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Stability Analysis and Controller Design for Repetitive Control System Based on 2D Hybrid Model ⋆

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Abstract: This paper concerns stability analysis and controller design for repetitive control. First, a two-dimensional (2D) continuous-discrete hybrid model of a repetitive control system is established. Next, new criteria for the asymptotic stability of the system are presented based on the model. Then, these criteria are extended to calculate lower bounds on stability margins to design a suitable controller. Unlike existing methods, the one in this paper employs a 2D hybrid model to independently handle the two different types of actions involved in repetitive control: continuous control and discrete learning. A numerical example demonstrates that this approach provides good performance.

1. INTRODUCTION

Tracking performance for a periodic reference input and rejection performance for a periodic disturbance can be improved by inserting a periodic-signal generator into the controller. This kind of control system is called a repetitive control system; it was first proposed by Inoue et al. [1981] and was subsequently developed by Hara et al. [1988] and many other researchers.

Over the last few decades, a considerable number of studies has been devoted to the theoretical development and practical application of repetitive control. For example, She et al. [2000] devised a discrete-time variable structure repetitive control algorithm; Dang and Owens [2004] used the Lyapunov method to investigate a multiperiod repetitive control scheme for a positive real system; and Chang et al. [2006] combined repetitive control with PID control.

On the other hand, considerable attention has also been focused on robust repetitive control. In order to deal with system uncertainties, Chen and Liu [2005] cascaded a repetitive controller with an H_∞ controller in the frequency domain; and Ramrath and Singh [2005] used a minimax problem formulation to investigate a robust repetitive control scheme. However, there is a trade-off between control performance and the robustness of the control system. Doh and Chung [2003] presented a robust stability condition in the form of a linear matrix inequality (LMI) for an uncertain plant in a repetitive control system based on a Lyapunov functional approach, and derived a method of designing a low-pass filter to achieve the best tracking performance. However, as pointed out in She et al. [2007], there is room to improve the tracking performance. Most design methods resolve the trade-off by exploiting the experience of operators and/or by trial and error because no effective and systematic way to solve this problem has yet been reported.

An analysis of this problem reveals that repetitive control actually involves two independent types of actions, control and learning, with completely different characteristics: Control within one repetition period is a continuous process, and learning between periods is discrete behavior. Clearly, information propagation occurs in both the continuous and discrete domains. However, all existing design methods for repetitive control systems ignore the difference between these types of actions, dealing with them only in the time domain and giving them equal treatment. As a result, they enable analysis and design based only of the overall effect of the control and learning actions, and cannot produce a sophisticated controller that adjusts each action independently.

This paper solves the problem with a new design method for repetitive control systems that employs two-dimensional (2D) system theory (e.g., Xie and Du [2002]). First, to improve stability conditions, a 2D continuous-discrete hybrid model is established to describe a repetitive control system. Next, an LMI is derived to obtain stability margins. Finally, the problem of designing a repetitive control system is converted into the problem of stabilizing a 2D continuous-discrete hybrid system. This paper presents a new design method for repetitive control that combines 2D system theory with the LMI technique (e.g., Boyd et al. [1994]).

Throughout this paper, $\mathbb{R}^n$ denotes n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real

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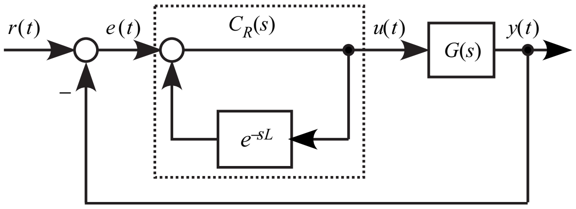

Fig. 1. Basic repetitive control system.

matrices. I_n means an $n \times n$ identity matrix, and the subscript is omitted when it is clear. $\mathbf{0}$ indicates a zero matrix with appropriate dimensions; and $\mathbf{0}_p$ is the $p \times p$ square zero matrix (the subscript is omitted if the dimension is clear.). $\star$ indicates the elements below the main diagonal of a symmetric block matrix. The notation $X > \mathbf{0}$ ($< \mathbf{0}$) means that the matrix X is positive (negative) definite. $\oplus$ denotes a direct sum, that is, $W_1 \oplus W_2 = \text{diag}\{W_1, W_2\}$. For any time-dependent variable $\xi(t)$, $\xi(t) = 0$ for $t < 0$.

2. DESCRIPTION OF REPETITIVE CONTROL SYSTEM AND ITS 2D REPRESENTATION

Fig. 1 shows the basic structure of a repetitive control system. The transfer function of a repetitive controller is

$$C_R(s) = \frac{1}{1 - e^{-sL}}, \tag{1}$$

where L is a constant and is equal to the known period of the periodic exogenous signal. Since

$$C_R(j\omega_k) = \frac{1}{1 - e^{-j\omega_k L}} = \infty,\ \omega_k = \frac{2k\pi}{L},\ k = 0, 1, \cdots, \tag{2}$$

the gain of the controller is infinite at the fundamental and harmonic angular frequencies of the periodic signal. So, when a controller contains $C_R(s)$, the control system perfectly tracks the periodic reference input.

As pointed out by Hara et al. [1988], a repetitive control system can be stabilized only when the plant has a direct path from the input to the output, or in other words, when the relative degree of the plant is zero. A repetitive control system is a neutral-type delay system; and since the repetitive controller contains an infinite number of poles on the imaginary axis, this type of system is very difficult to stabilize. Moreover, it has been proven that a repetitive control system cannot be stabilized when the relative degree of the plant is larger than zero.

To stabilize a plant that does not contain a direct path from the input to the output, Hara et al. [1988] presented the configuration of a modified repetitive control system with a low-pass filter, $q(s)$, inserted in the delay feedback line. Since the low-pass filter leaves only a finite number of poles of the repetitive controller in the low-frequency band on the imaginary axis and moves the others to the left in the s-plane, the modified repetitive control system is a retarded-type delay system, which is very easy to stabilize. However, since a modified repetitive control system contains only an approximate generation model of a periodic signal, perfect tracking is not possible. In other words, system stability comes at the cost of tracking precision. Since a repetitive control system for a plant with a relative degree of zero operates at the limit of control performance, we address this repetitive control problem in this paper.

This study considers the repetitive control system in Fig. 2. $r(t)$ is the periodic reference input, and K_e and K_p are the feedback gains to be determined.

Consider the following plant:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t), \end{cases} \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state of the plant, and $u(t), y(t) \in \mathbb{R}$ are the control input and output. For simplicity, we only consider the single-input single-output case, which means that $m = 1$. However, the results are easy to extend to the multiple-input multiple-output case.

The repetitive control law is

$$u(t) = F_e v(t) + F_p x(t), \tag{4}$$

where $v(t)$ is the output of the repetitive controller, which is given by

$$v(t) = v(t - L) + e(t). \tag{5}$$

Let

$$\begin{cases} \Delta x(t) = x(t) - x(t - L), \\ \Delta y(t) = y(t) - y(t - L), \\ \Delta e(t) = e(t) - e(t - L), \\ \Delta u(t) = u(t) - u(t - L). \end{cases}$$

Then, according to (3)-(5), we have

$$\begin{cases} \Delta\dot{x}(t) = A\Delta x(t) + B\Delta u(t), \\ \Delta e(t) = -C\Delta x(t) - D\Delta u(t), \end{cases} \tag{6}$$

and

$$\Delta u(t) = F_e \Delta v(t) + F_p \Delta x(t) = F_e e(t) + F_p \Delta x(t). \tag{7}$$

We introduce two domains, τ and k, to describe the two different types of actions involved in repetitive control, control and learning; τ is used to describe control within a period, and k is used to describe learning between periods. The time-domain variables are

$$\begin{cases} x(t) = x(kL + \tau) := x(k, \tau), \\ u(t) = u(kL + \tau) := u(k, \tau), \\ e(t) = e(kL + \tau) := e(k, \tau). \end{cases}$$

As a result, (6) becomes

$$\Delta\dot{x}(k, \tau) = A\Delta x(k, \tau) + B\Delta u(k, \tau), \tag{8}$$

and

$$e(k, \tau) = e(k - 1, \tau) - C\Delta x(k, \tau) - D\Delta u(k, \tau). \tag{9}$$

(8) and (9) constitute a 2D continuous-discrete hybrid model of the repetitive control system.

Note that (8) and (9) explicitly describe the control and learning actions, respectively, while the conventional model (3) only describes the combined effect of those actions. More specifically, (8) describes the control action during the k-th period, and (9) describes the learning action between the k-th and $(k-1)$-th periods. Since (8) does not contain the term $e(k, \tau)$, the control action during each period is independent of the learning action. In contrast, (9) shows that learning is strongly affected by the control action. This stands to reason because the faster the control converges, the less learning that is needed.

Remark 1. Studies have recently appeared on iterative learning control based on a 2D model. In this type of control, since the initial state is always the same at the beginning of each learning period, bounded control input always results in bounded output. In contrast, in repetitive

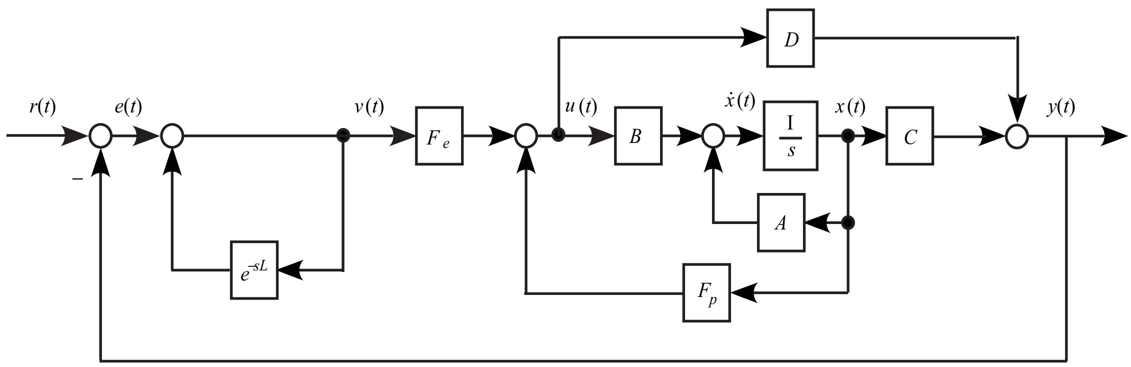

Fig. 2. Configuration of repetitive control system.

control, since the initial state is influenced by the control results of the previous period, bounded control input may result in unbounded output. So, the stability condition is much stricter for repetitive control than for iterative learning control. This difference is, in fact, expressed in the 2D model. Note that the term $e(k-1,\tau)$ appears in the 2D model for repetitive control, (9) but not in a 2D model for iterative learning control. This term is essential and indicates the difference between repetitive control and iterative learning control.

Combining (8) and (9) yields

$$\begin{bmatrix} \Delta\dot{x}(k,\tau) \\ e(k,\tau) \end{bmatrix} = \begin{bmatrix} A & \mathbf{0} \\ -C & 1 \end{bmatrix} \begin{bmatrix} \Delta x(k,\tau) \\ e(k-1,\tau) \end{bmatrix} + \begin{bmatrix} B \\ -D \end{bmatrix} \Delta u(k,\tau). \tag{10}$$

Then, the repetitive control law, (7), can be written

$$\Delta u(t) = F_e e(t) + F_p \Delta x(t).$$

That is,

$$\Delta u(k,\tau) = [K_p \quad K_e] \begin{bmatrix} \Delta x(k,\tau) \\ e(k-1,\tau) \end{bmatrix}, \tag{11}$$

where

$$K_e = \frac{F_e}{1+F_e D}, \quad K_p = \frac{F_p - F_e C}{1+F_e D}. \tag{12}$$

This shows that the design of a repetitive control law, (4), is equivalent to the design of a 2D state-feedback control law, (11). Therefore, if a 2D stabilizing control law, (11), is designed for the 2D system, (10), then the corresponding repetitive control system is also stable; and the gains in (4) are

$$F_p = \frac{K_p + K_e C}{1 - DK_e}, \quad F_e = \frac{K_e}{1 - DK_e}. \tag{13}$$

It is clear from (13) that the learning action depends only on K_e, while the control action depends on both K_e and K_p. Moreover, it is easy to independently adjust the control and learning actions by means of K_p and K_e, respectively; while it would be very difficult to do that by means of F_p and F_e.

3. STABILITY ANALYSIS

This section first gives stability conditions based on LMIs and then presents a method of calculating a stability margin for a repetitive control system.

3.1 LMI-based Stability Conditions

First, let us recall the following lemmas.

Lemma 1. (Rogers and Owens [1992]) The 2D system

$$\begin{bmatrix} \dot{x}(k,\tau) \\ y(k,\tau) \end{bmatrix} = \begin{bmatrix} \Phi & \Gamma_0 \\ \Theta & \Omega_0 \end{bmatrix} \begin{bmatrix} x(k,\tau) \\ y(k-1,\tau) \end{bmatrix} + \begin{bmatrix} \Gamma \\ \Omega \end{bmatrix} u(k,\tau), \tag{14}$$

where $x(k,\tau) \in \mathbb{R}^n$, $y(k,\tau) \in \mathbb{R}^m$, and $u(k,\tau) \in \mathbb{R}^l$, is stable if and only if the two-variable polynomial

$$\psi(s,\lambda) = \det \begin{bmatrix} sI_n - \Phi & -\Gamma_0 \\ -\lambda\Theta & I_m - \lambda\Omega_0 \end{bmatrix} \tag{15}$$

satisfies

$$\psi(s,\lambda) \neq 0, \quad \forall (s,\lambda) \in U^2_{s,\lambda}, \tag{16}$$

where $U^2_{s,\lambda} := \{(s,\lambda):\ \mathrm{Re}(s) \geq 0, |\lambda| \leq 1\}$.

Lemma 2. (Galkowski et al. [2003]) The 2D system, (14), is stable if there exist symmetrical matrices $W_1,\ W_2 > \mathbf{0}$ such that the following LMI holds:

$$\begin{bmatrix} -W^{11} & W^{11}\hat{\Phi}_2 \\ \star & -W^{01} + \hat{\Phi}_1^T W^{10} + W^{10}\hat{\Phi}_1 \end{bmatrix} < \mathbf{0}, \tag{17}$$

where

$$\hat{\Phi}_1 = \begin{bmatrix} \Phi & \Gamma_0 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \hat{\Phi}_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \Theta & \Omega_0 \end{bmatrix},$$

$W^{11} = W_3 \oplus W_2$, $W_3 > \mathbf{0}$ ($W_3 \in \mathbb{R}^{n\times n}$) is arbitrary, $W^{10} = W_1 \oplus \mathbf{0}_m$, and $W^{01} = \mathbf{0}_n \oplus W_2$.

Applying Lemmas 1 and 2 to the 2D model of the repetitive control system yields the following lemmas.

Lemma 3. The 2D system (10) is stable if and only if the two-variable polynomial

$$\psi_r(s,\lambda) = \det \begin{bmatrix} sI_n - A & \mathbf{0} \\ -\lambda C & 1-\lambda \end{bmatrix} \tag{18}$$

satisfies

$$\psi_r(s,\lambda) \neq 0,\ \forall (s,\lambda) \in U^2_{s,\lambda}. \tag{19}$$

Lemma 4. The 2D system (10) is stable if there exist symmetrical matrices $W_1,\ W_2 > \mathbf{0}$ such that the following LMI holds:

$$\begin{bmatrix} -W^{11} & W^{11}\hat{A}_2 \\ \star & -W^{01} + \hat{A}_1^T W^{10} + W^{10}\hat{A}_1 \end{bmatrix} < \mathbf{0}, \tag{20}$$

where

$$\hat{A}_1 = \begin{bmatrix} A & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix}, \quad \hat{A}_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -C & 1 \end{bmatrix}, \tag{21}$$

$W^{11} = W_3 \oplus W_2$, $W_3 > \mathbf{0}$ ($W_3 \in \mathbb{R}^{n\times n}$) is arbitrary, and $W^{10} = W_1 \oplus 0,\ W^{01} = \mathbf{0}_n \oplus W_2$.

For the repetitive control system in Fig. 2, we have the following result.

Theorem 1. If there exist symmetrical positive definite matrices $W_1, W_2, W_3 > \mathbf{0}$ such that the following matrix inequality

$$\begin{bmatrix} \pi_{11} & W_1 BK_e + (C + DK_p)^T W_2 (DK_e - 1) \\ \star & W_2(1 - DK_e)^2 - W_2 \end{bmatrix} < \mathbf{0} \quad (22)$$

holds, then the repetitive control system in Fig. 2 is stable under the repetitive control law (4) and (13). In (22),

$$\pi_{11} := W_1(A + BK_p) + (A + BK_p)^T W_1 \\ +(C + DK_p)^T W_2 (C + DK_p) - W_3.$$

Proof. Applying the control law (11) to (10) yields the state-space model of the closed-loop system:

$$\begin{bmatrix} \Delta\dot{x}(k,\tau) \\ e(k,\tau) \end{bmatrix} = \begin{bmatrix} A + BK_p & BK_e \\ -C - DK_p & 1 - DK_e \end{bmatrix} \begin{bmatrix} \Delta x(k,\tau) \\ e(k-1,\tau) \end{bmatrix}. \quad (23)$$

It is clear from Lemma 4 that the closed-loop system (23) is stable if there exist symmetrical matrices $W_1,\ W_2 > \mathbf{0}$ such that the following LMI holds:

$$\begin{bmatrix} -W^{11} & W^{11}\hat{A}'_2 \\ \star & -W^{01} + \hat{A}'^T_1 W^{10} + W^{10}\hat{A}'_1 \end{bmatrix} < 0, \quad (24)$$

where

$$\hat{A}'_1 = \begin{bmatrix} A+BK_p & BK_e \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \hat{A}'_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -C-DK_p & 1-DK_e \end{bmatrix},$$

and W^{11}, W^{10}, and W^{01} are the same as those in Lemma 4. A simple algebraic manipulation shows that LMI (24) is equivalent to (22). Finally, based on the definitions of $\Delta x(k,\tau)$, $\Delta v(k,\tau)$, and $\Delta e(k,\tau)$, and on (12) and (13), we obtain the repetitive control law (4) and (13). This completes the proof. □

3.2 Stability Margin

The stability margin of a 2D continuous-discrete hybrid system is now defined.

Definition 1. The lower bound on the stability margin of the 2D hybrid system (10) is defined to be the maximum of σ_1 and σ_2, subject to the condition

$$\psi_r(s,\lambda) \neq 0, \quad \forall (s,\lambda) \in \hat{U}^2_{s,\lambda}, \quad (25)$$

where $\hat{U}^2_{s,\lambda} := \{(s,\lambda) : \ \mathrm{Re}(s) > -\sigma_1, |\lambda| \le 1 + \sigma_2\}$.

In order to facilitate the finding of a margin, we introduce a new variable, σ, and a nonnegative number, η $(0 \le \eta \le 1)$, and we let

$$\sigma_1 = \eta\sigma, \quad \sigma_2 = (1-\eta)\sigma. \quad (26)$$

Then, the problem of finding a stability margin for the control system is converted into the problem of finding the maximum σ for a given positive η. The following theorem gives an LMI method of calculating σ.

Theorem 2. A lower bound on σ can be obtained by solving the following constrained optimization problem:

For a given $0 \le \eta \le 1$, maximize σ (> 0) in the following LMI:

$$\begin{bmatrix} \Pi_1 & \{1 + (1-\eta)\sigma\}\hat{A}_2^T W^{11} \\ \star & -W^1 \end{bmatrix} < \mathbf{0}, \quad (27)$$

where $\Pi_{11} = 2\eta\sigma W^{10} - W^{01} + \hat{A}_1^T W^{10} + W^{10}\hat{A}_1$; W^{10}, W^{01}, and W^{11} are given in Lemma 2; and $W_1, W_2, W_3 > \mathbf{0}$ are symmetrical positive definite matrices.

Proof. (25) is equivalent to

$$\psi_r(s',\lambda') \neq 0, \quad \forall (s',\lambda') \in \bar{U}^2_{s',\lambda'}, \quad (28)$$

where $\bar{U}^2_{s',\lambda'} = \{(s',\lambda') : \ \mathrm{Re}(s') > 0, |\lambda'| \le 1\}$, $s' = s - \sigma_1$, $\lambda' = \lambda - \sigma_2$. This and (26) allows us to use $\eta\sigma I_n + A$ and $1 + (1-\eta)\sigma$ to replace A and 1 in (21), respectively. A simple calculation yields the result. □

4. LMI-BASED DESIGN OF REPETITIVE CONTROLLER

The following lemma is first presented.

Lemma 5. (Galkowski et al. [2003]) Consider the 2D system (14), subject to the control law

$$u(k,\tau) = [K \ \ L] \begin{bmatrix} x(k,\tau) \\ y(k-1,\tau) \end{bmatrix}. \quad (29)$$

The resulting closed-loop system is stable if there exist symmetrical matrices $Y, Z > \mathbf{0}$ together with arbitrary matrices M and N such that the LMI

$$\begin{bmatrix} \Lambda & \Gamma_0 Z + \Gamma M & Y\Theta^T + N^T\Omega^T \\ \star & -Z & Z\Omega_0^T + M^T\Omega^T \\ \star & \star & -Z \end{bmatrix} < \mathbf{0} \quad (30)$$

holds, where $\Lambda = \Phi Y + Y^T\Phi + \Gamma N + N^T\Gamma^T$. Furthermore, if LMI (30) holds, then a stabilizing control law (29) is given by

$$K = NY^{-1}, \quad L = MZ^{-1}. \quad (31)$$

Applying Lemma 5 to the 2D system (10) yields the following theorem.

Theorem 3. If there exist symmetrical matrices $Y, Z > \mathbf{0}$ together with arbitrary matrices M and N such that the LMI

$$\begin{bmatrix} \Lambda & BM & -YC^T - N^T D \\ \star & -Z & Z - M^T D \\ \star & \star & -Z \end{bmatrix} < \mathbf{0} \quad (32)$$

holds, where $\Lambda = AY + Y^T A + BN + N^T B^T$, then the 2D system (10) is stable under the control law (11), where

$$K_p = NY^{-1}, \quad K_e = MZ^{-1}. \quad (33)$$

Remark 2. Theorem 3 provides an LMI-based method of designing repetitive control systems. It is easily implemented using the Robust Control Toolbox Boyd et al. [1994]. Compared with existing methods, this one is less computationally complex and more intuitive, and thus more practical.

When the stability margins σ_1^* and σ_2^* are stated explicitly in a design specification, we can obtain a suitable control law by combining Theorems 2 and 3.

Corollary 1. If there exist symmetrical matrices $Y, Z > \mathbf{0}$ together with arbitrary matrices M and N such that the following LMI

$$\begin{bmatrix} \Lambda & BM & (1+\sigma_2^*)(-YC^T - N^T D) \\ \star & -Z & (1+\sigma_2^*)(Z - M^T D) \\ \star & \star & -Z \end{bmatrix} < \mathbf{0} \quad (34)$$

holds, where $\Lambda = 2\sigma_1^* Y + AY + Y^T A + BN + N^T B^T$, then the 2D system (10) is stable under the control law (11), which is given by

$$K_p = NY^{-1}, \quad K_e = MZ^{-1}. \quad (35)$$

Table 1. Stability margins for various η.

η	σ	σ_1	σ_2
0.0	0.142	0.0000	0.1450
0.1	0.142	0.0142	0.1278
0.2	0.141	0.0282	0.1128
0.3	0.142	0.0426	0.0994
0.4	0.141	0.0564	0.0846
0.5	0.139	0.0695	0.0695
0.6	0.138	0.0828	0.0552
0.7	0.136	0.0952	0.0408
0.8	0.134	0.1072	0.0268
0.9	0.132	0.1188	0.0132
1.0	0.130	0.1300	0.0000

5. NUMERICAL EXAMPLE

This section gives a numerical example that illustrates the design procedure for the above method.

Consider the following plant:

$$\begin{cases} A = \begin{bmatrix} 0 & 1 \\ -1 & -5 \end{bmatrix}, \; B = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \qquad D = 1. \end{cases} \tag{36}$$

Let the periodic reference input be

$$r(t) = \sin\frac{2\pi t}{10} + 0.5\sin\frac{4\pi t}{10} + 0.5\sin\frac{6\pi t}{10}.$$

Solving the optimization problem (27) in Theorem 2 using the Robust Control Toolbox yields a suitable stability margin for a given η. The results are listed in Table 1.

Then, we design a repetitive controller of the form (4). If the stability margin is not explicitly stated, solving the feasible problem (32) yields

$$F_p = [-0.6443 \quad -0.7532], \quad F_e = 4.0. \tag{37}$$

On the other hand, if the stability margin is explicitly stated (for example, $\sigma_1 = 0.10$ and $\sigma_2 = 0.02$), then solving the feasible problem (34) yields

$$F_p = [-9.1686 \quad -0.4520], \quad F_e = 4.1616. \tag{38}$$

Figures 3 and 4 show simulation results for control laws (37) and (38), respectively. Note that the repetitive control system is stable, and the steady-state tracking error converges to zero.

6. CONCLUSION

This paper considered the stability analysis and controller design problems for a repetitive control system. First, unlike existing methods, the one in this paper employs a 2D continuous-discrete hybrid model of the system to independently handle the control and learning actions involved in repetitive control. Next, a stability condition and a stability margin were presented in terms of LMIs. Then, the results obtained were extended to the design of a repetitive control law. The design is carried out using LMIs, which are easily solved using the Robust Control Toolbox. Finally, a numerical example demonstrated the validity of the method.

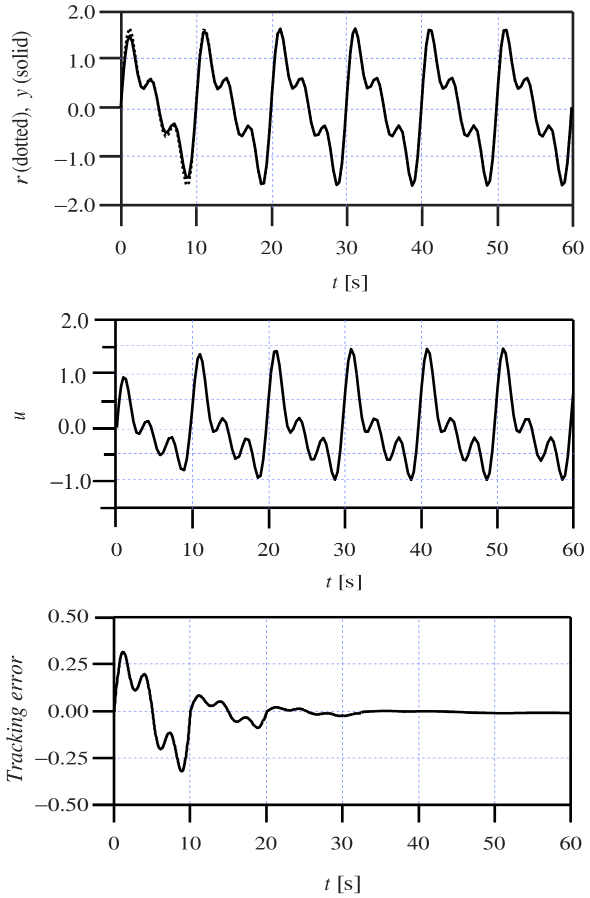

Fig. 3. Simulation results for repetitive control law (37) ($\sigma_1 = \sigma_2 = 0$).

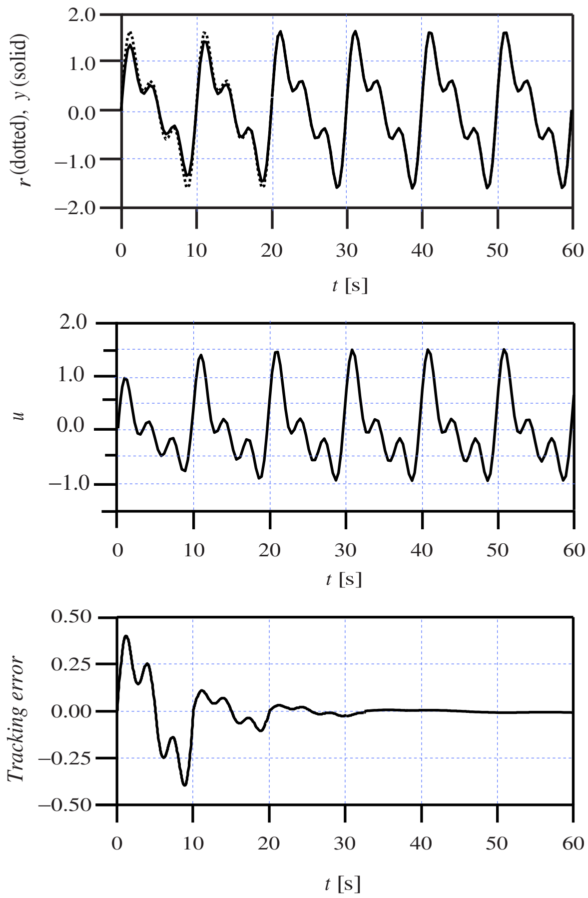

Fig. 4. Simulation results for repetitive control law (38) ($\sigma_1 = 0.10,\ \sigma_2 = 0.02$).

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