

# Model-Plant Mismatch Detection in MPC Applications using Partial Correlation Analysis

Abhijit S. Badwe\* Sirish L. Shah\*\*,<sup>1</sup> Sachin C. Patwardhan\*  
Rohit S. Patwardhan\*\*\*

\* *Department of Chemical Engineering, Indian Institute of Technology,  
Bombay, Mumbai 400076, India*

\*\* *Chemical and Materials Engineering Department, University of  
Alberta, Edmonton, Canada, T6G 2G6.*

\*\*\* *Matrikon Inc., Suite 1800, 10405 Jasper Avenue, Edmonton,  
Canada, T5J 3N4*

---

**Abstract:** In model predictive control of processes, the process model plays an important role. The performance of the controller depends on the quality of the model and hence on the model-plant mismatch. Although model-plant mismatch is inevitable, it is highly desirable to minimize it. For processes with large number of inputs and outputs, re-identification of the model is a costly exercise as keeping a large number of inputs in a perturbed or excited state for a long time means loss of normal production time. Hence, it would be highly desirable to detect the precise location of the mismatch so that only a few inputs would have to be perturbed and only the degraded portion of the model updated. In this work, a methodology is proposed for the detection of mismatch from closed-loop operating data. The proposed methodology is based on the analysis of partial correlations between the model residuals and the manipulated variables. Its efficacy is demonstrated on two simulation case studies as well as its application to data from an industrial process.

Keywords: Model Predictive Control, Performance Assessment, Model-Plant Mismatch, Partial Correlations Analysis

---

## 1. INTRODUCTION

Model predictive control (MPC) has been widely used in the process industry over the last two decades for controlling key unit operations in chemical plants. As a consequence, there has been significant research activity in the process control community with the aim of improving the analysis and synthesis of MPC controllers (Morari and Lee, 1999) and also towards developing techniques for performance assessment of existing controllers (Shah et al, 2002). The main motivation for this is economic as the existing MPCs are expected to cope with tight product specifications, improved quality and reduction in waste (Tsakalis and Dash, 2007).

The foundations of the research on controller performance assessment have been laid by Harris (1989), who proposed a performance benchmark based on the performance of a minimum variance controller (Astrom, 1970). Since then there have been significant research efforts in this area for which Kozub (1996), Qin (1998) and Huang and Shah(1999) have presented comprehensive surveys. Although several benchmarks for performance assessment have been developed, very few researchers have addressed the performance diagnostics aspect. The work of Patwardhan and Shah (2002) focusses on the performance

<sup>1</sup> Corresponding author. Tel +1 780 492 5162; Fax +1780 492 2881; Email sirish.shah@ualberta.ca

diagnostics of MPC controllers. They have tried to quantify the effect of constraints, modelling uncertainty and nonlinearity on the performance of linear MPCs.

MPC determines the optimal input moves by solving an optimization problem in which, the objective function makes use of predicted outputs over a finite horizon. Since a model of the process is used for generating these predictions, the quality of the model affects the closed-loop performance. The model used in MPC is usually identified at the commissioning stage of the plant. This model is never exact and contains some mismatch with the plant. Over time, several changes can occur in the process during the MPC operation. Moreover, most of the chemical processes are inherently nonlinear and linear models representing these processes are valid only over a limited range around the operating point. Thus, changes in the plant dynamics and nonlinearities in the process widen the gap between the model and the plant which in turn may lead to a degradation in MPC performance. Hence, it would be desirable to detect such a mismatch between the model and the plant and correct it by updating the model. However, updating the model requires re-identification which in turn asks for intrusive plant tests. Such tests disturb the normal operation of the plant and hence have economic repercussions. Therefore, it would be highly desirable to identify only that part or subsystem of the plant (or model), where significant mismatch occurs

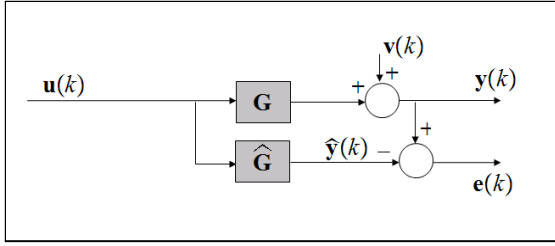


Fig. 1. Open Loop System

so that only inputs or setpoints related to that part of the plant will have to be perturbed if the need for re-identification arises.

In this work, we propose a methodology based on the analysis of partial correlations between the model residuals and manipulated variables (MVs) for the detection and isolation of model-plant mismatch. The methodology uses closed-loop operating data for the analysis and does not ask for intrusive tests on the plant. The methodology may be considered to be a part of a complete procedure for MPC controller performance assessment and diagnostics. The proposed methodology would be invoked after a deterioration in controller performance is detected and when it is confirmed that the root cause of poor MPC performance is significant model-plant mismatch.

The paper is organized as follows: In Section 2 the problem of mismatch detection and the associated challenges are presented. It also briefly gives the motivation for using partial correlation analysis for the problem at hand. A brief introduction to partial correlation analysis is presented in Section 3. In Section 4, the proposed methodology is discussed. The application of the proposed methodology is demonstrated on three case studies in Section 5. Initially, the efficacy of the methodology is evaluated on two simulation case studies followed by its application to an industrial case study. Section 6 gives concluding remarks and future directions.

## 2. PROBLEM DEFINITION

Suppose that  $\hat{\mathbf{G}}$  is a model representing the  $n \times m$  MIMO plant  $\mathbf{G}$ . Let  $\Delta = \mathbf{G} - \hat{\mathbf{G}}$  be the model-plant mismatch or model uncertainty. Now consider the open-loop situation of Figure 1, where  $\mathbf{y}(k)$  and  $\hat{\mathbf{y}}(k)$  are the plant and model output vectors respectively,  $\mathbf{u}(k)$  is the vector of uncorrelated inputs or manipulated variables (MVs),  $\mathbf{e}(k) = \mathbf{y}(k) - \hat{\mathbf{y}}(k)$  is the vector of model residuals and  $\mathbf{v}(k)$  is the vector of Gaussian disturbances acting on the process. The model residuals can then be written as,

$$\begin{aligned} \mathbf{e}(k) &= \mathbf{y}(k) - \hat{\mathbf{y}}(k) \\ &= \Delta \mathbf{u}(k) + \mathbf{v}(k) \end{aligned} \quad (1)$$

From the above expression it is clear that a correlation analysis between signals  $\mathbf{e}$  and  $\mathbf{u}$  would give the extent of mismatch  $\Delta$ . Thus, if  $\Delta_{i,j}$  was significant i.e. the mismatch in channel  $y_i-u_j$  was significant, then it would be seen through a significant correlation between  $e_i$  and  $u_j$ .

Now consider the closed-loop IMC structure of Figure 2. Here  $\mathbf{Q}$  is a multivariable controller, the design of which is

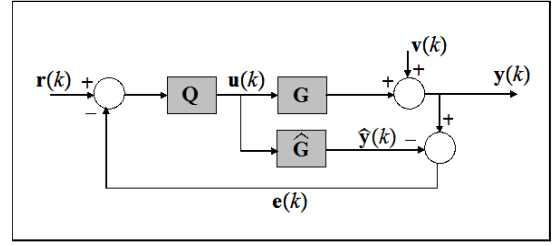


Fig. 2. Closed-Loop System: IMC Structure

based on the model  $\hat{\mathbf{G}}$  and  $\mathbf{r}(k)$  is the vector of setpoints. Then, the following expressions can be written

$$\mathbf{e}(k) = [\mathbf{I} + \Delta \mathbf{Q}]^{-1} \Delta \mathbf{Q} \mathbf{r}(k) + [\mathbf{I} + \Delta \mathbf{Q}]^{-1} \mathbf{v}(k) \quad (2)$$

$$\mathbf{u}(k) = \underbrace{\mathbf{Q} [\mathbf{I} + \Delta \mathbf{Q}]^{-1} \mathbf{r}(k)}_{\mathbf{S}_{ru}} - \underbrace{\mathbf{Q} [\mathbf{I} + \Delta \mathbf{Q}]^{-1} \mathbf{v}(k)}_{\mathbf{S}_{vu}} \quad (3)$$

where,  $\mathbf{S}_{ru}$  and  $\mathbf{S}_{vu}$  are the *input sensitivities* from  $\mathbf{r}$  and  $\mathbf{v}$  respectively. If from Eq.3,  $\mathbf{Q} \mathbf{r}(k)$  is substituted in Eq.2, the expression of Eq.1 is obtained.

In closed-loop operation, each MV in  $\mathbf{u}$  is computed at every instant by the multivariable controller  $\mathbf{Q}$ . The computation for each MV is based on the same error vector. Depending on the design of the controller, this may lead to correlations between the MVs. Such correlations amongst the MVs may confound the regular correlation analysis between the model residuals and the MVs. This in turn may lead to misleading information regarding the existence (or non-existence) of a significant correlation between the model residual and MV(s) under consideration and hence regarding the location of significant mismatch. To overcome this, we propose to use partial correlation analysis with an overall objective of isolating those input-output channels that contain significant mismatch.

*Remark 1.* If  $\mathbf{r} = 0$  i.e. there are no setpoint changes, then from Eqs. 2 and 3,  $\mathbf{e}(k)$  is related to  $\mathbf{u}(k)$  as,

$$\mathbf{e} = -\mathbf{Q}^{-1} \mathbf{u} \quad (4)$$

Thus, in the absence of setpoint changes, it would not be possible to determine the extent of mismatch. However, in a typical MPC operation, the targets (i.e. setpoints) are regularly computed by the upper LP (Linear Programming) layer (Shah et al, 2002). Hence, it can be safely assumed that sufficient setpoint excitation exists due to these specifications. Therefore, if data during periods of sufficient setpoint excitation is chosen, the channels containing mismatch can be effectively isolated.

## 3. PARTIAL CORRELATION ANALYSIS

Linear relationships between two or more variables can be detected by performing correlation analysis. However, if the causal or independent variables are correlated amongst themselves, correlation analysis may give misleading results - it may show correlation between two variables when none exists (spurious correlation due to the effect of other variables) or it may show zero correlation when one exists in reality (masking of true correlation).

Partial correlation analysis helps spot spurious correlations as well as to reveal hidden correlations. Partial correlation analysis has been extensively used in fields

ranging from social sciences to bio-informatics. Recently, Gudi and Rawlings (2006) have used partial correlation analysis for isolating interacting channels in identification for decentralized MPC. In principle, partial correlation analysis involves the determination of correlation between two variables with the effects from other variables removed (Smillie, 1966). Suppose that  $X_1, X_2, \dots, X_n$  are variables affecting a variable  $Z$  and suppose that the  $X_i$ 's are correlated with each other. Then, to evaluate the partial correlation between,  $Z$  and  $X_1$ , for example,  $Z$  and  $X_1$  are first linearly regressed on  $X_2, X_3, \dots, X_n$ ,

$$Z = \mathbf{X}\theta_Z + E_Z \quad (5)$$

$$X_1 = \mathbf{X}\theta_{X_1} + E_{X_1} \quad (6)$$

where,  $\mathbf{X} = [X_2 \ X_3 \ \dots \ X_n]$  is the matrix of regressors,  $\theta_Z$  and  $\theta_{X_1}$  are the vectors of regression coefficients. Next, the prediction errors for the models in Eqs. 5 and 6 are evaluated,

$$e_Z = Z - \mathbf{X}(\mathbf{X}\mathbf{X}^T)^{-1}\mathbf{X}^T Z \quad (7)$$

$$e_{X_1} = X_1 - \mathbf{X}(\mathbf{X}\mathbf{X}^T)^{-1}\mathbf{X}^T X_1 \quad (8)$$

$e_Z$  and  $e_{X_1}$  above are those components of  $Z$  and  $X_1$  respectively that are free of effects from  $\mathbf{X}$ . The partial correlation between  $Z$  and  $X_1$  is then given by the regular correlation between  $e_Z$  and  $e_{X_1}$ .

#### 4. PROPOSED METHODOLOGY FOR MODEL PLANT MISMATCH DETECTION

As discussed earlier, if the MVs are correlated with each other, a regular correlation analysis between the model residuals and the MVs would give misleading results. This would lead to detection of mismatch in channels that in reality do not contain any mismatch. Hence, we propose to use partial correlation analysis. However, for the problem at hand, partial correlations analysis in its usual form, as described in the previous section, cannot be used for two reasons:

- (1) We wish to analyze partial correlations in a dynamic sense because the variables under consideration (MV's and model residuals) are time series variables.
- (2) Also, as seen from Eq. 2 and Eq. 3, the model residuals and the MVs contain the effects of the unmeasured disturbances ( $\mathbf{v}$ ). This may confound the analysis of partial correlations as the effect of disturbance may get added up to the effect of model-plant mismatch. In particular, even if  $\Delta = 0$ , this may result in a non-zero partial correlation between the model residuals and the MVs.

One approach to resolve the first issue would be to make use of dynamic models in the regression step. For example models based on the Prediction Error Method (PEM) of Ljung (1999) may be used for this purpose. To address the second issue, however, the MVs first need to be freed from effects of the disturbances. This can be done by finding that component of each MV that contains effects of the setpoints only. This is discussed in more detail below. The various steps in the proposed methodology are as follows:

- (1) Choose data (model residuals and MVs) from the period where there is sufficient setpoint excitation in the process.
- (2) Find the disturbance free components of the MVs: The expression in Eq. 3 can be written as,

$$\begin{aligned} \mathbf{u}(k) &= \mathbf{S}_{\mathbf{r}\mathbf{u}}\mathbf{r}(k) - \mathbf{S}_{\mathbf{v}\mathbf{u}}\mathbf{v}(k) \\ &= \mathbf{u}^{\mathbf{r}}(k) + \mathbf{u}^{\mathbf{v}}(k) \end{aligned} \quad (9)$$

where  $\mathbf{u}^{\mathbf{r}}$  and  $\mathbf{u}^{\mathbf{v}}$  are those components of  $\mathbf{u}$  that contain effects of the setpoints and the disturbances respectively. The component  $\mathbf{u}^{\mathbf{r}}$  is free from disturbances because  $\mathbf{r}$  and  $\mathbf{v}$  are uncorrelated. Initially,  $\mathbf{S}_{\mathbf{r}\mathbf{u}}$  is identified as,

$$\mathbf{u}(k) = \widehat{\mathbf{S}}_{\mathbf{r}\mathbf{u}}\mathbf{r}(k) + \widehat{\mathbf{S}}_{\mathbf{v}\mathbf{u}}\mathbf{v}(k) \quad (10)$$

and  $\mathbf{u}^{\mathbf{r}}$  is reconstructed as,

$$\widehat{\mathbf{u}}^{\mathbf{r}}(k) = \widehat{\mathbf{S}}_{\mathbf{r}\mathbf{u}}\mathbf{r}(k) \quad (11)$$

- (3) Decorrelate  $\widehat{u}_i^{\mathbf{r}}$  and the rest of the MVs:

For this, we find that component of  $\widehat{u}_i^{\mathbf{r}}$  that is uncorrelated with the rest of the MVs. Initially, a model is identified between  $\widehat{u}_i^{\mathbf{r}}$  and all other MVs,

$$\widehat{u}_i^{\mathbf{r}}(k) = \mathbf{G}_{u_i}\widehat{\mathbf{u}}^{\mathbf{r}}(k) + \epsilon_{u_i}(k) \quad (12)$$

where  $\widehat{\mathbf{u}}^{\mathbf{r}}$  contains all MVs except  $\widehat{u}_i^{\mathbf{r}}$  and  $\epsilon_{u_i}$  is that component of  $\widehat{u}_i^{\mathbf{r}}$  that is uncorrelated with the MVs in  $\widehat{\mathbf{u}}^{\mathbf{r}}$ . An estimate of  $\epsilon_{u_i}$  is obtained as,

$$\widehat{\epsilon}_{u_i}(k) = \widehat{u}_i^{\mathbf{r}}(k) - \mathbf{G}_{u_i}\widehat{\mathbf{u}}^{\mathbf{r}}(k) \quad (13)$$

Essentially,  $\widehat{\epsilon}_{u_i}$  is that component of the MV  $u_i$  that is free of effects from disturbances (due to step 2) as well as the other MVs.

- (4) Similarly, decorrelate model residual,  $e_j$  and all MVs except  $u_i$ .

$$e_j(k) = \mathbf{G}_{e_j}\widehat{\mathbf{u}}^{\mathbf{r}}(k) + \epsilon_{e_j}(k) \quad (14)$$

and obtain an estimate of  $\epsilon_{e_j}$ ,

$$\widehat{\epsilon}_{e_j}(k) = e_j(k) - \mathbf{G}_{e_j}\widehat{\mathbf{u}}^{\mathbf{r}}(k) \quad (15)$$

- (5) Evaluate the correlation between  $\widehat{\epsilon}_{u_i}$  and  $\widehat{\epsilon}_{e_j}$ . A non-zero correlation between  $\widehat{\epsilon}_{u_i}$  and  $\widehat{\epsilon}_{e_j}$  indicates the presence of model-plant mismatch in the  $u_i$ - $y_j$  channels. The more significant this correlation, the more significant is the model-plant mismatch.

*Remark 2.* The analysis above considers data sets of finite lengths with random variations due to disturbances. This may lead to the partial correlation being non-zero even when no correlation exists. The partial correlations can then be tested for significance using the t-test, which gives the probability of getting a correlation as large as the observed value due to random errors or disturbances when the actual correlation is zero.

#### 5. CASE STUDIES

The methodology discussed in the previous section was applied to two simulation cases and an industrial process. Simple scenarios were considered in Case Study 1 such as mismatch being present in only one channel whereas the other channels contain zero mismatch. Simulation Case Study 2 is the benchmark Shell Control Problem. This case study was chosen for the large time delays and multi-variable interactions contained therein. In a real situation, mismatch would be present in all channels. The challenge for the proposed methodology would then be to isolate

all channels containing significant mismatch. Case Study 2 was used for simulating such situations. Finally, the proposed methodology was tested on data obtained from a Kerosene Hydrofiner Unit (KHU) at Suncor Energy's Oilsands plant in Fort McMurray, Alberta.

All results are presented in the form of partial correlation plots (and regular correlation plots in some cases). The probabilities associated with the computed correlations were computed for all cases. However, these probabilities are presented only for Case Study 1 because for all other cases these probabilities are zero because of presence of mismatch in all channels.

### 5.1 Simulation Case Study 1

This case study is an example of a 3CVs×4MVs×1FF system. For simulating the discrete process, a sampling period of 1 min. was used. Closed-loop simulations were carried out using MPC. A state-space formulation of MPC (Muske and Rawlings, 1993) was used. Prediction and control horizons of 30 and 10 respectively were used. All CVs were equally weighted (equal concern errors = 1) and all MVs were equally weighted (move suppression factor=0). MV4 was constrained at its lower limit effectively reducing the controller to a 3 × 3 MPC. The 3 × 3 model is as follows:

$$y(k) = \begin{bmatrix} \frac{4.05}{50s+1}e^{-6s} & \frac{1.77}{60s+1}e^{-7s} & \frac{5.88}{50s+1}e^{-6s} \\ \frac{5.39}{50s+1}e^{-4s} & \frac{5.72}{60s+1}e^{-3s} & \frac{6.9}{40s+1}e^{-3s} \\ \frac{4.38}{33s+1}e^{-5s} & \frac{4.42}{44s+1}e^{-5s} & \frac{7.2}{19s+1} \end{bmatrix} \quad (16)$$

Typical constraints on the MVs were (-20,20). Simulations were carried out for 1440 samples or 1 day's worth data. Various cases were simulated to depict realistic situations such as gain, delay and time constant mismatch of which two are presented here.

*Scenario 1: Gain mismatch* In this case, mismatch was added in such a way as to create a situation where the MV1-CV1 and MV1-CV2 gains are underestimated by 50%. Partial correlations between the model residuals and the MVs were evaluated as discussed in the previous section. These partial correlations are plotted in Figure 3. For the purpose of comparison, the regular correlations between each residual and each MV are also plotted in Figure 4. The significant partial correlations between  $e_1$  and MV1 and  $e_2$  and MV1 correctly point out the mismatch located in MV1-CV1 and MV1-CV2 channels (red boxes). On the other hand, the regular correlations seem to give misleading information regarding the location of mismatch as can be seen from the significant correlation values for all channels with respect to CV1 and CV2. The probabilities obtained from the t-test on the partial correlations and regular correlations, are presented for 1<sup>st</sup> lag along with the correlation coefficients in Tables 1 and 2 respectively. The numbers are the partial (or regular) correlation coefficients for 1<sup>st</sup> lag and the numbers in brackets are the associated probabilities. These probabilities were obtained using the `corrcoef` function in MATLAB. In Tables 1 and 2, the bold figures represent channels that contain significant correlations. Clearly for the channels

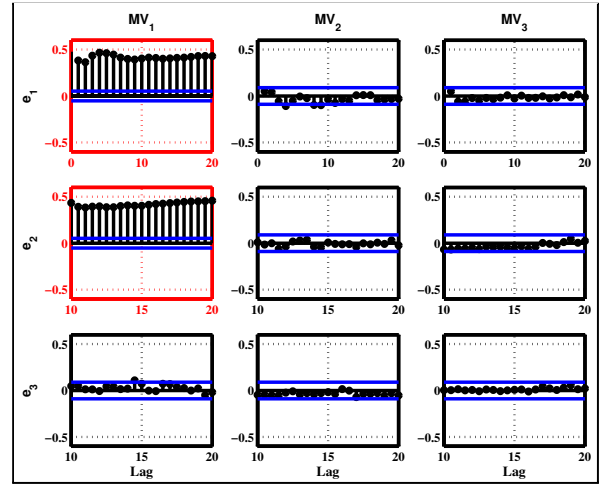


Fig. 3. Case Study 1: Gain Mismatch -Partial correlations plots - Mismatch correctly located (red boxes)

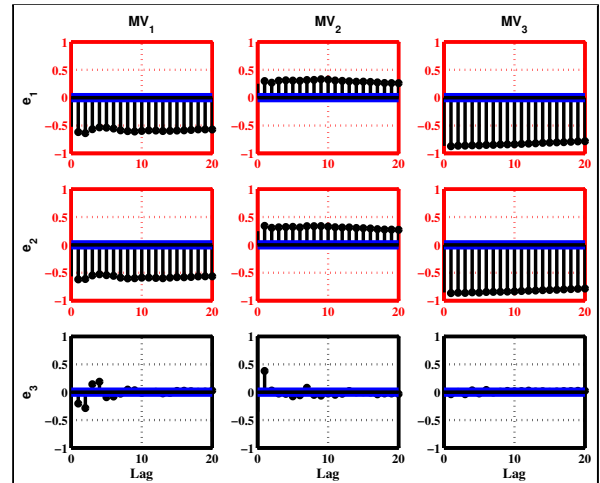


Fig. 4. Case Study 1: Gain Mismatch - Regular correlations between residuals and MVs

**Table 1. Case Study1-Gain Mismatch: Partial correlation coefficients and associated probabilities (in brackets) for 1<sup>st</sup> lag**

	MV1	MV2	MV3
$e_1$	<b>0.38 (0)</b>	0.05 (0.57)	0.075 (0.14)
$e_2$	<b>0.4(0)</b>	-0.025(0.33)	-0.039 (0.13)
$e_3$	0.011 (0.97)	-0.013(0.61)	0.01(0.53)

that contain mismatch, the probabilities associated with the partial correlation coefficients are zero and are large for the other channels, indicating that whatever small partial correlations are seen are spurious correlations. Also, for the regular correlation analysis the probabilities associated with the correlations for  $e_1$  and  $e_2$  are zero. This is a result of the correlation between the MVs.

*Scenario 2: Delay mismatch* Here, a (underestimated) delay mismatch of 2 samples was introduced in all channels from MV1 i.e. MV1-CV1,CV2,CV3. The partial correlation plots for this case are plotted in Figure 5. The 1<sup>st</sup> lag partial coefficients and their associated probabilities are presented in Table 3. The mismatches present in channels

**Table 2. Case Study1-Gain Mismatch: Regular correlation coefficients and associated probabilities (in brackets) for 1st lag**

	MV1	MV2	MV3
$e_1$	-0.63(0)	0.29(0)	-0.88(0)
$e_2$	-0.62(0)	0.28(0)	-0.87(0)
$e_3$	-0.006(0.81)	-0.02(0.42)	0.003(0.92)

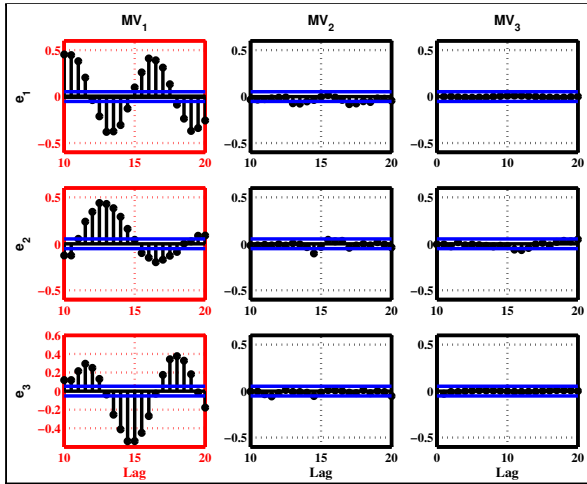


Fig. 5. Case Study 1: Delay Mismatch - Partial correlation plots

**Table 3. Case Study1-Delay mismatch: Partial correlation coefficients and associated probabilities for 1st lag**

	MV1	MV2	MV3
$e_1$	0.45(0)	-0.001(0.45)	7.5e-4(0.64)
$e_2$	-0.2(0)	-2e-4(0.43)	-2e-4(0.52)
$e_3$	0.18(0)	-1.3e-4(0.61)	-1.4e-3(0.39)

MV1-CV1,CV2,CV3 are revealed by the significant partial correlation values for these channels.

5.2 Simulation Case Study 2: The Shell Control Problem

The Shell Control Problem is a benchmark problem proposed at the Shell Process Control Workshop (Prett and Morari, 1987) and involves control of a heavy oil fractionator system characterized by large time delays in each input output pair. The heavy oil fractionator has three product draws, three side circulating loops and a gaseous feed stream. The system consists of seven measured outputs, three manipulated inputs and two unmeasured disturbances. Product specifications for top and side draws are determined by economic considerations. There is no product specification on bottom draw, however, there is an operating constraint on the bottom reflux temperature. Top draw, side draw and bottoms reflux duty can be used as manipulated variables to control the column while heat duties on the two other side loops (upper reflux duty and intermediate reflux duty) act as unmeasured disturbances to the column. A schematic of the fractionator is shown in Figure 6.

Since the controlled outputs of interest are top end point, side end point and bottoms reflux temperature, in this work we consider a subsystem consisting of only these

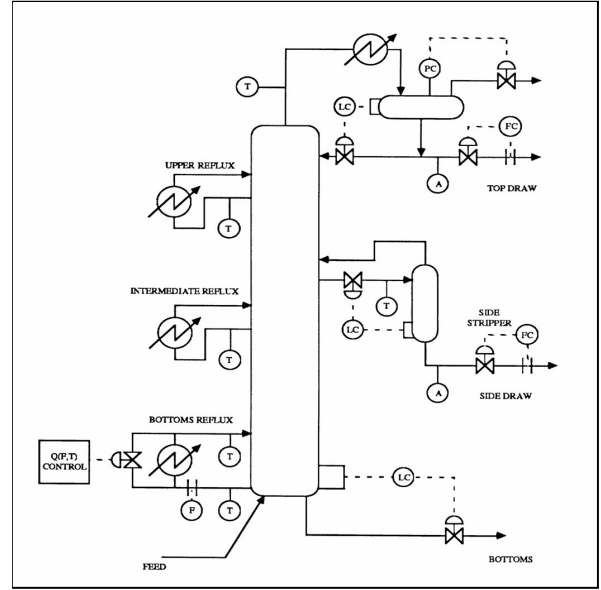


Fig. 6. A schematic of the Shell heavy oil fractionator three outputs. Further, the process dynamics are simulated under following assumptions (Patwardhan et al, 2006)

- Manipulated inputs are piecewise constant
- Disturbances entering the plant can be adequately represented using piecewise constant functions

Under these assumptions, a discrete dynamic model of the form

$$\hat{y}(z) = G_p(z)u(z) + G_d(z)d(z) \quad (17)$$

is developed with sampling time ( $T$ ) equal to 2 minutes. A minimal order state space realization of (17) of the form

$$\mathbf{X}(k+1) = \mathbf{A}\mathbf{X}(k) + \mathbf{B}_u\mathbf{u}(k) + \mathbf{B}_d\mathbf{d}(k) \quad (18)$$

$$\hat{y}(k) = \mathbf{C}\mathbf{X}(k) \quad (19)$$

with 51 state variables is used for simulation of process behavior. The stationary unmeasured disturbances  $d(z)$  are assumed to be generated by the following stochastic process

$$\mathbf{x}_w(k+1) = \mathbf{A}_w\mathbf{x}_w(k) + \mathbf{B}_w\mathbf{w}(k) \quad (20)$$

$$\mathbf{d}(k) = \mathbf{C}_w\mathbf{x}_w(k) + \mathbf{D}_w\mathbf{w}(k) \quad (21)$$

$$\mathbf{A}_w = \mathbf{C}_w = 0.95 I ; \mathbf{B}_w = \mathbf{D}_w = I \quad (22)$$

or equivalently by

$$\mathbf{d}(z) = \begin{bmatrix} z & 0 \\ z - 0.95 & z \\ 0 & z - 0.95 \end{bmatrix} \mathbf{w}(z) \quad (23)$$

where  $\mathbf{w} \in R^2$  is a zero mean normally distributed white noise process with  $\sigma_{w1} = \sigma_{w2} = 0.0075$ . In addition, the measured outputs are assumed to be corrupted with measurement noise

$$\mathbf{y}(k) = \hat{y}(k) + \mathbf{v}(k) \quad (24)$$

where  $\mathbf{v} \in R^3$  represents zero mean normally distributed white noise process with  $\sigma_{vi} = 0.005$  for  $i = 1, 2, 3$ . Closed loop simulations with MPC were carried out in MATLAB. A state-space formulation of MPC (Muske and Rawlings, 1993) was used. The model used in MPC was the same as the one used to simulate the plant dynamics except for the

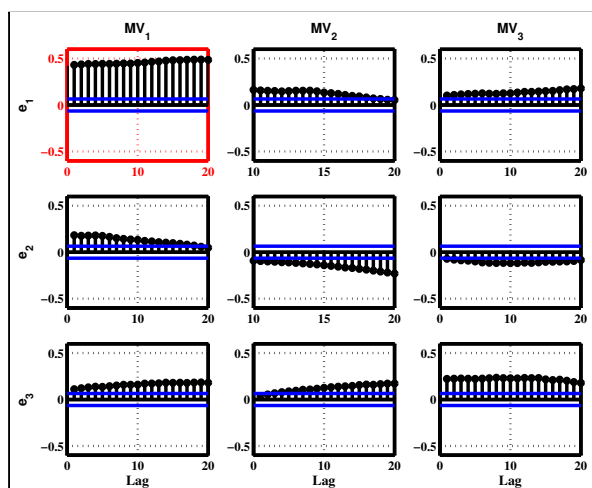


Fig. 7. Case Study 2: Gain Mismatch - Partial correlation plots

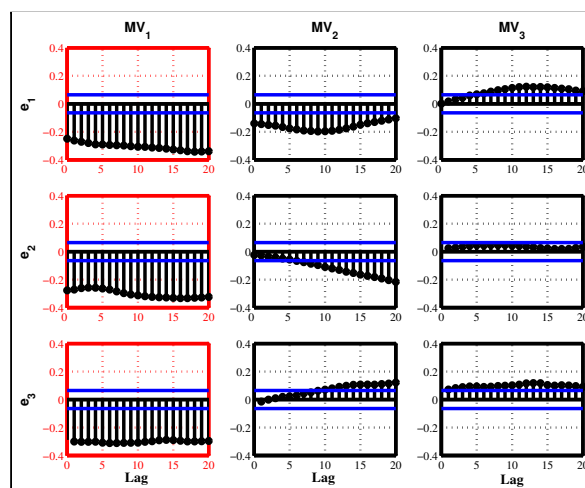


Fig. 9. Case Study 2: Delay Mismatch - Partial correlation plots

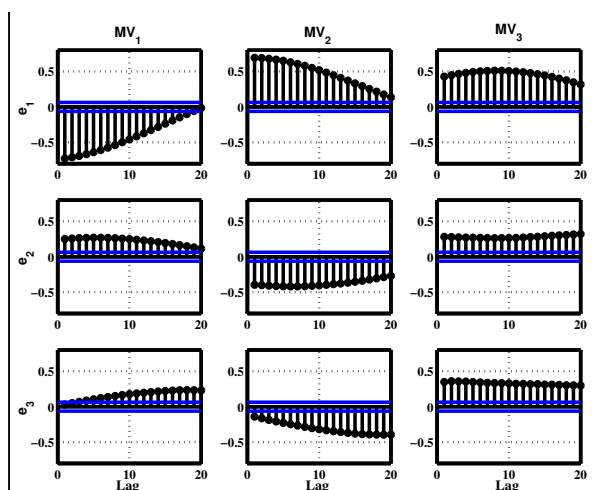


Fig. 8. Case Study 2: Gain Mismatch - Regular correlation plots

input-output channel where mismatch was introduced. As stated earlier, setpoint changes are a common occurrence in an industrial process. To simulate this scenario, setpoint excitation was introduced by applying PRBS signals of magnitude 0.1 and switching time of 20 minutes at the setpoints. Various scenarios of model-plant mismatch were simulated, two of which are presented here.

**Scenario-1: Gain mismatch** In this case, a more realistic scenario was created by introducing a 10% gain mismatch and 10% mismatch in time constant in all MV-CV channels except the MV1-CV1 channel. A larger (underestimated) gain mismatch of 50% was added in this channel. This was done to test whether the proposed methodology is able to detect the more significant mismatches when all channels contain some mismatch. The proposed methodology is in fact able to isolate the channel containing the most significant mismatch i.e. channel MV1-CV1. This can be seen from the significant partial correlation values for this particular channel (see Figure 7). Also it can be seen from Figure 8, that the full correlations between the model residuals and MVs give no useful information regarding the location of the most significant mismatch.

Table 4. KHU Unit - Details of CVs

CV	Description
CV1	Kero stripper feed valve position
CV2	Kero stripper pressure valve position
CV3	Condenser outlet temperature
CV4	Accumulator level valve position
CV5	Pressure of fuel gas to reboiler
CV6	Kero stripper bottom temperature
CV7	Kero product flash

Table 5. KHU Unit - Details of MVs

MV	Description
MV1	Kero stripper pressure
MV2	Kero stripper reflux flow
MV3	Reboiler outlet temperature

**Scenario-2: Delay Mismatch** In this case, a delay mismatch of 5 samples was introduced in channels MV1-CV1, MV1-CV2 and MV1-CV3. Also, all channels contained gain mismatches of 10%. The proposed methodology was able to isolate the channels containing delay mismatches even when all the channels contained some mismatch (gain mismatch). The partial correlation plots for this case are shown in Figure 9.

### 5.3 Industrial Case Study: Data from a Kerosene Hydrofiner Unit (KHU) at Suncor Energy's Upgrading Plant

The kerosene hydrofiner unit (KHU) at Suncor Energy Inc. is a standard hydrofining unit that desulphurizes the coker intermediate kerosene streams through a catalytic reaction with hydrogen. The KHU is controlled by an MPC controller which has 3 manipulated variables (MVs), 7 controlled variables (CVs) and 3 feedforward variables (FFs) or measured disturbance variables. The MPC computations take place every 1 minute. The details of the CVs and the MVs are given in Tables 4 and 5 respectively. A schematic of the KHU unit is shown in Figure 10. CV7 is not marked in the schematic because it is an inferred variable and is obtained through online computations on the process computer.

Since late 2006, the performance of the KHU MPC had deteriorated considerably (Jiang et al, 2007). The closed

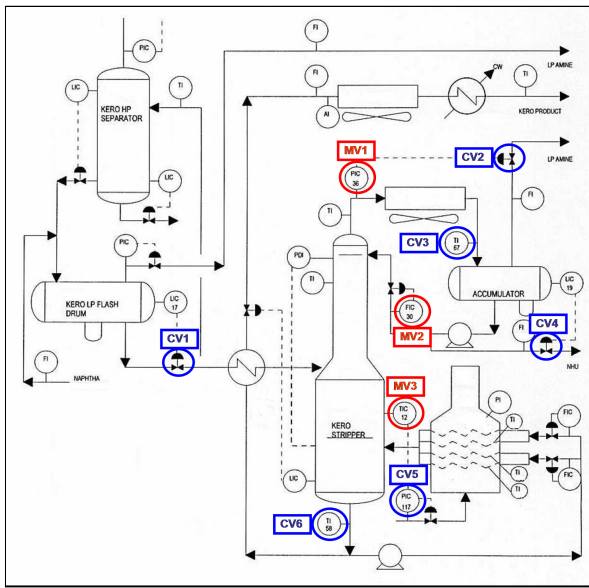


Fig. 10. Schematic of the KHU unit (Courtesy-Suncor Energy, Canada)

loop operating data was analyzed using the proposed methodology to investigate for model problems. The number of data points used for analysis was 2000. A regular correlation analysis between the model residuals and MVs cannot give any insight into the root-cause of the problem as can be seen from the regular correlation plots in Figure 11. If any conclusions were to be drawn based on these plots, the models between several MVs and CVs would have to be considered to be of a poor quality (red boxes). However, the proposed methodology, when applied to this data, reveals that the models between MV3-CV2 and MV2-CV4 contain a large mismatch with respect to the plant. This can be seen from the significant partial correlations between  $e_2$  and MV3 and  $e_4$  and MV2 (See Figure 12). The plots for  $e_1$ -MV2 and  $e_1$ -MV3 are blank in Figures 11 and 12 because MV2 and MV3 do not affect CV1 and hence there are no models for these channels. The results obtained using the proposed methodology are validated against those obtained by an independent analysis performed by Jiang et al (2007) on the same data. Their analysis involved re-identification of the models from closed-loop data using the Tai-Ji module of the CPM product from Matrikon Inc (Matrikon Inc., Tai-Ji Multivariable Identification Package, 2007), which is based on the ASYM method of Zhu (1998). The results of their analysis are shown in Figure 13. It is clearly seen that the current models (in the controller) for channels MV3-CV2 and MV2-CV4 are significantly different than those in the re-identified models. Moreover, models for other channels have also changed slightly (Eg. MV3-CV3). This may be due to process changes that have taken place in the unit since the time of commissioning. The blanks in Figure 13 are again those channels for which models do not exist.

## 6. CONCLUDING REMARKS

In this article, a methodology for the detection of model-plant mismatch in MPC applications has been proposed. The methodology is based on the analysis of partial correlations between the model residuals and manipulated

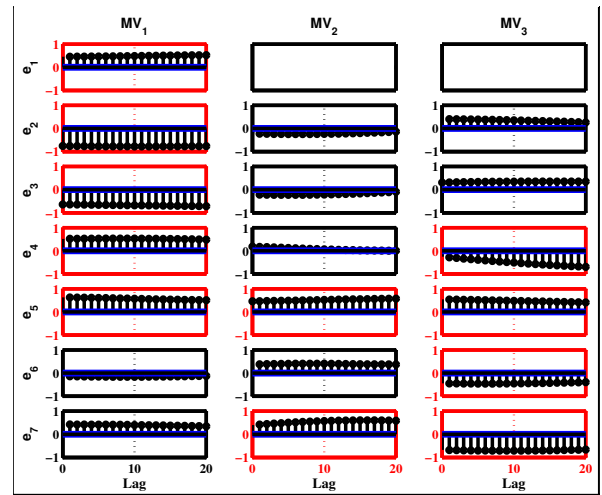


Fig. 11. KHU Unit: Regular correlation plots - Misleading information regarding location of significant mismatch (Red blocks)

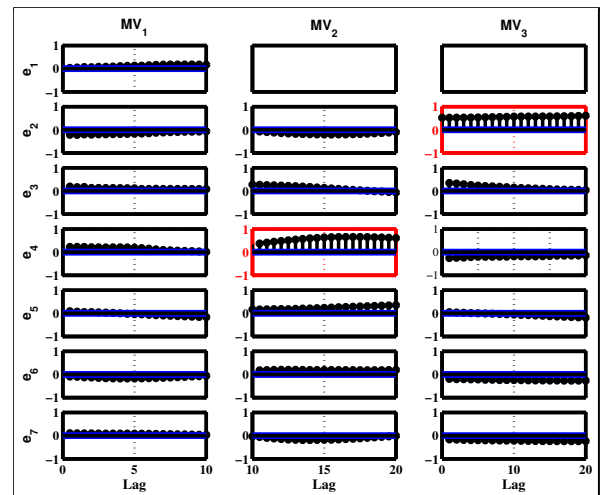


Fig. 12. KHU Unit: Partial correlation plots - Significant mismatch correctly located

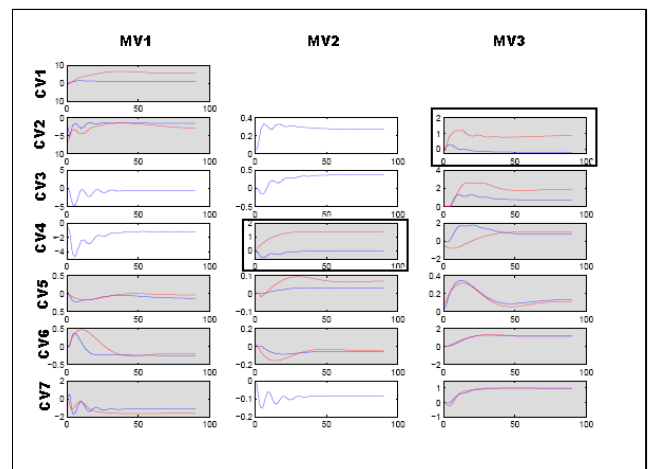


Fig. 13. KHU Unit: Step responses for model in MPC (Blue) and re-identified model (Red)

variables. The advantage of the proposed method is that it requires routine operating data for analysis. It has been shown that in the presence of correlations between the manipulated variables, it is not possible to identify the exact location of a mismatch when regular correlation analysis is used. Through simulations, the proposed methodology has been shown to successfully detect mismatch when the mismatch is contained in one channel as well as in multiple channels in the presence of unmeasured disturbances. For the KHU case study, the proposed methodology has been able to successfully detect those channels of the model that exhibit significant mismatch. The results are in exact agreement with those from an independent analysis which concluded that the gain signs in the model were incorrect. This was due to significant changes in the operation of the KHU unit since its commissioning two years ago.

Even though the poor models in the controller are detected, a more pertinent question to ask is: what is the quantitative effect of MPM on MPC performance? Is it significant in which case one can apply the proposed technique? Otherwise one should look at other causes for poor MPC performance. These questions would have to be answered by first analyzing how the existing controller handles the mismatch. A particular model may be of a very poor quality (significant mismatch with the plant) and yet depending on the design of the controller, it may or may not have a significant impact on the closed-loop performance. Recently, a non-invasive methodology for quantifying the impact of MPM on control performance has been proposed by Badwe et al (2008). However, there is still no complete solution available for this problem and this avenue is still open for research.

#### ACKNOWLEDGEMENTS

The authors wish to express sincere gratitude to Bruce Wilson and Foon Szeto of Suncor Energy Inc., Canada for allowing to use the KHU process data for analysis. Hailei Jiang is also acknowledged for providing the re-identification results for the KHU unit. Financial support from NSERC, Matrikon, Suncor and iCORE in the form of the Industrial Research Chair program at the University of Alberta is gratefully acknowledged.

#### REFERENCES

- Astrom, K.(1970). *Introduction to Stochastic Control Theory*, Academic Press, New York.
- Badwe, A.S., Patwardhan, R.S., Patwardhan, S.C. and Gudi, R.D. (2008) Quantifying the impact of model-plant mismatch on controller performance: A non-invasive approach., *Accepted for presentation at the 2008 International Symposium on Advanced Control of Industrial Processes (ADCONIP'08)*, Jasper, Canada
- Gudi, R.D. and Rawlings, J.B. (2006). Identification for decentralized model predictive control. *AIChE J.*, 52(6):2198-2210
- Harris, T.J. (1989) Assessment of control loop performance. *Can. J. Chem. Engng.* 67(10):856-861.
- Huang, B. and Shah, S.L. (1999) *Performance Assessment of Control Loops: Theory and Applications*, Springer, London.
- Jiang, H., Shah, S.L., Huang, B., Wilson, B., Patwardhan R.S. and Szeto, F. (2008) Performance assessment and model validation of two industrial MPC controllers. *Accepted for presentation at the 17<sup>th</sup> IFAC World Congress, Seoul, Korea.*
- Kozub, D.J. (1996) Controller performance monitoring and diagnosis experience and challenges, in: *Proc. 5th Int. Conf. on Chem. Process Control.*, Tahoe, CA AICHE and CACHE. pp:83-96.
- Ljung, L., (1999) *System Identification*, 2nd Edn., Prentice Hall PTR, NJ.
- Morari, M. and Lee, J.H. (1999) Model predictive control: Past, present and future. *Comput. Chem. Engng.*, 23:667-682.
- Muske, K.R. and Rawlings, J.B. (1993) Model predictive control with linear models. *AIChE J.*, 39:262-287.
- Patwardhan, R.S. and Shah, S.L. (2002) Issues in diagnostics of model-based controllers, *J. Process Contr.*, 12(3):413-427.
- Patwardhan, S.C., Manuja, S., Narasimhan, S. and Shah, S.L. (2006) From data to diagnosis and control using generalized orthonormal basis filters. Part II: Model predictive and fault tolerant control, *J. Process Contr.*, 16(2):157-175.
- Prett, D.M. and Morari, M. (1987) *Shell process control workshop*, Butterworth, NY
- Qin, S.J. (1998) Control performance monitoring – A review and assessment, *Comput. Chem. Engng.* 23:173-186.
- Shah SL, Huang B. Patwardhan R.S. (2002) Multivariate controller performance analysis: methods, applications and challenges. In: *CPC-VI, Proceedings of Sixth International Conference on Chemical Process Control*. Rawlings J.B., Ogunnaike BA, Eaton JW, eds. AICHE Symposium Series. 98:187-219.
- Smillie, K.W. (1966) *Introduction to regression and correlation*, Academic Press, Toronto.
- Tsakalis, K. and Dash, S.(2007) Multivariable controller performance monitoring using robust stability conditions. *J. Process Contr.*, 17(9):702-714.
- Zhu, Y. (1998) Multivariable process identification for MPC: The asymptotic method and its applications. *J. Process Contr.*, 8(2):101-115.