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# Iterative Identification and Control Satisfying Classical Robustness Measures ⋆

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**Abstract:** In this paper it is presented an iterative procedure for closed loop controller tuning applying a relay experiment. The phase margin is evaluated and a model is identified using constraints. This model is improved at high frequencies employing frequency data from the same experiment, the improved model is used to estimate the gain margin. The controller redesign is performed minimizing a frequency domain criterion based on gain and phase margins that are classical measures of robustness in addition to the crossover frequency. The procedure is applicable for a large number of processes types and safely approach specifications using a few experiments. Examples illustrate the properties of the design scheme. Copyright ©2008 IFAC

Keywords: Iterative Modelling and Control Design; Identification for Control; Closed Loop Identification.

## 1. INTRODUCTION

In many plants, process control usually is implemented through several control levels. The PID controller is often used in regulatory levels to provide the process robust stability and fast response to load disturbances (Skogestad and Postlewaite (2005)). In most systems, a simple PI controller is sufficient to handle the regulatory level functions. Frequently these controllers need to be redesigned under operation due to poor performance.

In this context, techniques for identification and controller redesign using closed-loop data have become very attractive. The closed-loop identification doesn't cause stops in system operation unlike open-loop identification. Other reasons which can be listed are demands on safety in process operation, unstable processes and restrictions in production. Its has also been argued that in closed loop it is possible to obtain representative restricted complexity process models in interesting frequency ranges which can be used to redesign controllers such as PI and PID (Albertos and Salas (2002)).

The redesigned controller specifications may be expressed by the gain and phase margins that are classical measures of robustness and together with the crossover frequency represent the time performance of the closed-loop as well. Several gain and phase margin tuning methods have been proposed in the literature. Some are based on graphical methods which are not suitable for PID autotuning whilst others are based on simple models using approximation which do not guarantee that the specification will be achieved (Ho et al. (1993) and Ho et al. (1997)). Model based tuning techniques that rely only on open loop simple dynamics may have poor performance when the process system is too complex. For example, decoupling that usually results in complicated diagonal elements with non-minimum phase behavior. There are some iterative procedures as the one presented in de Arruda and Barros (2003a) which uses an ad hoc iterative algorithm. Other techniques are based on numerical methods as the one presented in Karimi et al. (2003). Its major drawback is that is not suitable for non-minimum phase behavior including transport delay usually encountered in process systems. Besides that, this iterative methods always employ two relay experiments: Phase Margin and Gain Margin Experiments which consume operation time. The use of relevant information provided by a specific closed loop relay experiment together with simple models accurate on interesting frequencies can overcome these problems.

In this paper a method for iterative controller evaluation and redesign based on the knowledge of the gain and phase margins and the crossover frequency is proposed. The phase margin is estimated using a specific relay experiment. A restricted complexity model accurate on the relevant frequency range is identified using experiment data. The gain margin is estimated through a model based procedure that employs experiment harmonic information. It is established a frequency criterion that is optimized applying a gradient method. The numerical problem is solved using experiment information together with the model. Open loop experiments are not necessary. The proposed method can be applied to a large number of processes types including non-minimum phase and time delay dynamics given the relay feedback develops limit cycle. Convergence is achieved using few experiments.

The paper is organized as follows. Initially the relay experiment is discussed. After that the closed loop identification technique is reviewed. Following the gain margin estimate procedure is presented, it is explained how experiment

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information may be used to improve model parameters for the estimate. Then the controller optimization procedure is described. Finally simulations and an experimental example illustrate the effectiveness of the proposed method.

## 2. PROBLEM STATEMENT

Consider the closed loop shown in Fig. 1. The process transfer function is given by $G(s)$ while the controller is $C(s) = K_p(1 + \frac{1}{T_i s})$. The closed loop transfer function from the reference signal $r(t)$ to the process output $y(t)$ is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{L(s)}{1 + L(s)} \tag{1}$$

where $L(s) = G(s)C(s)$ is the *Loop Gain Transfer Function.*

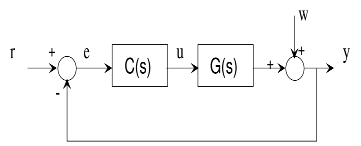


Fig. 1. The Closed Loop.

The crossover and critical frequencies are $\omega_g$ and $\omega_c$. The phase margin ($\phi$) is related to the frequency point where

$$\phi = \pi + \angle L(j\omega_g) \tag{2}$$

and $|L(j\omega_g)| = 1$. The gain margin ($A$) is defined as

$$A = \frac{1}{|L(j\omega_c)|} \tag{3}$$

where $\angle L(j\omega_c) = -\pi$.

The problem statement is: Given a closed loop system, evaluate robustness and performance through the gain and phase margins in addition to the crossover frequency estimate using only one closed loop experiment. If necessary, redesign the controller iteratively and safely to match the desired specifications.

## 3. RELAY EXPERIMENT

A general relay procedure to estimate the frequency point for which a given transfer function has a desired gain is presented in de Arruda and Barros (2003b). If the loop-gain is under test, the feedback structure is presented in Fig. 2 where $y_r'$ is the operation point during the test.

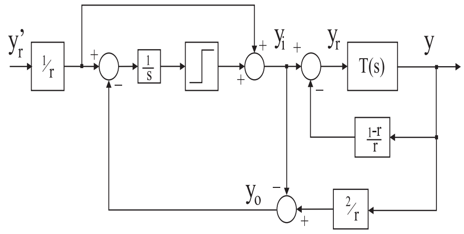


Fig. 2. Loop Gain Transfer Function Estimation.

This procedure allows the estimation of the frequency at which the loop transfer function magnitude is close to $r$. Selecting $r = 1$, the current gain crossover frequency $\omega_g$ and the phase margin can be estimated.

## 4. CLOSED LOOP IDENTIFICATION USING FREQUENCY DOMAIN CONSTRAINTS

Identification is performed using the relay experiment data to estimate a continuous-time model accurate close to the crossover frequency, i.e. $\hat{G}(j\hat{\omega}_g)$. The procedure solves a time least-squares problem subjected to a constraint in frequency. The constraint is obtained using the process frequency response on the first harmonic of the relay experiment signal. This technique was presented in Jr et al. (2006).

### *4.1 Optimization Using Equality Constraints*

In this section the general procedure for constrained minimization is reviewed. Assume the parameters to be optimized $\hat{\theta}$, that data is grouped in a vector form yielding matrices $Y$, $\Phi$ and the constraints are expressed as matrices $M$ and $\gamma$. Define the least-squares optimization problem as

$$\min_{\hat{\theta}} J = \left(Y - \Phi\hat{\theta}\right)^T \left(Y - \Phi\hat{\theta}\right)$$

subject to

$$M\theta = \gamma. \tag{4}$$

In order to find the solution for the least-squares optimization problem with constraints, one uses the equivalent minimization problem in relation to $\hat{\theta}$ and $\lambda$ (the Lagrange multiplier). Then the cost function is given by

$$J = \left(Y - \Phi\hat{\theta}\right)^T \left(Y - \Phi\hat{\theta}\right) + \lambda(\gamma - M\theta). \tag{5}$$

By defining

$$E = 2\Phi^T\Phi \tag{6}$$

$$F = 2\Phi^T Y \tag{7}$$

the optimal solution is

$$\lambda^T = \left\{ME^{-1}M^T\right\}^{-1}\left\{\gamma - M\,[E]^{-1}\,F\right\}$$

$$\hat{\theta} = [E]^{-1}\,(F + M^T\lambda^T).$$

### *4.2 Identification of FOPDT Models*

The used model is first-order plus dead-time (FOPDT) continuous-time represented by

$$G(s) = \frac{b}{s+a}e^{-\tau s}. \tag{8}$$

Define the regression vector with the available data being discrete-time

$$y(t) = \phi(t)\,\theta$$

where

$$\phi(t) = \left[\; -\int_0^t y(v)\,dv \;\; \int_0^t u(v)\,dv \;\; u(t) \right]^T, \theta = \left[\, a \;\, b \;\, \beta \,\right]^T.$$

The equality constraint is defined through the following regression vector which is obtained using the linear form (4) given by

$$\hat{z} = x^T(\hat{\omega}_g)\,\hat{\theta}$$

with

$$\hat{z} = j\hat{\omega}_g \hat{G}\left(j\hat{\omega}_g\right); x^T\left(j\hat{\omega}_g\right) = \left[\, -\hat{G}\left(j\hat{\omega}_g\right) \; 1 \; -j\hat{\omega}_g \,\right]$$

The final estimate obtained is $\left\{\hat{a}, \hat{b}, \hat{\tau} = \hat{\beta}/\hat{b}\right\}$

## 5. GAIN MARGIN ESTIMATE

Despite the fact that the relay experiments keep the process reference closed to the operation point, process variability is increased and employees time is still consumed during the tests. Thus, is desirable to reduce experiment time as possible. Gain and phase margins tuning methods usually employ the respective relay experiments, so that test time may be reduced excluding the gain margin experiment using model based estimates. The gain margin does not influence system performance as the phase margin and crossover frequency therefore an approximate solution is acceptable.

### *5.1 Model Improvement at Higher Frequencies*

The FOPDT model is identified using the presented relay experiment data. Closed loop stable systems controlled by PID have an interesting robustness property: $\omega_c > \omega_g$ (Skogestad and Postlewaite (2005)). During iterations it is considered that the closed loop system is stable and so on this condition is satisfied. Therefore it is possible to use higher frequency information provided by the relay experiment to improve the estimated model around the actual critical frequency. This improvement is done scaling model gain and delay using harmonics excitation to improve the gain margin estimate.

*Using Harmonic Information* The loop gain frequency response is computed at the harmonics of the relay excitation using the Discrete Fourier Transform. It is chosen the harmonic related to the loop gain frequency response function closest to the critical point and the real axis in the Nyquist Diagram. This choice aims conservatively improve the model at the critical frequency region.

*Model Gain Improvement* To improve the model in the harmonic frequency region is just necessary scale its gain $b$. Then, given an initial model

$$G_i(s) = \frac{b_i}{s+a} e^{-s\tau},$$

it is desired that the model gain be equal to the process response gain $G_d(j\omega_o)$ at the frequency $\omega_o$. The improved model gain is given by

$$b_d = b_i \frac{|G_d(j\omega_o)|}{|G_i(j\omega_o)|}.$$

*Model Phase Improvement* It is just necessary adjust the model delay to improve the model phase close to an arbitrary frequency $\omega_o$. The initial model is given by

$$G_i(s) = \frac{b}{s+a} e^{-s\tau_i},$$

and the phase improvement is given by

$$\angle G_d(j\omega_o) = \angle G_i(j\omega_o) + \Psi.$$

Then, it is possible to describe the improved model as

$$G_d(s) = \frac{b}{s+a} e^{-s(\tau_i + \psi)}$$

where $\Psi = \psi\omega_o$. Therefore, the necessary scale to adjust the model phase close to the interesting frequency point is computed

$$\psi = \frac{\angle G_d(j\omega_o) - \angle G_i(j\omega_o)}{\omega_o}.$$

The improved model obtained after the gain and phase adjustments is used to estimate the gain margin.

### *5.2 Computing the Gain Margin Estimate*

The gain margin estimate is computed solving for the frequency with zero loop gain imaginary part. Usually, this problem is solved using approximations or non-linear optimization. In this paper, another approach is presented employing specific Pade approximations. The improved model delay is approximated close to the chosen harmonic frequency $\omega_o$ using

$$e^{-j\omega\tau} \simeq e^{-j\omega_o\tau} \frac{1 - \frac{\tau}{2}(j\omega - j\omega_o)}{1 + \frac{\tau}{2}(j\omega - j\omega_o)}. \tag{9}$$

More details may be found in Fausett (1999). Therefore, it is obtained the following loop gain approximation

$$L(j\omega) \simeq K_p(1 + \frac{1}{T_i j\omega}) \frac{b}{j\omega + a} e^{-j\omega_o\tau} \frac{1 - \frac{\tau}{2}(j\omega - j\omega_o)}{1 + \frac{\tau}{2}(j\omega - j\omega_o)}. \tag{10}$$

Solving the equation (11)

$$\angle L(j\omega_c) = -\pi \tag{11}$$

using the loop gain in equation (10), it results a polynomial equation described by

$$\alpha_1\omega^4 + \alpha_2\omega^3 + \alpha_3\omega^2 + \alpha_4\omega + \alpha_5 = 0 \tag{12}$$

where

$$\alpha_1 = \cos(\omega_o L)\frac{\tau^2}{4},$$

$$\alpha_2 = \sin(\omega_o\tau)\left[\tau + (a - \frac{1}{T_i})\frac{\tau^2}{4}\right] + \cos(\omega_o\tau)\left[\omega_o\frac{\tau^2}{2}\right],$$

$$\alpha_3 = \sin(\omega_o\tau)\left[\omega_o\left(\frac{1}{T_i} - a\right)\frac{\tau^2}{2} - \omega_o\tau\right] +$$
$$\cos(\omega_o\tau)\left[\left(\frac{1}{T_i} - a\right)\tau - 1 + (\omega_o^2 + \frac{a}{T_i})\frac{\tau^2}{4}\right],$$

$$\alpha_4 = \sin(\omega_o\tau)\left[\frac{a\tau}{T_i} + \left(\frac{1}{T_i} - a\right)\left(1 - \frac{\omega_o^2\tau^2}{4}\right)\right] +$$
$$\cos(\omega_o\tau)\left[\omega_o\tau\left(a - \frac{1}{T_i}\right) - \frac{\omega_o a\tau^2}{2T_i}\right]$$

and

$$\alpha_5 = \sin(\omega_o\tau)\left[\frac{-a\tau\omega_o}{T_i}\right] + \cos(\omega_o\tau)\left[\frac{a}{T_i}(\frac{\omega_o^2\tau^2}{4} - 1)\right].$$

The real positive solution closest to the used harmonic frequency is chosen as the critical frequency estimate. Then, the gain margin is estimated using equations (10) and (3).

# 6. THE CONTROLLER OPTIMIZATION PROCEDURE

The controller redesign is based on the optimization of a frequency criterion that is defined as follows:

$$J(\rho) = \left[ \left( \frac{\omega_g - \omega_d}{\omega_d} \right)^2 + \left( \frac{\phi_e - \phi_d}{\phi d} \right)^2 + \left( \frac{K_u - K_d}{K_d} \right)^2 \right]$$

where $\rho = [K_p; \frac{K_p}{T_i}]$ is the controller parameter vector, $\omega_d$ and $\omega_g$ are the desired and measured crossover frequencies, $\phi_e$ and $\phi_d$ are the measured and desired phase margins, $K_u$ is the loop gain magnitude at the critical frequency and $K_d$ is the inverse of the desired gain margin $A_d$.

The controller parameters are obtained applying a gradient based optimization technique, the iterative Newton's formula

$$\rho_{i+1} = \rho_i - \gamma_i R^{-1} J'(\rho_i).$$

To solve this numerical problem is necessary to compute the gradient and the Hessian, $J'(\rho_i)$ and $R$ respectively. The algorithm will converge if the Hessian exists and is positive definite, even if the gradient is approximated. The gradient is given by

$$J'(\rho) = \left( \frac{\omega_g - \omega_d}{\omega_d^2} \right) \frac{\partial \omega_g}{\partial \rho} + \left( \frac{\phi_e - \phi_d}{\phi^2 d} \right) \frac{\partial \phi_e}{\partial \rho} + \left( \frac{K_u - K_d}{K_d^2} \right) \frac{\partial K_u}{\partial \rho}$$

and Hessian can be computed as

$$R = \frac{1}{\omega_d^2} \frac{\partial \omega_g}{\partial \rho} (\frac{\partial \omega_g}{\partial \rho})^T + \frac{1}{\phi_d^2} \frac{\partial \phi_e}{\partial \rho} (\frac{\partial \phi_e}{\partial \rho})^T + \frac{1}{K_d^2} \frac{\partial K_u}{\partial \rho} (\frac{\partial K_u}{\partial \rho})^T$$

where the second order derivatives have been suppressed to avoid a non-positive definite Hessian. The problem solution requires the computation of some derivatives considering the frequency response functions features at the critical and crossover frequencies.

The derivative $\frac{\partial \omega_g}{\partial \rho}$ is computed observing that between optimization iterations the magnitude of the loop gain is unity at the crossover frequency $\frac{\partial |L(j\omega_g)|}{\partial \rho} = 0$ what results in

$$\frac{\partial \omega_g}{\partial \rho} = - \frac{|G(j\omega_g)| \frac{\partial |C(j\omega_g)|}{\partial \rho}}{\left( |C(j\omega_g)| \frac{\partial |G(j\omega)|}{\partial \omega}|_{\omega_g} + |G(j\omega_g)| \frac{\partial |C(j\omega)|}{\partial \omega}|_{\omega_g} \right)}.$$

For the Phase Margin derivative $\frac{\partial \phi_e}{\partial \rho}$ it follows that at $\omega_g$

$$\frac{\partial \angle L(j\omega_g)}{\partial \rho} = \frac{\partial \angle C(j\omega_g)}{\partial \rho} + \left( \frac{\partial \angle C(j\omega)}{\partial \omega}|_{\omega_g} + \frac{\partial \angle G(j\omega)}{\partial \omega}|_{\omega_g} \right) \frac{\partial \omega_g}{\partial \rho}.$$

Finally, the computation of $\frac{\partial K_u}{\partial \rho}$ uses

$$\frac{\partial |L(j\omega_c)|}{\partial \rho} = |G(j\omega_c)| \frac{\partial |C(j\omega_c)|}{\partial \rho} + \left( |C(j\omega_c)| \frac{\partial |G(j\omega)|}{\partial \omega}|_{\omega_c} + |G(j\omega_c)| \frac{\partial |C(j\omega)|}{\partial \omega}|_{\omega_c} \right) \frac{\partial \omega_c}{\partial \rho}$$

and

$$\frac{\partial \omega_c}{\partial \rho} = - \left( \frac{\partial \angle C(j\omega)}{\partial \omega}|_{\omega_c} + \frac{\partial \angle G(j\omega)}{\partial \omega}|_{\omega_c} \right)^{-1} \frac{\partial \angle C(j\omega_c)}{\partial \rho}$$

where was used the fact that between iterations at the critical frequency $\frac{\partial \angle L(j\omega_c)}{\partial \rho} = 0$.

Process derivatives are computed using the FOPDT model as follows

$$\frac{\partial \angle G(j\omega)}{\partial w} = \frac{a}{\omega^2 + a^2} - \tau$$

and

$$\frac{\partial |G(j\omega)|}{\partial w} = \frac{-b\omega}{(\omega^2 + a^2)^{1.5}}$$

The derivatives evaluated at the crossover frequency are computed employing the identified model with constraints whilst the derivatives evaluated at the critical frequency are computed using the improved model.

# 7. SIMULATION EXAMPLES

In this section two representative simulation examples are shown which illustrate the use of the technique. The noise power applied during identification is 0.001 and the DFTs are computed evaluating just one period of the signals. The closed loop time response is simulated applying a step of magnitude 1 to the setpoint and a step disturbance of magnitude 0.1 to the process output.

## *7.1 Second Order Plus Dead Time Process*

The process is given by

$$G(s) = \frac{2s + 1}{(10s + 1)(0.5s + 1)} e^{-s}.$$

The initial PI controller is $C_i(s) = 1.68(1 + \frac{1}{13.53s})$.

*Evaluation* The phase margin and the crossover frequency are estimated through the relay experiment $\phi_e = 101^o$ and $\omega_{g_e} = 0.1269$. The model based gain margin estimate is $A_m = 18.28$. The model improvement is limited by the identified model dynamics. Actually, the gain margin, the phase margin and the crossover frequency are $A_r = 4.21$, $\phi_r = 101^o$ and $\omega_{g_r} = 0.16$ respectively. The new specifications are $A_d = 2.5$ and $\phi_d = 70^o$. It is desirable improve the system response by increasing the crossover frequency gradually during the iterations $\omega_{g_{i+1}} = 1.5\omega_{g_i}$.

*Iterative Redesign Procedure* Data between iterations are presented in table 1. The improved model at the third iteration is compared to the identified one and the real process in Fig. 3, the respective real gain margin is $A_r = 2.42$ and the model based estimated one is $A_m = 2.36$. The frequency cost function value $J$ has decreased along the iterations. The procedure has converged to the desired specifications as can also be noted in Fig. 4. The redesigned controller has improved the closed loop time response as shown in Fig. 5.

Table 1. Iteration Data

| | $K_p$ | $T_i$ | $\phi_e$ | $\omega_g$ | $A_m$ | $J$ |
|---|---|---|---|---|---|---|
| Initial | 1.68 | 13.53 | 101.43 | 0.1269 | 18.28 | 0.5290 |
| 1 | 2.21 | 8.25 | 90.08 | 0.2001 | 5.15 | 0.2292 |
| 2 | 3.48 | 6.79 | 88.27 | 0.3360 | 1.85 | 0.1521 |
| 3 | 2.84 | 2.80 | 66.24 | 0.3831 | 2.36 | 0.0587 |

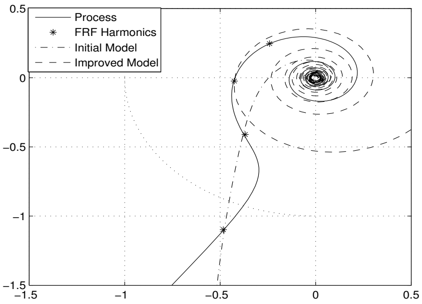


Fig. 3. Improved Model Nyquist Diagram

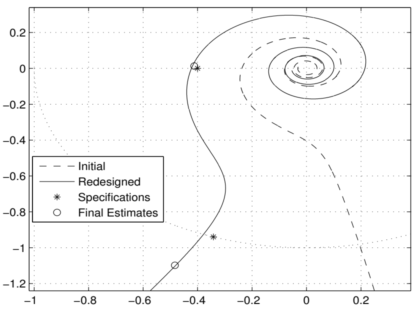


Fig. 4. Nyquist Diagram

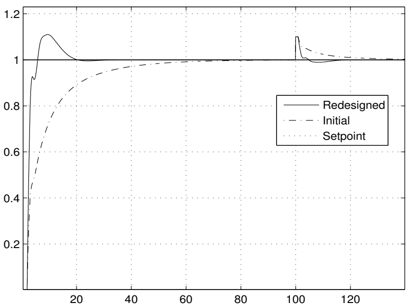


Fig. 5. Closed Loop Response

### *7.2 Eighth Order Process*

The process is now given by

$$G(s) = \frac{1}{(s+1)^8}$$

and the initial controller is $C_i(s) = 1.365(1 + \frac{1}{12.41s})$.

*Evaluation* The phase margin and the crossover frequency are estimated applying the relay experiment: $\phi_e = 39.43^o$ and $\omega_{g_e} = 0.278$. Stability is improved increasing the phase margin then $A_d = 2.5$ and $\phi_d = 70^o$. Due the small phase margin and its aggressive time response, it is desired to decrease the crossover frequency along the iterations $\omega_{g_{i+1}} = 0.8\omega_{g_i}$.

*Iterative Redesign Procedure* In Table 2 it is shown the iterations data. Even though the estimate procedure presents errors in the first iteration due the initial model dynamics, the algorithm converges to the specifications.

Table 2. Iteration Data

| | $K_p$ | $T_i$ | $\phi_e$ | $\omega_g$ | $A_m$ | $J$ |
|---|---|---|---|---|---|---|
| Initial | 1.36 | 12.41 | 39.43 | 0.2780 | 43.98 | 0.5075 |
| 1 | 1.21 | 22.44 | 115.45 | 0.0709 | 1.44 | 0,5152 |
| 2 | 0.57 | 4.94 | 72.67 | 0.0904 | 2.28 | 0.0366 |

The result is close to the specifications as noted in Fig. 6. Performance and stability have been improved, the system presents a smaller overshoot and rise time, damping has been improved also (Fig. 7).

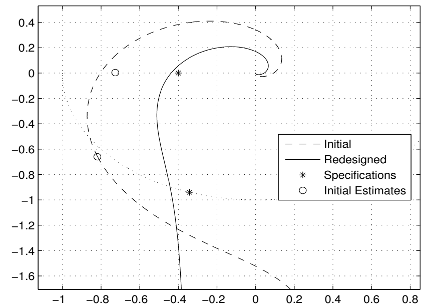


Fig. 6. Nyquist Diagram

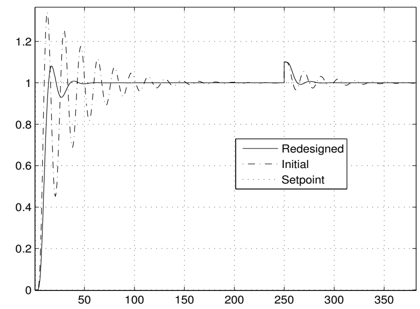


Fig. 7. Time Response

## 8. EXPERIMENTAL EXAMPLE

The laboratory scale process consists of a thermoelectric Peltier module acting as a heat pump on a flat metal plate load. An air cooler is used to extract heat from the opposite face of the Peltier module. The process temperature varies between $10^oC$ and $70^oC$ when operating at a room temperature of around $24^oC$. Power is applied using PWM actuators while the temperature is measured using LM35 sensors. The modelling of the thermoelectric module results in a complex model that is highly nonlinear as can be seen in Huang and Duang (2000). Linearization and model reduction results in a second order model without including the actuator and sensor dynamics. In this paper, the model is assumed to be unknown. The initial PI controller is $C_i(s) = 1(1 + \frac{1}{60s})$.

*Evaluation* The closed loop is evaluated using the relay experiment and it is estimated $\phi_e = 49^o$ and $\omega_{g_e} = 0.0143$. The relay experiment excitation is shown in Fig. 8 where can be observed the high noise power. The model based gain margin estimate is $A_m = 5.2$. Using another specific relay experiment, the gain margin experiment, it is estimated $A_e = 4$ (de Arruda and Barros (2003a)).

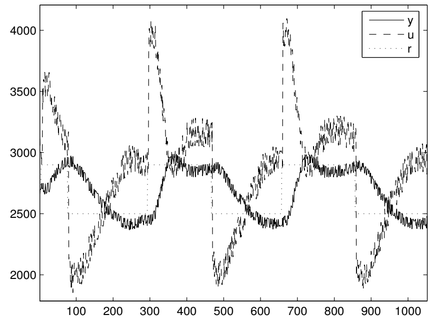


Fig. 8. Relay Experiment

Performance and stability may be improved with the new specifications $A_d = 3.5$ e $\phi_d = 70^o$. It is also desirable improve system response increasing the crossover frequency iteratively $\omega_{g_{i+1}} = 1.1\omega_{g_i}$.

*Iterative Redesign Procedure* The results obtained during iterations are presented in table 3. It can been noted that better values for the phase margin and the gain margin have been achieved.

Table 3. Iteration Data

| | $K_p$ | $T_i$ | $\phi_e$ | $\omega_g$ | $A_m$ | $J$ |
|---|---|---|---|---|---|---|
| Initial | 1 | 60 | 49.00 | 0.0143 | 5.2433 | 0.1028 |
| 1 | 1.6295 | 111.2903 | 61.59 | 0.0153 | 4.7306 | 0.0452 |
| 2 | 2.1069 | 150.5747 | 71.00 | 0.0153 | 4.6801 | 0.0362 |
| 3 | 2.5496 | 144.7354 | 70.72 | 0.0161 | 4.1876 | 0.0177 |

The system response has been improved, specifically overshoot, rise time, damping and variability as show in Fig. 9.

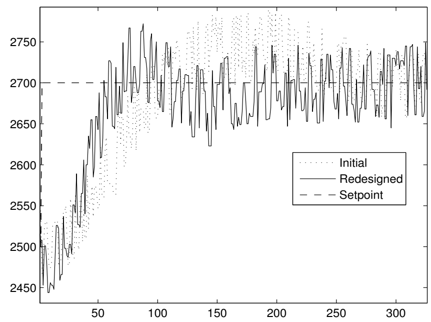


Fig. 9. Time Response

The parameters shown in Table 4 may be used to evaluate time response improvements also (Astrom and Hagglund (1995)). The redesigned controller has improved the majority of the parameters.

Table 4. Time Response Evaluation

| | Initial | Redesigned |
|---|---|---|
| IE | 6563 | 9823 |
| IAE | 19253 | 16239 |
| ITAE | 1923039 | 1693682 |
| ITE | -529837 | 453808 |
| ITSE | 130234109 | 88126600 |

## 9. CONCLUSIONS

In this paper is presented a novel procedure for controller evaluation and iterative redesign. The gain and phase margins that are classical robustness measures are evaluated then an optimization technique is applied to a frequency criterion. The technique uses a relay experiment to estimate the phase margin and the crossover frequency whilst a model is identified accurately close to the crossover frequency. Aiming reduce the number of experiments, the gain margin is estimated employing the model. Open loop experiments are not necessary. The procedure can be easily extended to PID controllers. It was shown how the technique can safely approach specifications using a few experiments. Simulation and experimental examples illustrate its effectiveness.

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