (9) as

$$T_{ss} = \left( \text{sgn}(v) \quad e^{-(v/v_s)^2} \text{sgn}(v) \quad v \right) \left( \alpha_0 \quad \alpha_1 \quad \alpha_2 \right)^T \quad (11)$$

and fill in the friction-velocity data:

$$\underbrace{\begin{bmatrix} T_{ss}^*(v_1) \\ \vdots \\ T_{ss}^*(v_n) \end{bmatrix}}_{\mathcal{T}_f} = \underbrace{\begin{bmatrix} 1 & e^{-(v_1/v_s)^2} & v_1 \\ \vdots & \vdots & \vdots \\ 1 & e^{-(v_n/v_s)^2} & v_n \end{bmatrix}}_{\mathcal{A}_f} \underbrace{\begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix}}_{\mathcal{P}_f} \quad (12)$$

The next step is to generate a set of discrete values of  $v_s$  in the range where  $v_s$  is expected to be. From the physical understanding of the friction effects, we know that the Stribeck velocity  $v_s$  is a velocity at which the friction torque drops before it starts to increase as the velocity increases, see Fig. 1 for an example. This figure shows the steady-state characteristic of the LuGre model for  $\alpha_0 = 0.0194$ ,  $\alpha_1 = 0.0055$ ,  $\alpha_2 = 0.0394$ ,  $\sigma_0 = 20$ ,  $\sigma_1 = 0.02$  and for three distinct values of  $v_s$ .

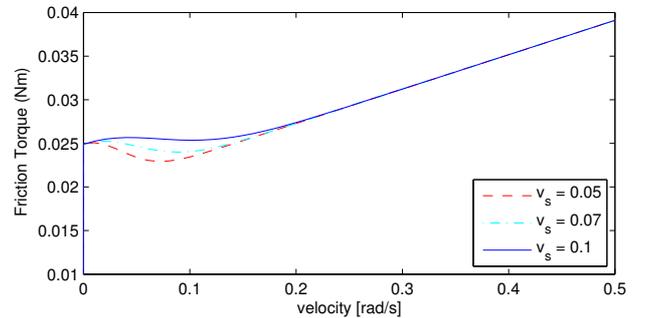


Fig. 1. The friction force as a function of the joint velocity according to the LuGre model (steady-state).

For each  $v_s$ , the parameter vector  $\mathcal{P}_f = (\alpha_0, \alpha_1, \alpha_2)^T$  is estimated by:

$$\mathcal{P}_f = (\mathcal{A}_f^T \mathcal{A}_f)^{-1} \mathcal{A}_f^T \mathcal{T}_f \quad (13)$$

For each estimate, we compute the cost function  $J_s$  and choose ( $\alpha_0, \alpha_1, \alpha_2, v_s$ ) corresponding to the lowest  $J_s$ .

*Estimation of Dynamic Parameters.* A method to estimate the dynamic parameters  $\sigma_0$  and  $\sigma_1$  was proposed in [Waiboer, 2007] and [Lischinsky et al., 1999]. Supply a very slow ramp input (voltage) to the open-loop system. This results in a simplification of (3) into the form  $T_f = \sigma_0 z$ . We take the first displacement measured by the sensor (denoted by  $x_1$ ) and the applied torque (denoted by  $T_1$ ) at that moment, so  $\sigma_0$  is approximated as

$$\sigma_0 = \frac{T_1}{x_1} \quad (14)$$

The  $\sigma_1$  parameter is estimated by using an empirical equation based on observations in practical experiments [Waiboer, 2007]:

$$\sigma_1 = 0.2\sqrt{J\sigma_0} \quad (15)$$

### 3.3 On-Line Parameter Adaptation

Friction characteristics can change due to changes in environmental factors such as temperature, normal force, lubricant viscosity, etc. These changes cause a mismatch between the friction model, with the parameters estimated off-line, and the actual friction. Therefore, the friction parameters should be updated on-line in order to have a reliable estimate of the friction torque.

Updating the friction model parameters on-line requires an adaptation mechanism. However, only those parameters that have significant influence on the LuGre model accuracy are updated. Sensitivity analysis revealed that parameters  $\alpha_0$  (Coulomb friction) and  $\alpha_2$  (viscous friction) have the most significant influence on the accuracy of the LuGre model. Therefore, we update these two parameters only. A modified friction model is formulated as follows:

$$T_f = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \theta_2 \alpha_2 v \quad (16)$$

$$\frac{dz}{dt} = v - \frac{\sigma_0}{g(v)} z |v| \quad (17)$$

$$g(v) = \theta_1 \alpha_0 + \alpha_1 e^{-(v/v_s)^2} \quad (18)$$

where the gains  $\theta_1$  and  $\theta_2$  are updated according to the following law:

$$\begin{aligned} \frac{d\theta_1}{dt} &= -\gamma_1 \frac{\partial T_f}{\partial \alpha_0} e \\ &= -\gamma_1 \frac{\sigma_0 \sigma_1 |v| z e^{-(v/v_s)^2}}{(\alpha_0 + \alpha_1 e^{-(v/v_s)^2})^2} e \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{d\theta_2}{dt} &= -\gamma_2 \frac{\partial T_f}{\partial \alpha_2} e \\ &= -\gamma_2 v e \end{aligned} \quad (20)$$

with the learning rates  $\gamma_1, \gamma_2 \in [0, 1]$ .

## 4. FRICTION COMPENSATION SCHEME

To compensate for friction, the friction torque is simply computed by the model and added to the control signal. There are two possibilities to compute the velocity which is needed as an input of the friction model: *feed-forward scheme*, in which the velocity is computed from the reference signal, and *feedback scheme*, in which the velocity is computed from the measured joint angle (or it is measured by a joint velocity sensor).

According to Alpeter [1999], better results were obtained with the feed-forward compensation scheme than with the feedback scheme. Therefore, we use the feed-forward scheme in this research. Combining the feed-forward compensation scheme and the adaptation law, the overall friction compensation scheme is obtained as shown in Fig. 2.

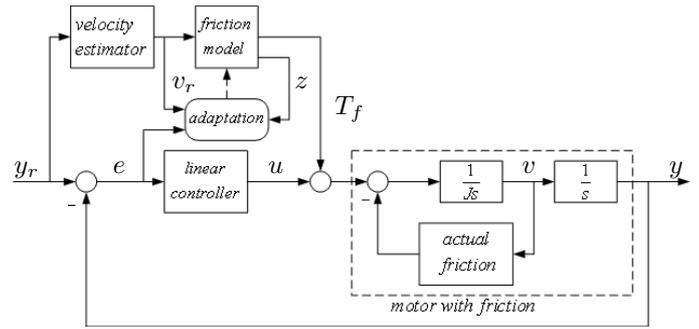


Fig. 2. Adaptive feed-forward friction compensation scheme.

## 5. EXPERIMENTAL RESULTS

The friction compensation method is applied to the Ed-Ro robotic manipulator. This lightweight robot has been designed at the Delft University of Technology as an experimental platform for research and education. This robot has five rotational joints as indicated in Fig. 3.

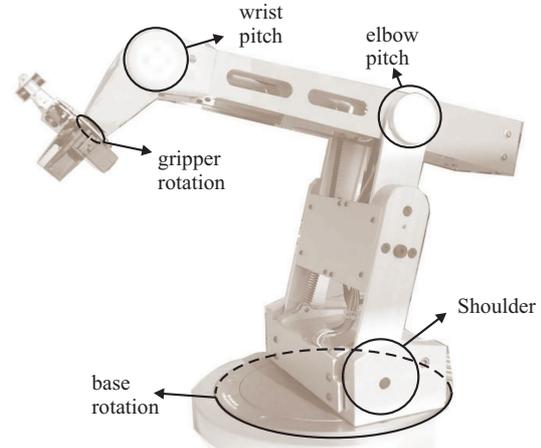


Fig. 3. Ed-Ro - a small experimental robotic manipulator.

By using the off-line parameter-estimation method described in Section 3, we gathered the friction-velocity data and calibrated the friction model for each joint. An example of the results obtained is given for the gripper rotation in Fig. 4.

The friction model parameters for all the joints of the Ed-Ro robot are given in Table 1.

The friction compensator is used in conjunction with the standard PID controller

$$C(s) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \quad (21)$$

with  $K_p = 3$ ,  $K_d = 0.2$ ,  $K_i = 1.2$ . The entire control scheme was implemented in Simulink, using continuous-time blocks and the ode45 integration method. The robot was interfaced to the computer via a serial port. Synchronization with real time was ensured by the `rtsync` block of the Real-Time Toolbox for Matlab by Humusoft.

Real-time experimental results with a sinusoidal reference signal for the gripper rotation are reported in Figures 5 through 7.

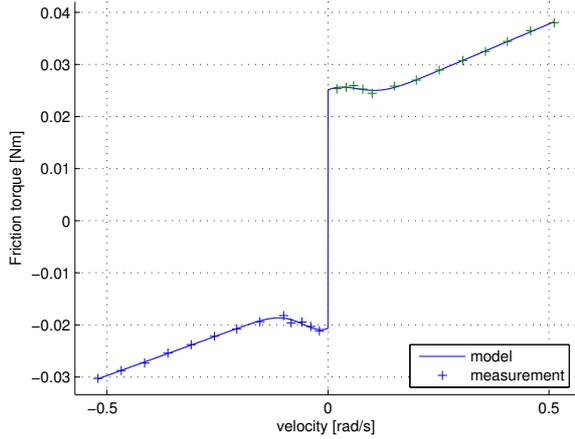


Fig. 4. Friction model for gripper rotation.

Table 1. Parameters of the LuGre model for the Ed-Ro robot for positive (+) and negative (-) velocity.

Friction Parameters		Gripper rotation	Wrist pitch	Elbow pitch	Base rotation
$\alpha_0$ (Nm)	(+)	0.0194	0.0704	0.1018	0.2371
	(-)	0.0148	0.1058	0.0957	0.1731
$\alpha_1$ (Nm)	(+)	0.0055	0.0145	0.0514	0.0285
	(-)	0.0040	0.0107	0.0209	0.1098
$\alpha_2$ (Nms/rad)	(+)	0.0394	0.1362	0.2216	0.1394
	(-)	0.0307	0.1200	0.1495	0.1338
$v_s$ (rad/s)	(+)	0.0720	0.0320	0.1230	0.0750
	(-)	0.0720	0.0710	0.0450	0.0620
$\sigma_0$ (Nm/rad)	(+)	6.1843	38.3236	59.3440	70.5845
	(-)	4.5356	40.0495	38.4564	50.2721
$\sigma_1$ (Nms/rad)	(+)	0.0184	0.1168	0.2579	0.4673
	(-)	0.0146	0.1162	0.1874	0.3908

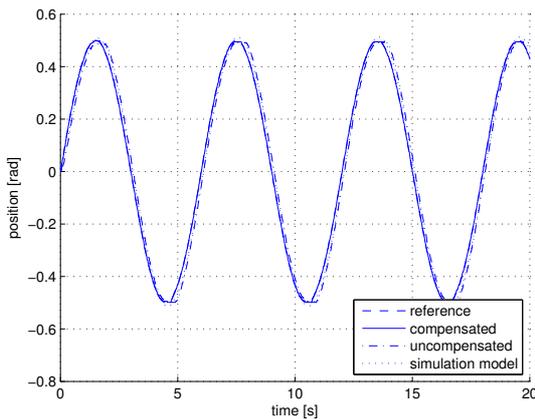


Fig. 5. Experiment with gripper rotation, position reference and output.

In Fig. 6, one can observe that friction compensation reduces the tracking error significantly. The same procedure was repeated for the other joints and the results are summarized in Fig. 8. Note that the RMS error between the reference and the actual output drops significantly under friction compensation.

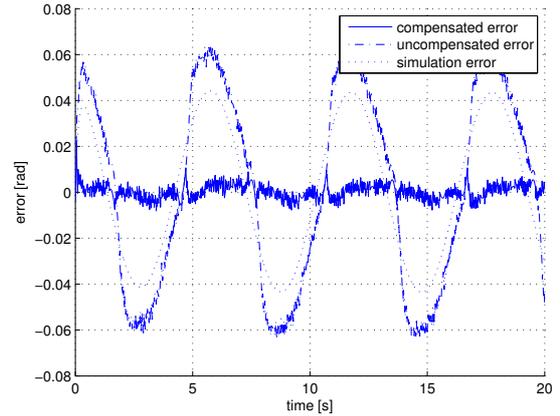


Fig. 6. Experiment with gripper rotation, position error.

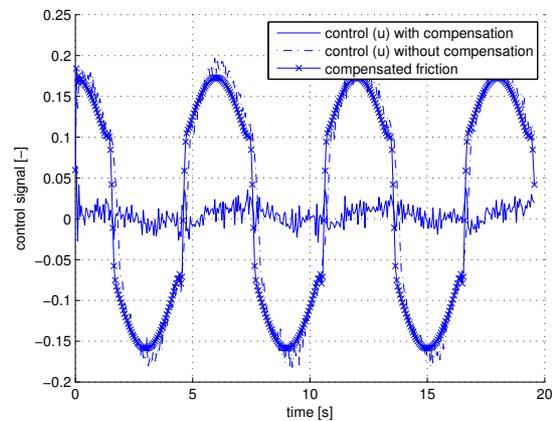


Fig. 7. Experiment with gripper rotation, control signal.

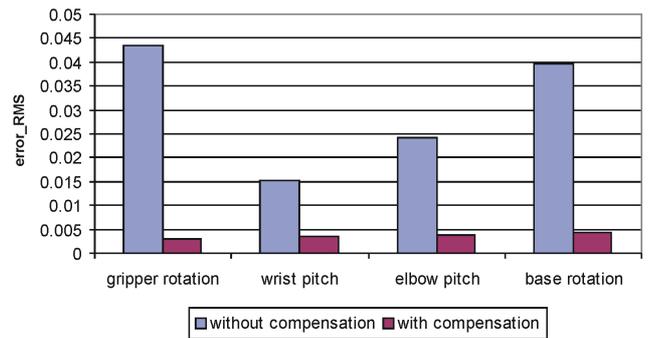


Fig. 8. An overview the RMS error for the different joints.

An experiment was designed to demonstrate the benefits of adaptive friction compensation. The quality of the friction model for the wrist rotation joint is purposely deteriorated by multiplying the initial values of its adaptable parameters by random constants as shown in Table 2.

Experimental results obtained with the adaptive compensation scheme are given in Figures 9 through 12. In Fig. 9, one can see that the adaptation gains converge to recover the true parameters. Figure 11 shows that the tracking error is initially large, but it rapidly decreases as the parameters converge within the first 4 seconds.

Table 2. Friction model parameter variation.

Original parameters	Initial parameters	Adapted parameters
$\alpha_0 = 0.0194$	$0.3\alpha_0$	$0.84\alpha_0$
$\alpha_1 = 0.0055$	$\alpha_1$	$\alpha_1$
$\alpha_2 = 0.0394$	$0.5\alpha_2$	$1.05\alpha_2$
$v_s = 0.0720$	$v_s$	$v_s$
$\sigma_0 = 6.1843$	$\sigma_0$	$\sigma_0$
$\sigma_1 = 0.0184$	$\sigma_1$	$\sigma_1$

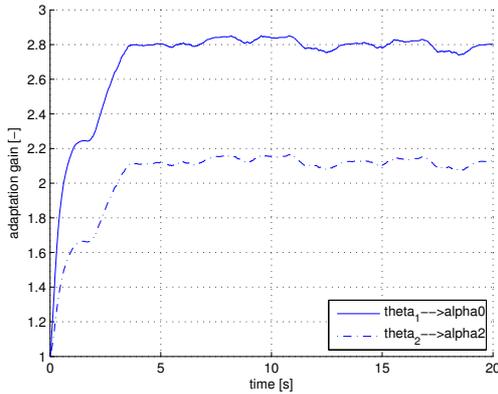


Fig. 9. Adapted gains  $\theta_1$  and  $\theta_2$ .

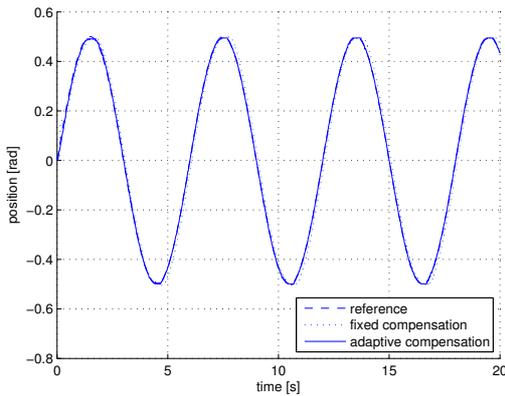


Fig. 10. Experiment with gripper rotation, position output.

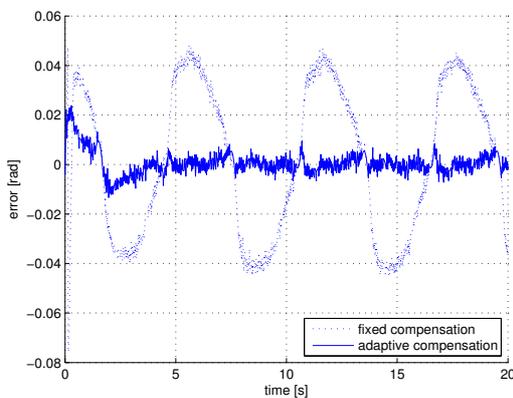


Fig. 11. Experiment with gripper rotation, position error.

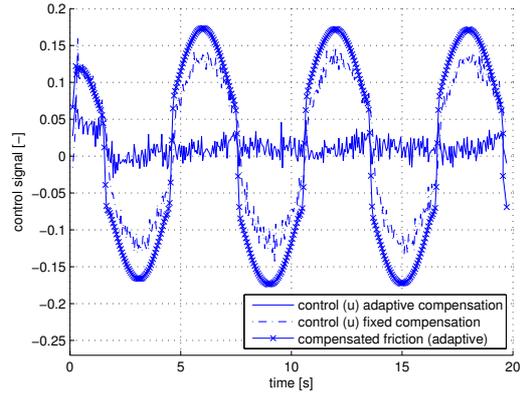


Fig. 12. Experiment with gripper rotation, control signal.

## 6. CONCLUSIONS

An application of model-based friction compensation to a robotic manipulator has been presented. The design procedure for the model-based compensation scheme involves the computation of the friction torque and off-line parameter estimation for the LuGre model structure. A straightforward on-line adaptation method for the Coulomb and viscous friction model coefficients has been presented to cope with environmental changes that may affect the friction characteristics. Real-time experimental results show that friction compensation yields a significant improvement, indicated by the reduction of the tracking error signal.

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