Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

On the feedback information in stabilization over unreliable channels

Hideaki Ishii *

* *Department of Computational Intelligence and Systems Science*
Tokyo Institute of Technology, Yokohama 226-8502, Japan
Email: ishii@dis.titech.ac.jp

Abstract: We consider the remote stabilization of linear systems using shared communication channels where transmitted messages may randomly be lost. We extend the results on the limitations arising in such systems and study the case where there is no feedback information from the actuator side to the controller side. The proposed control scheme utilizes a protocol shared by the actuator and the controller to carry out state and input estimation.

Keywords: Networked control, Message losses, Remote control, Stabilization.

1. INTRODUCTION

In the design of control systems that employ network channels, it is of importance to consider the characteristics of the communication available there. The control performance of such systems can heavily depend on the level of communication as well as on the capability of the control scheme to deal with the effects of communication. This viewpoint has motivated much research on networked control in recent years (Antsaklis and Baillieul 2007).

Here, we consider the problem of remote stabilization of a linear system over shared channels. The communication is unreliable in the sense that the messages transmitted may become lost due to time delay or errors. Since these losses are caused by other traffics in the channels and so on, they are assumed to be random. This problem has recently been studied in various works including (Elia 2005, Elia and Eisenbeis 2004, Hadjicostis and Touri 2002, Imer et al. 2006, Ishii 2007, Ishii 2008, Ishii and Hara 2006, Sinopoli et al. 2004, Xu and Hespanha 2005).

Most schemes in these works rely on the so-called acknowledgements in the communication. This enables the sender to know whether the messages that it transmitted have arrived at the receiver side or not. To achieve closed-loop stability, critical bounds on the loss rates for the channels have been obtained in, e.g., (Elia 2005, Elia and Eisenbeis 2004, Hadjicostis and Touri 2002, Ishii 2007); these bounds are characterized by the unstable plant dynamics.

In real-time control, however, such a feedback in the communication may not be practical because of delays and increase in the bandwidth. It is hence of interest to develop control schemes not utilizing acknowledgements. The difficulty there is that the remote controller is not aware of the actual control inputs applied at the actuator.

There are two approaches to remote control without acknowledgements. One is to simply let the controller transmit the control inputs with no regard to their arrival. This approach clearly limits the achievable performance. In (Imer et al. 2006), a sufficient condition on the loss rates is given. The case with one channel on the actuator side is studied in (Elia and Eisenbeis 2004).

The other approach is to equip the controller with an estimator that determines whether the control inputs were applied or not through the measurements received. The one channel case has recently been studied. In (Tatikonda and Mitter 2004, Sahai and Mitter 2006), stochastic control over a data rate limited, noisy channel is addressed; the controller however requires heavy computation at both the sensor/actuator. In (Epstein et al. 2006), the system setup is that all computation is at the sensor node. Alternatively, state observers with unknown inputs can be employed (e.g., (Hou and Müller 1994)) though this approach is not applicable to a plant whose inverse is unstable.

In this paper, we consider a remote control setup where the controller is connected to both the sensor and the actuator over unreliable channels. It is assumed that little computational resource is available at the sensor and the actuator. We develop a control scheme to achieve stabilization based on input estimation at the controller. One characteristic is that, on the actuator side, a component called the decoder is placed; it is capable of making some decisions regarding the input. We aim at clarifying the plants that can be stabilized if the loss rates in the channels meet the critical bounds determined by the unstable plant dynamics as mentioned above. In particular, we focus on plants with one unstable mode.

This paper is organized as follows: In Section 2, we formulate the problem. The control input estimation scheme is outlined in Section 3. In Section 4, we give the specifics of the proposed controller. This is followed by the main result in Section 5. We conclude the paper in Section 6.

2. SYSTEM SETUP

Consider the remote control system in Fig. 1. The plant denoted by G is a single-input single-output, linear time-

* This work was supported in part by the Ministry of Education, Culture, Sports, Science and Technology, Japan, under Grant-in-Aid for Scientific Research No. 17760344.

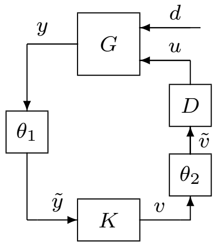

Fig. 1. Remote control over unreliable channels

invariant system. Its state-space equation is given by

$$x_{k+1} = Ax_k + Bu_k + d_k, \quad y_k = Cx_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}$ is the control input, $d_k \in \mathbb{R}^n$ is the disturbance, and $y_k \in \mathbb{R}$ is the measurement output.

Here, we make several assumptions on the plant as follows: (i) The pair (A, B) is stabilizable and (C, A) is detectable. (ii) The matrix A has one unstable eigenvalue, denoted by λ with $|\lambda| \geq 1$. Without loss of generality, we assume that $A = \mathrm{diag}(A_1, \lambda)$, where A_1 is a stable matrix, $B = [B_1^T \ b_2]^T$, and $C = [C_1 \ c_2]$ with $b_2, c_2 \in \mathbb{R}$. (iii) The transfer function of G has the relative degree ν, i.e., $CA^{i-1}B = 0$ for $i = 1, 2, \ldots, \nu - 1$ and $CA^{\nu-1}B \neq 0$; furthermore, $CA^{i-1}B \neq 0$ for $i \geq \nu$. (iv) The initial state x_0 is bounded as $\|x_0\| \leq \bar{x}_0$. (v) The disturbance d_k is bounded as $\|d_k\| \leq \bar{d}$ for all k.

The plant is remotely controlled by the controller K connected by shared communication channels. The channels are unreliable in that the transmitted messages are randomly lost. These losses are modeled by the i.i.d. random processes $\theta_{1,k}, \theta_{2,k} \in \{0, 1\}$, $k \in \mathbb{Z}_+$. A message in the channel i is lost at time k if $\theta_{i,k} = 0$ and is received otherwise. These processes are independent to each other. Let the probability of a loss be denoted by $\alpha_i \in [0, 1)$:

$$\alpha_i = \mathrm{Prob}\{\theta_{i,k} = 0\} \quad \text{for } i = 1, 2 \text{ and } k \in \mathbb{Z}_+.$$

Thus, the input to the controller is given by $\tilde{y}_k := \theta_{1,k} y_k$ while the decoder D receives the signal $\tilde{v}_k := \theta_{2,k} v_k$.

The controller K generates the control inputs v to be transmitted to the actuator side based on the measurement $\tilde{y}$. The decoder D receives $\tilde{v}_k$ at time k and applies the control u_k. In loose terms, it is assumed to have only low computational capability. This is to rule out the situation where the decoder can take over the controller tasks and thus the measurements are directly sent to the decoder.

The objective is to construct a control scheme such that the plant G is stabilized. Here, the stability criterion we employ is that in the mean-square sense defined by

$$\sup_{k \in \mathbb{Z}_+} E[\|x_k\|^2] < \infty, \tag{2}$$

where $E[\,\cdot\,]$ is the expectation.

In the current setup, the controller does not have the information regarding θ_2. As a consequence, the actual control input u is unknown to the controller.

One way to relax this situation is to allow acknowledgement messages sent from the decoder D to the controller K whenever the control input v_k is received. In this way, the information regarding $\theta_{2,k-1}$ is available at the controller; see, e.g., (Elia and Eisenbeis 2004, Ishii 2007). There, a necessary and sufficient condition to achieve closed-loop stability has been derived. The following proposition is a simple version restricted to the case with one unstable eigenvalue.

Proposition 2.1. Assume that the controller K is linear time varying and depends on $\theta_{1,k}$ and $\theta_{2,k-1}$ at each time k. There exists such a controller that stabilizes the plant (1) in the sense of (2) if and only if

$$\alpha_1, \alpha_2 < \frac{1}{|\lambda|^2}. \tag{3}$$

Since, in this paper, we consider the problem without acknowledgements, the information pattern on the controller is more restricted. Hence, the condition (3) should serve as a necessary condition. We will however show that, for a certain class of systems, a stabilizing controller indeed exists if the bounds in (3) are satisfied.

3. AN APPROACH TO INPUT ESTIMATION

In this section, we illustrate the ideas for the estimation of the control inputs through the measurements over the unreliable channel. The method is motivated by (Epstein et al. 2006, Sahai and Mitter 2006).

In the control scheme, the controller generates the state estimation, which is denoted by $\hat{x}_k$ at time k. As usual, the control scheme essentially determines the control inputs via $\hat{x}_k$. Let the estimation error be $e_k := x_k - \hat{x}_k$.

We consider the following scenario and see how the input estimation is possible at the controller K. Suppose that at time k_0, in K, the state estimate $\hat{x}_{k_0}$ is obtained. Further, suppose that the first measurement after time k_0 is received at k_1. Here, we assume $k_1 \geq k_0 + \nu$ and consider the time interval of $[k_0, k_1 - \nu]$. Note that due to the relative degree ν of the plant, the control input applied after $k_1 - \nu$ cannot be observed from the measurement y_{k_1}.

For $k \in [k_0, k_1 - \nu]$ during this interval, there are several possibilities that could occur at the plant. In the following, we consider two simple situations.

(i) At one time, say l_0, the control value v_{l_0} reaches the actuator side and is applied. Then, the measurement y_{k_1} received at time k_1 is

$$y_{k_1} = C\left[A^{k_1-k_0}x_{k_0} + A^{k_1-1-l_0}Bv_{l_0} + \sum_{h=k_0}^{k_1-1} A^{k_1-1-h}d_h\right].$$

(ii) None of the control input candidates v_k, $k \in [k_0, k_1 - \nu]$, arrives at the actuator side. In this case, the measurement is expressed as

$$y_{k_1} = CA^{k_1-k_0}x_{k_0} + \sum_{h=k_0}^{k_1-1} A^{k_1-1-h}d_h.$$

In (i) and (ii), in total, there are $k_1 - k_0 - \nu + 2$ possible cases. Now, we denote the control input estimate made at the controller by $\hat{u}_k$, $k \in [k_0, k_1 - \nu]$. In order to distinguish the $k_1 - k_0 - \nu + 2$ cases above, a simple and reasonable approach would be to follow the protocol as follows:

1. Generate the candidate measurements $\hat{y}_{k_1}^{(l)}$ at time k_1 assuming control being applied at time l or no control in the case of $l = -1$:

$$\hat{y}_{k_1}^{(l)} = \begin{cases} C\left[A^{k_1-k_0}\hat{x}_{k_0} + A^{k_1-1-l}Bv_l\right] \\ \qquad\qquad\qquad\text{if } l \in [k_0, k_1-1], \\ CA^{k_1-k_0}\hat{x}_{k_0} \qquad \text{if } l = -1. \end{cases}$$

2. Estimate the time of control input being applied via

$$l_{k_1} = \arg\min\left\{\left|y_{k_1} - \hat{y}_{k_1}^{(l)}\right| : \ l \in \{-1, k_0, \ldots, k_1-\nu\}\right\}. \qquad (4)$$

3. For $k \in [k_0, k_1-\nu]$, let the estimated inputs be

$$\hat{u}_k = \begin{cases} v_k & \text{if } k = l_{k_1}, \\ 0 & \text{otherwise.} \end{cases} \qquad (5)$$

Here, we emphasize that the time k_1 when the measurement arrives at the controller side after k_0 is a random variable. This means that at the point the control input v_k is generated, the controller is not aware of the time when its estimation takes place via (4) and (5). Also, the time l_0 when the first v_k reaches the decoder after k_0 is random.

We say that the input estimate scheme in (4) achieves perfect estimation if $\hat{u}_k = u_k$ for $k \in [k_0, k_1-\nu]$. To achieve this, the values of the inputs v_k must be properly chosen. The next lemma provides a sufficient condition for such estimation to be possible.

Lemma 3.1. If

$$\begin{aligned} &2\|CA^j\||e_{k_0}| + 2\sum_{h=0}^{j-1}\|CA^h\|\bar{d} \\ &\quad < \left|CA^{j-1}\left(A^{-m}Bv_{k_0+m} - A^{-l}Bv_{k_0+l}\right)\right|, \\ &\quad \forall j \geq \nu,\ \forall l, m \in \{0, 1, \ldots, j-\nu\},\ l \neq m, \end{aligned} \qquad (6)$$

and

$$\begin{aligned} &2\|CA^j\||e_{k_0}| + 2\sum_{h=0}^{j-1}\|CA^h\|\bar{d} < \left|CA^{j-1-l}Bv_{k_0+l}\right|, \\ &\qquad \forall j \geq \nu,\ \forall l \in \{0, 1, \ldots, j-\nu\}, \end{aligned} \qquad (7)$$

then, from the input estimation scheme (4) and (5), we have perfect estimation as

$$\hat{u}_k = u_k, \quad \forall k \in [k_0, k_1-\nu].$$

There are a few remarks. (i) The two conditions (6) and (7) must hold for infinite combinations of (i, l, m) because, as mentioned above, the precise arrival times of the control inputs and those of the measurements are unknown a priori. (ii) One issue that has implicitly been assumed is that there is only one control applied before a measurement is received. (iii) As we have observed, to ease the input estimation, we will permit only one input after the controller makes a measurement.

4. CONTROL ALGORITHM

Consider the system in Fig. 2. This is the setup in Fig. 1, where the controller structure is shown in detail. The overall controller consists of three components: The input estimator, the state estimator, and the control input generator.

We first provide the outline of the control algorithm. Let $k_0, k_1, \ldots \in \mathbb{Z}_+$ be the times at which the measurements y_k arrive at the controller ($\theta_{1,k} = 1$). Also, denote by $\hat{u}_k$ the control input estimate made at the controller. Then, let σ_k be the time index l of the most recent estimate $\hat{u}_l$ that has been made by the time k. Its initial value is $\sigma_k = -1$ for $k = -1, 0, 1, \ldots, k_0 - 1$.

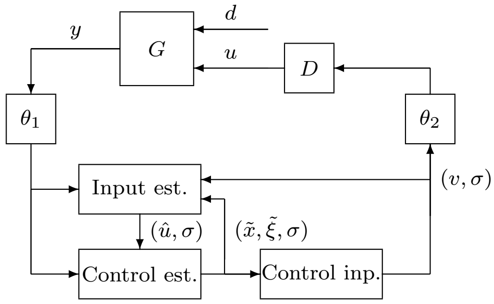

Fig. 2. Remote control scheme

The controller transmits the pair (v_k, σ_k), which is received by the decoder D if $\theta_{2,k} = 1$. At D, the control input is applied following the rule as follows: First, let l_k be the time when the last control was applied before the time k; more specifically, let $l_k := \max\{l < k : \ u_l \neq 0\}$ or $l_k := -1$ if the maximization has no solution (when no input is applied yet). We also introduce the so-called dwell-time parameter $\delta \in \mathbb{Z}_+$. This is the time that the controller must wait before applying the next control. Assume that $\delta \geq \nu + 2$. The control input is determined as

$$u_k = \begin{cases} v_k & \text{if } \theta_{2,k} = 1 \text{ and if } l_k \leq \min\{\sigma_k, k-\delta\} \\ & \qquad\qquad\qquad \text{or } l_k = -1, \\ 0 & \text{otherwise.} \end{cases} \qquad (8)$$

In words, this says that if the last control was applied before or at time σ_k and if that time was at least δ steps before, or if the control arrives for the first time, then the new control is used. Otherwise, the control is zero.

At each time k_i, the overall controller functions as follows. First, at the input estimator, the estimates $\hat{u}_k$ are obtained by the scheme from the previous section. Hence,

$$\hat{u}_k, \quad k = \sigma_{k_i-1}+1, \sigma_{k_i-1}+2, \ldots, k_i-\nu.$$

Note that due to the relative degree ν, the inputs after $k_i - \nu$ cannot be estimated. If there exists a time k where $\hat{u}_k \neq 0$, then, according to the rule (8) at the decoder, no control has been applied after this time k up to $k_i - 1$. Therefore, further input estimation can be made as

$$\hat{u}_k = 0, \quad k = k - \nu + 1, \ldots, k_i - 1.$$

Next, at the state estimator, two types of state estimates are generated: The true estimate $\hat{x}_k$ for $k \in \mathbb{Z}_+$ and the tentative estimate $\tilde{x}_k$ for $k = k_0, k_1, \ldots$. These estimates are made based on the following information: $\left\{\hat{u}_0, \ldots, \hat{u}_{\sigma_k},\ \tilde{y}_0, \ldots, \tilde{y}_k\right\}$. The true estimate $\hat{x}_k$ is made only up to time $\sigma_k + 1$ using the estimated inputs. On the other hand, the tentative one $\tilde{x}_{k_i}$ is generated assuming that no control has been applied since σ_{k_i-1}, i.e., during the time where the actual input is still unknown to the controller; this estimate is made at every k_i when a measurement is received. In general, the two state estimates are not equal.

At time k, the estimates most recently obtained are denoted by $\hat{x}_{\xi(k)}$ and $\tilde{x}_{\tilde{\xi}(k)}$, where $\xi(k), \tilde{\xi}(k) \in \mathbb{Z}_+$ are the time indices. It holds that $\xi(k) = \sigma_k + 1 \leq \tilde{\xi}(k)$, $k \in \mathbb{Z}_+$. Their initial values are set as $\hat{x}_0 = \tilde{x}_0$ and $\xi(0) = \tilde{\xi}(0) = 0$. By definition, $\tilde{\xi}(k)$ can be written as

$$\tilde{\xi}(k) = \begin{cases} \max\{k_i \le k : i \in \mathbb{Z}_+\} & \text{if the maximization has a solution,} \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Finally, in Fig. 2, at the control input generator, the tentative estimate $\tilde{x}_{k_i}$ at time k_i is received. It then outputs the control input candidate v_k. The use of the tentative estimate is justified since, at the decoder, v_k will be applied only when no control has been applied since σ_k.

Now, the details of the algorithms employed in the components in the control scheme are described.

4.1 Input estimator

At each time k_i when the measurement y_{k_i} from the sensor side is received, the input estimator generates the estimates $\hat{u}_k$. There are two cases depending on k_i.

1. If $\sigma_{k_i-1} + 1 > k_i - \nu$, no new estimate can be made because of the relative degree ν of the plant. In this case, set the time index σ_k of the most recently estimated input to be

$$\sigma_k = \sigma_{k_i-1}, \quad k = k_i, \ldots, k_{i+1} - 1.$$

Then, proceed to the state estimation step in Section 4.2 given below.

2. If $\sigma_{k_i-1} + 1 \le k_i - \nu$, then new estimates are computed as follows. First, set

$$\begin{aligned} k_i^{(0)} &= \sigma_{k_i-1} + 1, \\ k_i^{(j)} &= \min\{k_l : \ k_i^{(j-1)} < k_l < k_i - \nu + 1, \ l \in \mathbb{Z}_+\}, \\ &\qquad j = 1, 2, \ldots, m_i, \qquad (10) \\ k_i^{(m_i+1)} &= k_i - \nu + 1, \end{aligned}$$

where m_i denotes the largest j such that the minimization in (10) has a solution. The times $k_i^{(j)}$ are when the measurements were received during the time between σ_{k_i-1} and $k_i - \nu + 1$; see Fig. 3.

In an increasing order, for $j = 0, 1, \ldots, m_i$, follow the steps (a)–(c) below.

(a) Check whether a nonzero control was applied during the interval $[k_i^{(j)}, k_i^{(j+1)} - 1]$ by the input estimation:

$$l_i^{(j)} = \arg\min\big\{\big|y_{k_i} - \hat{y}_{k_i}^{(l)}\big| : l \in \{-1, k_i^{(j)}, \ldots, k_i^{(j+1)} - 1\}\big\},$$

where

$$\hat{y}_{k_i}^{(l)} = \begin{cases} C\left[A^{k_i - \tilde{\xi}(k_i^{(j)})}\tilde{x}_{\tilde{\xi}(k_i^{(j)})} + A^{k_i-1-l}Bv_l\right] \\ \qquad\qquad \text{if } l \in \{k_i^{(j)}, \ldots, k_i^{(j+1)} - 1\}, \\ CA^{k_i - \tilde{\xi}(k_i^{(j)})}\tilde{x}_{\tilde{\xi}(k_i^{(j)})} \quad \text{if } l = -1. \end{cases}$$

We remark that $\hat{y}_{k_i}^{(l)}$ is the estimate of the output assuming that control was applied at time l if $l \ge k_i^{(j)}$ and $\hat{y}_{k_i}^{(-1)}$ is the estimate assuming no control since the time $k_i^{(0)}$. Here, the state estimate $\tilde{x}_{\tilde{\xi}(k_i^{(j)})}$ is used because, as shown in (9), this is the most recent tentative estimate for the time in $[k_i^{(j)}, k_i^{(j+1)} - 1]$.

(b) If $l_i^{(j)} \ne -1$, then for $k \in \{k_i^{(j)}, \ldots, k_i - 1\}$, set

$$\hat{u}_k = \begin{cases} v_k & \text{if } k = l_i^{(j)}, \\ 0 & \text{otherwise,} \end{cases}$$

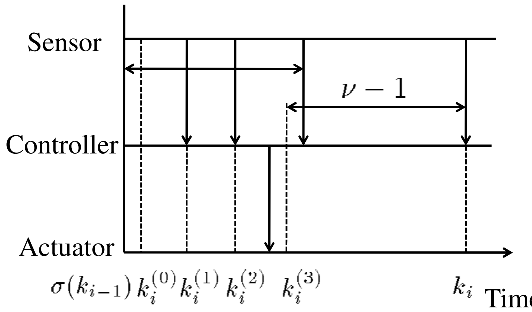

Fig. 3. Timings of the messages received: From the sensor to the controller, and from the controller to the actuator

and also set

$$\sigma_k = k_i - 1, \quad k = k_i, k_i + 1, \ldots, k_{i+1} - 1.$$

Proceed to the state estimation given in Section 4.2.

(c) If $l_i^{(j)} = -1$, then let

$$\hat{u}_k = 0, \quad k = k_i^{(j)}, \ldots, k_i^{(j+1)} - 1.$$

Now, check whether $j = m_i$. If so, set $\sigma_k = k_i - \nu$ for $k = k_i, \ldots, k_{i+1} - 1$ and proceed to the state estimation step. If $j < m_i$, then set j to $j + 1$ and follow the steps (a)–(c) above.

The next lemma provides a sufficient condition for perfect input estimation based on Lemma 3.1.

Lemma 4.1. (i) Suppose that for each k_i with $i \in \mathbb{Z}_+$, the control input v_{k_i+m} satisfies the following inequality:

$$\begin{aligned} |v_{k_i+m}| > \sup_{l,j \in \mathbb{Z}_+,\ l<m\le j-\nu} \frac{1}{|CA^{j-1-m}B|} \\ \times \Bigg[|CA^{j-1-l}B| \cdot |v_{k_i+l}| + 2\|CA^j\| \cdot |e_{k_i}| \\ + 2\sum_{h=0}^{j-1}\|CA^h\| \cdot \bar{d}\Bigg] \end{aligned} \tag{11}$$

for $m \in \{1, 2, \ldots, k_{i+1} - k_i - 1\}$. Then, the input estimator achieves perfect estimation: $\hat{u}_k \equiv u_k$.

(ii) There exist scalars $\gamma_1, \gamma_2, \gamma_3 > 0$ such that for each $j, l, m \in \mathbb{Z}_+$ with $l < m \le j - \nu$, it holds that

$$\begin{aligned} \frac{|CA^{j-1-l}B|}{|CA^{j-1-m}B|} &\le \gamma_1\ |\lambda|^{m-l}, \qquad (12) \\ \frac{2\|CA^j\|}{|CA^{j-1-m}B|} &< \gamma_2\ |\lambda|^m, \\ \frac{2\sum_{h=0}^{j-1}\|CA^h\| \cdot \bar{d}}{|CA^{j-1-m}B|} &< \gamma_3\ |\lambda|^m. \end{aligned}$$

Here, we make the technical assumption on the plant that $\gamma_1 \le 1$. We have comments on this condition later.

4.2 State estimator

At time k_i, the state estimator receives the measurement y_{k_i} and the estimated inputs $\{\hat{u}_l\}$ and then generates the two state estimates, the true one $\hat{x}_k$ and the tentative one $\tilde{x}_{k_i}$. Depending on the results of the input estimates, there are two cases as follows.

1. If a nonzero input is detected in $\hat{u}_k$, then the true state estimate $\hat{x}_k$ is updated as

$$\hat{x}_{k+1} = A\hat{x}_k + B\hat{u}_k - \theta_{1,k+1}L\left[\tilde{y}_{k+1} - C(A\hat{x}_k + B\hat{u}_k)\right],$$
$$k = \xi(k_i - 1), \ldots, k_i - 1,$$
$$\xi(k) = k_i, \quad k = k_i, \ldots, k_{i+1} - 1,$$

where $L \in \mathbb{R}^n$ is the observer gain. The tentative estimate $\tilde{x}_{k_i}$ is set equal to the true one as

$$\tilde{x}_{k_i} = \hat{x}_{k_i},$$
$$\tilde{\xi}(k) = k_i, \quad k = k_i, \ldots, k_{i+1} - 1.$$

2. If all the inputs just estimated are zero or no estimate is made, then the true state estimate is not updated. Hence,

$$\xi(k) = \xi(k_i - 1), \quad k = k_i, \ldots, k_{i+1} - 1.$$

In the update of the tentative estimate, zero control input is assumed for the times where $\hat{u}_k$ are not available yet:

$$\tilde{x}_{k_i} = A^{k_i - k_{i-1}}\tilde{x}_{k_{i-1}} - L\left[\tilde{y}_{k_i} - CA^{k_i - k_{i-1}}\tilde{x}_{k_{i-1}}\right],$$
$$\tilde{\xi}(k) = k_i, \quad k = k_i, \ldots, k_{i+1} - 1.$$

The state estimator has the following properties.

Lemma 4.2. (i) Suppose $\hat{u}_k = u_k$ for all $k \in \mathbb{Z}_+$. Then, the estimation error $e_k = x_k - \hat{x}_k$ (with respect to the real estimate) satisfies

$$e_{k+1} = (A + \theta_{1,k+1}LCA)e_k + (I + \theta_{1,k+1}LC)d_k. \quad (13)$$

Moreover, there exists an observer gain L such that $\sup_{k\in\mathbb{Z}_+} E[\|e_k\|^2] < \infty$ if and only if $\alpha_1 < 1/|\lambda|^2$.

(ii) At each k such that $u_k \neq 0$, the two state estimates coincide as $\tilde{x}_{\tilde{\xi}(k)} = \hat{x}_{\tilde{\xi}(k)}$.

It may appear redundant to use a full order state estimator for stabilization of a plant with one unstable mode. However, for the input estimation, the stable modes cannot be ignored; the controller does not know when the transients in such modes sufficiently decay after a control is applied.

4.3 Control input generator

At all times k, the control input generator transmits the input v_k satisfying the condition (11) in Lemma 4.1 (i) and the time index σ_k of the most recent input estimate.

We introduce a system that provides a bound on the state estimation error e_k appearing in (11). Let $\zeta_0^{(1)}, \ldots, \zeta_0^{(2(n-1))} \in \mathbb{R}^n$ be the vertices of the hypercube $\{\zeta : \|\zeta\| \leq \bar{x}_0\}$ containing the initial states such that $\zeta_0^{(i)} \neq -\zeta_0^{(j)}$ for $i \neq j$. For each i, let $\zeta_k^{(i)}$ be determined by the recursion

$$\zeta_{k+1}^{(i)} = (A+\theta_{1,k+1}LCA)\zeta_k^{(i)} + \|I+\theta_{1,k+1}LC\|\,\bar{d}\,s_k^{(i)}, \quad (14)$$

where

$$s_k^{(i)} := \frac{(A + \theta_{1,k+1}LCA)\zeta_k^{(i)}}{\|(A + \theta_{1,k+1}LCA)\zeta_k^{(i)}\|}.$$

Then, let $\bar{\zeta}_k := \max_i \|\zeta_k^{(i)}\|$, $k \in \mathbb{Z}_+$. It is clear from (13) and (14) that if $\hat{u}_k = u_k$ for all k, then $\|e_k\| \leq \bar{\zeta}_k$. Also, $\bar{\zeta}_k$ can be computed on the controller side at the time k since $\theta_{1,k}$ is known at this time.

The control input v_k is determined by the most recent tentative state estimate $\tilde{x}_{\tilde{\xi}(k)}$. Specifically, it is given by

$$v_k = FA^{k-\tilde{\xi}(k)}\tilde{x}_{\tilde{\xi}(k)} + w_k, \quad (15)$$

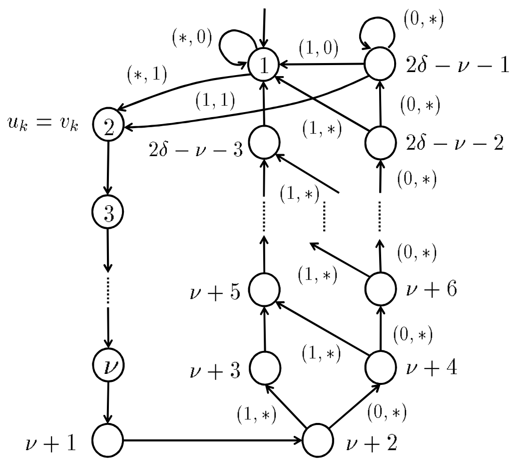

Fig. 4. Transition diagram

where w_k is given by

$$w_k = \mathrm{sgn}\big(FA^{k-\tilde{\xi}(k)}\tilde{x}_{\tilde{\xi}(k)}\big)\frac{1}{\gamma_1}|\lambda|^{k-\tilde{\xi}(k)}(k-\tilde{\xi}(k))[\gamma_2\bar{\zeta}_{\tilde{\xi}(k)}+\gamma_3] \quad (16)$$

and the feedback gain $F \in \mathbb{R}^{1\times n}$ takes the form $F = [0 \cdots 0 \ -\lambda/b_2]$. In (15), the control consists of two terms. The first is for the deadbeat control of the unstable mode while the second enables the input estimation.

The following properties hold for the control inputs.

Proposition 4.3. (i) With the control input v_k in (15), the input estimator achieves perfect estimation, $\hat{u}_k = u_k$, $\forall k$.

(ii) For the random process w_k in (16), it holds that $\sup_{k\in\mathbb{Z}_+} E[w_k^2] < \infty$ if and only if $\alpha_1 < 1/|\lambda|^2$.

5. TIMING OF CONTROLS AND STABILITY

We consider the timings when control inputs are applied at the actuator and then present the main result on closed-loop stability.

According to the decoder rule in (8), the control input is applied at time k if $l_k \leq \min\{\sigma_k, k-\delta\}$, or if $\theta_{2,k}$ is equal to 1 for the first time. This rule can be described by a Markov chain model whose transition diagram is shown in Fig. 4. In the model, there are $N := 2\delta - \nu - 1$ modes, each of which is represented by a node; they are labeled from 1 to $2\delta-\nu-1$. We denote by η_k the mode at time k. Each edge is labeled with the message loss modes $(\theta_{1,k}, \theta_{2,k})$ that trigger the transition. Here, the entry with $*$ indicates that it can be either 0 or 1, and the edges that are deterministic are not labeled. From this diagram, the transition probability matrix $P = [p_{ij}] \in \mathbb{R}^{N\times N}$ can easily be formed, where p_{ij} is the probability of transition from mode i to mode j.

The diagram in Fig. 4 can be explained as follows. Mode 1 is the initial mode. When the decoder receives a message ($\theta_{2,k} = 1$), the state moves to mode 2, and the control is applied. The following modes, from 3 to $\nu + 2$, correspond to the relative degree of the plant; the input estimator must wait before finding out about the control just applied. Then, after mode $\nu + 2$, when the controller receives a measurement ($\theta_{1,k} = 1$), the control input is estimated. However, the controller must wait for the dwell time δ before applying the next input. Hence, the remaining modes from $\nu + 3$ to $2\delta - \nu$ constitute two rows corresponding to whether a measurement is received or not. It takes at least $\delta - 1$ steps to go from mode 2 back to the initial mode.

In what follows, we focus on the problem of stabilization and thus assume perfect estimation at the controller: $\tilde{x}_k = x_k$, $\forall k$. Under the Markov chain model in Fig. 4, the control input is determined by its state η_k as

$$u_k = \begin{cases} Fx_k & \text{if } \eta_k = 2, \\ 0 & \text{otherwise.} \end{cases}$$

As a consequence, we can describe the system as

$$x_{k+1} = \bar{A}_{\eta_k} x_k + d_k, \tag{17}$$

where

$$\bar{A}_i = \begin{cases} A + BF & \text{if } i = 2, \\ A & \text{otherwise.} \end{cases}$$

The stability of the system in (17) is characterized in the next proposition. Note that this system is a Markovian jump linear system; see, e.g., (Costa et al. 2005).

Proposition 5.1. (i) The system (17) is stable in the sense of (2) if and only if there exist positive-definite matrices $Q_1, Q_2, Q_3 \in \mathbb{R}^{n\times n}$ satisfying the following inequalities:

$$\alpha_2 A^T Q_1 A + (1-\alpha_2)(A+BF)^T Q_2 (A+BF) - Q_1 < 0,$$
$$(A^T)^{\delta-1}\big[(1-\alpha_1^{\delta-\nu-1})Q_1 + \alpha_1^{\delta-\nu-1}Q_3\big]A^{\delta-1} - Q_2 < 0.$$
$$(1-\alpha_1)\alpha_2 A^T Q_1 A + (1-\alpha_1)(1-\alpha_2)$$
$$\times (A+BF)^T Q_2 (A+BF) + \alpha_1 A^T Q_3 A - Q_3 < 0,$$

(ii) If $\alpha_1, \alpha_2 < 1/|\lambda|^2$, then by taking δ large enough, there exist Q_1, Q_2, Q_3 satisfying the inequalities above.

We fix the parameter δ following (ii) in the proposition. Now, we are in the position to show the main result on the stability of the closed-loop system in Fig. 2.

Theorem 5.2. In the remote control system in Fig. 2, assume that γ_1 in (12) satisfies $\gamma_1 \le 1$. Then, under the control scheme described above, the plant is stabilized in the mean-square sense as in (2) if

$$\alpha_1, \alpha_2 < \frac{1}{|\lambda|^2}. \tag{18}$$

Notice that the bounds (18) coincide with those in Proposition 2.1, which is for the case using acknowledgements.

We comment on the class of plants that can be handled by the theorem. The assumption $\gamma_1 \le 1$ can be stated as

$$\big|C(A/\lambda)^{k+i}B\big| \le \big|C(A/\lambda)^k B\big|, \quad \forall k \ge \nu - 1, \quad i > 0. \tag{19}$$

This means that the system specified by $(A/\lambda, B, C)$ has an impulse response whose absolute value is nonincreasing for $k \ge \nu - 1$. Notice that this system is marginally stable. Clearly, the condition holds for scalar systems. It is noted that the remote control problem without acknowledgements is studied in (Imer et al. 2006). There, for the scalar case, a sufficient condition on the loss probabilities is derived, but α_1 and α_2 cannot be chosen independently.

One class of plants satisfying (19) is given in the transfer function form as $G(z) = \sum_{i=0}^{n-1} c_i/(z - p_i)$, where one pole is unstable $p_0 \ge 1$ and the rest satisfy $p_i \in [0,1)$ for $i \ne 0$, and also all c_i corresponding to nonzero poles take the same sign. Plants with nonminimum phase zeros can be found in this class such as $G(z) = 1/(z-2) + 1/(z-0.5) = 2(z-1.25)/((z-2)(z-0.5))$.

6. CONCLUSION

In this paper, we have considered the remote control problem over unreliable channels when no acknowledgement is employed. We have shown that, for a certain class of plants, the system can be stabilized if the loss probabilities satisfy a condition that has been known for the case when acknowledgements are used. Future research will deal with extensions to more general cases.

REFERENCES

Antsaklis, P. J. and J. Baillieul, Guest Editors (2007). Special Issue on the Technology of Networked Control Systems. *Proceedings of the IEEE.*

Costa, O. L. V., M. D. Fragoso and R. P. Marques (2005). *Discrete-Time Markov Jump Linear Systems.* Springer. London.

Elia, N. (2005). Remote stabilization over fading channels. *Systems & Control Letters* **54**, 238–249.

Elia, N. and J. N. Eisenbeis (2004). Limitations of linear remote control over packet drop networks. In: *Proc. IEEE Conf. on Decision and Control.* pp. 5152–5157.

Epstein, M., L. Shi and R. M. Murray (2006). An estimation algorithm for a class of networked control systems using UDP-like communication schemes. In: *Proc. IEEE Conf. on Decision and Control.* pp. 5597–5603.

Hadjicostis, C. N. and R. Touri (2002). Feedback control utilizing packet dropping network links. In: *Proc. IEEE Conf. on Decision and Control.* pp. 1205–1210.

Hou, M. and P. C. Müller (1994). Disturbance decoupled observer design: A unified viewpoint. *IEEE Trans. Autom. Control* **39**, 1338–1341.

Imer, O. Ç., S. Yüksel and T. Başar (2006). Optimal control of LTI systems over unreliable communication links. *Automatica* **42**, 1429–1439.

Ishii, H. (2007). Utilizing multiple unreliable channels in remote stabilization of linear systems. In: *Proc. American Control Conf.* pp. 1723–1728.

Ishii, H. (2008). H^∞ control with limited communication and message losses. *Systems & Control Letters* **57**, 322–331.

Ishii, H. and S. Hara (2006). A subband coding approach to networked control. In: *Proc. MTNS.* pp. 2906–2911. Also, to appear, *Automatica*, 2008.

Sahai, A. and S. Mitter (2006). The necessity and sufficiency of anytime capacity for stabilization of a linear system over a noisy communication link – Part I: Scalar systems. *IEEE Trans. Inform. Theory* **52**, 3369–3395.

Sinopoli, B., L. Schenato, M. Franceschetti, K. Poolla, M. I. Jordan and S. S. Sastry (2004). Kalman filtering with intermittent observations. *IEEE Trans. Autom. Control* **49**, 1453–1464.

Tatikonda, S. and S. Mitter (2004). Control over noisy channels. *IEEE Trans. Autom. Control* **49**, 1196–1201.

Xu, Y. and J. P. Hespanha (2005). Estimation under uncontrolled and controlled communications in networked control systems. In: *Proc. IEEE Conf. on Decision and Control and European Control Conf.* pp. 842–847.