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Design of continuous and discontinuous output regulators for a MAGLEV system ⋆

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Abstract: This work presents the design of continuous and discontinuous output regulators for a magnetic levitation (MAGLEV) system, for asymptotic output trajectory tracking and disturbance rejection. The nonlinear full information case is considered for both regulators. Then by numerical simulations one compares the performance of both control strategies, under criteria as transient response, steady-state accuracy, feasibility of control implementation and parameter sensitivity. The superior performance of the discontinuous regulator is then put in evidence by the obtained results.

Keywords: Regulation; Nonlinear system control.

1. INTRODUCTION

Magnetic levitation systems are used in several applications such as frictionless bearings (Allaire et al. (1998)), high-speed maglev passenger trains (Ono et al. (1998)), levitation of wind tunnel models (Muscroft et al. (2006)), levitation of molten metal (Im et al. (2005)) and the levitation of metal slabs during manufacture (Jayawant et al. (1965)) in industrial process. These systems have intrinsical unstable nonlinear dynamics, requiring of closed–loop control designs for the stabilization of such systems. Several control techniques has been applied to the stabilization of MAGLEV systems, for example, feedback linearization (Barie et al. (1996)), sliding mode control (Muthairi et al. (2004)), backstepping (Mahmoud (2003)), among others. In the case of tracking sinusoidal signals, feedback linearization (Barie et al. (1996), Hajjaji et al. (2001)), and backstepping (Pranayanuntana et al. (2000)) are reported in literature. Although the tracking of sinusoidal signal is more natural for the output regulation technique, there are not works reported in the literature about output regulation for MAGLEV systems.

Recently, output regulation has been combined with other control techniques such sliding modes (Utkin et al. (2004)), fuzzy control (Castillo et al. (2004)) and artificial neural networks (Castillo et al. (2005)), in order to improve the output regulation strategy. The output regulation problem consist in finding if possible a control law for which the output can asymptotically track a signal and at the same time reject perturbations signals. The main feature that distinguishes the output regulation problem from conventional tracking and disturbance rejection problems is that in the output regulation problem, the class of reference signals and disturbances consists of solutions of some autonomous system of differential equations. This system is called an exosystem. Reference signals and/or disturbances generated by the exosystem are called exosignals. The control law must also asymptotically stabilize the system even if the exosystem is absent.

In the linear setting a complete solution of the problem was presented in Francis (1977), based on the existence of a solution for a set of algebraic matrix equations. In the nonlinear framework, it was shown in Isidori (1990) that the solution can be posed in terms of the solution of a set of nonlinear differential equations, which represents a generalization of the Francis conditions. This set of equations become known as the Francis-Isidori-Byrnes (FIB) equations. Basically, the regulator solution can be viewed as finding a steady-state surface on which the output tracking error map is zero, and which can be made attractive and invariant by feedback.

An alternative approach to deal with this problem is the use of the sliding mode technique to decompose and simplify the regulator design procedure and impose robustness properties as in Utkin (1992) and Elmail (1992). The underlying idea is to design a sliding surface on which the dynamics of the system are constrained to evolve by means of a discontinuous control law, instead of designing a continuous stabilizing feedback, as in the case of the classical regulator problem. The sliding manifold contains the steady-state surface, where the dynamic of the system tends asymptotically along the sliding manifold to the steady-state behavior.

In the full information case, a static state feedback sliding mode regulator design have been investigated in Elmail

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(1992), Gopalswamy et al. (1993) and Castillo et al. (1995). To overcome the limitation of the full information knowledge, a dynamic discontinuous error feedback strategy have been designed in Edwards et al. (1998) for linear systems, and in Sira (1993) for a certain class of nonlinear systems. Considering that the state of the exosystem is accessible, a dynamic error feedback regulator has been proposed in Bonivento et al. (2000) for a class of nonlinear systems with unitary relative degree. In Utkin et al. (2004), it is considered that the exosystem is not accessible, therefore proposing a dynamic discontinuous error feedback regulator for the general case of linear and nonlinear systems, including as well a class of dynamic systems presented in the so-called Regular and Block Controllable forms.

The objective of this work is then to compare the design and performance of the classical output regulator and sliding mode output regulator by means of simulations for asymptotic output trajectory tracking where the nonlinear full information case is considered. Then, issues like transient response, steady-state accuracy, feasibility of control implementation and parameter sensitivity are discussed.

The rest of this work is organized as follows. In Section 2 the classical and discontinuous output regulation theories are briefly revisited. In Section 3, the continuous and discontinuous output regulation for the MAGLEV system are designed. Section 4 deals with simulations where the results of both regulators are compared, and finally some commentaries conclude the work in Section 5.

2. RECALLS ON OUTPUT REGULATION THEORY

In this section the main ideas behind classical output regulation theory are briefly revisited. For the discontinuous output regulator only conditions are stated where details can be found in Utkin et al. (2004).

2.1 Classical Output Regulation

Let us consider a nonlinear system with error signal as output:

$$\dot{x} = f(x) + g(x)u + d(x)w \tag{1}$$

$$e = h(x) - q(w) \tag{2}$$

$$\dot{w} = s(w) \tag{3}$$

where x is the state vector defined on a neighborhood X of the origin of $\Re^n$, with $u \in \Re^m$ as input and $e \in \Re^p$ as the output tracking error signal. The vectors $f(x)$, $h(x)$, $q(w)$, $s(w)$ and the columns of $g(x)$ and $d(x)$ are smooth vector fields of class $\mathcal{C}^{\infty}_{[t,\infty\}}$, and in addition it is assumed that $f(0) = 0$ and $h(0) = 0$. The output tracking error e is the difference between the output $h(x)$ of the system and a reference signal $q(w)$ generated by a given exosystem (3) with state w, defined on a neighborhood W of the origin of $\Re^s$. This system is characterized by the following assumption.

(H_1) *The Jacobian matrix $S = \left[\frac{\partial s}{\partial w}\right]_{w=0}$ at the equilibrium point $w = 0$ has all its eigenvalues on the imaginary axis.*

Now, the formal characterization of the output regulation problem will be presented as in Isidori (1990).

Problem 1. State Feedback Output Regulation Problem (SFORP). Given a nonlinear system of the form (1)-(2) and a neutrally stable exosystem (3), find, if possible, a mapping $u = \alpha(x, w)$ such that

(S_{SF}) the equilibrium $x = 0$ of

$$\dot{x} = f(x) + g(x)\alpha(x, 0)$$

is asymptotically stable in the first approximation,

(R_{SF}) there exists a neighborhood $V \subset X \times W$ of $(0, 0)$ such that, for each initial condition $(x(0), w(0)) \in V$, the solution of the closed-loop system

$$\dot{x} = f(x) + g(x)\alpha(x, w) + d(x)w$$
$$\dot{w} = s(w)$$

satisfies

$$\lim_{t\to\infty} h(x) - q(w) = 0.$$

The solvability of the SFORP under assumption (H_1), can be stated in terms of the existence of a pair of mappings $x = \pi(w)$ and $u = c(w)$, with $\pi(0) = 0$ and $c(0) = 0$, that solve the FIB equations

$$\frac{\partial \pi(w)}{\partial w} s(w) = f(\pi(w)) + g(\pi(w))c(w) + d(\pi(w))w \tag{4}$$

$$0 = h(\pi(w)) - q(w). \tag{5}$$

The continuous controller $\alpha(x, w)$ can be chosen as follows

$$\alpha(x, w) = c(w) + K\Big(x - \pi(w)\Big) \tag{6}$$

where K is a matrix which places the eigenvalues of the linear approximation of the closed–loop system (1) and (6) at the equilibrium point $x = 0$, namely $(A + BK) \in C^-$ whith $A = \left[\frac{\partial f}{\partial x}\right]_0$, $B = g(0)$.

2.2 Sliding Mode Output Regulator

Analogously to the SFORP, the *State Feedback Sliding Mode Output Regulator Problem* (Utkin et al. (2004)) is defined as the problem of finding a sliding manifold

$$\sigma(x, w) = 0, \qquad \sigma = \Big(\sigma_1, \cdots, \sigma_m\Big)^T \tag{7}$$

and a discontinuous controller

$$u_i = \begin{cases} u_i^+(x, w) & \text{if } \sigma_i(x, w) > 0 \\ u_i^-(x, w) & \text{if } \sigma_i(x, w) < 0 \end{cases} \qquad i = 1, \cdots, m \tag{8}$$

where $u = \Big(u_1 \quad \cdots \quad u_m\Big)^T$.

Here $u_i^+(x, w)$, $u_i^-(x, w)$ and the sliding manifold (7) are chosen such that the following conditions holds

- (SMS_{SF}) (Sliding Mode Stability). The control (8) is designed to induce sliding mode motion on the sliding manifold (7) in finite time;
- (S_{SF}) The equilibrium $(x, \xi) = (0, 0)$ of the sliding mode dynamic

  $$\dot{x} = f(x) + g(x)u_{eq}|_{\sigma(x,w)=0}$$

  is asymptotically stable, where u_{eq} is the equivalent control defined as solution of $\dot{\sigma} = 0$;

- (R_{SF}) There exists a neighborhood $V \subset X \times W$ of $(0,0)$ such that, for each initial condition $(x_0, w_0) \in V$, the output tracking error (2) goes asymptotically to zero, i.e. $\lim_{t\to\infty} e(t) = 0$.

3. OUTPUT REGULATION OF A MAGLEV SYSTEM

In the following, the mathematical model of a maglev system is presented in sub–section 3.1, the classical regulator is designed in sub–section 3.2, while in sub–section 3.3 the proposed discontinuous regulator is designed.

3.1 Mathematical model and problem formulation for the MAGLEV system

Figure 1 shows an schematic diagram of a maglev system.

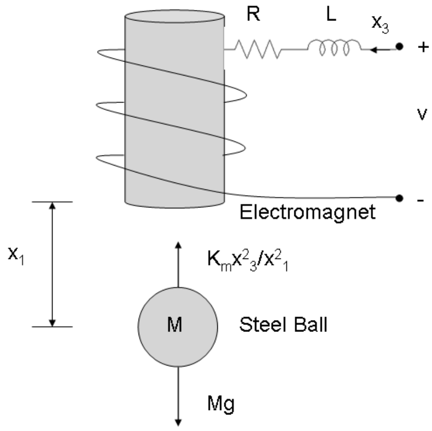

Fig. 1. Schematic diagram of a MAGLEV system.

The mathematical model of the MAGLEV system is given by the following equations (Barie et al. (1996)):

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= g - \frac{k_m}{M}\frac{x_3^2}{x_1^2} \\ \dot{x}_3 &= -\frac{R}{L}x_3 + \frac{1}{L}v \\ y &= x_1 \end{aligned} \tag{9}$$

with state vector defined as $x = (x_1,\ x_2,\ x_3)^T$, where x_1 represents the position of the steel ball of mass M which is positively incrementing in the downward position, x_2 is the velocity of the steel ball, x_3 is the current trough the coil, v is the input voltage applied to the coil, y the output of the system. The constant parameters are the resistance of the coil denoted by R, the inductance denoted by L, g is the gravitational constant and is considered as a known perturbation term, finally k_m is the magnetic constant of the electromagnet.

The control problem consist in forcing the output $y = x_1$ to track a reference signal $x_{1,r}$. Therefore one can consider the following output tracking error

$$e = x_1 - x_{1,r}. \tag{10}$$

The reference and perturbation signals are supposed to be generated by an autonomous exosystem (3) given by

$$\begin{aligned} \dot{w}_1 &= -\alpha w_2 \\ \dot{w}_2 &= \alpha w_1 \\ \dot{w}_3 &= 0 \\ \dot{w}_4 &= 0 \end{aligned} \tag{11}$$

with initial conditions $w_1(0) = w_2(0) = a$, $w_3(0) = b$ and $w_4(0) = c$ and outputs

$$x_{1,r} = q(w) = w_1 + w_3,\ g = d(w) = w_4. \tag{12}$$

3.2 Continuous Regulation of a Maglev System

To design the classical continuous regulator (6), the mappings $x = \pi(w)$ and $u = c(w)$ are calculated from the FIB equations (4) and (5). Using the mathematical model of the MAGLEV (9), (10) and the proposed exosystem (11), the FIB equations takes the following form:

$$\frac{\partial \pi_1(w)}{\partial w}s(w) = \pi_2(w) \tag{13}$$

$$\frac{\partial \pi_2(w)}{\partial w}s(w) = d(w) - \frac{k_m}{M}\frac{\pi_3^2(w)}{\pi_1^2(w)} \tag{14}$$

$$\frac{\partial \pi_3(w)}{\partial w}s(w) = -\frac{R}{L}\pi_3(w) + \frac{1}{L}c(w) \tag{15}$$

$$0 = \pi_1(w) - q(w) \tag{16}$$

with $s(w) = (-\alpha w_2,\ \alpha w_1,\ 0,\ 0)^T$.

The steady state mappings $\pi(w) = (\pi_1(w),\ \pi_2(w),\ \pi_3(w))^T$ and $c(w)$ are obtained from the previous equations, yielding to the following expressions

$$\begin{aligned} \pi_1(w) &= w_1 + w_3 \\ \pi_2(w) &= -\alpha w_2 \\ \pi_3(w) &= (w_1 + w_3)\sqrt{\frac{M}{k_m}(w_4 + \alpha^2 w_1)} \\ c(w) &= (R(w_1 + w_3) - \alpha L w_2)\sqrt{\frac{M}{k_m}(w_4 + \alpha^2 w_1)} \\ &\quad - \frac{ML\alpha^3}{2k_m}\frac{w_2(w_1 + w_3)}{\sqrt{\frac{M}{k_m}(w_4 + \alpha^2 w_1)}}. \end{aligned} \tag{17}$$

Finally the control action can be chosen as in (6), with $c(w)$ defined in (17) and K such that the matrix $(A + BK)$ is Hurwitz.

3.3 Discontinuous Regulation of a Maglev System

Let us define the steady state error as

$$z = x - \pi(w) = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} x_1 - \pi_1(w) \\ x_3 - \pi_3(w) \\ x_3 - \pi_3(w) \end{pmatrix} \tag{18}$$

the dynamic equation for (18) with tracking error e (10) can be obtained from (9) as

$$\dot{z}_1 = z_2 + \pi_2(w) - \frac{\partial \pi_1(w)}{\partial w}s(w) \tag{19}$$

$$\dot{z}_2 = d(w) - \frac{k_m}{M}\frac{(z_3 + \pi_3(w))^2}{(z_1 + \pi_1(w))^2} - \frac{\partial \pi_2(w)}{\partial w}s(w) \tag{20}$$

$$\dot{z}_3 = -\frac{R}{L}(z_3+\pi_3(w)) + \frac{1}{L}u - \frac{\partial\pi_3(w)}{\partial w}s(w) \qquad (21)$$
$$e = z_1 + \pi_1 - q(w).$$

where π_1, π_2 and π_3 are solutions of equations (16), (13) and (14) respectively.

Now, one defines the sliding function and control as

$$u = -kLsign(\sigma),\ \sigma = z_3 + \Sigma_1 z^1,\ k > 0 \qquad (22)$$

with $\Sigma_1 = (\Sigma_{1,1}\ \Sigma_{1,2})$ and $z^1 = (z_1,\ z_2)^T$.

In order to prove the convergence of the state-vector of the closed loop system (19)-(21) to $\sigma = 0$, let us consider the following Lyapunov candidate function

$$V = \frac{1}{2}\sigma^2 \qquad (23)$$

and let us define the level sets

$$\Omega_c = \{\sigma \in \Re | V \le c\}, \quad c > 0$$

which contain the origin. For any given closed bounded set $\Omega \subset \Re$ one can find a $\bar{c}$ such that $\Omega \subset \Omega_{\bar{c}}$. Taking the derivative of (23) along the trajectories of the closed-loop system (19)-(21)

$$\dot{V} = \sigma(-ksign(\sigma) + f_v) = -k|\sigma| + \sigma f_v$$

where $f_v = -(1/L)v_{eq}$. If the control gain k is chosen such that $k > |(1/L)v_{eq}(z,w)|$, where $v_{eq}(z,w)$ is a solution of $\dot{\sigma} = 0$, namely

$$\begin{aligned} v_{eq} &= R(z_3+\pi_3) + L(\partial\pi_3/\partial w)s(w) \\ &\quad -L\Sigma_{1,1}\left(z_2 + \pi_2 - (\partial\pi_1/\partial w)s(w)\right) \\ &\quad -L\Sigma_{1,2}\begin{pmatrix} d(w) - (k_m/M)(z_3+\pi_3)^2/(z_1+\pi_1)^2 \\ -(\partial\pi_2/\partial w)s(w) \end{pmatrix} \end{aligned}$$

then

$$\dot{V} \le -|\sigma|(k - |f_v|) = -\lambda|\sigma| = -\lambda\sqrt{2V}$$

with $\lambda = k - sup_{\Omega_{\bar{c}}}|f_v|$. Therefore, condition (SMS_{SF}) holds, i.e., the sliding mode exists, and the convergence in finite time of the state vector of the closed loop system (19)-(21) to $\sigma = 0$ is guaranteed.

Making use of the comparison principle (Khalil (2002)), one can consider the following differential equation

$$\dot{\mathcal{V}} = -\lambda\sqrt{2\mathcal{V}} \qquad (24)$$

with $V(t_0) = (1/2)\sigma^2(t_0) \le \mathcal{V}(t_0)$. Therefore,

$$V(t) \le \mathcal{V}(t) = \begin{cases} \left(\sqrt{\mathcal{V}(t_0)} - \frac{\lambda}{\sqrt{2}}(t-t_0)\right)^2 & \text{for } t_0 \le t \le t_s \\ 0 & \text{for } t > t_s \end{cases}$$

with $t_s = t_0 + \sqrt{2\mathcal{V}(t_0)}/\lambda$, and thus $\sigma = 0$ is reached in finite time, forcing sliding mode motion in the system, with a region of attraction $\Omega_{\bar{c}}$ containing Ω (See Utkin (1992) for more details). After the sliding mode occurs, one has $z_3 = -\Sigma_1 z^1$ (see (22)), and considering only the linear part of (19) and (20), then, the motion of the linearized closed–loop system (sliding mode motion) will be governed by

$$\begin{aligned} \dot{z}^1 &= (A_{11} - A_{12}\Sigma_1)z^1 + R_1 w + \phi_{1,s}(z,w) \qquad (25) \\ \dot{w} &= Sw \\ e &= z_1 + \pi_1(w) - q(w) \end{aligned}$$

with $\phi_{1,s}(z,w)$ as a function of higher order terms that vanish at the origin with their first derivative,

$$R_1 = A_{11}\Pi_1 + A_{12}\Pi_2 - \Pi_1 S + D$$

where

$$\Pi_i = \left[\frac{\partial\pi_i}{\partial w}\right]_{(0,0)},\ i = 1,2\ ,\ S = \left[\frac{\partial s(w)}{\partial w}\right]_{(0)},$$
$$D = \left[\frac{\partial d(w)}{\partial w}\right]_{(0)}.$$

In the work of Utkin et al. (1978) has been demonstrated that if the pair (A, B) is controllable, then, the pair (A_{11}, A_{12}) is controllable as well. In such case, one can always find Σ_1 such that the matrix $(A_{11} - \Sigma_1 A_{12})$ is Hurwitz.

Moreover if equations (13) and (14) holds, then,

$$R_1 w + \phi_{1,s}(z,w) = \begin{pmatrix} \pi_2 - \frac{\partial\pi_1}{\partial w}s(w) \\ d(w) - \frac{k_m}{M}\frac{\pi_3^2(w)}{\pi_1^2(w)} - \frac{\partial\pi_2}{\partial w}s(w) \end{pmatrix} = 0$$

that under the property of centre manifolds, we have $z_i(t) \to 0 \Rightarrow x_i(t) \to \pi_i(w(t))$, $i = 1,2$, and $z_3 \to 0 \Rightarrow x_3(t) \to \pi_3(w(t))$ with $t \to \infty$. Thus, the requirement (S_{SF}) is fulfilled. By continuity, if condition (16) holds, then the output tracking error (10) converges to zero and condition (R_{SF}) holds too.

It is worth to mention that in this case, the term $c(w)$ is not required, so, equation (15) is not necessary.

4. SIMULATION RESULTS

Simulations are carried out in order to compare the performance of both regulators. The nonlinear model (9) is linearized around the operating point $x_{op} = \left(b,\ 0,\ \sqrt{gM/k_m}b\right)$, $v_{op} = R\sqrt{gM/k_m}b$. The initial conditions for the MAGLEV are set as $(0.045,\ \ 0,\ \ 0)^T$ and for the exosystem (11), $a = 0.0070716$, $b = 0.05$, $c = 9.8$. Taking the nominal parameters of the MAGLEV system as $k_m = 3\ k_g m^3/s^2 A^2$, $M = 0.14\ Kg$, $g = 9.8m/s^2$, $R = 1.2\ \Omega$, $L = 1\times10^{-3}\ H$; and $b = 0.05\ m$, the following pairs of matrices are reckoned:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 392 & 0 & -579.6551 \\ 0 & 0 & -1200 \end{pmatrix},\ B = \begin{pmatrix} 0 \\ 0 \\ 1000 \end{pmatrix},$$
$$A_{11} = \begin{pmatrix} 0 & 1 \\ 392 & 0 \end{pmatrix},\ A_{12} = \begin{pmatrix} 0 \\ -579.6551 \end{pmatrix}.$$

The matrix K in (6) and the matrix Σ_1 in (25) are calculated using the LQR function provided in Matlab, resulting as follows

$$K = (-19.2903 \quad -10.0346 \quad 9.4335),$$
$$\Sigma = (-12.9423 \quad -12.2492).$$

To verify the robustness properties, some plant parameter variations are introduced as can be appreciated in Figure

2, where R and k_m may change up to 100 % from their nominal values. It is worth to mention that the perturbation term generated by the variation of R satisfies the matching condition (Utkin (1992)), but not the variations on k_m.

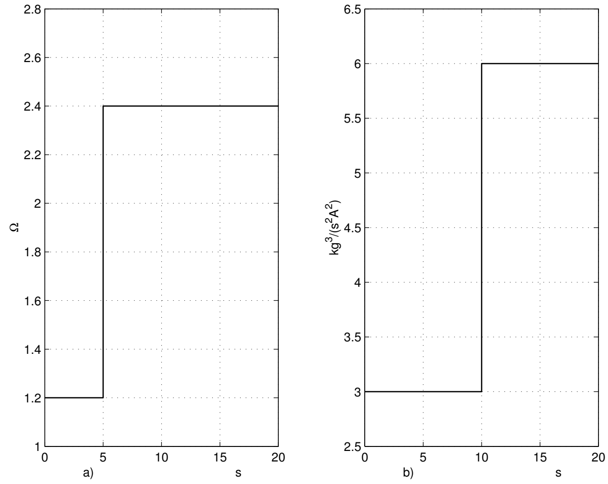

Fig. 2. a) Resistance variation. b) Magnetic constant variation

Figure 3 compares the tracking of the output signal for the continuous output regulator (COR) and the discontinuous output regulator (DOR), where can be appreciated that for $0 \leq t < 5$ both output signals shows a good performance, but for $5 \leq t < 10$ where the perturbation term due to the variation in R is present, the DOR shows superior performance over the COR due to the matching condition. Finally, the unmatched perturbation term due to the variation of k_m, appearing at $t \geq 10$ affects adversely the maglev system in both cases, but the DOR still performs better than the COR.

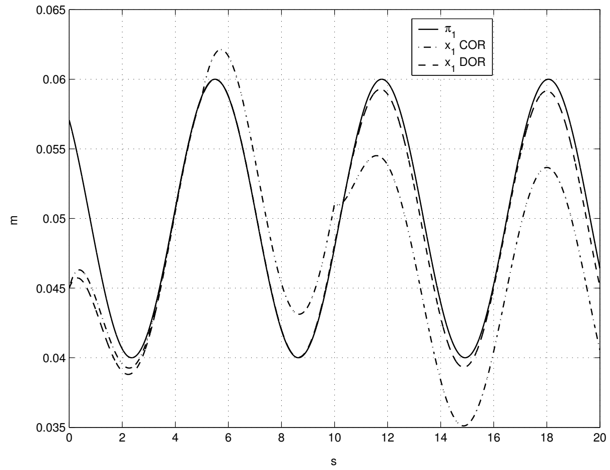

Fig. 3. Comparison of the output signals.

Figure 4 shows the output tracking error for both regulators, where can be appreciated the transient and steady–state responses.

Figure 5 shows π_3, which represents the ideal steady-state behavior of the current. Note that the current obtained with the COR is not equal to π_3 for $t \geq 5$ yielding to unsatisfactory results. With respect to the current

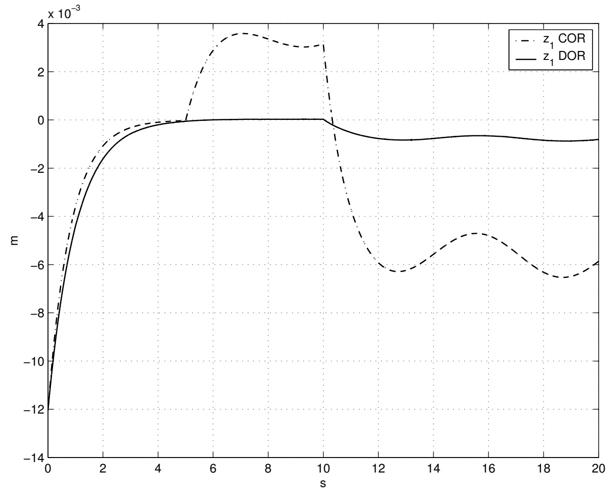

Fig. 4. Comparison of the output error signals.

obtained with the DOR, it becomes different to π_3 for $t \geq 10$, but it is more approximated to π_3, therefore yielding to more satisfactory results with respect to the COR.

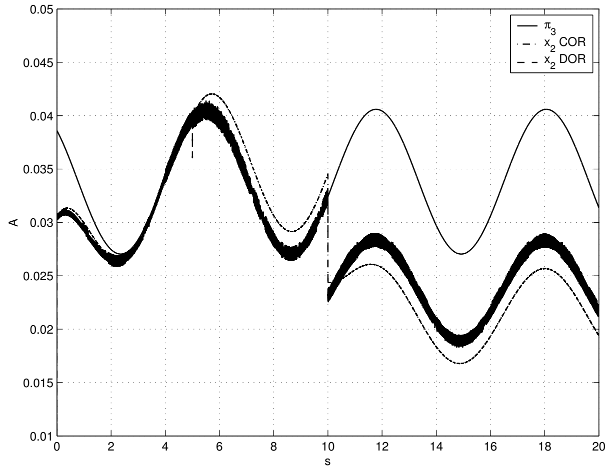

Fig. 5. Comparison of the current signals.

Finally, Figure 6 shows the voltage input signals where the continuous and discontinuous natures of the COR and the DOR can be appreciated. The main advantage of having discontinuous control signals is that it avoids the use of PWM as mentioned in Utkin (1992), therefore, facilitating a straightforward implementation of the control action. In the case of the COR, a PWM is still needed, implying an additional stage in the control–loop that can introduce unmodeled dynamics. Moreover in the case of the COR, an undesirable peak voltage appears at time $t = 0$.

5. CONCLUSIONS

In this work two related control laws were designed and compared on a simulation basis for a MAGLEV system. Table I summarizes the main results. From the simulations, one can conclude that both controllers performs well under ideal conditions, but the DOR has demonstrated a superior performance over the COR when parameter variations are introduced to the MAGLEV system. Another advantage of the DOR is that the control design requires

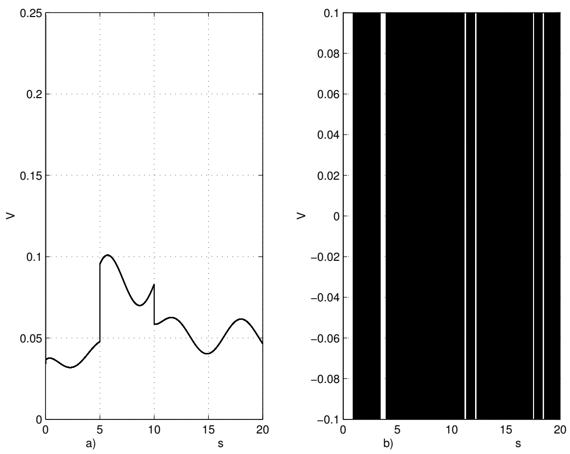

Fig. 6. Comparison of the voltage signals.

of less calculations than the COR, as one can see that $c(w)$ is not required in the DOR design and that the pair to stabilize the system in the first approximation in the DOR is one dimension lower than in the COR case. Moreover, sliding mode control has the advantage of not requiring PWM by generating a discontinuous control action signal capable of driving switching power devices.

Table 1. Comparison of DOR and COR

Comparison criteria	**DOR**	**COR**
Transient response	Good	Good
Steady-State accuracy	Good	Bad
Parameter sensitivity	Low	High
Implementation complexity	Less equations and no PWM	More equations and PWM

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