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Starting Speed Control of SI Engine Based on Extremum Seeking Control

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Abstract: The extremum seeking controller (ESC) that minimizes the unknown performance index is applied to the control of SI (spark ignition) engines in the idle speed region. The proposed control system consists in the three parts, fuel-jet mount controller, ignition-timing controller and engine speed controller. The ESC applied to the engine speed controller should be modified in order to improve the efficiency of the perturbation signal. Moreover, to decrease the overshoot of the engine speed in starting time, the new control technique of the ignition timing is proposed. Finally to show the effectiveness of the proposal method, the numerical simulations are shown using the mathematical model of a six cylinders SI engine that admission is implemented intermittently.

1. INTRODUCTION

Recently, the regulations on the automobile performance become severe to solve the global environmental problem and the energy depletion problem. One is the exhaust emission regulation, which is begun in 1966 in Japan, and strengthened year by year. The test method of the exhaust emission of the gasoline vehicle and diesel vehicle was reviewed in 2005. Furthermore the strengthen emission standard that is the severest in the world is executed for the exhaust emission decrease. The automakers are positively working on the research and development of the technology to achieve a high-quality engine. On the other hand, the advanced control of the power train is possible according to remarkable evolution of the electronic mounting technology. For such a background, the need of new control method and theory construction for the engine and power train system is increased.

The purpose of Extremum seeking control (ESC) is to seek and to keep the optimal states that keep the performance index the extremum. To achieve this purpose, extremum-seeking controller observes only output of the performance index (for example, Efficiency, cost, and so on) and seeks the optimal state based on only the output signal. Then it is necessary to be stabilized the system, but not necessary to know the system and the performance index forms. In this thesis, ESC that minimizes the performance index of engine output is applied to the control of throttle plate. And from a simple fuel behavior model, the fuel injection controller is constructed to control the air-fuel ratio. To decrease the overshoot of the starting speed, the control technique of the ignition timing is proposed. Finally to show the effectiveness of the proposal method, the simulation is shown using the six cylinders reciprocating engine model that intake and exhaust are implemented intermittently.

2. ENGINE CONTROL PROBREM

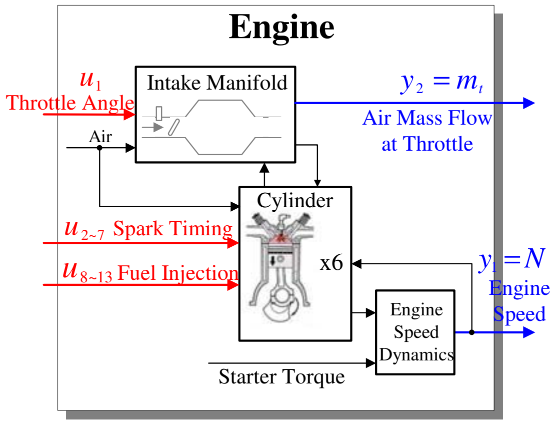

Fig. 1. Engine model

The entire engine model is described by Fig.1[1]. In this paper we conside the six-cylinders SI engine as a plant . The system is described by the following equation that has $(1 + 2 \times 6)$-inputs, 2-outputs and 37-states.

$$\frac{d}{dt}\boldsymbol{x}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)),\ \boldsymbol{x}(0) = \boldsymbol{x}_0 \quad (1)$$

$$\boldsymbol{y}(t) = \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \quad (2)$$

where, $\boldsymbol{x}(t) \in R^{37}$ is state, $\boldsymbol{u}(t) \in R^{(1+2\times6)}$ input, and $\boldsymbol{y}(t) \in R^2$ output. The input is constructed by throttel opening angle, fuel injection, and spark timing whereas the output engine speed, and air mass flow.

The control performance required is stated below.

- Engine speed should reach 650 ± 50rpm asymptotically less than 1.5s after engine started.
- Reduce the overshoot as much as possible.
- Minimize the integrating amount of fuel injection as much as possible.

And we set variation for three parameters as follows.

- Friction torque T_f
$$\left\|\frac{T_f - T_{fs}}{T_{fs}}\right\| \leq 0.2(\pm 20\%)$$
T_{fs}:Friction torque of standard model
- Initial crank angle $CA(0)$
$$CA(0) = 120i \longrightarrow 1.28\%$$
$$CA(0) = 60 + 120i \longrightarrow 15.4\%$$
$$i = 0, 1, 2, 3, 4, 5$$
- Cranking speed n_c
$$\|n_c(t) - 250\| \leq 50[rpm]$$

3. CONTROLLER DESIGNE

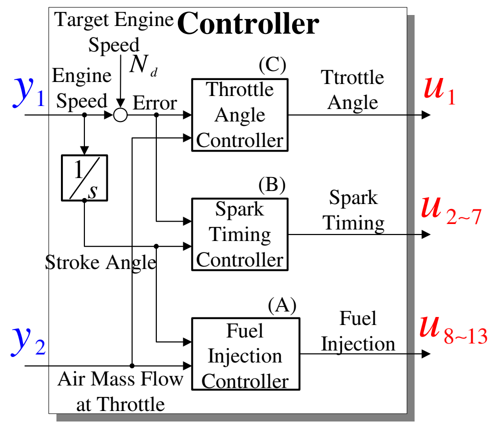

Fig. 2. Control system

3.1 FUEL INJECTION CONTROLLER

The amout of fuel injection is determened by following steps. First, predict the cylinder-in-air mass in one intake stroke by using the measurement of air mass flow through the throttle plate. Then, fuel is injected such that the ratio of predicted air mass and fuel mass satisfys stoichiometric air fuel ratio.

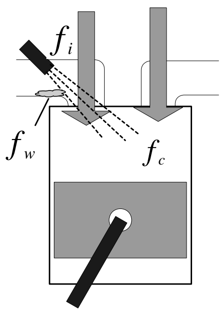

Fig. 3. Fuel injection model

Fig.3 shows the brief fuel model. Where, f_i is the fuel mass injected into the corresponding intake port, f_c the fuel mass aspirated into the cylinder, and f_w the mass of fuel stored in the fuel film. According the model a mass balance can be expressed as follows.

$$f_w(k+1) = Pf_w(k) + Rf_i(k) \tag{3}$$
$$f_c(k) = (1-P)f_w(k) + (1-R)f_i(k) \tag{4}$$

Where, coefficiencts P and R depend on the the engine speed, intake pressure, port and valve tempreture, and on many other variables. Since we assume fuel is injected instantaneously, fuel dynamics is expressed by the discrete-event representation.

Moreover, we define air mass in the cylinder as $\hat{M}_c$, stoichimetric air fuel ratio as $\alpha_r (= 14.5)$, then the reference fuel mass into the cylinder can be expressed as following equation

$$f_{cr}(k) = \frac{\hat{M}_c(k)}{\alpha_r} \tag{5}$$

and by equations (3), (4) and (5), we obtain the inverse model below.

$$f_i(k) = \frac{f_{cr}(k) - (1-P)\hat{f}_w(k)}{1-R} \tag{6}$$
$$\hat{f}_w(k) = P\hat{f}_w(k-1) + Rf_i(k-1) \tag{7}$$

By implementing fuel injection according to equation (6) and (7), air fuel ratio can be controled near the stoichiometric A/F[1]. However, there is a probrem that how we predict air mass in the cylinder. To solve this problem we predict air mass in the cylinder from intake manifold pressure. Unfortunately, in this case we can not use mesured intake manifold pressure, so we predict intake manifold pressure by MVEM (Mean Value Engine Model) which is proposed by Elbert Hendricks[2].

$$\dot{\hat{P}}_m = \hat{P}_m \frac{\kappa R T_0}{V_m \hat{P}_m}(y_2 - \frac{\hat{T}_m}{T_0} m_c) \tag{8}$$

$$\dot{\hat{T}}_m = \hat{T}_m \frac{\kappa R T_0}{V_m \hat{P}_m}\{y_2(1 - \frac{1}{\kappa}\frac{\hat{T}_m}{T_0}) - \frac{\hat{T}_m}{T_0} m_c (1 - \frac{1}{\kappa})\} \tag{9}$$

$$m_c = \frac{N_{cyl} y_1}{4\pi \hat{T}_m}(a_{m1}\hat{P}_m + a_{m2}) \tag{10}$$

$$\hat{M}_c = \frac{1}{\hat{T}_m}(a_{m1}\hat{P}_m + a_{m2}) \tag{11}$$

Where, P_m, T_m, $\dot{m_c}$, $\hat{M}_c$, κ, R, T_0, V_m, N_{cyl} and a_m are intake manifold pressure, intake manifold temperature, air mass flow in the cylinder, air mass in the cylinder, gas constant, atmosphere tempareture, intake manifold volume, number of cylinders and constant derived from energy conservation law during intake stroke respectively.From equations (8), (9), (10), (11), we can predict P_m and M_c.

3.2 SPARK TIMING CONTROLLER

Mordern SI engines have rather low idle speed limits in order to minimize fuel consumption and pollutant emission.

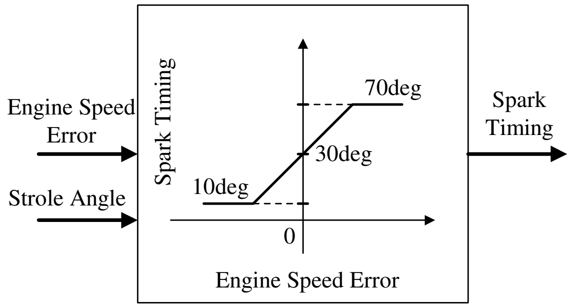

Fig. 4. Spark timing controller

The drawback of such low limits is that sudden changes in engine torque may stall the engine since combustion becames unstable easily. Hence, though maximum engine torque generates at the sparking timing near 10deg after TDC, spark timing is chosen larger in idling speed operating point[1]. Fig.4 shows structure of spark timing controller. If engine speed is smaller than desired engine speed, then chose smaller spark timing, and if engine speed is larger than desired engine speed, then chose lager spark timing. When engine speed reachs desired engine speed (i.e. idle speed), spark timing is designed to be apploximately 30deg. Moreover, we switch u_2 to u_7 accoding to stroke angle.

3.3 THROTTLE ANGLE CONTROLLER

Control purpose of throttle angle controller is to minimize the performance index z by changing the throttle angle as a control input. Performance index is given as follows.

$$z = h(y) = R(y_1 - N_d)^2 + Qy_2^2 \tag{12}$$

Where, N_d is desired engine speed, and R, Q are weight respectively. Controller is designed by extremum seeking control by purturbation method. Fig.2 shows proposal method. In ordinary ESC, correlation is given by multiplying $\sin\omega t$ by ϕ in Fig.2, however influence of input perturbation to engine speed is not appear apparently. Hence, we can not get correlation by multiplying ϕ by $\sin\omega t$.

The engine under consideration has six cylinders and each two adjoin cylinders differ 120deg crank angle each other, hence value of engine speed oscillates small. In proposal method, we get correlation by passing engine speed through the high pass filter in order to substitute high frequency of output z_1 for $\sin\omega t$.

1-input 1-output nonlinear system is defined by following equation.

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t), u_1(t)) \tag{13}$$

$$z(t) = h \circ p(\boldsymbol{x}(t)) \tag{14}$$

Where, $\boldsymbol{x} \in \Re^{51}$, $u_1 \in \Re^1$, $z \in \Re^1$ is state, input, and output respectively. $f(\boldsymbol{x}, u)$ and $p(\boldsymbol{x})$ are smooth function. There exists the smooth control input that stabilizes nonlinear system equation (13) as follows.

$$u_1(t) = \theta \tag{15}$$

Where, θ is the parameter that determine only extreme. By this control input nonlinear system equation(13) is expressd below.

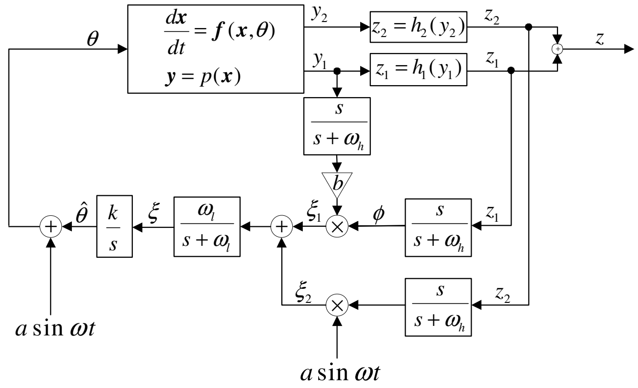

Fig. 5. Block diagram of proposed method

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}(t), \theta) \tag{16}$$

We make the following assumptions about the closed-loop system.

Assumption2.1 There exists following smooth functions $\boldsymbol{\ell} : \Re \to \Re^n$.

$$f(\boldsymbol{x}, \theta) = 0 \qquad \leftrightarrow \qquad \boldsymbol{x} = \boldsymbol{\ell}(\theta) \tag{17}$$

Assumption2.2 For each $\theta \in \Re$, the equilibrium $\boldsymbol{x} = \boldsymbol{\ell}(\theta)$ is locally exponentially stable.

Assumption2.3 The dynamics of θ is much slower than system dynamics, hence the performance index is apploximately expressed as $z(t) = F(\theta(t))$.

Assumption2.4 There exist θ^*, θ_1^*, θ_2^* such that

$$\frac{\partial F}{\partial \theta}(\theta^*) = 0, \frac{\partial^2 F}{\partial \theta^2}(\theta^*) > 0 \tag{18}$$

$$\frac{\partial z_1}{\partial \theta}(\theta_1^*) = 0, \frac{\partial^2 z_1}{\partial \theta^2}(\theta_1^*) > 0, \frac{\partial y_1}{\partial \theta}(\theta_1^*) > 0 \tag{19}$$

$$\frac{\partial z_2}{\partial \theta}(\theta_2^*) = 0, \frac{\partial^2 z_2}{\partial \theta^2}(\theta_2^*) > 0. \tag{20}$$

Thus, the output equilibrium $z = F(\theta)$ has a maximum at θ^*. The objective of extremum seeking method is to designe control input that minimizes performance index equation (14). Where, θ^* is unkouwn.

From Assumption 2.4

$$z_1(\theta) = z_1^* + \frac{z_1''}{2}(\theta - \theta_1^*)^2 \tag{21}$$

$$z_2(\theta) = z_2^* + \frac{z_2''}{2}(\theta - \theta_2^*)^2 \tag{22}$$

$$y_1(\theta) = y_1^* + y_1'(\theta - \theta_1^*) + \frac{y_1''}{2}(\theta - \theta_1^*)^2. \tag{23}$$

Where, we difine

$$\tilde{\theta}(t) \triangleq \theta^* - \hat{\theta}(t) \tag{24}$$

$$\tilde{\theta}_1(t) \triangleq \theta_1^* - \hat{\theta}(t) \tag{25}$$

$$\tilde{\theta}_2(t) \triangleq \theta_2^* - \hat{\theta}(t). \tag{26}$$

Then, from $\theta(t) = \hat{\theta}(t) + a\sin\omega t$, equation (25) and (26) are expressed as

$$\theta - \theta_1^* = a\sin\omega t - \tilde{\theta}_1 \tag{27}$$

$$\theta - \theta_2^* = a\sin\omega t - \tilde{\theta}_2. \tag{28}$$

From equation (27) and (28), (21), (22), and (23) are rewritten as

$$z_1(t) = z_1^* + \frac{z_1''}{2}(\tilde{\theta_1} - a\sin\omega t)^2 \tag{29}$$

$$z_2(t) = z_2^* + \frac{z_2''}{2}(\tilde{\theta_2} - a\sin\omega t)^2 \tag{30}$$

$$\begin{aligned} y_1(t) &= y_1^* + y_1'(a\sin\omega t - \tilde{\theta}_1) \\ &\quad + \frac{y_1''}{2}(\tilde{\theta}_1 - a\sin\omega t)^2. \end{aligned} \tag{31}$$

Using relation

$$2\sin^2\omega t = 1 - \cos 2\omega t \tag{32}$$

then equation (29), (30), and (31) are debeloped as follows.

$$\begin{aligned} z_1(t) &= z_1^* + \frac{z_1''}{2}\tilde{\theta}_1{}^2 - az_1''\tilde{\theta}_1\sin\omega t + \frac{a^2 z_1''}{4} \\ &\quad - \frac{a^2 z_1''}{4}\cos 2\omega t \end{aligned} \tag{33}$$

$$\begin{aligned} z_2(t) &= z_2^* + \frac{z_2''}{2}\tilde{\theta}_2{}^2 - az_2''\tilde{\theta}_2\sin\omega t + \frac{a^2 z_2''}{4} \\ &\quad - \frac{a^2 z_2''}{4}\cos 2\omega t \end{aligned} \tag{34}$$

$$\begin{aligned} y_1(t) &= y_1^* + y_1'(a\sin\omega t - \tilde{\theta}_1) + \frac{y_1''}{2}\tilde{\theta}_1{}^2 \\ &\quad - ay_1''\tilde{\theta}_1\sin\omega t + \frac{a^2 y_1''}{4} - \frac{a^2 y_1''}{4}\cos 2\omega t \end{aligned} \tag{35}$$

Outputs $z_1(t)$, $z_2(t)$, and $y_1(t)$ are passed through the high pass filter

$$\frac{s}{s+\omega_h} \tag{36}$$

and the first term of constants are removed.

$$\begin{aligned} \frac{s}{s+\omega_h}z_1(t) &\approx \frac{z_1''}{2}\tilde{\theta}_1{}^2 - az_1''\tilde{\theta}_1\sin\omega t \\ &\quad - \frac{a^2 z_1''}{4}\cos 2\omega t \end{aligned} \tag{37}$$

$$\begin{aligned} \frac{s}{s+\omega_h}z_2(t) &\approx \frac{z_2''}{2}\tilde{\theta}_2{}^2 - az_2''\tilde{\theta}_2\sin\omega t \\ &\quad - \frac{a^2 z_2''}{4}\cos 2\omega t \end{aligned} \tag{38}$$

$$\begin{aligned} \frac{s}{s+\omega_h}y_1(t) &\approx y_1'(a\sin\omega t - \tilde{\theta}_1) + \frac{y_1''}{2}\tilde{\theta}_1{}^2 \\ &\quad - ay_1''\tilde{\theta}_1\sin\omega t \\ &\quad - \frac{a^2 y_1''}{4}\cos 2\omega t \end{aligned} \tag{39}$$

Thus,

$$\xi_1 = \left(\frac{s}{s+\omega_h}z_1\right)\cdot\left(\frac{bs}{s+\omega_h}y_1\right) \tag{40}$$

$$\xi_2 = a\sin\omega t\left(\frac{s}{s+\omega_h}z_2\right) \tag{41}$$

Terms of $\tilde{\theta}$ larger than 2th are neglected since we now consider local analysis. More over, terms that have high frequency is removed by low pass filter $\frac{\omega_l}{s+\omega_l}$. Using relations equation (32),

$$2\cos 2\omega t\sin\omega t = \sin 3\omega t - \sin\omega t, \tag{42}$$

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}}, \tag{43}$$

and

$$\hat{\theta} = \frac{k}{s}\frac{\omega_l}{s+\omega_l}(\xi_1 + \xi_2) \tag{44}$$

following equation is derive.

$$\begin{aligned} \dot{\tilde{\theta}} &= -\dot{\hat{\theta}} = -k\frac{\omega_l}{s+\omega_l}(\xi_1+\xi_2) \\ &\approx -k\left[-\frac{z_1''y_1'a^2b}{2}\tilde{\theta}_1 + \frac{z_1''y_1''a^4b}{32} - \frac{z_2''a^2}{2}\tilde{\theta}_2\right] \end{aligned} \tag{45}$$

From equation (24), (25), (26), (45) is expressed as

$$\begin{aligned} \dot{\tilde{\theta}} = -k\Bigg[&-\frac{z_1''y_1'a^2b}{2}\{(\theta_1^* - \theta^*) + \tilde{\theta}\} + \frac{z_1''y_1''a^4b}{32} \\ &-\frac{z_2''a^2}{2}\{(\theta_2^* - \theta^*) + \tilde{\theta}\}\Bigg]. \end{aligned} \tag{46}$$

Since

$$\begin{aligned} z_1(\theta) + z_2(\theta) &= z_1^* + z_2^* + \frac{z_1''}{2}(\theta - \theta_1^*)^2 \\ &\quad + \frac{z_2''}{2}(\theta - \theta_2^*)^2 \end{aligned} \tag{47}$$

has a minimum at $\theta = \theta^*$,thus

$$\begin{aligned} \left.\frac{\partial(z_1+z_2)}{\partial\theta}\right|_{\theta=\theta^*} &= z_1''(\theta^* - \theta_1^*) + z_2''(\theta^* - \theta_2^*) \\ &= 0. \end{aligned} \tag{48}$$

Let $y_1' = 1$ in equation (46), then

$$\dot{\tilde{\theta}} = -k\left[-\frac{z_1''a^2}{2}\tilde{\theta} + \frac{z_1''y_1''a^4b}{32} - \frac{z_2''a^2}{2}\tilde{\theta}\right]. \tag{49}$$

Further, by making $a > 0$ sufficiently small, we obtain following equation.

$$\dot{\tilde{\theta}} = -k\left[-\frac{z_1''a^2}{2} - \frac{z_2''a^2}{2}\right]\tilde{\theta} + O(a^4) \tag{50}$$

Since, $k < 0$, the system is stable and $\hat{\theta}$ converges to near θ^* if $\tilde{\theta} \to O(a^4)$.

4. NUMERICAL SIMULATIONS

From Fig.6 to Fig.11 shows the simulation of the starting speed contorl of SI engine based on the proposal method. Parameters of perturbation method are set as $a = 0.5$, $\omega = 30$rad/s, $\omega_h = 60$, $\omega_l = 10$, $k = -1$. And the performance index is

$$z = 1/1500(y_1 - 650)^2 + 0.5y_2^2 \tag{51}$$

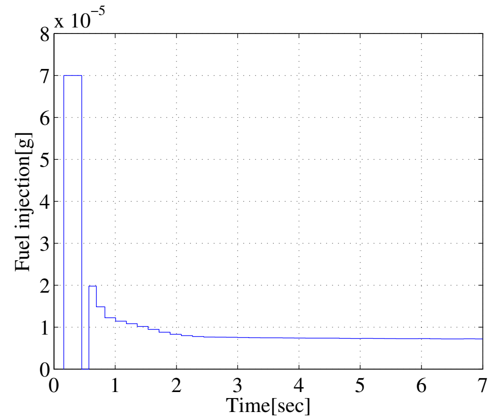

Fig. 6. Iutput(Fuel injection)

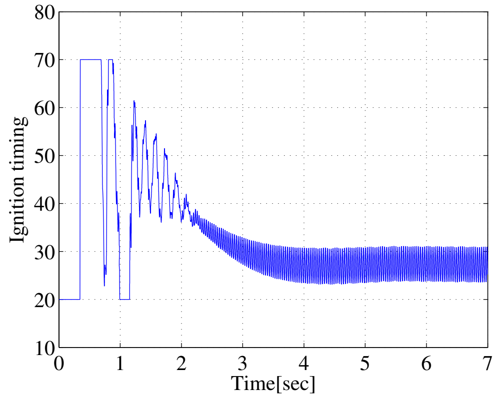

Fig. 7. Iutput(Spark timing)

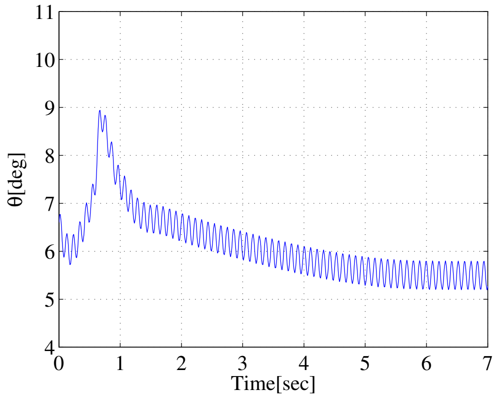

Fig. 8. Iutput(Throttle angle)

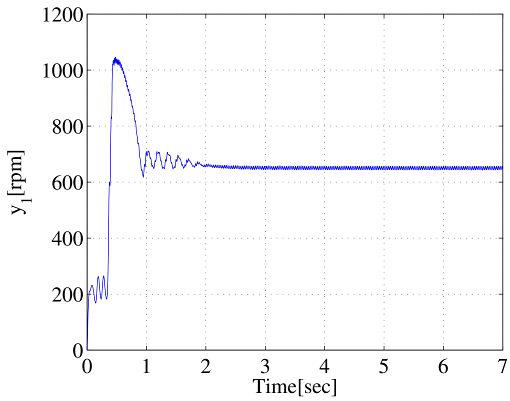

Fig. 9. Output(Engine speed)

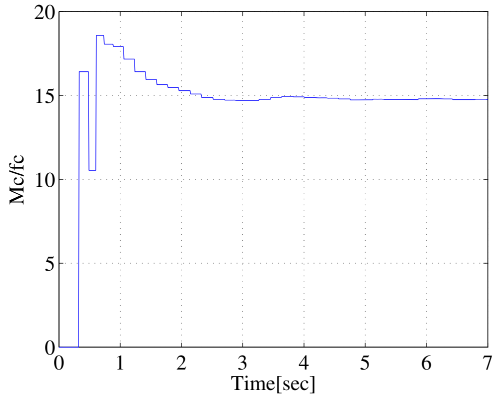

Fig. 10. Air/Fuel ratio of cylinder 1

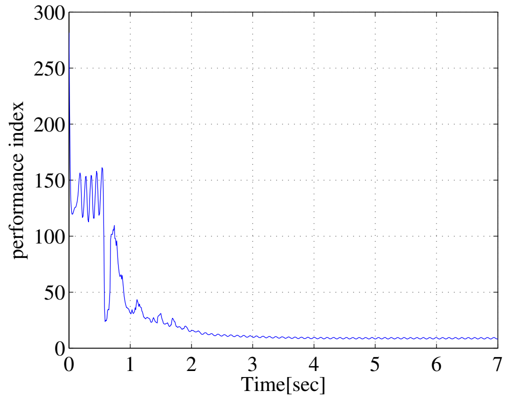

Fig. 11. Performance index(equation (51))

5. CONCLUSION

In this paper, we proposed 3 subcontrollers (i.e. fuel injection controller, spark timing controller, throttle angle controller) for a six-cylinders reciprocating SI engine. Fuel injection controller is designed by constructig the invers fuel model to keep air fuel ration stoich. Spark timing controller is designed by using the engine speed error and adjusting the spark timing to reduce the overshoot of engine speed. Finally Throttle angle contoroller based on extremum seeking contorol is proposed. This method is realized by minimizing performance index that is caluculated by engine output. In ordinary perturbation method, extremum seekig is implemented by adding a perturbation signal to input. For a engine model under consideration, however infuluence of a perturbation signal is small. For this reason, we use the oscilatory signal of engin speed to seek the extrem.

Finally, effectiveness is confirmed by the simulations. These simulation shows that effective performance is attained in terms of air fuel ratio, engine speed overshoot, convergence of desired engine speed, and reduction of fuel consumption.

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