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A new robust-control-oriented system identification method

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Abstract: This paper introduces a new robust-control-oriented system identification method, which consists of the following three steps: 1. High-order ARX model identification; 2. Loop shaping weighting functions design based on the high-order ARX model; 3. Control-oriented model reduction by minimizing the weighted L_2-gap between the high-order ARX model and the low-order model. This method truly integrates the control objective into the identification step. A robust controller can be readily designed as a result of the identification. Simulation examples are given to show that smaller weighted ν-gap can be achieved by using the proposed method.

1. INTRODUCTION

A typical goal of system identification is to obtain a model G_{id}, from the testing data u and y, based on which an advanced model-based controller can be designed for the true plant G_0. Over the past two decades, the robust control theory has established practical design methods for this purpose, see McFarlane and Glover (1990); Zhou and Doyle (1998). Invariably, these methods use the nominal model G_{id} as well as some model uncertainty information ΔG for robust controller design.

Traditional way of describing model uncertainty is by using additive and multiplicative model uncertainty bounds, or parametric uncertainty regions. On the other hand, a different and more general model uncertainty is the coprime factor uncertainty, see Vidyasagar (1985); Georgiou (1988), which turns out to be appropriate for controller design, see McFarlane and Glover (1990), and optimal for continuity of loop properties, see Vinnicombe (2001). A method to estimate coprime uncertainty factors directly from experimental data was proposed in Tsakalis et al. (2002), based on the uncertainty model unfalsification philosophy in Kosut (1995). Worst case ν-gap between the nominal model and the parametric PEM uncertainty region is estimated in Bombois et al. (2001); Gevers et al. (2003). In Date and Vinnicombe (2004), an algorithm is proposed to minimize the ν-gap between the identified model and the true system, assuming frequency testing data is available.

For most of the existing identification methods, the model uncertainty is only estimated after the nominal model is obtained. In Zhan and Tsakalis (2007), we proposed a system identification method aiming to minimize the weighted NCF model uncertainty directly. The control performance requirements were explicitly considered at the identification step with the use of the weighting functions. Motivated by these results, in this paper we consider an alternative robust-control-oriented system identification procedure to address issues that were unresolved in Zhan and Tsakalis (2007). These issues concerned the selection of the weighting functions and the contamination of the data by relatively large exogenous stochastic disturbances. To alleviate the problems caused by such disturbances, we use an intermediate high-order-ARX identification step (as in Zhu and Backx (1993)) and then perform a model order reduction to minimize the NCF uncertainty, using the techniques of Zhan and Tsakalis (2007). Only single input single output (SISO) systems are considered in this paper.

The rest of the paper is organized as follows: The system identification for robust control problem is reviewed in Section 2. The proposed identification procedure is described in Section 3. Simulation examples are given in Section 4.

2. SYSTEM IDENTIFICATION FOR ROBUST CONTROL

The uncertainty description arising in coprime factor models has been proven to be a convenient way of describing the model uncertainty for robust controller design. For a system in a left coprime factorization (CF), the uncertainty G_Δ has the following expression:

$$G_\Delta = (M + \Delta_M)^{-1}(N + \Delta_N) \tag{1}$$

where $G = M^{-1}N$ is the nominal model, (N, M) is the left coprime factorization of G, and (Δ_N, Δ_M) is the coprime factor uncertainty. The controller K stabilizes the uncertainty system G_Δ provided that K stabilizes the nominal system G and:

$$\left\| \begin{bmatrix} K(I - GK)^{-1}M^{-1} \\ (I - GK)^{-1}M^{-1} \end{bmatrix} \right\|_\infty \| [\Delta_N \ \Delta_M] \|_\infty \leq 1 \tag{2}$$

Normalized coprime factorizations (NCF) of the model have been particularly useful in robust controller design. Assuming (N, M) is an NCF of the nominal system G and (Δ_N, Δ_M) is the corresponding NCF uncertainty, an optimal controller K can be calculated directly to maximize the stability margin:

$$\epsilon_{max} = \left(\inf_K \left\| \begin{bmatrix} K(I-GK)^{-1}M^{-1} \\ (I-GK)^{-1}M^{-1} \end{bmatrix} \right\|_\infty \right)^{-1} \quad (3)$$

With optimal controller, maximum NCF uncertainty can be tolerated.

The loop shaping optimal controller design method introduced in McFarlane and Glover (1990) has been shown to be both a simple and powerful approach to incorporate frequency weighted performance objectives in the NCF controller design. It consists of three steps. First a pre-weighting function W_1 and a post-weighting function W_2 are used to shape the nominal plant so that the open loop shape $G_s = W_2 G W_1$ meets the control performance objectives. Then the optimal controller K_s is calculated for the shaped plant G_s. The final controller K is calculated as $W_1 K_s W_2$ for the original plant.

After introducing the weights, the weighted NCF uncertainty system becomes:

$$G_{\Delta s} = W_2 G_\Delta W_1 = (M_s + \Delta_{M_s})^{-1}(N_s + \Delta_{N_s}) \quad (4)$$

The relationships between the weighted NCF uncertainty $(\Delta_{N_s}, \Delta_{M_s})$ and the unweighted CF uncertainty (Δ_N, Δ_M) are: (Zhan and Tsakalis (2007))

$$\begin{aligned} \Delta_{N_s} &= M_s W_2 M^{-1} \Delta_N W_1 \\ \Delta_{M_s} &= M_s W_2 M^{-1} \Delta_M W_2^{-1} \end{aligned} \quad (5)$$

It is apparent that weighted NCF uncertainty $\|[\Delta_{N_s}\ \Delta_{M_s}]\|_\infty$ determines how much robust stability margin the controller must have to guarantee in addition to nominal performance. Using a model unfalsification philosophy (e.g., Kosut (1995)), a system identification method was formulated in Zhan and Tsakalis (2007) by directly minimizing the NCF uncertainty of the weighted system, under the constraint that the model and the model uncertainty can fully explain the experiment data:

$$\begin{aligned} &\min_\theta \|[\,\Delta_{N_s}\ \ \Delta_{M_s}\,]\|_\infty \\ &s.t.: M(\theta)y - N(\theta)u = \Delta_N u - \Delta_M y \end{aligned} \quad (6)$$

where θ is the vector of unknown model parameters to be identified. The difference from the traditional system identification methods like PEM is in shifting the focus from minimizing an open loop prediction error to maximizing the predictability of closed loop behavior. For this formulation, the unfalsification framework is necessary (in some form) because there is never a guarantee that the data contain information about the worst-case uncertainty. The key concept here is to recognize that the first term of the stability condition (2), or its weighted versions, can be made all-pass by the optimal controller. It is then up to the identification to minimize the uncertainty, as uniformly as possible in the frequency domain. If the product is larger than unity, the model is inadequate to ensure that the control objective is met. (In the unfalsification framework, this means that there exists a system that fits the data and yields an unstable closed loop.) Of course, the potential remedies for this problem are to improve the model and/or change the control objectives. If feasible, the former is achieved by an increase in the number of parameters so as to reduce the error spectral peaks. The latter involves a change of the weighting functions to reduce the norm of the closed loop sensitivities.

A detailed algorithm to solve (6) using an iterative LMI optimization procedure is presented in Zhan and Tsakalis (2007), using a bank of band-pass filters to approximate $\|[\Delta_{N_s}\ \Delta_{M_s}]\|_\infty$ from time-domain data. In the absence of exogenous disturbances in the data, it was shown that, by solving (6), the identified model G has the minimum weighted L_2-gap to true system G_0 among all the models in the specified model structure S. So identification problem (6) is equivalent to:

$$\min_{G \in S} \delta_{L2}(W_2 G W_1, W_2 G_0 W_1) \quad (7)$$

with the L_2-gap of two plants G_1, G_2 being defined as (e.g., see Vinnicombe (2001)):

$$\delta_{L2}(G_1, G_2) = \|\tilde{M}_2 (G_1 - G_2) M_1\|_\infty \quad (8)$$

where $(\tilde{N}, \tilde{M})$, (N, M) denote the normalized left coprime factorization (NLCF) and the normalized right coprime factorization (NRCF) respectively. For the SISO case considered in this paper, there is no difference between the two and we simply use (N, M) to represent both. The L_2-gap is closely related to the ν-gap as:

$$\delta_\nu(G_1, G_2) = \begin{cases} \delta_{L_2}(G_1, G_2) & W(G_1, G_2) = 0 \\ 1 & \text{otherwise} \end{cases} \quad (9)$$

where $W(G_1, G_2) = 0$ is the winding number (wno) condition. Detailed discussions of L_2-gap, ν-gap, and winding number can be referred to Vinnicombe (2001).

Define closed loop transfer function H(G,K) as:

$$H(G, K) = \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1} [I\ \ -G] \quad (10)$$

Apply controller K to two systems G_1 and G_2, the difference in the closed loop transfer function is bounded by, (see Vinnicombe (2001)):

$$\delta_\nu(G_1, G_2) \le \|H(G_1, K) - H(G_2, K)\|_\infty \le \frac{\delta_\nu(G_1, G_2)}{b_{G_1,K} \sin\phi} \quad (11)$$

where $\phi = \arcsin b_{G_1,K} - \arcsin \delta_\nu(G_1, G_2)$. Assuming the wno condition is satisfied, smaller $\delta_{L_2}(G_1, G_2)$ means smaller closed loop distance between models and thus more consistent and predictable closed loop behavior.

3. ROBUST SYSTEM IDENTIFICATION THROUGH HIGH-ORDER MODEL REDUCTION

A key issue in the application of the procedure of Zhan and Tsakalis (2007) in practical problems is its difficulty in handling exogenous disturbances. For example, in the presence of additive disturbance d, the model G identified from (6) minimizes the following (assuming the weighting functions are equal to identity for simplicity, see Zhan and Tsakalis (2007)):

$$\min_{G \in S} \max_\omega \frac{|M(jw)|^2 (|G_0(jw) - G(jw)|^2 + \frac{\Phi_d(w)}{\Phi_u(w)})}{\frac{1}{|M_0(jw)|^2} + \frac{\Phi_d(w)}{\Phi_u(w)}} \quad (12)$$

where Φ_d and Φ_u are the power spectrum of the disturbance d and the input signal u, respectively. It is clear that the identification is biased in the existence of additive disturbance and the perturbation $\frac{\Phi_d(w)}{\Phi_u(w)}$ is not attenuated with increasing number of experiment data. The solution of (12) will approach the solution of (7) only when $\frac{\Phi_d}{\Phi_u}$ goes to zero.

In the sequel, we present an approach to alleviate this problem by using high-order ARX identification to reduce the size of the effective Φ_d. In addition to this, a lesser issue is the assumed knowledge of the performance weighting functions, which often requires a preliminary model. The automated design of weighting function is important to make the identification truly control-oriented.

To address these issues, the following robust-control-oriented system identification procedure is proposed:

(1) Identify a high-order ARX model G_h from the input and output data.
(2) Design the weighting functions W_1,W_2 based on G_h.
(3) Identify a low-order model G through robust-control-oriented high-order model reduction.

It goes without saying that the feasibility of the controller design problem may introduce the need to iterate some of these steps.

3.1 High-order ARX identification

The most appealing property of the ARX model structure is simplicity. However, the low-order ARX is known to be a biased model structure, generally leading to inconsistent model results. On the other hand, ARX models are capable of representing any linear system arbitrarily well, provided that the model order is high enough, see Ljung (1987); Zhu and Backx (1993).

Since the high-order ARX model is effectively an unbiased model structure, its model error only comes from variance. The asymptotic variance of the high-order model G_h is given in Ljung (1985):

$$var(G_h(jw)) \approx \frac{n}{N}\frac{\Phi_d(w)}{\Phi_u(w)} \tag{13}$$

where n is the model order, N is the length of the experiment data. Based on this expression, Zhu and Backx (1993) define a 3σ upper bound for the error of G_h:

$$|G_h(jw) - G_0(jw)| \leq 3\sqrt{\frac{n}{N}\frac{\Phi_d(w)}{\Phi_u(w)}} \text{ w.p. } \geq 99\% \tag{14}$$

While this approach can yield a system identification procedure in L_2, our interest in it is in the ability to discriminate between exogenous (stochastic) disturbances and unmodeled dynamics.

3.2 Weighting Functions Design

The general guidelines of choosing the weighting functions can be referred to Vinnicombe (2001). In this paper, the weighting functions are to be designed based on G_h to meet the following requirements:

- Performance requirement: The gain of the weighted plant $W_2G_hW_1$ should be large at frequencies where significant noise attenuation is required.
- Robustness requirement: $\delta_{L_2}(W_2G_hW_1, W_2G_0W_1)$ is sufficiently small.

Since the weighted high-order model $G_{hs} = W_2G_hW_1$ can be readily calculated, a check of the performance requirement is straightforward. For robustness requirement, notice that

$$\delta_{L2}(G_{hs}, G_{0s}) = \|M_{hs}W_2(G_h - G_0)W_1M_{0s}\|_\infty \tag{15}$$

where (N_{hs}, M_{hs}) is the NCF of G_{hs} and (N_{0s}, M_{0s}) is the NCF of G_{0s}. Without knowing the true system, (15) can only be evaluated approximately. It is noted that the asymptotic result (14) can be used to get an upper bound of $|G_h - G_0|$.

The remaining problem is to estimate M_{0s}, for which we have

$$M_{0s}^*M_{0s} = (I + G_{0s}G_{0s}^*)^{-1} \tag{16}$$

We then have the following power spectrum relationship of the weighted system G_{0s}:

$$\Phi_{ys}(w) = |G_{0s}(jw)|^2\Phi_{us}(w) + \Phi_{ds}(w) \tag{17}$$

From (16) and (17), it follows that $|M_{0s}(jw)|$ can be simply estimated as $\sqrt{\frac{\Phi_{us}(w)}{\Phi_{us}(w)+\Phi_{ys}(w)}}$. Assuming the signal to noise ratio is large, the error in this estimate will be small.

The design of weighting functions is itself a topic that attracts a lot of research activity, and it can involve very complicated algorithms. In the paper, we assume the basic format of the weighing functions is known with a few tunable parameters. The appropriate value of the parameters can be determined by validating through the performance and robustness requirements as discussed. In the example in Section 4, we simply use $W = w_c/s$ as the weighting function, with w_c being a tunable parameter affecting the closed-loop bandwidth. The objective of the weighting function design is to find the desired bandwidth that the controller can push to with sufficient robust stability margin.

3.3 High-order model reduction

The standard weighted l_2-norm model reduction problem is posed as:

$$\min_{G \in S} \int_{-\pi}^{\pi} |[G(jw) - G_h(jw)]W(\omega)|^2 d\omega \tag{18}$$

Essentially, (18) tries to find a low-order model G by minimizing the frequency-weighted open-loop distance between G and G_h. However, systems with similar open-loop behavior in l_2 sense, may have very different closed-loop behavior. In the proposed method, G is obtained by minimizing the weighted L_2-gap to the high-order model:

$$\min_{G \in S} \delta_{L_2}(W_2G_hW_1, W_2GW_1) \tag{19}$$

While the ν-gap is a more rigorous choice for model order reduction (19), to keep the problem tractable the wno condition is not enforced for the high-order model identification (G_h). There is no additional benefit to enforce it between $W_2 G_h W_1$ and $W_2 G W_1$ in the model reduction step.

No results have been reported on the analytical solution of (19). Based on Hankel norm model reduction, upper and lower bound of the ν-gap model approximation is given in Vinnicombe (2001). A numerical approach is taken to solve (19) in this paper. A white noise input sequence u_h is generated, the high-order model output y_h is simulated using G_h and u_h. A low-order model is identified with the regenerated data u_h, y_h and weighting functions W_1, W_2, by solving (6), using the procedure developed in Zhan and Tsakalis (2007). Since there is no additive disturbance in the regenerated data, the identification procedure will find a low-order model that minimizes (19).

3.4 Error Analysis

The model G is identified by minimizing $\delta_{L_2}(G_s, G_{hs})$ in the model reduction step. What we really would like to minimize is $\delta_{L_2}(G_s, G_{0s})$. Since G_h approaches G_0 asymptotically, it is obvious that G from (19) is an unbiased estimate of the true optimal solution (7). A bound on the additional error incurred due to the use of the high-order ARX step is obtained by using (14)

$$|\delta_{L_2}(G_s, G_{0s}) - \delta_{L_2}(G_s, G_{hs})| \le \delta_{L_2}(G_{hs}, G_{0s})$$
$$\le 3\sqrt{\frac{n}{N}}\left\| M_{hs} W_2 \sqrt{\frac{\Phi_d(w)}{\Phi_u(w)}} W_1 M_{0s} \right\|_\infty \quad \text{w.p.} \ \ge 99\% \quad (20)$$

From (20), it is expected that $|\delta_{L_2}(G_s, G_{0s}) - \delta_{L_2}(G_s, G_{hs})|$ approaches zero with the experiment data length N in the rate of $\sqrt{\frac{1}{N}}$. It should also be noted that the weighting functions are explicitly chosen to make the estimated $\delta_{L_2}(G_{hs}, G_{0s})$ small, as stated in Section 3.2. This analysis indicates that for high enough order, the δ_{L_2} distance between low and high order estimates is approximately the same as the distance between low order and true system. The advantage over a single-step estimation of the low-order system is that these estimates are unbiased.

4. SIMULATION RESULTS

Let us consider the following system to be identified:

$$G_0(s) = \frac{0.0001 \cdot (0.001875 s^6 - 0.002946 s^5 + 0.01265 s^4}{s^7 + 3.47 s^6 + 4.044 s^5 + 1.8895 s^4}$$
$$\frac{-0.03552 s^3 + 0.03498 s^2 - 0.2259 s + 0.7685)}{0.4251 s^3 + 0.0496 s^2 + 0.0029 s + 0.0000677}$$

The step response and bode plot of $G_0(s)$ are shown in Fig. 3. The system identification objective is to identify a 2nd-order model, based on which, a robust controller is obtained using the loop shaping controller design procedure in McFarlane and Glover (1990).

Denoting our method as robust control identification method (RCID), the results are compared to other standard methods: output error (OE), box-jenkins (BJ), and asymptotic method (ASYM). OE and BJ are PEM methods that use the output error model structure and Box-Jenkins model structure respectively, see Ljung (1987).

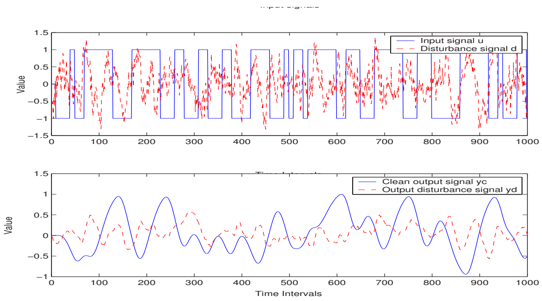


Fig. 1. Input and output signal. Upper figure: solid,input signal u; dashed, unknown disturbance d. Lower figure: solid, clean output signal y; dashed, output disturbance $G_d d$.

The ASYM method can be referred to (Zhu and Backx (1993)).

4.1 Simulation Setup

The generalized binary noise (GBN) (Tulleken (1990)) is used as the input testing signal u to the process, with a switching time of 10 seconds and the probability of switch 0.2. A disturbance is added to the output as:

$$y = G_0(s)u + G_d(s)d \quad (21)$$

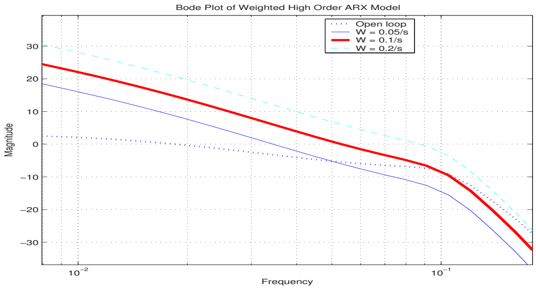
where $G_d(s) = \frac{50}{1000s+1}$. The energy of random disturbance d is bounded by $W_I(j\omega) = \frac{10j\omega+0.2}{5j\omega+1}$ to the input signal u: $|d(\omega)| \le |W_I(jw)||u(\omega)|$. A total of 1000 experiment data points are collected with the sampling rate 1 sample per second. An example of the input signal u and disturbance signal d, the corresponding clean output signal $y_c = G_0(s)u$ and output disturbance signal $y_d = G_d(s)d$ are shown in Fig. 1.

4.2 Model Identification Example

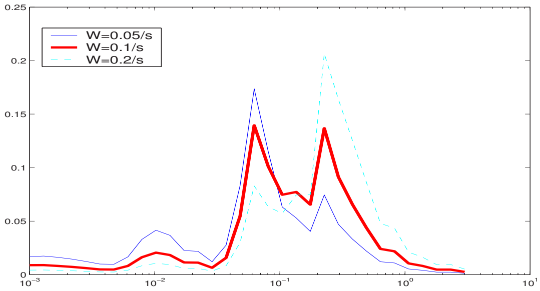
We identify a 30th-order ARX model in the first step. The step response and bode plot are included in the subsequent model responses for comparison (Fig. 3).

In a typical application, a feasible estimate of the desired closed-loop bandwidth is known a priori. Following the optimal loop shaping controller design approach, the initial weighting functions W_1 and W_2 are designed to shape G_h so that the control performance requirements can be met. For example, suppose the bandwidth requirement is that $\omega_B > 0.03$ and the simple weighting function $W = w_c/s$ is used. We need to find the ω_c so that the gain crossover frequency of WG_h satisfies the bandwidth requirement. The bode plots of WG_h with $W = 0.2/s, 0.1/s, 0.05/s$ are shown in Fig. 2(a), from which $w_c \ge 0.05$ satisfies the performance requirement. For the robustness requirement, the weighted NCF uncertainty is estimated with W and G_h using the approach described in Section 3.2. The estimated weighted NCF uncertainty of G_h with $w_c = 0.2, 0.1, 0.05$ are plotted in Fig. 2(b). In this case, $w_c = 0.1$ can be comfortably chosen since it gives a smaller estimated uncertainty.

It should be pointed out that increasing the order of the weighting function provides a possible avenue to maximize bandwidth but it should be treated as a last resort

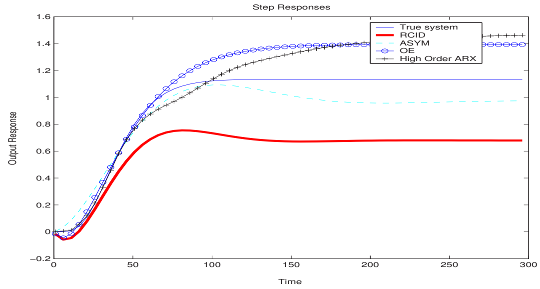


(a) Bode plot of weighted high-order ARX model

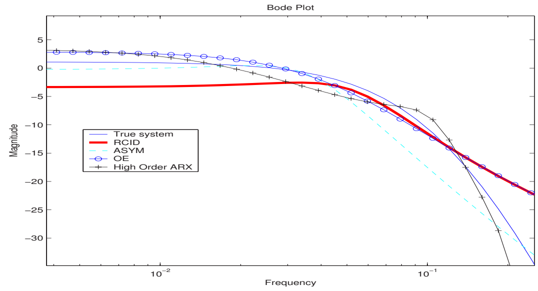


(b) Est. NCF error of weighted high-order ARX model

Fig. 2. Comparison of weighting functions. dotted: no weighting; solid: $W = 0.05/s$; thick solid: $W = 0.1/s$; dashed: $W = 0.2/s$.



(a) Step responses of identified models



(b) Bode plots of identified models

Fig. 3. Comparison of model responses. solid: true system; thick solid: RCID, dashed: ASYM; o: OE; +: high order model.

and only for cases where long data records are available, because the exact shape of the uncertainty estimates is data dependent. Furthermore, uncertainty constraints may enter both in terms of a maximum and a minimum bandwidth. Intuitively, increasing the loop gain will be constrained by the ever-present high frequency uncertainty. Its low frequency counterpart is observed in unstable or marginally stable systems (integrators) where the low frequency information is uncertain and a minimum controller bandwidth is required to achieve the stabilization of the nominal system.

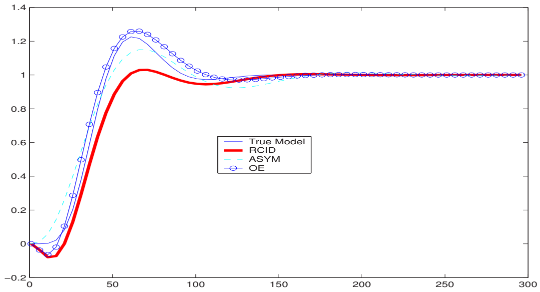
Next, the model reduction problem (19) is solved to get a $2nd$-order model, denoted as G_{rcid}. As a comparison, models are also identified using OE and ASYM methods, denoted as G_{oe} and G_{asym}, respectively. The step responses and bode plots of all the identified models are shown in Fig. 3.

For these low order model candidates, it is instructive to observe how the modeling error is addressed using our knowledge of the actual system. Looking at the output disturbance first, the norm of the actual disturbance signal is 6.7439. The open loop prediction error residue is calculated as $e = y - G_{id}u$. The norms of the prediction error residue of the identified models are: $norm(e_{rcid}) = 8.4497$, $norm(e_{oe}) = 7.0228$, $norm(e_{asym}) = 8.5853$. Obviously, the output error method gives the smallest output prediction error, which is precisely the optimization objective of this method.

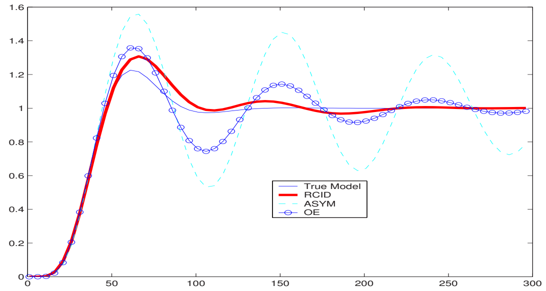
On the other hand, the actual ν-gap can also be calculated between the weighted identified model and weighted real system. The results are: $\delta_\nu(WG_0, WG_{rcid}) = 0.0999$, $\delta_\nu(WG_0, WG_{oe}) = 0.14$, and $\delta_\nu(WG_0, WG_{asym}) = 0.3089$. Here, the RCID method yields the smallest weighted ν-gap of all the three methods for this example. It is also interesting to see that although the prediction error norms of the G_{rcid} and G_{asym} are similar, G_{rcid} gives a significantly smaller weighted ν-gap to the true system.

The significance of the ν-gap results becomes apparent when the closed-loop responses are considered. We design a loop shaping optimal controller in (3) for each low-order model with the weighting $W = 0.1/s$. We denote the controllers as: K_{oe}, K_{rcid}, and K_{asym}, corresponding to the identified models G_{oe}, G_{rcid}, and G_{asym}. An "ideal" controller is also calculated based on the true system and the weighting, denoted as K_0, and the corresponding closed loop system is referred to as the "ideal" closed-loop system. All the designed controllers are first applied to the corresponding identified nominal models. The setpoint tracking responses of the different nominal closed-loop systems are shown in Fig. 4(a). Notice that all nominal systems exhibit similar behavior since they are designed for the same loop shaping weight, with the small variations being due to the model differences.

The remaining question now is how close these responses will be to the true system. That is, we now use each controller to compute the closed-loop response with the original (true) system $G_0(s)$. The setpoint tracking of the different closed-loop systems is shown in Fig. 4(b). Comparing to Fig. 4(a), we notice that the controller K_{rcid} gives the most consistent performance when applied to G_0. The advantage of the proposed approach to do identification by minimizing the weighted coprime factor uncertainty, is best illustrated by this comparison. The closed-loop systems with controller K_{rcid} matches the ideal closed-loop system very well, while there are significant deviations for the systems with controllers K_{oe} and K_{asym}.



(a) Nominal Responses



(b) True Responses

Fig. 4. Comparison of closed-loop responses with different models. solid: true system; thick solid: RCID, dashed: ASYM; o: OE.

Table 1. Monte carlo simulation results

Method	$\delta_\nu(WG_{id}, WG_0)$		$\epsilon_{max} - \delta_\nu$	
	Mean	Std	Mean	Std
RCID	0.1129	0.02536	0.2107	0.02977
ASYM	0.2161	0.03502	0.1188	0.04112
BJ	0.2334	0.06915	0.1146	0.06409
OE	0.1522	0.05657	0.1588	0.05722

4.3 Monte Carlo Simulation Results

We run 100 simulations with different realizations of output disturbances. In each simulation, the excitation input signal u and disturbance signal d are generated randomly as described in Section 4.1. The weighting function is fixed as $W(s) = 0.1/s$ and $2nd$-order models are identified by using the four methods: OE, ASYM, BJ, and the proposed RCID. In the BJ method, a $4th$-order noise model structure is used so that it has enough flexibility to model the noise.

The ν-gap between the weighted identified low-order model (denoted by G_{id}) and weighted true process model is calculated for each simulation. The adjusted stability margin, defined as $\epsilon_{max}(WG_{id}) - \delta_\nu(WG_0, WG_{id})$, is also calculated. Obviously $\epsilon_{max}(WG_{id})$ should be greater than $\delta_\nu(WG_0, WG_{id})$ to guarantee controller stability in the presence of model mismatch. Larger values of $\epsilon_{max}(WG_{id}) - \delta_\nu(WG_0, WG_{id})$ yield smaller values for ϕ in (11), implying in turn smaller values for the closed-loop deviation from the ideal case $\|H(WG_{id}, K_{opt}) - H(WG_0, K_{opt})\|_\infty$. The average and standard deviation of the weighted ν-gap and the adjusted stability margin of all the simulations are summarized in Table 1.

Our general observation from this limited Monte Carlo study is that the RCID method gives the smallest average ν-gap and the biggest adjusted stability margin. The RCID method also gives the smallest variance in both values, indicating more consistent estimation results.

In this simulation study, 2nd-order models are identified by different methods to approximate the true 7th-order dynamics. It is demonstrated that the RCID method can find the most control-relevant low-order model, with smallest weighted ν-gap and thus most consistent closed-loop control behavior. The advantage may be smaller when a higher order model structure is used, assuming all the methods can converge to their global minimum.

5. CONCLUSIONS

In this paper, we presented a new method for robust-control-oriented system identification, based on a high-order model reduction approach with an objective of minimizing weighted coprime factor uncertainty. Simulation examples for SISO systems illustrate the application of the method and yield very encouraging results in terms of the closed-loop behavior.

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