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Observability of Complex Systems: Minimal Cost Sensor Network Design ⋆

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Abstract:
Plants instrumentation is a crucial issue due to the importance of sensors in allowing the observability and in increasing the redundancy and the reliability. Designing a sensor network becomes complicated when the complexity of the system increases. In this paper, a strategy is proposed to design a minimal cost sensor network ensuring the observability of complex systems. The strategy is based on the decomposition of complex systems into subsystems. This decomposition helps in treating each subsystem separately and allows the use of reduced order observers rather than one observer for the whole system.

Keywords:
Sensor network design, plants instrumentation, complex systems, minimal cost

1. NOMENCLATURE

SS_j	A subsystem
IM_{ij}	Interconnection matrix: the influence of SS_i on SS_j
C_j	Minimal cost output matrix of SS_j
J_j	Sensor set corresponding to C_j
$\bar{C}_j$	Minimal cost output matrix verifying the local observability of SS_j
$\bar{J}_j$	Sensor set corresponding to $\bar{C}_j$
$\bar{\bar{C}}_j$	Minimal cost output matrix matching subsystem interconnection
$\bar{\bar{J}}_j$	Sensor set corresponding to $\bar{\bar{C}}_j$
δ_j^i	Sensor measuring variable x_j^i of SS_j
$cost(J_j)$	Cost of sensor set J_j
co_j^i	Cost of sensor δ_j^i

2. INTRODUCTION AND PROBLEM STATEMENT

The problem of plants instrumentation has been studied over the last thirty years: the criteria of observability (Luong et al. (1994)), maximum estimation accuracy (Sen et al. (1998)) and minimum cost (Madron and Veverka (1992)) were considered in the design of sensor networks. Later, other criteria like maximizing fault detectability and isolability (Carpentier et al. (1997)), redundancy (Luong et al. (1994)) and reliability (Ali and Narasimhan (1996)) were introduced in the design objectives.

The focus was mainly on chemical plants (Ali and Narasimhan (1996), Madron and Veverka (1992)). Other studies treated steady-state (Luong et al. (1994)), linear (Sen et al. (1998)), bilinear processes (Ali and Narasimhan (1996)) and structured systems (Commault et al. (2005b), Commault et al. (2005a)).

In (Staroswiecki et al. (2004)), Staroswiecki *et al* addressed the sensor fault tolerant estimation and the associated sensor network design problem for linear systems. Their objective consists in designing a sensor network that ensures system's observability and at the same time verifies an *a priori* defined criteria of reliability and redundancy degree. This strategy requires testing system's observability for a number of sensor sets. Observability test is quite simple for linear systems and for relatively small scale nonlinear systems.

However, for large scale nonlinear complex systems, testing observability may not be feasible because of the large memory space required for calculations. In addition, the design of an observer for the overall system may be a hard task and the on line estimation may impose heavy calculations.

In (Chamseddine et al. (2007)), the authors extend the work of Staroswiecki *et al* (Staroswiecki et al. (2004)) to nonlinear large scale complex systems. The complex system is decomposed into interconnected subsystems. The decomposition enables the use of reduced order observers for subsystems. This simplifies observers design and reduces the calculation requirement. For each subsystem, the minimum set of sensors allowing its observability is determined while taking into consideration its interconnection with the other subsystems.

In some cases, the cost is an important criterion of the design objectives. The aim of this paper is to show how to design a minimal cost sensor network ensuring the observability of complex systems consisted of interconnected subsystems.

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The paper structure is as follows: in section 3, some preliminaries for observability of systems subjected to external disturbances are given. Section 4 presents the system decomposition which is the first step of the sensor network design strategy. This strategy is detailed then in section 5 and an academic example illustrating the proposed strategy is presented in section 6. Finally the conclusion and future works are given.

3. PRELIMINARIES

The preliminaries given in this section concern systems observability in the presence of unknown inputs. Consider the following system affected by unknown inputs $\tilde{f}(t)$:

$$\begin{cases} \dot{x}(t) = f(x(t),u(t)) + \tilde{F}\tilde{f}(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

System (1) is locally weakly observable if $rank(OM) = n$ where OM is the observability matrix given by:

$$OM = (dL_f^0 h(x)^T \cdots dL_f^{n-2}h(x)^T \; dL_f^{n-1}h(x)^T)^T \quad (2)$$

where, for any vector V, $dV = \left(\frac{\partial V}{\partial x_1}, \frac{\partial V}{\partial x_2}, \ldots, \frac{\partial V}{\partial x_n}\right)$, $L_f h(x) = \sum_{i=1}^{n} f_i \frac{\partial h(x)}{\partial x_i}$ is the Lie derivative of $h(x)$ and $L_f^k h(x) = L_f(L_f^{k-1}h(x))$.

However, due to the presence of unknown inputs $\tilde{f}(t)$, an additional condition should be verified. This second condition is known as matching condition. It is given by: $rank(C\tilde{F}) = rank(\tilde{F})$ (Trinh and Ha (2000)).

The sensor set J to be used can then be viewed as the union of two sets $\bar{J}$ and $\bar{\bar{J}}$ ($J = \bar{J} \cup \bar{\bar{J}}$). The first set $\bar{J}$ verifies the first condition. The second set $\bar{\bar{J}}$ verifies the second condition. There may be sensors common to the two sets. $\bar{J}$ depends on the dynamics $f(x,u)$ of (1). $\bar{\bar{J}}$ depends on the structure of $\tilde{F}$. Set J (respectively $\bar{J}$ and $\bar{\bar{J}}$) is the sensor set correspondent to C (respectively $\bar{C}$ and $\bar{\bar{C}}$).

Remark 1. *In the rest of the paper, the first condition will be referred to as '**verifying the local observability**'. The second condition will be referred to as '**matching the unknown inputs**' or '**interconnection matching**'. The two conditions will be referred to as '**verifying system's observability**'.*

The preliminaries given in this section are important because, as will be shown in the next section, the decomposition of systems results in subsystems with external inputs.

4. DECOMPOSITION OF COMPLEX SYSTEMS

A common approach to solve the complexity of complex systems is system decomposition. In this work, object-oriented decomposition (Fradkov et al. (1999)) is used. This decomposition is based on the system physical structure. It implies the separation of the system into simpler subsystems to be considered individually. If the complex system is composed of N subsystems, then the dynamics of subsystem SS_j can be written as:

$$SS_j \begin{cases} \dot{x}_j(t) = f_j(x_j(t),u(t)) + \tilde{F}_j\tilde{f}_j(\tilde{x}_j(t),u(t)) \\ y_j(t) = h_j(x_j(t)) = C_j x_j(t) \end{cases} \quad (3)$$

where $x_j \in \Re^{n_j}$ ($x_j = (x_j^1, x_j^2, ..., x_j^{n_j})^T$, $\sum_{j=1}^{N} n_j = n$) and $y_j \in \Re^{p_j}$ are respectively, the state vector and the output vector of subsystem SS_j. $\tilde{x}_j$ is a subset of x, it represents all the variables (other than x_j) that affect SS_j. f_j is the subsystem SS_j dynamics. $\tilde{f}_j$ is the unknown inputs vector. u is the control inputs vector. $\tilde{F}_j$ and C_j are constant matrices.

Remark 2. *The representation considered in (3) is a special case of:*

$$SS_j \begin{cases} \dot{x}_j(t) = f_j(x_j(t),u(t)) + \tilde{F}_j\tilde{f}_j(x(t),u(t)) \\ y_j(t) = h_j(x_j(t)) = C_j x_j(t) \end{cases} \quad (4)$$

where $\tilde{f}_j$ is function of x and not only $\tilde{x}_j$. In this case, the observability of SS_j depends of f_j and $\tilde{f}_j$. This more complicated case is not treated in this paper.

Remark 3. *The unknown inputs $\tilde{f}_j$ of (3) are supposed to be upper bounded, i.e. $|\tilde{f}_j(t)| < \rho_j$. This requires the stability of interconnected subsystems. This problem is treated in the literature and is not studied here.*

Assumption 1. *The output matrices C_j ($j = \{1,...,N\}$) are assumed to be diagonal matrices. They consist of zeros and ones. If x_j^i is measured then $C_j(i,i)$ = 1; else $C_j(i,i)$ = 0.*

The objective is to find the structure of C_j ($j = 1,...,N$) so that subsystems are observable and the cost of sensors is minimal.

The interconnection between two subsystems SS_i and SS_j can correspond to one of the four following cases: SS_i is affected by SS_j (Fig. 1.a), SS_j is affected by SS_i (Fig. 1.b), SS_i and SS_j affect each other (Fig. 1.c) and SS_i and SS_j are independent (Fig. 1.d). The two cases (a) and (b) will be called *simple interconnection*, case (c) will be called *double interconnection*.

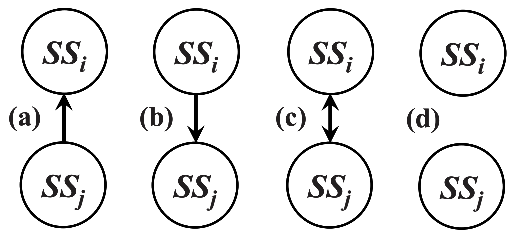

Fig. 1. Interconnection of two subsystems

In the next section the minimal cost sensor network design for complex systems observability is presented and explained.

5. MINIMAL COST SENSOR NETWORK DESIGN

For the sake of simplification, the Sensor Network Design Strategy (SNDS) will be explained for two doubly interconnected subsystems SS_1 and SS_2. This strategy can be generalized easily for more than two interconnected subsystems. The simple interconnection is a special case of the double interconnection and thus it will not be treated here.

The dynamics of SS_1 and SS_2 are given by:

$$SS_1 \begin{cases} \dot{x}_1(t) = f_1(x_1(t),u_1(t)) + \tilde{F}_1\tilde{f}_1(\tilde{x}_1(t),u_1(t)) \\ y_1(t) = C_1 x_1(t) \end{cases} \quad (5)$$

and

$$SS_2 \begin{cases} \dot{x}_2(t) = f_2(x_2(t),u_2(t)) + \tilde{F}_2\tilde{f}_2(\tilde{x}_2(t),u_2(t)) \\ y_2(t) = C_2 x_2(t) \end{cases} \quad (6)$$

As noted before, $\tilde{x}_j$ represents all the variables (other than x_j) that affect the subsystem SS_j $(j = 1,2)$. Since SS_1 and SS_2 are doubly interconnected, then $\tilde{x}_1$ is a subset of x_2 $(\tilde{x}_1 \subseteq x_2)$ and $\tilde{x}_2$ is a subset of x_1 $(\tilde{x}_2 \subseteq x_1)$.

5.1 Interconnection matrix representation

Before explaining the SNDS, the Interconnection Matrix (*IM*) is presented. The *IM* is used to represent in a matrix form the affected and the affecting variables. For SS_1 and SS_2, $IM_{12} \in \Re^{n_2 \times n_1}$ represents the influence of SS_1 on SS_2. Matrix $IM_{21} \in \Re^{n_1 \times n_2}$ represents the influence of SS_2 on SS_1. The elements of IM_{12} and IM_{21} consist of '0' and '1'. These two matrices can be constructed as follows:

If $\dfrac{\partial \dot{x}_2^i}{\partial x_1^j} \neq 0$ then $IM_{12}(i,j) = 1$, else $IM_{12}(i,j) = 0$ $(i = 1,...,n_2; j = 1,...,n_1)$.
If $\dfrac{\partial \dot{x}_1^i}{\partial x_2^j} \neq 0$ then $IM_{21}(i,j) = 1$, else $IM_{21}(i,j) = 0$ $(i = 1,...,n_1; j = 1,...,n_2)$.

5.2 Minimal cost SNDS: local observability

The local observability of SS_j depends on the dynamics f_j. It is independent of the interconnection $\tilde{f}_j$ between subsystems. Thus, $\bar{J}_j$ can be determined for each subsystem SS_j independently of the other subsystem $(j = 1,2)$ (see section 3). The problem of determining $\bar{J}_j$ can be formulated as:

$$Pr \begin{cases} \text{Minimize } cost(\bar{J}_j) \\ \text{Subject to } rank(OM_j) = n_j \end{cases} \qquad (7)$$

with OM_j is the observability matrix of SS_j. The observability test is easier to perform for each subsystem separately, rather than testing overall system's observability. Problem (7) may have multiple solutions so_j for each subsystem SS_j. Thus, it is necessary to determine set $\bar{J}_j$ to be considered among the possible solutions $\bar{J}_j^{l_j}$ $(j = 1,2$ and $l_j = 1,...,so_j)$.

5.3 Minimal cost SNDS: interconnection matching

The next step of the SNDS is to determine $\bar{\bar{J}}_1$ and $\bar{\bar{J}}_2$ that match subsystem interconnections (see section 3). In this particular case, two paths can be followed. This is shown in Fig. 2. One can start on SS_1 and end on SS_2 (solid line), or start on SS_2 and end on SS_1 (dashed line). In the first path P_1, $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_2$ are determined to match the unknown inputs of SS_1, i.e. to match IM_{21}. Once the observability of SS_1 is ensured, the external inputs affecting SS_2 will become known as they will be measured/estimated in SS_1. Same logic for path P_2.

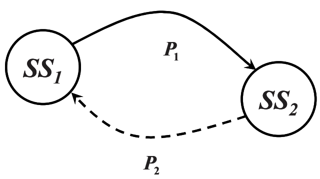

Fig. 2. Possible paths to sensor network design

In the general case, if n_D is the number of the double interconnections between subsystems then the total number n_P of possible paths is $n_P = 2^{n_D}$. The SNDS is explained in the sequel for both paths P_1 and P_2.

Path P_1 This path consists in starting by SS_1 and in finding $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_2$ so that to match the elements of matrix IM_{21} (the influence of SS_2 on SS_1).

For a given $\bar{\bar{C}}_1$, a matrix $I\bar{M}_{21}$ is defined as:

$$I\bar{M}_{21} = IM_{21} - \bar{\bar{C}}_1 IM_{21} \qquad (8)$$

Matrix $\bar{\bar{C}}_1 IM_{21}$ represents the elements of IM_{21} matched by $\bar{\bar{C}}_1$. Matrix $I\bar{M}_{21}$ represents the elements of IM_{21} which are not matched by $\bar{\bar{C}}_1$.

A matrix $I\bar{\bar{M}}_{21}$ is defined as:

$$I\bar{\bar{M}}_{21} = I\bar{M}_{21} - I\bar{M}_{21}\bar{\bar{C}}_2 \qquad (9)$$

Matrix $I\bar{M}_{21}\bar{\bar{C}}_2$ represents the elements of $I\bar{M}_{21}$ matched by $\bar{\bar{C}}_2$. Matrix $I\bar{\bar{M}}_{21}$ represents the elements of $I\bar{M}_{21}$ that are not matched by $\bar{\bar{C}}_2$.

In other words, matrix $I\bar{\bar{M}}_{21}$ represents the elements of IM_{21} which are not matched by $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_2$. From (8) and (9), $I\bar{\bar{M}}_{21}$ can be written as:

$$I\bar{\bar{M}}_{21} = IM_{21} - \bar{\bar{C}}_1 IM_{21} - IM_{21}\bar{\bar{C}}_2 + \bar{\bar{C}}_1 IM_{21}\bar{\bar{C}}_2 \qquad (10)$$

The objective is to find the structure of $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_2$ such that IM_{21} is matched and $cost(\bar{\bar{J}}_1 \cup \bar{\bar{J}}_2)$ is minimal. This problem can be interpreted as being an optimization problem Pr_1:

$$Pr_1 \begin{cases} \text{Minimize } cost(\bar{\bar{J}}_1) + cost(\bar{\bar{J}}_2) \\ \text{Subject to} \\ IM_{21} - \bar{\bar{C}}_1 IM_{21} - IM_{21}\bar{\bar{C}}_2 + \bar{\bar{C}}_1 IM_{21}\bar{\bar{C}}_2 \equiv 0 \end{cases} \qquad (11)$$

An example is given here to clarify the matrices $I\bar{M}_{21}$ and $I\bar{\bar{M}}_{21}$ defined in this section. Consider two subsystems SS_1 and SS_2 doubly interconnected with $n_1 = 3$ and $n_2 = 2$. Suppose that matrix IM_{21} is given by :

$$IM_{21} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

Matrix IM_{21} means that variable x_1^1 is affected by x_2^1 and x_2^2 and that variable x_1^2 is affected by x_2^2.

If the variable x_1^1 is measured, matrices $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_1 IM_{21}$ will then have the following structure:

$$\bar{\bar{C}}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ and } \bar{\bar{C}}_1 IM_{21} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix};$$

Remark 4. *The zero lines in matrix $\bar{\bar{C}}_1$ have no physical meaning. However, they are kept to facilitate matrix manipulation.*

Matrix $\bar{\bar{C}}_1 IM_{21}$ represents the affecting variables which lie in the channel of the measured variable x_1^1. Matrix $I\bar{M}_{21}$ will then be:

$$I\bar{M}_{21} = IM_{21} - \bar{\bar{C}}_1 IM_{21} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix};$$

This matrix represents the variables affecting SS_1 which do not lie in the channel of the measured variable x_1^1. In other words,

$I\bar{M}_{21}$ represents the elements of IM_{21} which are not matched by the output matrix $\bar{\bar{C}}_1$.

Furthermore, if the variable x_2^2 is measured, then matrix $\bar{\bar{C}}_2$ will be given by:

$$\bar{\bar{C}}_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix};$$

and $I\bar{M}_{21}\bar{\bar{C}}_2$ will be:

$$I\bar{M}_{21}\bar{\bar{C}}_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix};$$

Matrix $I\bar{M}_{21}\bar{\bar{C}}_2$ represents the measured variables among the variables which are not matched by $\bar{\bar{C}}_1$. Since x_2^2 is measured, it is no longer unknown. Thus, it is eliminated from $I\bar{M}_{21}$. Matrix $I\bar{\bar{M}}_{21}$ is given by:

$$I\bar{\bar{M}}_{21} = I\bar{M}_{21} - I\bar{M}_{21}\bar{\bar{C}}_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix};$$

$I\bar{\bar{M}}_{21}$ represents the elements of IM_{21} that are not matched by $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_2$. $I\bar{\bar{M}}_{21} \equiv 0$ because measuring x_1^1 and x_2^2 matches the interconnection matrix IM_{21}.

Matrix $I\bar{\bar{M}}_{21}$ can be directly expressed as:

$$I\bar{\bar{M}}_{21} = IM_{21} - \bar{\bar{C}}_1 IM_{21} - IM_{21}\bar{\bar{C}}_2 + \bar{\bar{C}}_1 IM_{21}\bar{\bar{C}}_2$$

but the steps are detailed here for explanation.

Path P_2 Similarly to path P_1, the objective is to find $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_2$ such that IM_{12} is matched and $cost(\bar{\bar{J}}_1 \cup \bar{\bar{J}}_2)$ is minimal. The optimization problem Pr_2 is:

$$Pr_2 \begin{cases} \text{Minimize } cost(\bar{\bar{J}}_1) + cost(\bar{\bar{J}}_2) \\ \text{Subject to} \\ IM_{12} - \bar{\bar{C}}_2 IM_{12} - IM_{12}\bar{\bar{C}}_1 + \bar{\bar{C}}_2 IM_{12}\bar{\bar{C}}_1 \equiv 0 \end{cases} \quad (12)$$

The solutions $(\bar{\bar{C}}_1, \bar{\bar{C}}_2)$ obtained for each problem may be different. In addition, each problem may have multiple solutions. Suppose that problem Pr_i has s_i solutions. Thus, it is necessary to find which path P among the paths P_i $(i = 1,2)$ and which solution $(\bar{\bar{C}}_1, \bar{\bar{C}}_2)$ among the solutions $(\bar{\bar{C}}_1, \bar{\bar{C}}_2)^k$ $(k = 1, ..., s_i)$ should be considered.
Problems (11) and (12) are known as binary or zero-one nonlinear optimization problem. This is because the variables can only be zeros or ones. Many software programs are available for solving such problems. One may cite for example: MATLAB®, MAPLE®, GAMS® and LINDO®. It should be noted that none of these programs is able to solve (11) and (12) in their matrix form. However, it is easy to transform them to a scalar optimization. This is shown in the example.

5.4 Minimal cost SNDS: observability

After the determination of $\bar{J}_j$ and $\bar{\bar{J}}_j$, the last step is to determine J_1 and J_2. Sets J_1 and J_2 that verify the observability of SS_1 and SS_2 and minimize the network cost are the sets that, for all possible solutions $\bar{J}_j^{l_j}$ $(l_j = 1, ..., so_j)$ and all possible solutions $(\bar{\bar{J}}_1, \bar{\bar{J}}_2)^k$ $(k = 1, ..., s_i)$ of all possible paths P_i $(i = 1,2)$, minimize the following:

$$cost(\bar{J}_1^{l_1} \cup \bar{\bar{J}}_1^k) + cost(\bar{J}_2^{l_2} \cup \bar{\bar{J}}_2^k) \quad (13)$$

with $(i = 1,2;\ j = 1,2;\ k = 1, ..., s_i$ and $l_j = 1, ..., so_j)$.

6. EXAMPLE

In this section, an academic example is given to clear up the proposed SNDS. Two doubly interconnected subsystems SS_1 and SS_2 are supposed to have the following state representation:

$$SS_1 \begin{cases} \overbrace{\begin{pmatrix} \dot{x}_1^1 \\ \dot{x}_1^2 \\ \dot{x}_1^3 \end{pmatrix}}^{\dot{x}_1} = \overbrace{\begin{pmatrix} f_1^1(x_1) \\ f_1^2(x_1) \\ f_1^3(x_1) \end{pmatrix}}^{f_1(x_1,u_1)} + \overbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}^{\tilde{F}_1} \overbrace{\begin{pmatrix} \tilde{f}_1^1(x_2^1, x_2^2) \\ \tilde{f}_1^2(x_2^1, x_2^2) \\ \tilde{f}_1^3(x_2^1) \end{pmatrix}}^{\tilde{f}_1(\tilde{x}_1,u_1)} \\ y_1 = C_1 x_1 \end{cases} \quad (14)$$

$$SS_2 \begin{cases} \overbrace{\begin{pmatrix} \dot{x}_2^1 \\ \dot{x}_2^2 \\ \dot{x}_2^3 \end{pmatrix}}^{\dot{x}_2} = \overbrace{\begin{pmatrix} f_2^1(x_2) \\ f_2^2(x_2) \\ f_2^3(x_2) \end{pmatrix}}^{f_2(x_2,u_2)} + \overbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}}^{\tilde{F}_2} \overbrace{\begin{pmatrix} \tilde{f}_2^1(x_1^1) \\ \tilde{f}_2^3(x_1^1, x_1^2) \end{pmatrix}}^{\tilde{f}_2(\tilde{x}_2,u_2)} \\ y_2 = C_2 x_2 \end{cases} \quad (15)$$

Staroswiecki *et al* (Staroswiecki et al. (2004)) proposed an automaton in order to quantify system redundancy. The automaton for SS_1 is supposed to be given in Fig. 3. It represents all possible sensor network configurations for the subsystem. Three sensors δ_1^1, δ_1^2 and δ_1^3 can be used.

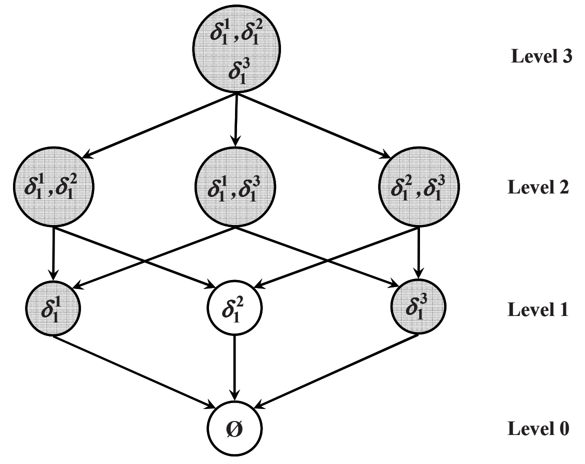

Fig. 3. Automaton of SS_1

A circle (or a node) represents a sensors network's state and an arrow represents the transition from a state to another. The transition occurs when a sensor breaks down. For example, if the initial sensor network state is $\{\delta_1^1, \delta_1^2\}$ and sensor δ_1^1 breaks down, sensors network passes from state $\{\delta_1^1, \delta_1^2\}$ to state $\{\delta_1^2\}$.

A node is gray if the corresponding sensor set verifies the system local observability. It is white if it does not. As example, for node $\{\delta_1^1, \delta_1^3\}$, the system is locally observable. For node $\{\emptyset\}$ (no sensors), the system is not observable.

The automaton for SS_2 is supposed to be given in Fig. 4. Three sensors δ_2^1, δ_2^2 and δ_2^3 can be used.

Sensors prices are given in Table 1. The objective is to find the structure of C_1 and C_2 so that the two subsystems SS_1 and SS_2 are observable and that $cost(J_1) + cost(J_2)$ is minimal.

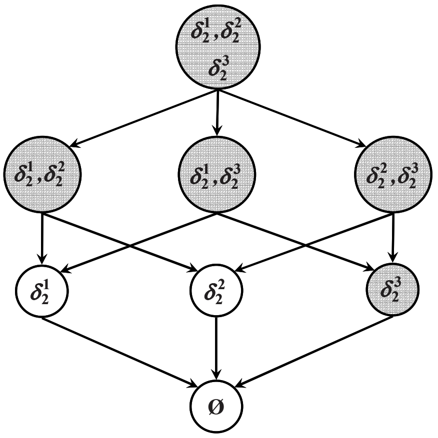

Fig. 4. Automaton of SS_2

Table 1. Sensors prices

Sensor	δ_1^1	δ_1^2	δ_1^3	δ_2^1	δ_2^2	δ_2^3
Price	70 €	40 €	60 €	30 €	50 €	90 €

6.1 *Interconnection Matrices IM*

The interconnection matrices IM_{21} and IM_{12} are given by:

$$IM_{21} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \text{ and } IM_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

6.2 *Local observability: determination of $\bar{C}_j$*

This step tends to resolve the problem given by (7). By investigating the automaton of SS_1 and SS_2 (respectively Fig. 3 and Fig. 4), one can notice that the sensor sets $\bar{J}_1$ and $\bar{J}_2$ verifying the local observability of SS_1 and SS_2 and minimizing sensors cost are $\bar{J}_1 = \{\delta_1^3\}$ and $\bar{J}_2 = \{\delta_2^1, \delta_2^2\}$ respectively. Thus :

$$\bar{C}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } \bar{C}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

6.3 *Interconnection matching: determination of $\bar{\bar{C}}_j$*

After the determination of $\bar{C}_1$ and $\bar{C}_2$, the next step toward designing C_1 and C_2 is finding $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_2$. This can be obtained by solving the two problems (11) and (12). This point is addressed in the sequel.

Optimization problem resolution method In this section, the method to solve the optimization problems given by (11) and (12) is explained.

Consider first the problem Pr_1 given by (11). It is quite interesting to investigate the structure of $I\bar{\bar{M}}_{21} = IM_{21} - \bar{\bar{C}}_1 IM_{21} - IM_{21}\bar{\bar{C}}_2 + \bar{\bar{C}}_1 IM_{21}\bar{\bar{C}}_2$. Supposing that $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_2$ are given by:

$$\bar{\bar{C}}_1 = \begin{pmatrix} \delta_1^1 & 0 & 0 \\ 0 & \delta_1^2 & 0 \\ 0 & 0 & \delta_1^3 \end{pmatrix}; \bar{\bar{C}}_2 = \begin{pmatrix} \delta_2^1 & 0 & 0 \\ 0 & \delta_2^2 & 0 \\ 0 & 0 & \delta_2^3 \end{pmatrix}$$

with δ_1^i and $\delta_2^j \in \{0,1\}$ $(i, j = 1,2,3)$.

Matrix $I\bar{\bar{M}}_{21}$ will then be given by:

$$I\bar{\bar{M}}_{21} = \begin{pmatrix} 1-\delta_1^1-\delta_2^1+\delta_1^1\delta_2^1 & 1-\delta_1^1-\delta_2^2+\delta_1^1\delta_2^2 & 0 \\ 1-\delta_1^2-\delta_2^1+\delta_1^2\delta_2^1 & 1-\delta_1^2-\delta_2^2+\delta_1^2\delta_2^2 & 0 \\ 1-\delta_1^3-\delta_2^1+\delta_1^3\delta_2^1 & 0 & 0 \end{pmatrix}$$

It is clear that $I\bar{\bar{M}}_{21}$ has the same structure as IM_{21} and the nonzero elements of this matrix have the following form: $1 - \delta_1^i - \delta_2^j + \delta_1^i\delta_2^j$. The index j of δ_2^j is the same as the affecting variables of SS_2 whereas the index i of δ_1^i is the same as the affected variables of SS_1 $(i, j = 1,2,3)$.

Furthermore, because the elements δ_1^i and δ_2^j of $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_2$ are binary (i.e. δ_1^i and $\delta_2^j \in \{0,1\}$), one can say that:

$$cost(\bar{\bar{J}}_1) + cost(\bar{\bar{J}}_2) = \sum_{i=1}^{3} co_1^i \delta_1^i + \sum_{j=1}^{3} co_2^j \delta_2^j$$

with co_1^i and co_2^j are the costs of δ_1^i and δ_2^j respectively. Thus, problem (11) can be reformulated as following:

$$Pr_1 \begin{cases} \text{Minimize } \sum_{i=1}^{3} co_1^i \delta_1^i + \sum_{j=1}^{3} co_2^j \delta_2^j \\ \text{Subject to} \\ \quad 1-\delta_1^1-\delta_2^1+\delta_1^1\delta_2^1 = 0 \\ \quad 1-\delta_1^2-\delta_2^1+\delta_1^2\delta_2^1 = 0 \\ \quad 1-\delta_1^3-\delta_2^1+\delta_1^3\delta_2^1 = 0 \\ \quad 1-\delta_1^1-\delta_2^2+\delta_1^1\delta_2^2 = 0 \\ \quad 1-\delta_1^2-\delta_2^2+\delta_1^2\delta_2^2 = 0 \\ \quad \delta_1^i \in \{0,1\}, (i = 1,2,3) \\ \quad \delta_2^j \in \{0,1\}, (j = 1,2,3) \end{cases} \quad (16)$$

Similarly, the problem (12) can be reformulated as:

$$Pr_2 \begin{cases} \text{Minimize } \sum_{i=1}^{3} co_1^i \delta_1^i + \sum_{j=1}^{3} co_2^j \delta_2^j \\ \text{Subject to} \\ \quad 1-\delta_2^1-\delta_1^1+\delta_2^1\delta_1^1 = 0 \\ \quad 1-\delta_2^3-\delta_1^1+\delta_2^3\delta_1^1 = 0 \\ \quad 1-\delta_2^3-\delta_1^2+\delta_2^3\delta_1^2 = 0 \\ \quad \delta_1^i \in \{0,1\}, (i = 1,2,3) \\ \quad \delta_2^j \in \{0,1\}, (j = 1,2,3) \end{cases} \quad (17)$$

Problem resolution Using LINDO®, the two problems (16) and (17) are solved.

Problem Pr_1 has one solution $(s_1 = 1)$. The matrices $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_2$, are:

$$\bar{\bar{C}}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \bar{\bar{C}}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

This solution means that the interconnection matrix IM_{21} can be compensated by measuring the affecting variables x_2^1 and x_2^2 of SS_2. The cost of this sensor set: $cost(\bar{\bar{J}}_1 \cup \bar{\bar{J}}_2) = cost(\{\delta_2^1, \delta_2^2\}) = 80$ €.

Problem Pr_2 has one solution $(s_2 = 1)$ which is:

$$\bar{\bar{C}}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \bar{\bar{C}}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

This solution means that the affecting variables x_1^1 and x_1^2 of SS_1 are measured. The cost of this sensor set: $cost(\bar{\bar{J}}_1 \cup \bar{\bar{J}}_2) = cost(\{\delta_1^1, \delta_1^2\}) = 110$ €.

6.4 Subsystems observability: determination of C_j

The last step of the SNDS consists in determining path P, sets $(\bar{J}_1, \bar{J}_2)$ and $(\bar{\bar{J}}_1, \bar{\bar{J}}_2)$ that minimize the following criteria:

$$cost(J_1) + cost(J_2) = cost(\bar{J}_1 \cup \bar{\bar{J}}_1) + cost(\bar{J}_2 \cup \bar{\bar{J}}_2)$$

Path P_1 For the first path P_1: $J_1 = \bar{J}_1 \cup \bar{\bar{J}}_1 = \{\delta_1^3\}$ and $J_2 = \bar{J}_2 \cup \bar{\bar{J}}_2 = \{\delta_2^1, \delta_2^2\}$.

The correspondent output matrices are:

$$C_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

For path P_1, the sensor network cost is: $cost(J_1) + cost(J_2) = 140$ €.

Path P_2 For the second path P_2: $J_1 = \bar{J}_1 \cup \bar{\bar{J}}_1 = \{\delta_1^1, \delta_1^2, \delta_1^3\}$ and $J_2 = \bar{J}_2 \cup \bar{\bar{J}}_2 = \{\delta_2^1, \delta_2^2\}$.

The correspondent output matrices are:

$$C_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

For this path, the sensor network cost is: $cost(J_1) + cost(J_2) = 250$ €.

By comparing the solutions obtained for each path, it can be found that the minimal cost sensor network is obtained by following path P_1. Hence using three sensors that measure the variables x_1^3, x_2^1 and x_2^2.

7. CONCLUSION

In this paper, a SNDS for complex systems is presented. The sensor network is minimal cost and verifies the system observability. As an illustration, an academic example of a system consisted of two subsystems is used. As shown, two possible paths exist for the sensor network design. As the number of the subsystems increases, the number of the possible paths increases. Due to the numerous feasible paths and the possible solutions for each one, the strategy should be programmed to make easier the determination of the solution.

The designed sensor network depends on the subsystems and their interconnections. In this work, a complex system is decomposed into subsystems by using a physical decomposition method. This technique consists in splitting the system into subsystems while respecting its physical structure. However, other decomposition techniques may be used. This problem is not considered in this paper.

The minimal cost sensor network is determined by using two separate optimization problems: the first for local observability and the second for interconnection matching. The solution obtained for each problem is optimal. However, there is no guarantee that the combination of the two solutions is optimal. Thus, it is necessary to encapsulate the two problems in a unique optimization problem to guarantee the optimality of the network cost. This problem will be considered in future works. In addition, other criteria such as sensor network reliability of complex systems should be investigated.

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