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Multi-Scale Distributed Port-Hamiltonian Representation of Ionic Polymer-Metal Composite ⋆

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Abstract: This paper shows that one of soft actuators, Ionic Polymer-Metal Composite (IPMC) can be modeled in terms of distributed port-Hamiltonian systems with multi-scale. The physical structure of IPMC consists of three parts. The first part is an electric double layer at the interface between the polymer and the metal electrodes. The frequency response of the polymer-metal interface shows a fractal degree of gain slope. Then we adopt a black-box circuit model to this part and give considerations for distributed impedance parameters. The second part is an electrostress diffusion coupling model with bending and relaxation dynamics. This part is represented by an electro-osmosis, which is a water transport by an electric field, and a streaming potential, which is an electric field created by a water transport. We discuss the relationship of stress and bending moment induced by swelling. The third part is a mechanical system modeled as a flexible beam with large deformations. The representation has the capability extracting the control structure based on passivity from distributed parameter systems possessing a complex behavior.

Keywords: Modeling, Design methodologies

1. INTRODUCTION

Polyelectrolyte gels that deform under electric field has been expected as soft actuators and sensors (Osada [2004]). A new type gel, called an ionic polymer metal composite (IPMC) (Shahinpoor and Jim [2001], Asaka et al. [2004]), is a swollen polyelectrolyte gel of certain fluorocarbon networks that is plated with metal electrodes and includes a counterion such as Na^+. The IPMC demonstrates the quick response to small electric fields and the robustness for a large number of bending cycles. The characteristic of the deformation strongly depends on the kind of counterions in the gel. Various modeling methods of IPMC have been proposed in the viewpoint of both a black box model with a system identification (Takagi et al. [2007]) and a white box model using an actual physical structure (Yamaue et at. [2005]).

Generally, soft materials such as IPMC are modeled by partial differential equations as a distributed parameter system. The most general way of introducing a system model for the control of the distributed system is the method of adopting classical types of partial differential equations classified by the existence of analytical solutions (Sobolev [1964]). In this method, some approximations, that is linearization, discretization, and finite dimensional reduction are considered. These are used in the field of numerical simulation mainly.

On the other hand, a new modeling method for distributed systems, called a distributed port-Hamiltonian systems has been presented (Van der Schaft and Maschke [2002]), which is a geometric system representation based on pas-

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sivity (Van der Schaft [2000]). The port-Hamiltonian representation is based on pairs of power variables, called a port, and the product of the pair is equal to its power. The system is connected to each subsystem through the port with preserving the total energy of connected systems, and then the connected system can be written as a port-Hamiltonian system. Two control strategies for such a system there exists (Van der Schaft [2000], Stramigioli [2001], Ortega et at. [1999]). One of them is stabilization by a damping injection on the port. The other is an energy shaping, which means the arrangement of the global minimum point of energy functions, by connecting another port-Hamiltonian system as a dynamical compensator to ports of plants. Additionally, the distributed port-representation has the capability dealing with a boundary observation of an internal energy change by using the boundary port. The applications of the distributed port-Hamiltonian system have been studied in the viewpoint of multi-scale (Couenne et at. [2005], Eberard et at. [2005]).

In this paper, we extend the electrostress diffusion coupling model for polyelectrolyte gels expressing the relaxation phenomenon of IPMC (Yamaue et at. [2005]) to a distributed parameter multi-physics system in terms of the distributed port-Hamiltonian systems with multi-scale as a control system.

2. MODELING

In this section, we assume that a distributed parameter model for IPMC lies on 3-dimensional space. Let the x-axis be the longitudinal direction of the film, the y-axis be the width of the film, and the z-axis be along the cross section of the film.

2.1 Equivalent model of electric double layer

The electrical impedance of IPMC is capacitive, because there exists electric double layer at the interface between the polymer and the metal electrodes (Osada [2004]). The polymer-metal interface has a complex structure and its frequency response shows a fractal degree of gain slope (Takagi et al. [2005]). Then, a black-box circuit modeling of the electrical system of IPMC has been discussed (Fig.1, Takagi et al. [2007]).

Consider a virtual coordinate $\xi \in [0, L]$ for the black-box model of distributed systems. Let $v(\xi,t), i(\xi,t)$ be an electrical potential and a current, respectively. Now, we assume that the series impedance is $R_1(\xi)$ as an electrode resistance and the parallel impedance is $R_2(\xi)+1/(sC_2(\xi))$ with the polymer resistance $R_2(\xi)$ and the electric double layer capacitance $C_2(\xi)$. Then, we have

$$\begin{cases} R_2C_2\dfrac{\partial^2 i}{\partial t\partial\xi} + \dfrac{\partial i}{\partial \xi} + C_2\dfrac{\partial v}{\partial t} = 0\,, \\ \dfrac{\partial v}{\partial \xi} + R_1 i = 0\,, \end{cases} \tag{1}$$

where $i(0,t) = j_e(t)$, $v(0,t) = v_a(t)$, and $i(L,t) = 0$.

A) The case of constant coefficients R_1, R_2, C_2: Assuming R_1, R_2, C_2 are constant, the system (1) can be transformed into

$$C_2\frac{\partial v}{\partial t} = \frac{1}{R_1}\frac{\partial^2 v}{\partial \xi^2} + \frac{R_2C_2}{R_1}\frac{\partial^3 v}{\partial t\partial\xi^2}\,. \tag{2}$$

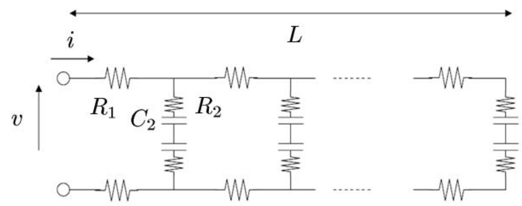


Fig. 1. Black-box model of electric double layer of IPMC

In the port-Hamiltonian setting, one introduces as state variable, a conserved quantity. In the model of the double layer of equation (2), this is the distributed charge $Q(\xi,t)$ which is related to the voltage by: $v(\xi,t) = \frac{Q(\xi,t)}{C_2(\xi)}$. The voltage is the co-energy variable: $v(\xi,t) = \delta_Q\mathcal{H}(Q)$ where $\mathcal{H}(Q,\xi) = \int_0^L \frac{1}{2}\frac{Q^2}{C_2}d\xi$ is the electrical energy of the model and δ is a variational derivative.

The evolution equation is actually a conservation law:

$$\frac{\partial Q}{\partial t} = -\frac{\partial f_2}{\partial \xi}\,, \tag{3}$$

where f_2 is the flux variable (current). In turn, we define the right-hand side of (3) as $-f_1$. Then, f_2 is generated by the phenomenological law:

$$f_1 = \frac{\partial f_2}{\partial \xi}\,, \quad f_2 = -\frac{1}{R_1}e_2 \tag{4}$$

where e_2 is a "generalized thermodynamic force" (here a voltage density) which is constituted by two parts through e_1:

$$e_1 = v - R_2 f_1\,, \quad e_2 = \frac{\partial e_1}{\partial \xi} \tag{5}$$

consisting in a reversible voltage due to the charge and a voltage due to dissipation. Then, we obtain

$$\begin{aligned} \frac{\partial Q}{\partial t} &= -\frac{\partial f_2}{\partial \xi} = -\frac{\partial}{\partial \xi}\left(-\frac{1}{R_1}e_2\right) = -\frac{\partial}{\partial \xi}\left(-\frac{1}{R_1}\frac{\partial}{\partial \xi}e_1\right) \\ &= -\frac{\partial}{\partial \xi}\left(-\frac{1}{R_1}\frac{\partial}{\partial \xi}\left(v + R_2\frac{\partial Q}{\partial t}\right)\right) = -f_1\,. \end{aligned} \tag{6}$$

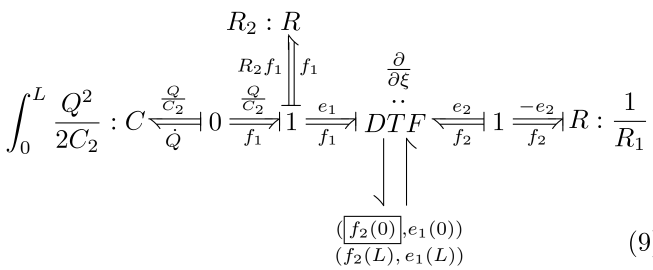
One recovers the relations in the domain:

$$\begin{bmatrix} f_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{\partial}{\partial \xi} \\ \dfrac{\partial}{\partial \xi} & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ f_2 \end{bmatrix} \tag{7}$$

and if we complete with the port boundary variables:

$$\begin{bmatrix} f|_\partial \\ e|_\partial \end{bmatrix} = \begin{bmatrix} f_2(0) \\ f_2(L) \\ e_1(0) \\ e_1(L) \end{bmatrix} = \begin{bmatrix} \text{current} \\ \text{voltage} \end{bmatrix}\,, \tag{8}$$

then we have the distributed port-Hamiltonian system (Van der Schaft and Maschke [2002]). The bond-graph representation of this system is as follows:



(9)

The pair of boundary-ports at $\xi = 0$ consists of the input: $e_1(0,t) = v_a(t)$ and the conjugate output: $f_2(0,t) = j_e(t)$ with $\dot{Q} \to 0$ for $t \gg T$ where T is a large integer, that is

$$j_e(t) = -\frac{1}{R_1}\frac{dv}{d\xi}\bigg|_{\xi=0} \tag{10}$$

connecting to an electro-mechanical system in the following. The another boundary is terminated by the condition: $f_2(L,t) = 0$.

B) The case of distributed coefficients $R_1(\xi), R_2(\xi), C_2(\xi)$: The system (1) can be transformed into

$$\begin{aligned} &C_2\frac{\partial v}{\partial t} - \frac{1}{R_1}\frac{\partial^2 v}{\partial \xi^2} + \frac{1}{R_1^2}\frac{\partial R_1}{\partial \xi}\frac{\partial v}{\partial \xi} \\ &\qquad - \frac{R_2C_2}{R_1}\frac{\partial^3 v}{\partial t \partial \xi^2} + \frac{R_2 C_2}{R_1^2}\frac{\partial R_1}{\partial \xi}\frac{\partial^2 v}{\partial t \partial \xi} = 0\,, \\ &C_2\frac{\partial v}{\partial t} + (1 + R_2C_2\partial_t)\left(-\frac{1}{R_1}\frac{\partial^2 v}{\partial \xi^2} + \frac{1}{R_1^2}\frac{\partial R_1}{\partial \xi}\frac{\partial v}{\partial \xi}\right) = 0\,. \end{aligned} \tag{11}$$

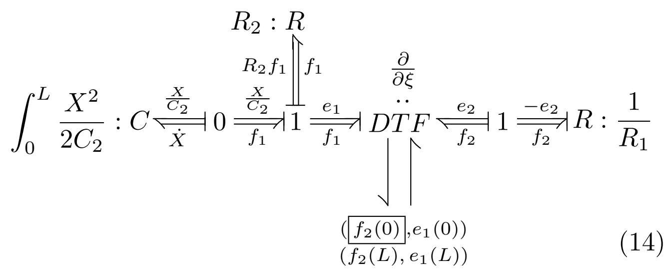
Introducing the new state variable: $X = C_2 v + R_2 C_2 f_1$, we obtain

$$\frac{\partial X}{\partial t} = -\left(-\frac{1}{R_1}\frac{\partial^2 v}{\partial \xi^2} + \frac{1}{R_1^2}\frac{\partial R_1}{\partial \xi}\frac{\partial v}{\partial \xi}\right) := -f_1\,, \tag{12}$$

where we defined the following power variables:

$$\begin{aligned} &f_1 = \frac{\partial f_2}{\partial \xi}\,, \quad f_2 = -\frac{1}{R_1}e_2\,, \\ &e_1 = v = \frac{Q}{C_2} = \frac{X}{C_2} - R_2 f_1\,, \quad e_2 = \frac{\partial e_1}{\partial \xi}\,. \end{aligned} \tag{13}$$

This system can be illustrated as follows:



(14)

Remark 1. The structure of the system corresponds with the conservation law

$$\frac{\partial X}{\partial t} = -\frac{\partial f_2}{\partial \xi} \tag{15}$$

coupled to the closure equation:

$$f_2 = -\frac{1}{R_1}\frac{\partial}{\partial \xi}\left(\delta_X \mathcal{H}(X) - R_2\frac{\partial f_2}{\partial \xi}\right)\,, \tag{16}$$

where $\mathcal{H}(X) = \int_0^L \frac{1}{2}\frac{X^2}{C_2}d\xi$. If $R_2 = 0$, the state X changes to Q and the system is reduced to an ordinary diffusion equation. Then, the system can be considered as a "diffusion-like equation" with the modulated conserved quantity X. Generally speaking, it is difficult to construct control systems for such an equation in terms of analytical methods. However, the port-representation provides us an intuitive synthesis framework without theoretical difficulties.

2.2 Electrostress diffusion coupling model with bending and relaxation dynamics

The electric current density j_e and the water flux density j_s in the polyelectrolyte gels are expressed by the coupling the gradient of pressure p and the electric field ψ as follows

$$\begin{cases} j_e = -\sigma_e \nabla\psi - \lambda\nabla p\,, \\ j_s = -\kappa\nabla p - \lambda\nabla\psi\,, \end{cases} \tag{17}$$

where σ_e is the conductance, κ is the Darcy's permeability and λ is the Onsager's coupling constant (De Gennes et at. [2000]). The first equation of (17) expresses an electro-osmosis, which is a water transport by an electric field. The second equation of (17) is a streaming potential, which is an electric field created by a water transport.

Next, we introduce the model expressing relaxation phenomenon of IPMC (Yamaue et at. [2005]). Now, we assume that there is no electro chemical reaction in the electrode of IPMC. The film is considered as a strip with thickness h. Let $R(t)$ be a radius of a curvature of the film at the time t. When the film bends with curvature $1/R(t)$, the displacement $u(x,y,z,t)$ of the film is given by

$$u_x = \frac{z}{R(t)}x,\ u_y = \frac{z}{R(t)}y,\ u_z = u_z(z,t)\,. \tag{18}$$

Thus, the swelling ratio $f_s(z,t) = \nabla u$ is given by

$$f_s(z,t) = \frac{\partial u_z}{\partial z} + \frac{2z}{R(t)}\,. \tag{19}$$

Consider the linearized stress tensor:

$$\sigma_{ij} = K\sum_k \frac{\partial u_k}{\partial k}\delta_{ij} + G\left(\frac{\partial u_i}{\partial j} + \frac{\partial u_j}{\partial i} - \frac{2}{3}\sum_k \frac{\partial u_k}{\partial k}\delta_{ij}\right)\,, \tag{20}$$

where $i,j,k = \{x,y,z\}$, K is the bulk modules and G is the shear modulus of gels. Then, we have

$$\sigma_{zz}(z,t) = \left(K + \frac{4}{3}G\right)f_s(z,t) - \frac{4G}{R(t)}z\,, \tag{21}$$

$$\sigma_{xx}(z,t) = \left(K - \frac{2}{3}G\right)f_s(z,t) + \frac{2G}{R(t)}z\,. \tag{22}$$

From the force balance equation $\partial(\sigma_{zz} - p)/\partial z = 0$ and the boundary condition $\sigma_{zz} - p = 0$ at $z = \pm h/2$, the pressure $p(z,t)$ is calculated by

$$p(z,t) = \left(K + \frac{4}{3}G\right)f_s(z,t) - \frac{4G}{R(t)}z\,. \tag{23}$$

From (17), it follows that

$$\begin{aligned} j_s(z,t) &= -\left(\kappa - \frac{\lambda^2}{\sigma_e}\right)\left(K + \frac{4}{3}G\right)\frac{\partial f_s}{\partial z} \\ &\qquad + \left(\kappa - \frac{\lambda^2}{\sigma_e}\right)\frac{4G}{R(t)} + \frac{\lambda}{\sigma_e}j_e(z,t)\,, \\ &= -D'\frac{\partial f_s}{\partial z} + \Phi + \frac{\lambda}{\sigma_e}j_e(z,t)\,, \end{aligned} \tag{24}$$

where

$$D' := \left(\kappa - \frac{\lambda^2}{\sigma_e}\right)\left(K + \frac{4}{3}G\right)\,, \Phi := \left(\kappa - \frac{\lambda^2}{\sigma_e}\right)\frac{4G}{R(t)}\,. \tag{25}$$

Substituting (24) into the continuity equation

$$\frac{\partial f_s}{\partial t} = -\frac{\partial j_s}{\partial z}\,, \tag{26}$$

the time evolution of the swelling ratio can be written by the diffusion equation

$$\frac{\partial f_s}{\partial t} = D' \frac{\partial^2 f_s}{\partial z^2}\,, \tag{27}$$

where we used the charge conservation law $\partial j_e / \partial z = 0$. If we assume that the electrode is impermeable $j_s|_{\pm h/2} = 0$, then we obtain the boundary condition

$$\left.\frac{\partial f_s}{\partial z}\right|_{\pm \frac{h}{2}} = \frac{\lambda}{D'\sigma_e} j_e(z,t) \pm \frac{\Phi}{D'}\,. \tag{28}$$

Writing the conservation law for the swelling ratio $f_s(z,t)$ directly as a "*diffusion equation*":

$$\frac{\partial f_s}{\partial t} = -\frac{\partial}{\partial z}\left(-D' \frac{\partial}{\partial z}\delta_{f_s}\mathcal{H}_{sw}\right) \tag{29}$$

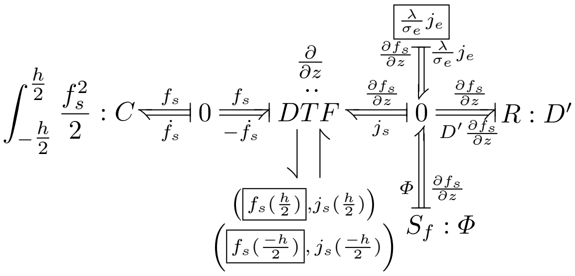
where $\mathcal{H}_{sw}(f_s) = \int_0^L \frac{f_s^2}{2}dz$. As in the preceding section, this makes appear a Dirac structure which may be represented by a DTF-element. Then the boundary port variables are:

- the swelling ratio at the boundary: $f_s|_\partial$
- the flux variable at the boundary: $\beta_{sw} = -D' \frac{\partial}{\partial z} f_s$

which defines a dissipative port-Hamiltonian system (Maschke and Van der Schaft [2005]). Writing a model under the assumption of "charge conservation", the boundary conditions are:

$$\beta_{sw} = -\Phi(t) + \frac{\lambda}{\sigma_e} j_e(t) \tag{30}$$

The following is the bond graph of the above system:



(31)

where $f_s|_\partial$ are connected to the following mechanical part through (36).

Remark 2. In bond graph terms, the boundary port is connected to a source element. Writing the boundary condition in terms of the port variable, here the flux variable has the advantage to avoid to parameterize the relation by an "internal" parameter D'. However this writing has the disadvantage to be not flexible to changes in the modeling assumptions. To be more general the coupling with the flux of charges has to be written *in* the domain according to the equation (24).

2.3 Modulated coupling on stress and bending moment induced by swelling

We assume that the stress (22) on the edge of x axis can be decompose into an active part σ_a generated by electrical fields and a passive part σ_p caused by the restoring force of the curvature $1/R(t)$. Then, we have

$$\sigma_{xx} = \sigma_a + \sigma_p\,, \tag{32}$$

$$\sigma_a = \left(K - \frac{2}{3}G\right) f_s(z,t)\,,\ \sigma_p = \frac{2G}{R(t)} z\,. \tag{33}$$

First, a passive bending moment M_p is calculated by

$$M_p(t) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_p(z,t) bz\, dz = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{2G}{R(t)} bz^2\, dz = \frac{2Gbh^3}{12R(t)} = \frac{YI}{1+\nu}\frac{1}{R(t)}\,, \tag{34}$$

where Y is Young's modulus such that $G = Y/2(1+\nu)$, $I = bh^3/12$ is a moment of inertia of area, and ν is Poisson's ratio. Let $w(x,t)$ be a shearing position. Now, we assume the curvature $1/R(t)$ to be a strain $\partial\theta(x,t)/\partial x$. Then, we obtain

$$M_p(t) = -\frac{YI}{1+\nu}\frac{\partial \theta(x,t)}{\partial x} = -\Psi \frac{\partial\theta(x,t)}{\partial x}\,. \tag{35}$$

Let us consider an active bending moment M_a induced from σ_a as follows:

$$M_a(t) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_a(z,t) bz\, dz = \left(K - \frac{2}{3}G\right) b \int_{-\frac{h}{2}}^{\frac{h}{2}} f_s(z,t) bz\, dz\,, \tag{36}$$

where b is a coefficient of moment of inertia of cross section.

Remark 3. Note that the boundary ports $j_x|_\partial$ are terminated by the impermeable condition $j_x|_\partial = 0$ of electrode in the coupling between the electro-mechanical part and the mechanical part. Then, this port connection has no reaction from the mechanical part, which this means that efforts $j_x|_\partial$ are fixed to zero. The effect of the flows $f_s|_\partial$ in the mechanical part is expressed as a modulated source, which is an element of bound-graph. Thus, the input to the mechanical part should be calculated by (36).

2.4 Mechanical part with large deformations

First, we introduce the equation of flexible beams under large overall motions on the plane presented (Simo and Vu-Quoc [1986]). This model carries the advantage that drastic simplification of the inertia temporal part is obtained by the linear uncoupled inertia operator. After that, we show two reduced model of the flexible beams with large deformations.

A) The case of large deformations: Let A_ρ be a mass per unit length and let I_ρ be a mass moment of inertia of a cross section. Let EA, GA and EI be an axial, a shear and a flexural stiffness of beams, respectively. Let us consider the equations of motion

$$\begin{cases} A_\rho \begin{bmatrix} y_{tt} \\ w_{tt} \end{bmatrix} - \partial_x(\Lambda C \Gamma) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \\ I_\rho\, \theta_{tt} - EI\, \theta_{xx} - \Xi \Lambda C \Gamma = \partial_x M_a\,, \end{cases} \tag{37}$$

where $\partial_x M$ is the input from the electro-mechanical system, $x \in [0, M]$ is the spatial coordinate along the equilibrium position, $(x+y)$ is the axial position, w is the shearing position, θ is the rotation of the cross section along the unchangeable length of the beam and the matrices: Λ, C, Γ_1, Γ_2 and Ξ, are given by

$$\Lambda := \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \; C := \begin{bmatrix} EA & 0 \\ 0 & GA \end{bmatrix},$$
$$\Gamma := \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} = \Lambda^\top \begin{bmatrix} 1 + y_x - \cos\theta \\ w_x - \sin\theta \end{bmatrix},$$
$$\Xi := [-w_x \;\; 1 + y_x]. \tag{38}$$

The kinetic energy $\mathcal{T}$ and the potential energy $\mathcal{U}$ are expressed as

$$\mathcal{T} = \frac{1}{2}\int_0^M \left[A_\rho\, {y_t}^2 + A_\rho\, {w_t}^2 + I_\rho\, {\theta_t}^2\right] dx, \tag{39}$$

$$\mathcal{U} = \frac{1}{2}\int_0^M \left[EA\, {\Gamma_1}^2 + GA\, {\Gamma_2}^2 + EI\, {\theta_x}^2\right] dx. \tag{40}$$

Next, the variational differential of the Hamiltonian density $\mathcal{H} = \mathcal{T} + \mathcal{U}$ is obtained as the functional 1-form:

$$\delta\mathcal{H} = \int_0^M \Bigg[A_\rho\, y_t\, dy_t + A_\rho\, w_t\, dw_t + I_\rho\, \theta_t\, d\theta_t - \Xi\Lambda C\Gamma\, d\theta + \left(\Lambda C\Gamma\right)^\top \begin{bmatrix} dy_x \\ dw_x \end{bmatrix} + EI\, \theta_x\, d\theta_x\Bigg] dx. \tag{41}$$

From (41) the energy variables p, ϵ and the co-energy variables ν, σ are defined by the following.

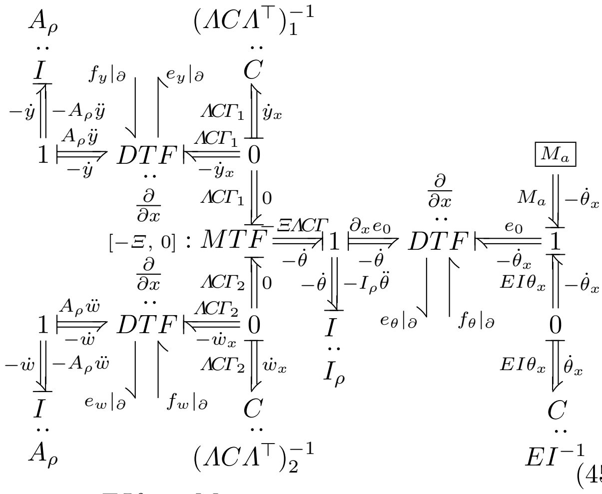
$$\begin{array}{ll} p_1 = -A_\rho\, y_t\, dx, & \nu_1 = -y_t; \\ p_2 = -A_\rho\, w_t\, dx, & \nu_2 = -w_t; \\ p_3 = -I_\rho\, \theta_t\, dx, & \nu_3 = -\theta_t; \\ \epsilon_1 = \theta\, dx, & \sigma_1 = -\Xi\Lambda C\Gamma; \\ \epsilon_2 = y_x\, dx, & \sigma_2 = \left(\Lambda C\Gamma\right)_1; \\ \epsilon_3 = w_x\, dx, & \sigma_3 = \left(\Lambda C\Gamma\right)_2; \\ \epsilon_4 = \theta_x\, dx, & \sigma_4 = EI\, \theta_x, \end{array} \tag{42}$$

where $(\,\cdot\,)_i$ means an extracted i-th element. Setting

$$f_{p_i} = -\frac{\partial p_i}{\partial t}, \quad f_{\epsilon_j} = -\frac{\partial \epsilon_j}{\partial t}, \tag{43}$$

$$e_{\nu_k} = \nu_k, \quad e_{\sigma_l} = \sigma_l, \tag{44}$$

the energy variables (42) can be connected to the Stokes-Dirac structure and defines a field port-Lagrangian systems (Nishida and Yamakita [2005]), which is a kind of dual representation of the distributed port-Hamiltonian system. The above system can be modeled as a bond graph.



(45)

where $e_0 := EI\theta_x + M_a$.

B) The case of small strains: First, introducing the following infinitesimal strain assumption in (37)

$$\begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} = \Lambda^\top \begin{bmatrix} 1 + y_x - \cos\theta \\ w_x - \sin\theta \end{bmatrix} \approx \begin{bmatrix} y_x \\ w_x - \theta \end{bmatrix}, \tag{46}$$

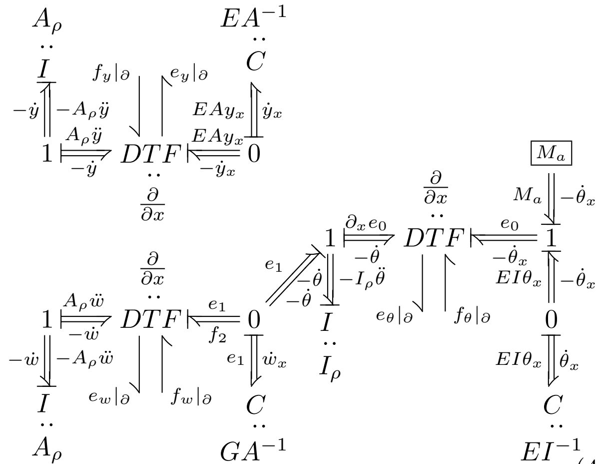
we obtain a Timoshenko beam model.

$$\begin{cases} A_\rho\, y_{tt} - EA\, y_{xx} = 0, \\ A_\rho\, w_{tt} - GA\left(w_{xx} - \theta_x\right) = 0, \\ I_\rho\, \theta_{tt} - EI\, \theta_{xx} - GA\left(w_x - \theta\right) = \partial_x M_a. \end{cases} \tag{47}$$

Thus, we define the energy variables:

$$\begin{array}{ll} p_1 = -A_\rho\, y_t\, dx, & \nu_1 = -y_t; \\ p_2 = -A_\rho\, w_t\, dx, & \nu_2 = -w_t; \\ p_3 = -I_\rho\, \theta_t\, dx, & \nu_3 = -\theta_t; \\ \epsilon_1 = (w_x - \theta)\, dx, & \sigma_1 = -GA\, \theta; \\ \epsilon_2 = y_x\, dx, & \sigma_2 = EA\, y_x; \\ \epsilon_3 = (w_x - \theta)\, dx, & \sigma_3 = GA\, w_x; \\ \epsilon_4 = \theta_x\, dx, & \sigma_4 = EI\, \theta_x. \end{array} \tag{48}$$

which defines a port-representation also (Golo et at. [2002]). The reduced system can be illustrated as follows:



(49)

where $e_1 := GA(w_x - \theta)$ and $f_2 := -\dot{w}_x + \dot{\theta}$.

C) The case of infinitesimal strains: Moreover, an Euler-Bernoulli model

$$A_\rho\, w_{tt} + EI\, w_{xxxx} = -\partial_{xx} M_a \tag{50}$$

is obtained by assuming a shear deformation is negligible: $(w_x - \theta) \to 0$ and $GA \to \infty$ in addition to the Timoshenko beam model.

3. DISCUSSION

In the viewpoint of multi-scale distributed port-Hamiltonian systems (Eberard et at. [2005]), this system has three levels regarding physical scale. That is, the electrical part (1), the electro-mechanical part (17) and the mechanical part (37), and then state variables for each level can be defined by $\{x(t), z(x,t), \xi(z,x,t)\} \in \mathcal{X}_{el}$, $\{x(t), z(x,t)\} \in \mathcal{X}_{em}$, and $x(t) \in \mathcal{X}_{me}$, respectively. The essential concept of the multi-scale port-representation is that we consider the relation $\mathcal{X}_{el} \ll \mathcal{X}_{em} \ll \mathcal{X}_{me}$ as a fiber bundle structure with fiber coordinates z, ξ and a base coordinate x, where $\ll$ means that there exists a large enough difference between two scales. Actually, we assumed $\partial j_e/\partial z = 0$ in the contact of two lower levels, then this means the situation that same fibers regarding ξ-axis are defined on each point of the domain of z uniformly.

The multi-scale factors of IPMC can be classified as follows:

1–A Fractal impedance based on the structure of polymer and metal electrode in the electrical system,
1–B Response of electrical charge in the double layer in the electrical system,
2 Relaxation in the mechanical-electrical system,
3 Deformation magnitude from stationary position in the mechanical system,
4 Parameter changes in coefficients depending on varieties of counterions in the chemical system.

The scale of above factors is changeable according to a scope of observation. There are several models according to the factors.

1–A { (i) Lumped parameter black-box model [Sec. 2.1-A]; (ii) Distributed parameter model [Sec. 2.1-B]

1–B { (i) Fast system; uniform charge conservation law $\partial j_e/\partial z = 0$ [Sec. 2.1]; (ii) Slow system; distributed current density j_e regarding z-axis

2 { (i) Fast system; e.g. 2nd order delay system; (ii) Slow system; extended diffusion equation for relaxation phenomena [Sec. 2.2]

3 { (i) Large deformation beam model [Sec. 2.4-A]; (ii) Timoshenko beam model [Sec. 2.4-B]; (iii) Euler-Bernoulli beam model [Sec. 2.4-C]

4 { (i) Single property system; (ii) Chemical property varying system

where there are the relations: 1–A $\ll$ 3 in the spatial scale, 4 $\ll$ 2 $\ll$ 1–B in the time scale. Then, the scale can be selected according to observed phenomena in an actual model. This situation indicates that such a set of complex and different scale partial differential equations can be formalized by the port-representation and the observed scale can be changed with the multi-scale connection through the boundary ports. The port-represented model is based on passivity, which is a familiar concept with engineers, and then it is easy to understand its essential physical network structure. Moreover, a lot of control strategies has been proposed for port-represented systems (Stramigioli [2001], Ortega et at. [1999]). These methods can be adapted to the models without further considerations.

4. CONCLUSION

This paper shows the modeling of Ionic Polymer-Metal Composite (IPMC) with the distributed port-Hamiltonian systems with multi-scale. As a result, we can use a port-Hamiltonian model with an appropriate scale case by case.

REFERENCES

M. Shahinpoor and K. W. Jim, “Ionic polymer-metal composites: I. fundamentals”, *Smart Materials and Structures 10*, pp. 819-833, 2001.

K. Asaka, N. Mori, K. Hayashi, Y. Nakabo, T. Mukai and Z. W. Luo, “Modeling of the electromechanical response of ionic polymer metal composites (IPMC)”, *Pro. of SPIE, Smart Structures and Materials 2004: Electroactive Polymer Actuators and Devices (EAPAD)*, Vol. 5385, pp. 172-181, 2004.

K. Takagi, Z. W. Luo, K. Asaka, and K. Tahara, “Limited-angle motor using ionic polymer metal composite”, *Proc. of SPIE, Smart Structures and Materials 2005: Electroactive Polymer Actuators and Devices (EAPAD)*, Vol. 5759, pp. 487-496, 2005.

K. Takagi, Y. Nakabo, Z.W. Luo and K. Asaka, “On a distributed parameter model for electrical impedance of ionic polymer”, *Proc. of SPIE*, Vol. 6524, pp. 652416-1-652416-8, 2007.

T. Yamaue, H. Mukai, K. Asaka and M. Doi, “Electrostress Diffusion Coupling Model for Polyelectrolyte Gels”, *Macromolecules*, Vol. 38, pp. 1349-1356, 2005.

B.J. Yi and B. Vishniac, “Modeling of EAPs as Multiple Energy Domain Systems: A Bond Graph Approach”, *Proc. of SPIE, Smart Structures and Materials 2006, Electroactive Polymer Actuators and Devices (EAPAD)*, Vol. 6168, pp. 1-13, 2006.

P.G. de Gennes, K.Okumura, M.Shahinpoor, K.J.Kim, “Mechanoelectric effects in ionic gels”, *Europhysics Letters*, vol. 50, no. 4, pp. 513-518, 2000.

J.C. Simo and L. Vu-Quoc, “On the Dynamics of Flexible Beams under Large Overall Motions - The plane Case: Part I”, *J. of Applied Mechanics*, Vol. 53, pp. 849-854, Dec., 1986.

A.J. van der Schaft and B.M. Maschke, “Hamiltonian formulation of distributed-parameter systems with boundary energy flow”, *Journal of Geometry and Physics*, Vol. 42, pp. 166-194, 2002.

G. Nishida and M. Yamakita, “Distributed Port Hamiltonian Formulation of Flexible Beams Under Large Deformations”, *Proc. of the 2005 IEEE Conf. on Control Applications*, Tronto, pp. 589-594, 2005/8.

F. Couenne, D. Eberard, L. Lefévre, C. Jallut and B. Maschke, “Multi-scale distributed parameter model of an adsorption column using a bond graph approach”, *ESCAPE 15*, Barcelona, 2005.

D. Eberard, L. Lefévre and B. Maschke, “Multiscale coupling in Heterogeneous Diffusion Processes: A Port-Based Approach”, *Proc. Int. Conf. PhysCon 2005*, Saint Petersburg, Russia, August 24-26, 2005.

G. Golo, V. Talasila and A. van der Schaft, “A Hamiltonian formulation of the Timoshenko Beam Model”, *Proc. of Mechatronics 2002*, Univ. of Twente, 2002.

A.A. Franco, P. Schott, C. Jallut, and B.M. Maschke, “In Multi-scale Bond graph model of the electrochemical dynamics in a fuel cell”, *5th MATHMOD*, Vienna, Austria, February 8-10, 2006.

B. Maschke and A.J. van der Schaft, *Advanced Topics in Control Systems Theory*, Lecture Notes from FAP 2004, chapter Compositional modeling of distributed-parameter systems, pp. 115-154, Springer, 2005.

Y. Osada, ed., *Soft Actuators*, (in Japanese), NTS, Tokyo, 2004.

A.J. van der Schaft, *L_2-Gain and Passivity Techniques in Nonlinear Control*, 2nd revised and enlarged edition, Springer-Verlag, London, 2000.

S. Stramigioli, *Modeling and IPC Control of Interactive Mechanical Systems: A Coordinate-free Approach*, Lecture Notes in Control and Information Sciences 266, Springer-Verlag, London, 2001.

R. Ortega, A.J. van der Schaft and B.M. Maschke, *Stability and Stabilization of Nonlinear Systems*, chap. 13, Lecture Notes in Control and Information Sciences 246, Springer-Verlag, London, pp. 239-260, 1999.

S.L. Sobolev, *Partial Differential Equations of Mathematical Physics*, Dover Pub., 1964.