Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Nonlinear observer and output feedback design for a combustion engine test bench ★

Dina Shona Laila * **Engelbert Grünbacher** **

* *Electrical & Electronic Department, Imperial College London, London SW7 2AZ, UK. d.laila@imperial.ac.uk*
** *Johannes Kepler University, Altenbergerstrasse 69, A-4020 Linz, Austria. engelbert.gruenbacher@jku.at*

Abstract: Combustion engine control depends strongly on the availability and the quality of the signals involving in the controller construction. In general, not all signals are available through measurement, and therefore an observer is necessary to realize the controller. This paper proposes an observer design for a combustion engine test bench. The observer is used to estimate the torque and the rotation angle of the engine, based on the measurement of the engine and the dynamometer speeds. The convergence of the observer is proved, and separation principle is also shown. The observer is then used to construct an output feedback controller for set point tracking of the test bench. Numerical simulations are performed, showing the performance of the observer and comparing the performance of the output feedback with the state feedback controller. Moreover, the effect of combustion oscillation which causes a vibration noise is taken into account, and the use of internal model based filter to handle the noise is presented.

Keywords: Combustion engine test bench control; Observer design; Internal model based filter; Setpoint tracking.

1. INTRODUCTION

Most of the feedback stabilization problems for nonlinear systems are solved using state feedback approach, assuming that all states are available from measurement. However, this assumption is often unrealistic in practice. In this situation, state feedback cannot be realized and hence output feedback or dynamic feedback control becomes necessary. While design tools mainly aim at designing a state feedback controller [8], designing an observer is a useful solution to provide the estimates of the unmeasured states to be used for constructing an output feedback controller.

In other cases even when the states may be available from measurement, observer may be useful in reducing the number of sensors applied to the plant, and hence reducing the data acquisition complexity and the cost especially when the required sensors are complicated and expensive. Another problem in real control implementation is the measurement noise. While in theory we often assume all signals involve in the operation are ideal, in reality noise commonly appears and affects the measurement signals.

In this paper, we study an output feedback control design for a combustion engine test bench. As combustion engines are widely used in automotive as well as industrial applications, the topic has attracted many researchers to study the control problems of the engine as well as the engine test bench (see [1, 5, 10] and references therein).

The issue of partially available state measurements and the noise are addressed. To handle the problem of unmeasured signals, an observer design is proposed in this paper. The observer is a partial state nonlinear observer, and the design follows the Luenberger observer approach, applied to nonlinear systems. We prove the convergence of the observer by showing the convergence of the observation error. We also show that separation principle holds. This is very important as we will use the observer to build an output feedback controller for the test bench.

In practice, the batch behavior of the combustion, which depends on the crankshaft angle [11] causes the combustion oscillation which is considered as a periodic noise to the engine speed. An internal model based filter is also designed to get rid of the effect of this periodic noise. Some simulations are done to test the performance of the observer and the filter to solve a setpoint tracking problem of the speed and the torque of the test bench.

2. NOTATION AND DEFINITIONS

The set of real numbers is denoted respectively by $\mathbb{R}$. A function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$ if it is continuous, strictly increasing and zero at zero. It is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and unbounded. Functions of class $\mathcal{K}_\infty$ are invertible. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}L$ if $\beta(\cdot, t)$ is of class $\mathcal{K}$ for each $t \geq 0$ and $\beta(s, \cdot)$ is decreasing to zero $\forall s > 0$. We often drop the arguments of a function whenever they are clear from the context.

Consider a general input affine nonlinear system

$$\dot{x} = f(x) + g(x)u \ , \quad y = h(x) \ , \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input and $y \in \mathbb{R}^p$ is the output. The functions f, g and h are smooth and f is zero at zero. If the input u is a state feedback controller, we write the closed loop system of (1) as

$$\dot{x} = \tilde{f}(x) \tag{2}$$

★ This work was performed under the grant of 230100-LCM.

We use the following definitions throughout the paper.

Definition 2.1. (Asymptotic stability): A continuous and differentiable function $V : \mathbb{R}^n \rightarrow \mathbb{R}$ is called an asymptotic stability (AS) Lyapunov function for the continuous-time system (2) if there exist class $\mathcal{K}_\infty$ functions $\alpha_1(\cdot)$, $\alpha_2(\cdot)$ and $\alpha_3(\cdot)$ such that the following holds

$$\alpha_1(|x|) \le V(x) \le \alpha_2(|x|) \ , \tag{3}$$

$$\frac{\partial V}{\partial x}\tilde{f}(x) \le -\alpha_3(|x|) \ , \tag{4}$$

for all $x \in \mathbb{R}^n$. ■

Definition 2.2. (Asymptotic stabilizability): A nonlinear system (1) is asymptotically stabilizable by means of a state feedback if there exists a state feedback controller $u = u(x)$, such that the closed-loop system (2) with control u is asymptotically stable. ■

Consider another dynamical system

$$\dot{z} = \Gamma_T(z, y, u); \quad \hat{x} = \gamma(z) \ , \tag{5}$$

where $z \in \mathbb{R}^l$.

Definition 2.3. (Asymptotically stable observer): The system (5) is an asymptotic observer for (1) if for any $x \in R^n$ and $z \in R^l$ the estimation state $\hat{x}$ asymptotically converges to the estimated state x. If $\hat{x} = z$, the system (5) is called an identity observer. Moreover, the system (1) is called asymptotically observable if it possesses an asymptotic observer. ■

3. OBSERVER DESIGN FOR THE ENGINE TEST BENCH

3.1 Engine test bench model

A simple schematic diagram of the combustion engine test bench is illustrated in Figure 1.

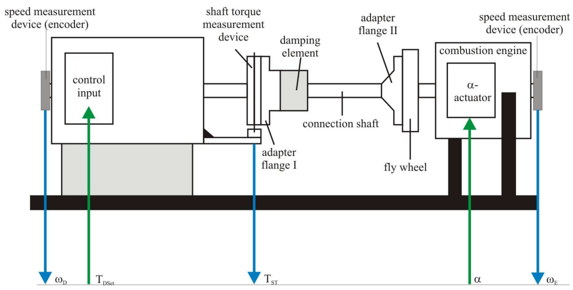

Figure 1. The combustion engine test bench system

The main parts of such a dynamical engine test bench are the dynamometer, the connection shaft and the combustion engine itself. One of the control design objectives for a dynamical engine test bench control is to stabilize the engine torque and the engine speed.

Considering the torque of the combustion engine and the air gap torque of the dynamometer as the inputs to the mechanical part of the engine test bench system, the dynamical model of the engine can be represented by a two mass oscillator

$$\dot{\psi}_\Delta = \omega_E - \omega_D \tag{6}$$

$$\dot{\omega}_E = \frac{1}{\theta_E}(T_E - c\psi_\Delta - d(\omega_E - \omega_D)) \tag{7}$$

$$\dot{\omega}_D = \frac{1}{\theta_D}(c\psi_\Delta + d(\omega_E - \omega_D) - T_{DSet}) \ , \tag{8}$$

where ψ_Δ is the torsion angle, ω_E and ω_D are respectively the engine and the dynamometer angular velocity, T_E is the engine's torque, T_{DSet} is the air gap torque of the dynamometer, θ_E and θ_D are the inertia of the engine and the dynamometer, respectively. The dynamical model of the combustion engine test bench is described by

$$\dot{T}_E = -\rho(T_{Estat}, \omega_E)T_E + \rho(T_{Estat}, \omega_E)T_{Estat} \ ,$$

where T_{Estat} is the output of the static engine map and $\rho(T_{Estat}, \omega_E)$ is the nonlinear state and input depending eigenvalue. Under some assumptions this dynamical model is approximated by the class of the extended Hammerstein systems (see [3] for more details)

$$\dot{T}_E = -(c_0 + c_1\omega_E + c_2\omega_E^2)T_E + m(\omega_E, T_E, \alpha) \ . \tag{9}$$

with c_0, c_1 and c_2 are some positive coefficients and $m(\omega_E, T_E, \alpha)$ is a continuous nonlinear function. From the continuity of m, without lose of generality, we assume that it is locally Lipschitz with respect to T_E.

3.2 Observer design

From the previous subsection we have obtained the dynamic model of the test bench (6)-(9), which is a fourth order system. In practice, from the four state variables of the system, only the engine angular velocity ω_E and the dynamometer angular velocity ω_D are available through measurement. Hence, we can write the output equation for the system as

$$y_1 = \omega_E \qquad y_2 = \omega_D \ . \tag{10}$$

As the control problem of the test bench usually involves the torque control, in order to design a feedback controller the knowledge of the torque T_E becomes necessary as it is needed for constructing the feedback controller. For that, an observer is required to estimate the unmeasured states T_E, as well as ψ_Δ. In this subsection, we propose an observer design that functions to estimate these two quantities. The constructed observer is a reduced order observer instead of a full order one.

The following theorem provides the observer construction, and the procedure of how to construct the observer is given in the proof of the theorem.

Theorem 3.1. Given a continuous-time model of an engine test bench (6)-(9), with measured output (10). The following reduced order observer

$$\begin{aligned} \dot{\hat{T}}_E &= -(c_0 + c_1\omega_E + c_2\omega_E^2)\hat{T}_E + m(\omega_E, \hat{T}_E, \alpha) + L_1 e_1 \\ \dot{\hat{\psi}}_\Delta &= \omega_E - \omega_D + L_2 e_2 \end{aligned} \tag{11}$$

where $L_1 > 0$, $L_2 > 0$ and

$$\begin{aligned} e_1 &= \theta_E\dot{\omega}_E + \theta_D\dot{\omega}_D + T_{DSet} - \hat{T}_E \\ e_2 &= \frac{1}{c}(\theta_D\dot{\omega}_D - d(\omega_E - \omega_D) + T_{DSet} - c\hat{\psi}_\Delta) \end{aligned} \tag{12}$$

is an asymptotically stable observer for the system. ■

Proof of Theorem 3.1: Given the system (6)-(9) with output (10) and the observer (11). We define the estimation errors as $e_1 := T_E - \hat{T}_E$ and $e_2 := \psi_\Delta - \hat{\psi}_\Delta$. First, we will show that the error terms satisfy (12). It is straight forward that from (8) we can obtain

$$\psi_\Delta = \frac{1}{c}\Big(\theta_D\dot{\omega}_D - d(\omega_E - \omega_D) + T_{DSet}\Big) \ . \tag{13}$$

Moreover, from (7) and (13) we have

$$\begin{aligned} T_E &= \theta_E\dot{\omega}_E + c\psi_\Delta + d(\omega_E-\omega_D) \\ &= \theta_E\dot{\omega}_E + \theta_D\dot{\omega}_D - d(\omega_E-\omega_D) + T_{DSet} + d(\omega_E-\omega_D) \\ &= \theta_E\dot{\omega}_E + \theta_D\dot{\omega}_D + T_{DSet} \ . \end{aligned} \quad (14)$$

Therefore using (14) we obtain

$$e_1 = T_E - \hat{T}_E = \theta_E\dot{\omega}_E + \theta_D\dot{\omega}_D + T_{DSet} - \hat{T}_E \ ,$$

and using (13), we obtain

$$e_2 = \psi_\Delta - \hat{\psi}_\Delta = \frac{1}{c}\Big(\theta_D\dot{\omega}_D - d(\omega_E-\omega_D) + T_{DSet} - c\hat{\psi}_\Delta\Big).$$

Now, we can write the error dynamics

$$\begin{aligned} \dot{e}_1 = \dot{T}_E - \dot{\hat{T}}_E &= -(c_0 + c_1\omega_E + c_2\omega_E^2)e_1 \\ &+ m(\omega_E, T_E, \alpha) - m(\omega_E, \hat{T}_E, \alpha) - L_1 e_1 \ , \end{aligned} \quad (15)$$

and $\dot{e}_2 = -L_2 e_2$. To show the asymptotic stability of the error system, we choose $V = \frac{1}{2}e^\top e$ as the Lyapunov function. The derivative of V is

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 \ . \quad (16)$$

From the local Lipschitzity of m w.r.t T_E, we can write

$$m(\omega_E, T_E, \alpha) - m(\omega_E, \hat{T}_E, \alpha) \leq L_m(T_E - \hat{T}_E) = L_m e_1 \ ,$$

with $L_m > 0$. Hence, we can write

$$\begin{aligned} \dot{V} &< -(c_0 + c_1\omega_E + c_2\omega_E^2)e_1^2 + L_m e_1^2 - L_1 e_1^2 - L_2 e_2^2 \\ &= -\Big(C(\omega_E) + L_1 - L_m\Big)e_1^2 - L_2 e_2^2 < -L e_1^2 - L_2 e_2^2. \end{aligned} \quad (17)$$

The existence of $L > 0$ is guaranteed by choosing L_1 large enough so that $C(\omega_E) + L_1 > L_m$ for all ω_E. Therefore the Lyapunov derivative is negative definite. Hence, it is proved that the observer (11) is an asymptotically stable observer for the system (6)-(9) with output (10). ∎

3.3 Separation Principle

Separation principle needs to hold when an observer is used for designing an output feedback controller. For the separation principle to hold, we require asymptotic stabilizability and uniform observability of the system with respect to the observer.

Given a state feedback control u_k for the system (6)-(9). To guarantee that the estimated state $\hat{T}_E$ and $\hat{\psi}_\Delta$ can be used to replace the unmeasured state T_E and ψ_Δ in a feedback control construction, the separation principle must hold. The separation principle required to solve the stabilization problem is stated in the following result.

Proposition 1. (Separation Principle) Consider the system (6)-(9). Suppose there exists a controller $u_k = u_k(T_E, \psi_\Delta, \omega_E, \omega_D)$ that asymptotically stabilizes the system. Assume that u_k is continuous, and zero at zero. The asymptotic stabilization for the system using an output feedback $u_k = \hat{u}_k(\hat{T}_E, \hat{\psi}_\Delta, \omega_E, \omega_D)$ from the observer (11) is solvable if the closed-loop system is uniformly observable. ∎

Remark 3.1. Note that in practice the signals $\dot{\omega}_E$ and $\dot{\omega}_D$ are not measured. Although theoretically it is possible to use a differentiator to obtain these signals from the output ω_E and ω_D, it is not practical as a differentiator needs two input signals. The common practice is by approximating the derivatives as follows

$$\dot{\omega}_E \approx \frac{\omega_E(t) - \omega_E(t-T)}{T}, \quad \dot{\omega}_D \approx \frac{\omega_D(t) - \omega_D(t-T)}{T},$$

with $T > 0$ sufficiently small. For digital implementation of the observer using computer, T may be taken equal to the sampling period of the process. ∎

4. SET POINT TRACKING USING OUTPUT FEEDBACK

4.1 Output feedback controller design

As separation principle is valid for the state feedback controller and the observer, we can use the state estimate to substitute the original state to construct an output feedback controller for the engine. In [9] we have designed a controller that guarantees asymptotic stability for a setpoint tracking problem of the engine. The controller is designed via a model transformation approach as briefly describe in the followings.

We define the state normalization as follows

$$\begin{aligned} x_1 &= \frac{T_E - T_{E0}}{\Delta T_E} \ , \qquad x_2 = \frac{\psi_\Delta - \psi_{\Delta 0}}{\max(\psi_\Delta)} \ , \\ x_3 &= \frac{\omega_E - \omega_{E0}}{\Delta\omega_E} \ , \qquad x_4 = \frac{\omega_D - \omega_{D0}}{\Delta\omega_D} \ , \end{aligned} \quad (18)$$

with T_{E0}, $\psi_{\Delta 0}$, ω_{E0} and ω_{D0} defines the operating point and ΔT_E, $\max(\psi_\Delta)$, $\Delta\omega_E$ and $\Delta\omega_D$ the maximum expected distance from the equilibrium point. With this scaling and taking $c\max(\psi_\Delta) = \Delta T_E$, the system (6)-(9) can now be represented in its normalized model as follows

$$\begin{aligned} \dot{x}_1 &= -(\tilde{c}_0 + \tilde{c}_1 x_3 + \tilde{c}_2 x_3^2)x_1 - \gamma_1 x_3 - \gamma_2 x_3^2 + u_1 \\ \dot{x}_2 &= b(x_3 - x_4) \\ \dot{x}_3 &= \frac{1}{\theta_E}\Big(\frac{c}{b}x_1 - \frac{c}{b}x_2 - d(x_3 - x_4)\Big) \\ \dot{x}_4 &= \frac{1}{\theta_D}\Big(\frac{c}{b}x_2 + d(x_3 - x_4)\Big) + u_2 \ , \end{aligned} \quad (19)$$

with the inputs

$$u_1 = \frac{m(x_1, x_3, \alpha) - m(0,0,\alpha_0)}{\Delta T_E} \ , \ u_2 = -\frac{T_{DSet} - T_{D0}}{\theta_D\Delta\omega_D} \ ,$$

and $\tilde{c}_0$, $\tilde{c}_1$, $\tilde{c}_2$, b, γ_1, γ_2 are positive constants.

In [9] a continuous-time controller has been constructed to satisfy some robust optimal design criteria. The control Lyapunov function used for designing the controller is

$$V(x_1, x_2, x_3, x_4) = k_1 x_1^2 + k_2 x_2^2 + k_3 x_3^2 + k_4 x_4^2 + k_5 x_2 x_4 \ ,$$

with $k_i \in \mathbb{R}^+$, $i = 1 \cdots 4$ and $k_5 \in \mathbb{R} - \{0\}$. The positive definiteness of $V(\cdot)$ is guaranteed for some k_5 with $|k_5|$ sufficiently small. The controller takes form

$$u = -[R(x)g(x)]^\top\left[\frac{\partial V(x)}{\partial x}\right]^\top = -\begin{bmatrix} 2r_1 k_1 x_1 \\ r_2(2k_4 x_4 + k_5 x_2) \end{bmatrix} \quad (20)$$

with a positive matrix $R = \mathrm{diag}[r_1, 0, 0, r_2]$. The controller has been proved to asymptotically stabilized the system.

Note that the controller (20) is designed to asymptotically stabilize the normalized model (19) of the engine. As our main objective is to apply the controller to the engine test bench, we need to transform back the normalized model of the test bench and test the stability of tracking of the original system. From the state transformation (18), we have the following relations

$$\begin{aligned} m(\omega_E, T_E, \alpha) &= u_1\Delta T_E + T_{E0}(c_0 + c_1\omega_{E0} + c_2\omega_{E0}^2) \\ T_{DSet} &= -u_2\theta_D\Delta\omega_D + T_{D0} \ , \end{aligned} \quad (21)$$

where we have chosen $\psi_{\Delta 0} = \frac{T_{E0}}{c}$, $T_{D0} = T_{E0}$ and $\omega_{E0} = \omega_{D0}$. The setpoint tracking aims to follow the changing of operating points (T_{E0}, ω_{E0}) of the engine.

Replacing the unmeasured states with their estimate value, and applying the transformation (18), the output feedback controller takes the form

$$\begin{aligned} m(\omega_E, \hat{T}_E, \alpha) &= -2r_1k_1(\hat{T}_E - T_{E0}) + T_{E0}(c_0 + c_1\omega_{E0} + c_2\omega_{E0}^2) \\ T_{DSet} &= 2k_4r_2\theta_D(\omega_D - \omega_{D0}) + k_5r_2\theta_D\Delta\omega_D\frac{c\hat{\psi}_\Delta - T_{E0}}{\Delta T_E} + T_{D0}. \end{aligned} \tag{22}$$

4.2 Simulation results

In this subsection, by simulation we first show the convergence of the observer in estimating the states T_E and ψ_Δ. Further, we will apply the output feedback controller (22) to control the engine test bench (6)-(9). The performance of the output feedback controller (22) is compared to the state feedback controller (20) for a setpoint tracking assignment. The parameters of the test bench are given on Table 1. The engine parameters are based on a dynamic test bench with a production BMW M47D diesel engine. The coefficients of the approximate dynamic model after scaling are $\tilde{c}_0 = 6.3466$, $\tilde{c}_1 = 3.2096$, $\tilde{c}_2 = 2.7744$, $b = 1.8264 \times 10^3$, $\gamma_1 = 4.8143$ and $\gamma_2 = 4.1616$. For the controller we have chosen the parameters $k_1 = 1.5686$, $k_2 = 0.00174$, $k_3 = 0.88$, $k_4 = 1.05$, $k_5 = -0.0145$ and $R = \text{diag}[1, 2]$. We apply the controller for a setpoint tracking when changing the operating point (T_E, ω_E) of the engine each following a square wave reference signal. The initial condition of the engine (50, 50/c, 3000, 3000) and the initial condition of the observer (100, 100/c). We have assumed that $\dot{\omega}_E$ and $\dot{\omega}_D$ are not measured, and we use the dirty derivative as in Remark 3.1 with $T = 0.1$ sec.

Table 1. The engine's parameters

Parameter	Value	Unit
Engine inertia (θ_E)	0.32	kgm^2
Dynamometer inertia (θ_D)	0.28	kgm^2
Damping constant (d)	3.5505	Nms/rad
Stiffness of the shaft (c)	1.7441×10^3	Nm/rad

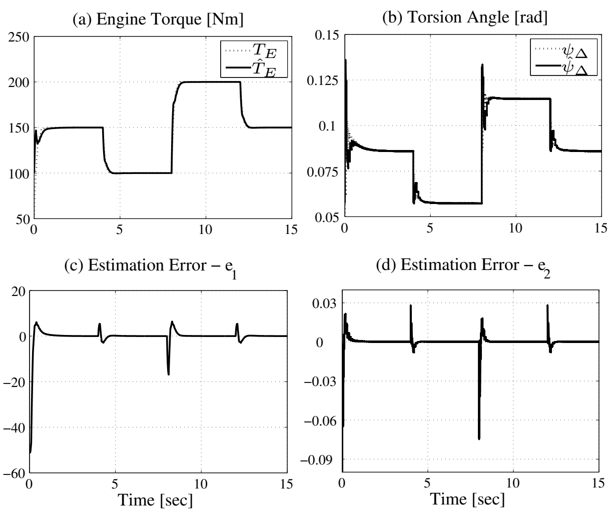

Figure 2. Convergence test of the observer.

Figure 2 shows that the observer can estimate the unmeasured states T_E and ψ_Δ very well as the observer converges very quickly to the engine system, even when the initial condition of the observer is very different from the initial condition of the engine. The response of the system with the output feedback is shown in Figure 3 which appears almost overlapped with the response with state feedback.

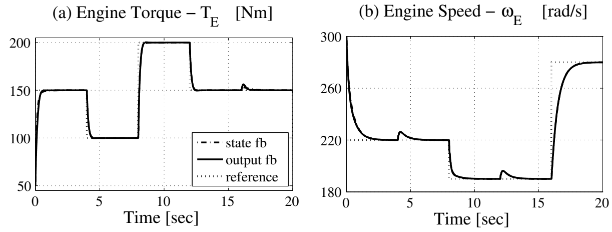

Figure 3. Tracking using output feedback controller.

5. FILTERING THE PERIODIC NOISE

Although it has been shown that the observer can converge quickly and it performs well to construct an output feedback controller for the engine test bench, this has not fully solved the implementation problem. In fact the combustion engine model is just an approximation of a highly nonlinear system, that the performance limits of the actuators have to be considered and furthermore that the measured signals, i.e. ω_E and ω_D are affected by the batch behavior of the combustion, which depends on the crankshaft angle [11]. Since each cylinder fires every 720° crankshaft angle (720°CA), it means for a four strokes engine a combustion occurs in every 180°CA that causes the combustion oscillation which is considered as a periodic noise to the engine speed.

For the considered control task the latter is much more complicated. In order to achieve a robust estimator it is necessary to increase the observer gains L_1 and L_2, whereas this will also increase the effect of the noisy speed measurement to the estimated signals, particularly as the error terms (15) depend on the derivatives of the measured speed signals. Neglecting this noise will cause the generated output feedback very noisy and not implementable due to the performance limits of the actuators. As a result the full control loop will not perform well. To get rid of this periodic noise, we use a fast filter. The frequency of the fundamental oscillation of the noise is directly related to the engine speed and therefore it is known.

For the control point of view we are only interested in the mean value of the signals (T_E and ψ_Δ), hence we have to separate the periodical part and the mean value part of the signal. We apply a frequency varying internal model filter to reconstruct the estimated signal including the periodical signals. From the state of the internal model it is then possible to calculate the mean value of the reconstructed signals. In the following we will only sketch the method and for further details we refer to [4].

5.1 Modeling the combustion oscillation via parameter varying exosystem

It can be seen that the combustion oscillations can be described by linear but frequency depending harmonic oscillators

$$\dot{\omega}_i = S_i(\eta(t))\omega_i \ , \quad d_{hi} = c'_{Si}\omega_i \tag{23}$$

where

$$S_i(\eta) = \begin{pmatrix} 0 & -i\eta(t) \\ i\eta(t) & 0 \end{pmatrix} \quad \forall i = 1 \cdots 6 \tag{24}$$

and $\eta(t)$ defines the frequency of the first harmonic of the combustion oscillations. The output map is

$$c_{Si} = (\alpha_i \ \ 0) \ \text{ or } \ c_{Si} = (0 \ \ \alpha_i) \ . \tag{25}$$

For a simple integrator we can assume the following offset $\dot{\omega}_0 = 0$ and $d_{h0} = \alpha_0\omega_0$. Hence the full periodic signal (considering up to the 6th harmonics) with $\omega = (\omega_0 \ \omega_{11} \ \omega_{12} \ \cdots \ \omega_{61} \ \omega_{62})'$ becomes

$$\dot{\omega} = S(\eta(t))\omega \ , \quad d_h = c'_S\omega \tag{26}$$

with

$$S(\eta(t)) = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & S_1(\eta(t)) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_6(\eta(t)) \end{pmatrix} \tag{27}$$

and from the second output map

$$c'_S = (\alpha_0 \ 0 \ \alpha_1 \ 0 \ \alpha_2 \ \cdots \ 0 \ \alpha_6) \ . \tag{28}$$

Note that we have chosen the second output map given by (25) for the same reason as given in [2, proof of Lemma 1].

The internal model principle will be utilized to reconstruct the combustion oscillation. Commonly the internal model principle is only considered for constant frequencies [7], thus the structure of the internal model description of the actual problem has to be rearranged slightly, by taking the exosystem to be parameter dependent and the internal model controller to be parameter varying. The structure of the internal model based filter is shown by Figure 4. In our case, the observer is acting as the exosystem as shown by Figure 5, when using $G(s) = 1$.

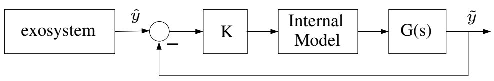

Figure 4. Internal model based filter structure

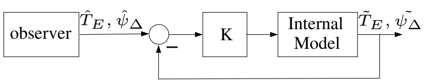

Figure 5. Connection of the observer and the internal model filter

Remark 5.1. Note that in reality the estimate $\hat{T}_E$ and $\hat{\psi}_\Delta$ contain the oscillation component d_h which involves the periodical part. As we are interested in the mean value of the signals, in the rest of the discussion we only consider the periodic combustion oscillation d_h since T_E and ψ_Δ are not periodic. Also, as $\hat{T}_E$ and $\hat{\psi}_\Delta$ are treated in the same way, due to limited space, we only present the result for filtering $\hat{T}_E$. ■

5.2 Design of the frequency dependent internal model

In the standard case when the oscillation frequency is constant, the error $\hat{T}_E - \tilde{T}_E$ tends to zero if the poles of the internal model includes all the eigenvalues of the exosystem and it is controllable. If the exosystem is a nonlinear oscillator, the internal model is constructed by searching for the normal form of the exosystem in order to get the observable modes of the nonlinear exosystem which would be necessary for designing the control input and therefore also to design the (controllable) internal model. However for this application a normal form is not necessary, since the oscillation model (26) can be simply extended to get a controllable system which has the same eigenvalues (IM principle) as the exosystem (26) itself.

Therefore the integrator subsystem has to be extended by the control input u_0

$$\dot{\xi}_0 = u_0 \ , \quad \hat{d}_{h0} = \xi_0 \tag{29}$$

and to the oscillator subsystems we add an input vector $b = (0 \ 1)'$ such that the internal submodel takes the form

$$\begin{aligned} \dot{\xi}_i &= A_i(\eta)\xi + b_i u_i = \begin{pmatrix} 0 & -i\eta(t) \\ i\eta(t) & 0 \end{pmatrix} \xi_i + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_i \\ \hat{d}_{hi} &= c_{IM_i}\xi = (0 \ 1)\xi \ . \end{aligned} \tag{30}$$

Note that in (30) the gains α_0 to α_6 are set equal to 1 since for the modeling purpose the magnitudes of the oscillations may be assumed constant. The magnitude of the oscillation can also be defined by the initial states of the exosystem. Thus the composite internal model is

$$\begin{aligned} \dot{\xi} &= A(\eta)\xi + Bu = A(\eta)\xi + B(u_1 \ \cdots \ u_6)' \\ \hat{d}_h &= c_{IM}\xi \end{aligned} \tag{31}$$

with

$$A(\eta) = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & A_1(\eta) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_6(\eta) \end{pmatrix} \ , \quad B = \begin{pmatrix} b_0 & 0 & \cdots & 0 \\ 0 & b_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_6 \end{pmatrix} \ ,$$

$$c_{IM} = (1 \ c_{IM_1} \ \cdots \ c_{IM_6})' \ .$$

5.3 Stabilizing parameter varying feedback controller

We will design a converging, stabilizing controller for the internal model, aiming to get the steady state response of the internal model equal to the measured oscillation [6]. For a static but parameter varying feedback control law $u = K(\eta)e$ the closed loop system becomes

$$\dot{\xi} = A(\eta)\xi + BK(\eta)e = (A(\eta) - BK(\eta)\tilde{c}'_{IM})\,\xi + BK(\eta)d_h .$$

Referring to [2], convergence is achieved if and only if the parameter varying closed loop system is asymptotically stable. With $\tilde{c}'_{IM_i} = (0 \ 1)$ and $K = (k_0(\eta) \ \ldots \ k_6(\eta))'$ the closed loop of the oscillator subsystems becomes

$$\dot{\xi}_i = (A_i(\eta) - b_i k_i(\eta)\tilde{c}_{IM_i})\xi_i + k_i(\eta)\tilde{c}_{IM_i} d_h . \tag{32}$$

For stability analysis d_h is set equal to zero and the system matrix of the closed loop oscillator subsystems is

$$A_{i_{cl}}(\eta) = \begin{pmatrix} 0 & -i\eta \\ i\eta & -k_i(\eta) \end{pmatrix} \quad \forall i = 1, \ldots, 6 \ . \tag{33}$$

For the second order subsystems we further choose a constant controller $k_i = \tilde{k}_i$, $\forall i = 1, \ldots, 6$, and $A_{i_{cl}}(\eta)$ in (33) becomes

$$A_{i_{cl}}(\eta) = \begin{pmatrix} 0 & -\eta \\ \eta & -\tilde{k}_i \end{pmatrix} \quad \forall i = 2, \ldots, 6 \ . \tag{34}$$

For the integrator subsystem the constant feedback $u_0 = \tilde{k}_0 e$ is used, such that the subsystem becomes

$$\dot{\xi}_0 = -\tilde{k}_0\xi_0 + k_0 d_h \ . \tag{35}$$

Hence the controller that stabilizes the internal model is

$$K = (\tilde{k}_0\eta \; \tilde{k}_1 \; \cdots \; \tilde{k}_6)' \tag{36}$$

where $\tilde{k}_1$ to $\tilde{k}_6$ are positive constants. These constants influence the convergence rate of the oscillator and someone might desire fast convergence without overshoot. Here the characteristic polynomial $\Delta_i(s) = s^2 + \tilde{k}_i s + (i\eta)^2$ of the closed loop subsystem (34) will be considered. One possibility for fastest convergence without overshoot is at $\tilde{k}_i = 2i\eta$. However for a general application where the measured signal is always noisy and the output of the observer (and particularly the predicted output) is noisy too, if k_i has too high values and the convergence rate is too fast (the observer tends to learn the noise).

It is well known for LPV systems that fast changing parameters can cause stability problems. Therefore, we need to make sure that the filter is stable in a given parameter range. The proof of stability of the filter follows exactly the same steps as in [4] and due to limited space is not presented in this manuscript.

5.4 Simulation results

The final control structure now consists of the observer of the noise filter and of the controller. In Figure 6 we can see the engine torque, the engine speed, the control input 1 and the control input 2 for the test bench control problem. In Figure 7 we show the comparison of the filtered signal using the internal model observer and a comparable Butterworth filter. Hence it can be seen that even in dynamic operation the estimation error of the mean value combustion engine test bench is quite small.

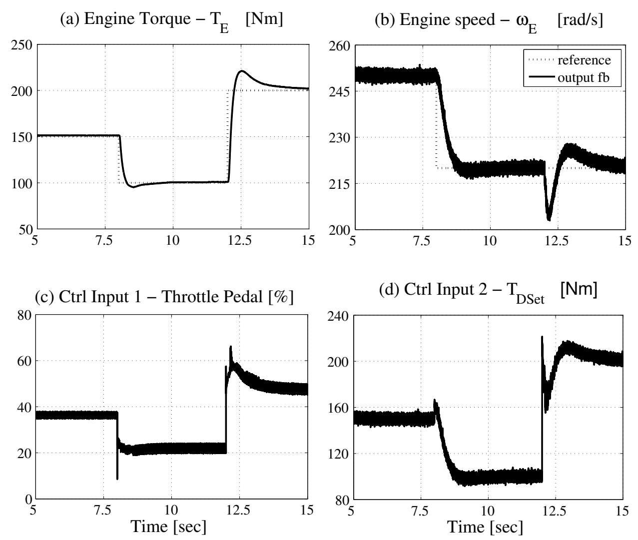

Figure 6. Tracking using output feedback and noise filter.

6. SUMMARY

In this paper we have presented a partial state observer design for a combustion engine test bench system. We have shown that the observer is asymptotically convergent to the system. We have also shown that separation principle is satisfied. We have demonstrated some simulation results showing the performance of an output feedback controller developed using the proposed observer.

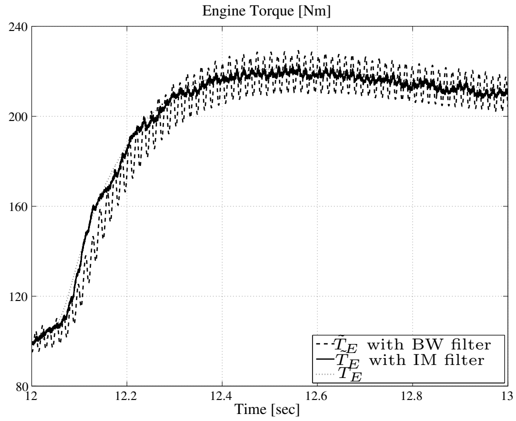

Figure 7. Comparison of the measured and the filtered estimated torque.

Moreover, as noise always involves in the real measurement, we also discuss the issue on vibration noise. The effect of the noise to the estimation and the use of filter to solve the problem is also presented. Some simulation results are also provided.

As this study is done only based on simulation, the next step will be to implement the observer and output feedback controller as well as the filter design to the real engine test bench for solving the setpoint tracking problem.

REFERENCES

[1] D. Carlucci, F. Donati, and M. Peisino. Digital control of a test bench for a diesel engine. In *Modelling, Identification and Control. Proc. IASTED Int. Symp.*, pages 248–251, 1984.

[2] C. Furtmüller and E. Grünbacher. Suppression of periodic disturbances in continuous casting using and internal model predictor. In *Proc. IEEE Int. Conf. Control Applications*, Munich, Germany, 2006.

[3] E. Gruenbacher. *Robust Inverse Control of A Class of Nonlinear Systems.* PhD Thesis. Johannes Kepler Universität, 2005.

[4] E. Grünbacher, C. Furtmüller, and L. del Re. Suppression of frequency varying periodic disturbances in continuous casting using and internal model predictor. In *Proc. IEEE ACC*, New York, USA, 2007.

[5] L. Guzzella and A. Amstutz. Control of diesel engines. *Control Syst. Mag.*, 18:53–71, 1998.

[6] A. Isidori. *Nonlinear Control Systems.* Springer Verlag, 1995.

[7] C. D. Johnson. *Theory of Disturbance-Accommodating Controllers, Control and Dynamic Systems: Advances in Theory and Appl., Vol. 12.* Acad. Press, 1976.

[8] H. K. Khalil. *Nonlinear Control Systems 2nd Ed.* Prentice Hall, 1996.

[9] D. S. Laila and E. Grünbacher. Discrete-time control design for setpoint tracking of a combustion engine test bench. In *Proc. 46th IEEE Conf. Decision and Control*, New Orleans, LA, 2007.

[10] R. Outbib, X. Dovifaaz, A. Rachid, and M. Ouladsine. A theoretical control strategy for a diesel engine. *Trans. ASME Dynamic Systems, Meas. and Control*, 128:453–457, 2006.

[11] M. Schmidt and J.-A. Kessel. Casma – crank angle synchronous moving average filtering. In *Proc. American Control Conference*, pages 1339–1340, 1999.