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Optimal input signals for bandlimited scanning systems ⋆

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Abstract: Periodic scanning trajectories are designed with minimal harmonic content and fixed linear regions. While minimum harmonic content reduces vibration in mechanical scanners, fixed linear regions eliminate curvature in the scan area. Simulated and experimental results demonstrate less induced vibration than presently used techniques.

Keywords: Microscopes; Optimal Trajectory; Trajectory Planning.

1. INTRODUCTION

Many scientific and industrial machines contain mechanical scanners driven with periodic trajectories, for example, beam steering scanners, manufacturing robots, cam motion generators, and Scanning Probe Microscopes (SPMs) (Meyer et al. [2004]). In this work, periodic input signals are designed to maximize the speed and accuracy of bandlimited scanners without explicit knowledge of the systems dynamics. We focus on designing inputs for scanning probe microscope nano-positioning stages, as reviewed in Zou et al. [2004], Abramovitch et al. [2007] and Devasia et al. [2007].

A new input trajectory is proposed that achieves perfectly linear scanning in a certain range with minimum bandwidth requirements. While a region of the scan range is fixed, the remaining part, consisting of the turning points, is optimized to minimize high frequency content.

Other techniques for input shaping have been popular in industry for some time. The most straight-forward of which is the minimum acceleration technique. This involves replacing the turning points of trajectory with a smooth quadratic curve. Although this minimizes inertial force, it does not lead to optimal tracking performance. Minimum acceleration signals were used in Rost et al. [2005] to achieve SPM imaging rates of up to 200 frames per second.

Better performance than the minimum acceleration signal can be achieved by convolving the desired trajectory with a signal that minimizes induced vibration, see Masterson et al. [2000], Singhose et al. [1995] and Singer and Seering [1990]. The foremost disadvantages of convolution techniques are: the significant filter length, sensitivity to parameter variation, and increased control signal magnitude (Masterson et al. [2000]).

In the following section, a signal optimization scheme is proposed that allows parts of the trajectory to be fixed as linear or otherwise. In Section 3, a range of cost functions are described that minimize properties such as the acceleration and signal power. The *out-of-bandwidth power cost function* is used in Section 4 to generate optimal scanning signals for a low-bandwidth piezoelectric nanopositioning stage. Experimental results are followed by conclusions in Section 5.

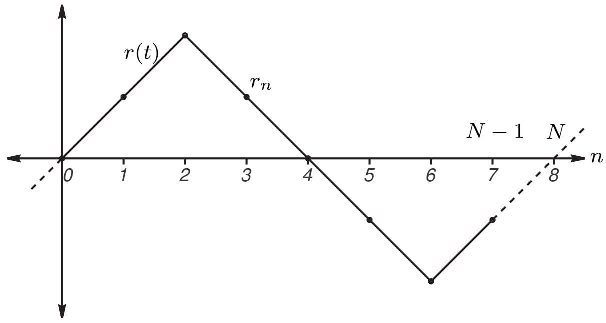

Fig. 1. A periodic signal $r(t)$ and its samples r_n

2. SIGNAL OPTIMIZATION

In this Section, the optimization associated with the objective is defined. The method begins with an ideal scanning trajectory, this is split into regions that are fixed, and regions to be modified. The variable parts are then redesigned to satisfy a quadratic cost function. In the next Section, cost functions are described for various time and frequency domain objectives.

Consider the triangular waveform $r(t)$ plotted in Figure 1.

The samples of $r(t)$ are denoted $r_n = r(\Delta n)$ where Δ is the sampling interval, $n \in \{0, 1, 2, \cdots, N-1\}$ and N is the number of samples per period. In the illustration, the sampling frequency $F_s = \frac{1}{\Delta}$ is equal to eight times the triangle frequency F_T.

The samples of $r(t)$ over one period can be written in vector notation:

$$r = \begin{bmatrix} r_1 \\ r_2 \\ r_2 \\ \vdots \\ r_{N-1} \end{bmatrix} = \begin{bmatrix} r(0) \\ r(\Delta) \\ r(2\Delta) \\ \vdots \\ r((N-1)\Delta) \end{bmatrix}. \quad (1)$$

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In general, one period of a waveform $x(t)$ will be denoted x where $x \in \mathcal{R}^{N\times 1}$.

In this work, we seek a new signal y that is equal to r at an arbitrary set of sample indexes S and free to vary elsewhere. The free part of the signal is varied to minimize the quadratic cost $y^T H y$. That is, we seek y that is the solution to

$$y = \arg\min_x \; x^T H x, \quad \text{subject to } x_k = r_k \;\; k \in S, \qquad (2)$$

where $x \in \mathcal{R}^{N\times 1}$ and $H \in \mathcal{R}^{N\times N}$. Equation (2) is equivalent to the linearly constrained convex quadratic optimization problem (Fletcher [1987])

$$y = \arg\min_x \; x^T H x + 2f^T x, \quad \text{subject to } Ax = r(S), \quad (3)$$

where A is the selection matrix representing S and $r(S)$ is a row vector containing the samples of r_n indexed by the values of S.

Equation (3) can be restated in matrix notation as (Fletcher [1987])

$$\begin{bmatrix} H & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} y \\ \lambda \end{bmatrix} = \begin{bmatrix} -f \\ r(S) \end{bmatrix}, \qquad (4)$$

where λ are the Lagrange multipliers (Fletcher [1987]). Equation (4) has a solution (Fletcher [1987])

$$\begin{bmatrix} y \\ \lambda \end{bmatrix} = \begin{bmatrix} H & A^T \\ A & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} -f \\ r(S) \end{bmatrix}. \qquad (5)$$

3. COST FUNCTIONS

The weighting matrix H can be chosen so that $x^T H x$ represents a wide variety of frequency domain objectives. In the following subsections, cost functions are described for signal power, frequency weighted power, velocity and acceleration.

3.1 Background: Discrete Fourier Series

The discrete Fourier series c_k of a periodic signal r_n is described by the analysis function (Proakis and Manolakis [2007])

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} r_n \; e^{-jn\frac{2\pi k}{N}}. \qquad (6)$$

The synthesis function is (Proakis and Manolakis [2007])

$$r_n = \sum_{k=0}^{N-1} c_k \; e^{jn\frac{2\pi k}{N}}, \qquad (7)$$

where $\widehat{\omega} = \frac{2\pi k}{N}$ is the normalized frequency, and $\frac{2\pi}{N}$ is the normalized fundamental frequency. The real frequency in Hertz is related to $\widehat{\omega}$ by $f = \frac{\widehat{\omega}}{2\pi\Delta}$. As an example, the discrete Fourier components of an 8 sample signal are shown in Figure 2.

The discrete Fourier coefficients of r can be written in matrix notation:

$$c = \frac{1}{N} E \quad r \qquad (8)$$

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_{N-1} \end{bmatrix} = \frac{1}{N} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j\frac{2\pi 1}{N}} & \cdots & e^{-j(N-1)\frac{2\pi 1}{N}} \\ 1 & e^{-j\frac{2\pi 2}{N}} & \cdots & e^{-j(N-1)\frac{2\pi 2}{N}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\frac{2\pi(N-1)}{N}} & \cdots & e^{-j(N-1)\frac{2\pi(N-1)}{N}} \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \\ \vdots \\ r_{N-1} \end{bmatrix} \qquad (9)$$

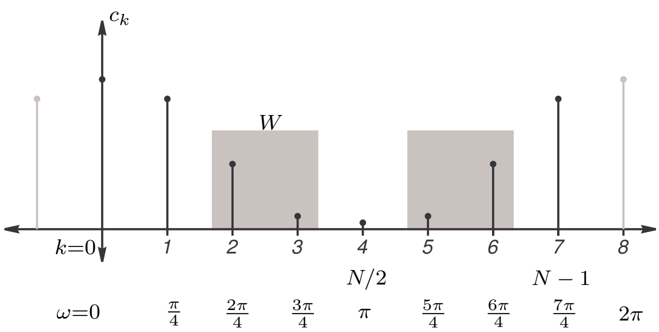

Fig. 2. The Fourier components c_k of r.

3.2 Minimizing signal power

By Parseval's equality, the average power P_r of a discrete time signal r is [1]

$$P_r = \frac{1}{N}\sum_{n=0}^{N-1} |r_n|^2 = \sum_{k=0}^{N-1} |c_k|^2 = \|c_k\|_2^2, \qquad (10)$$

where the sequence $|c_k|^2$ for $k \in \{0, 1, 2, \cdots, N-1\}$ is the distribution of power as a function of frequency, or the power spectral density. This can be written in vector form:

$$\begin{aligned} P_r &= c^* c \qquad (11) \\ &= \frac{1}{N^2} r^T E^* E r, \end{aligned}$$

thus, referring to equation 2, minimum power is achieved by

$$H = \frac{1}{N^2} E^* E. \qquad (12)$$

3.3 Minimizing frequency weighted power

In Figure 2, a frequency dependent weighting W is shown. The power resident in the shaded bandwidth can be calculated by summing only these components. W must be symmetric around π.

We wish to specify a cost function in equation 2 that represents power above a certain frequency or harmonic. This allows complete freedom in signal power up to the K^{th} harmonic while imposing a minimum power constraint on higher frequencies. The frequency weighted power P_r^W of r is:

$$P_r^W = \frac{1}{N^2} r^T E^* W E r, \qquad (13)$$

where $W = diag(Q)$ and

$$Q = \begin{cases} 0 & k \in [0 \cdots K] \\ 1 & k \in [K{+}1 \cdots N\text{-}K\text{-}1] \\ 0 & k \in [N\text{-}K \cdots N\text{-}1] \end{cases},$$

thus, referring to equation 2, minimum out-of-band power is achieved by

$$H = \frac{1}{N^2} E^* W E. \qquad (14)$$

3.4 Time domain cost functions

In addition to the frequency domain objectives discussed in the previous subsections, the quadratic cost in Equation (2) can also represent a function of time. In this work, we limit the cost function definition to the power at the output of an FIR filter whose input is y. That is, we seek to minimize:

$$\frac{1}{N}\sum_{n=0}^{N-1} \left|B(q^{-1}) y_n\right|^2 \qquad (15)$$

where $B(q^{-1})$ is an FIR filter of order N_B and length N_B+1.

In matrix form, $z_n = B(q^{-1})y_n$ can be written

$$z = \mathbf{B} \quad y \tag{16}$$

$$\begin{bmatrix} z_{0+N_b} \\ z_{1+N_b} \\ \vdots \\ z_{N-1} \end{bmatrix} = \begin{bmatrix} b_{N_B} & \cdots & b_1 & b_0 & 0 & 0 & 0 & 0 \\ 0 & b_{N_B} & \cdots & b_1 & b_0 & 0 & 0 & 0 \\ & & & & & & & \\ 0 & 0 & 0 & 0 & b_{N_B} & \cdots & b_1 & b_0 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{bmatrix} \tag{17}$$

The power in z is

$$\frac{1}{N}\sum_{n=0}^{N-1}\left|B(q^{-1})y_n\right|^2 = \frac{1}{N}\left\|B(q^{-1})y_n\right\|_2^2 \tag{18}$$
$$= \frac{1}{N}z^T z$$
$$= \frac{1}{N}y^T\mathbf{B}^T\mathbf{B}y.$$

Referring to equation 2, $\frac{1}{N}\left\|B(q^{-1})y_n\right\|_2^2$ is minimized when

$$H = \frac{1}{N}\mathbf{B}^T\mathbf{B}. \tag{19}$$

3.5 Minimum velocity

The discrete velocity of y_n is the first-order time derivative

$$\frac{dy_n}{dt} = \frac{y_n - y_{n-1}}{\Delta}. \tag{20}$$

Thus, the FIR filter that represents differentiation is

$$B(q^{-1}) = \frac{1}{\Delta}(1 - 1q^{-1}). \tag{21}$$

In the time-domain cost function (19), $b_0 = 1$ and $b_1 = -1$.

3.6 Minimum acceleration

The discrete acceleration of y_n is the second-order time derivative

$$\frac{d^2y_n}{dt^2} = \frac{1}{\Delta}\left(\frac{dy_n}{dt} - \frac{dy_{n-1}}{dt}\right) \tag{22}$$
$$= \frac{(y_n - y_{n-1}) - (y_{n-1} - y_{n-2})}{\Delta^2}$$
$$= \frac{y_n - 2y_{n-1} + y_{n-2}}{\Delta^2}$$

thus,

$$B(q^{-1}) = \frac{1}{\Delta^2}(1 - 2q^{-1} + 1q^{-2}) \tag{23}$$

In the time-domain cost function (19), $b_0 = 1$, $b_1 = -2$ and $b_2 = 1$.

3.7 Frequency weighted objectives

A frequency weighting on the power in y_n can also be specified with an FIR filter. The quadratic cost H representing power at the output of the filter is described in Equations (18) and (19).

Frequency weighting of the velocity, acceleration or other can be achieved by designing an FIR weighting filter and convolving the coefficients with the coefficients of the original analysis filter, for example, the acceleration filter.

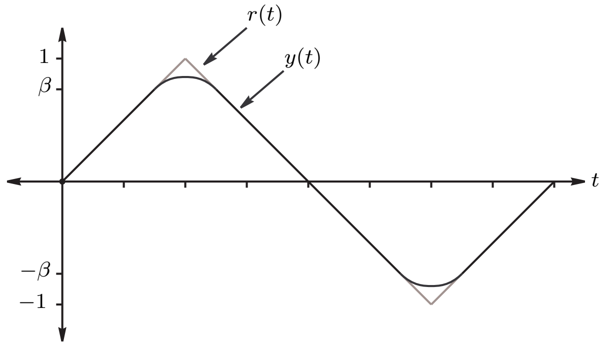

Fig. 3. The reference and optimal trajectory, $r(t)$ and $y(t)$. The optimal signal is equal to $r(t)$ when $r(t) < |\beta|$, otherwise there is no restriction.

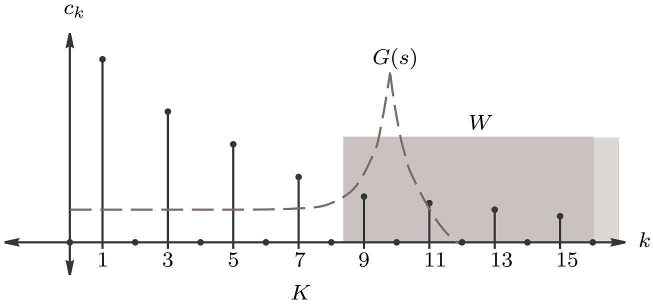

Fig. 4. The Fourier components of a triangular scanning signal plotted against harmonic number k

4. PRACTICAL APPLICATION

The desirable characteristics of periodic scanning trajectories are optimized using the frequency weighted power objective described in Section 3.3. The optimal trajectory contains the least possible power above a certain frequency while maintaining perfect linearity over a subset of the range. By varying the amount of signal left free for manipulation, the trade-off between scan-range and induced vibration can be continuously varied (for a constant scan rate).

For triangular and sawtooth scanning waveforms the linear range is easily specified by a single parameter β. Referring to Figure 3, the optimal trajectory y_k is equal to r_k when $r_k < |\beta|$, otherwise there is no restriction. Using the notation in Section 2, the previous statement can be rewritten as $y(S) = r(S)$ where S is the set of sample indexes for which $r_k < |\beta|$.

To specify the frequency weighting, it is convenient to stipulate the number of unrestricted low-frequency harmonics that may appear in the optimal signal. The spectrum of a triangular scanning signal is shown in Figure 4. The frequency components of the optimal signal are unrestricted between DC and the K^{th} harmonic. All harmonics greater than K are penalized equally.

A Matlab function, `generateTriangle`, is available by contacting the first author. It contains the functionality to generate and simulate optimal scanning signals.

4.1 Simulated Performance

In this example, a unit amplitude, 1 Hz triangle signal, sampled at 1 kHz is considered for optimization `generateTriangle(1000,1,.5,7,1)`. With a linear scan

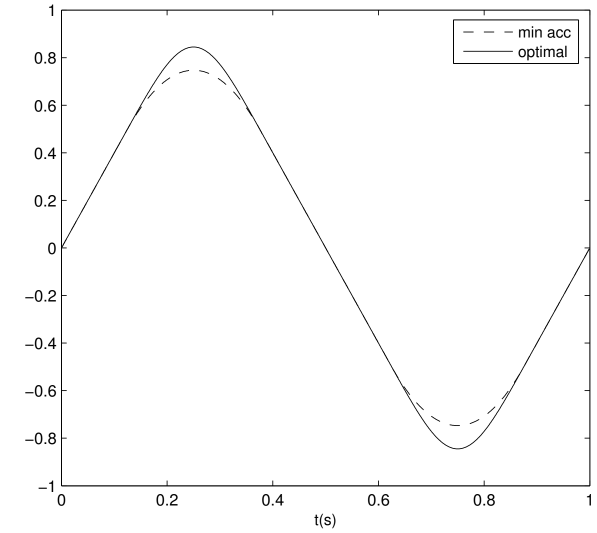

Fig. 5. The minimum acceleration (- -) and optimal scanning trajectory (—). Both signals have a linear range of ±0.5.

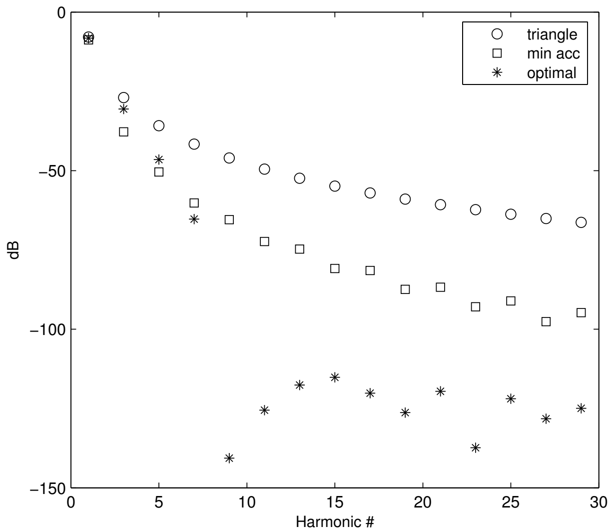

Fig. 6. The Fourier coefficients of the triangle, minimum acceleration and optimal signals (in dB).

range of β=0.5 and K=7, the minimum acceleration and optimal signals are plotted in Figure 5. As the signals are similar in shape, it is reasonable to expect the Fourier coefficients to be comparable, but this is not the case.

The Fourier coefficients of the minimum acceleration and optimal trajectory are compared to a triangle wave in Figure 6. The optimized signal has larger low-frequency components but vastly reduced harmonics above k=7. Undesirable harmonics are reduced by between 75 dB (at k=9) and 30dB at higher frequencies.

The improvement in time domain performance can be demonstrated by applying the optimized signal to a resonant second-order system, similar to a one-degree-freedom mechanical system. The system is chosen to have unity

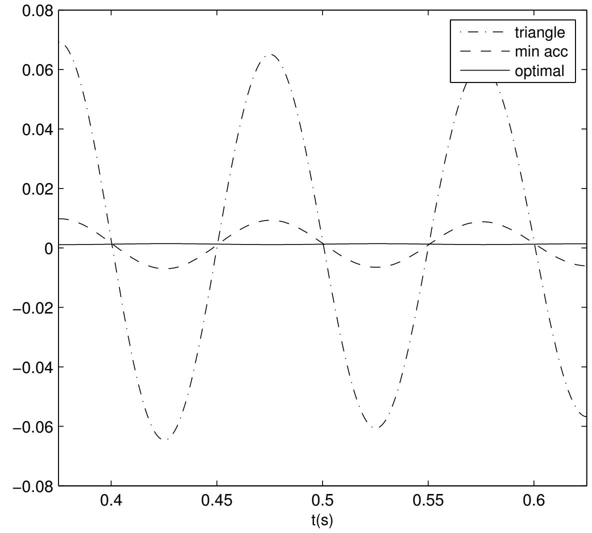

Fig. 7. The simulated vibration during the linear region of the scan. Triangle (-.-), minimum acceleration (- -) and optimal signal (—).

Signal	Peak-to-Peak Error	Percentage of Range
Triangle	0.13	13%
Min. Acc.	$8{\times}10^{-3}$	0.8%
Optimal	$0.32{\times}10^{-3}$	0.032%

Table 1. Induced Oscillation

gain, 10 Hz resonance frequency and a damping ratio of 0.01, the transfer function is:

$$G(s) = \frac{\omega_n^2}{s + 2\omega_n\xi_n s + \omega_n^2} \tag{24}$$

where ω_n=2π10 and ξ_n=0.01. The resonance frequency occurs between the 9$^{\rm th}$ and 11$^{\rm th}$ harmonic of the input. The output error signals for an unfiltered triangle signal, minimum acceleration signal and optimized signal are plotted in Figure 7. The peak-to-peak magnitude of oscillation during the linear region is tabulated in Table 1. The optimized signal induces only 4% the oscillation of the minimum acceleration trajectory.

4.2 Choosing β and K

When using the frequency weighted power objective, frequency content above the cutoff is minimized by decreasing β and increasing K. If either parameter is fixed, the other can be varied to reduce scan error to an arbitrary value.

If scan range is valued highly, a good choice for β is 0.7, this provides approximately the maximum scan range before high-frequency content significantly increases. Beyond β=0.8 there is little difference between the optimal, and minimum acceleration signals. In practice however, the optimal signal out-performs minimum acceleration as there is less power in harmonics just above K. This is important because K is usually chosen close to the mechanical resonance frequency so the first few harmonics above K are critical. If β is chosen large (β ≥0.7), the scan error must be minimized by including a large number of harmonics. For example, if the scan frequency is one twentieth the mechanical resonance frequency, K can be chosen up to 19.

If scan speed is highly valued, K must be small. If the scan speed is 10% of the resonance frequency, K must be 9 or less. The smallest reasonable value for K is 5, which allows only 3 sine waves in the optimal signal and scan speeds up to 20% of the resonance frequency. In such cases, β must be severely reduced to minimize induced vibration.

The authors recommend two general purpose choices for β and K:

- $\beta = 0.7$ and $K = 9$. This provides good scan range, operation up to 10% of the mechanical resonance frequency and a reasonable minimum of induced vibration. Slower scan speeds with higher K improve performance.
- $\beta = 0.5$ and $K = 7$. This is more suitable for high performance scanning where induced vibration is to be minimized or scan speed increased up to 14% of the resonance frequency.

4.3 Improving feedback and feedforward controllers

Feedback In addition to improving the performance of open-loop scanners, minimum bandwidth input signals are also beneficial when using feedback. Feedback systems with tracking controllers typically have a bandwidth less than one-tenth the open-loop bandwidth. Furthermore, integral tracking controllers provide significant error reduction up to only one-tenth the closed-loop bandwidth. Thus, systems with tracking controllers have extremely limited bandwidth. In such cases, an optimized reference trajectory utilizes the available bandwidth by ensuring that all input harmonics lie within the frequency range where controller loop-gain is high.

In more general circumstances, a lower bandwidth input signal relaxes the controller bandwidth specification. This, in turn, requires less controller gain, resulting in greater robustness and less noise feedthrough from the sensors to the regulated variable.

Feedforward In systems using inversion based feedforward control, the choice of reference signal is critical. Wide bandwidth input signals have frequency components in ranges where the inversion filter has a significant response. At these frequencies, small modeling errors can result in large errors in the filter response and inverted signal (Devasia [2002]). Sensitivity to modeling error can be reduced if the reference signal has negligible harmonic content in the bandwidth where inversion is required, (Devasia [2002]).

Optimal scanning signals also reduce control signal magnitude by avoiding frequencies where the plant response is small. This is highly advantageous for iterative systems that achieve near perfect inversion, for example Wu and Zou [2007]. If the internal reference signal contains frequency components at, or near, plant zeros, extremely large inputs are generated. Optimal reference signals that do not contain high frequency content avoid this problem.

4.4 Experimental Application

Two-axis micro- and nano-positioning stages are used extensively in many forms of scanning probe microscope. They typically comprise a pair of piezoelectric actuators, mechanical displacement amplifiers and a flexure guided sample platform. Although these configurations can achieve high precision with millimeter range motion; the internal displacement amplifiers, large piezoelectric stacks and platform mass contribute to a low mechanical resonance frequency. An example of such a stage is the Physik Intrumente P-734, which has a range of 100 microns but a resonance frequency of only 410 Hz. At 200 Hz, there is a 12 degree phase shift and 2 dB gain fluctuation from DC. Above 200 Hz the phase and magnitude response degrade rapidly. Therefore, to avoid phase and magnitude distortion, the input signal spectra must be contained to within 200 Hz.

Without using model-based inversion, the fastest practical scan speed for the platform under consideration is around 27 Hz, thus the 3rd, 5th and 7th harmonic occur at 85, 135

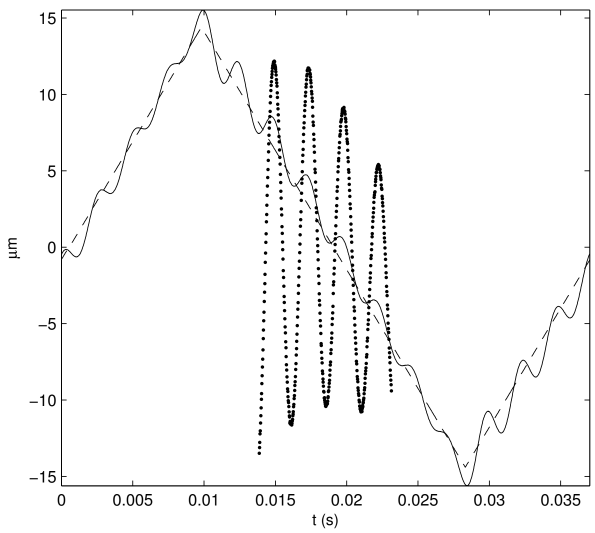

Fig. 8. The response of the positioner to a 27 Hz triangle scan. Displacement (—), Triangle (- -), Error magnified by 10 (· · ·)

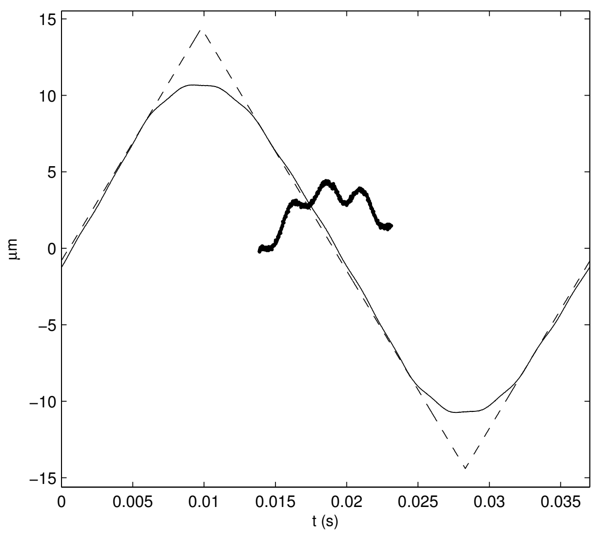

Fig. 9. The response of the positioner to a 27 Hz minimum acceleration scan (β=0.7). Displacement (—), Ideal Scan (- -), Error magnified by 10 (· · ·)

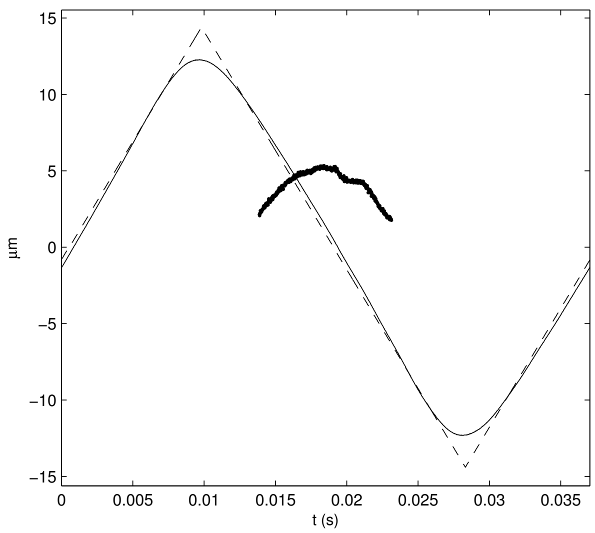

Fig. 10. The response of the positioner to a 27 Hz optimal scan (β=0.7, K=7). Displacement (—), Ideal Scan (- -), Error magnified by 10 ($\cdots$)

and 189 Hz. An optimal signal can be designed to achieve high scan range with minimal harmonic content above 189 Hz, this implies β=0.7 and K=7. With a sampling rate of 54 kHz (2000 points per period), the 27 Hz optimal input signal can be generated with the command: `y = generateTriangle(54000,27,0.7,7)`. This signal was applied to generate a scan with 18 micron linear range. The resulting displacement for a triangular trajectory, minimum acceleration trajectory and optimal trajectory are shown in Figures 8, 9 and 10. Although the dominant cause of scan error is piezoelectric nonlinearity, the oscillation induced by the optimal signal is one-tenth that of the minimum acceleration signal and one-hundredth the triangle. This can be further improved by reducing the scan speed (increasing K) and/or reducing β.

5. CONCLUSIONS

The foremost speed limitation of electromechanical scanners is the first resonance frequency. In this work, scanning trajectories for bandlimited systems are designed that minimize a frequency or time domain quadratic cost function while enforcing linearity over a certain range. Specific cost functions include minimum velocity, acceleration and power. These are easily combined for multiple objectives and/or subjected to frequency domain weightings.

The frequency-weighted-power objective was developed to maximize scanning performance of bandlimited systems. It enforces linearity over a certain range ($\pm\beta$) while minimizing signal power above a certain frequency. It is convenient to specify the cut-off frequency by the number of allowable signal harmonics K. The key advantages of the frequency-weighted-power signal are:

- Perfect linearity over a certain range ($\pm\beta$)
- Minimum frequency content above the K^{th} harmonic
- β and K can be varied to achieve arbitrarily low oscillation
- Simplifies and improves the performance of feedforward and feedback control systems.

The frequency-weighted-power signal outperforms present techniques in simulation and experiment on a two-axis nanopositioning platform. Even with conservative values of β and K, an order of magnitude improvement in induced oscillation can be achieved. The improvement increases dramatically as scan range is sacrificed, or more harmonics are allowed.

The goals of this work have been focussed mainly on improving scan accuracy by reducing high-frequency signal content and hence oscillation. Present investigation includes assessing the speed limit of electromechanical systems. With low values of β, scan speed can be increased to 20% of the resonance frequency with no feedback or feedforward control required.

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