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Setpoint Servo Problem for Symmetric Affine Systems -Asymptotical Stabilization by PI Control-

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Abstract: This paper is concerned with control of nonholonomic systems. As is well known, symmetric affine system is unable to control with continuous time-invariant state feedback. In this paper we apply PI Control to a setpoint servo problem for the symmetric affine system. PI control possesses two adjustable parameters K_P, K_I, and in addition the so-called manual reset quantity $\boldsymbol{m}_0$. It is remarked that adjusting $\boldsymbol{m}_0$ is equivalent to adjusting an initial condition $\boldsymbol{z}_0$ of integrator $\dot{\boldsymbol{z}} = \boldsymbol{e}$. By the PI control with the manual reset $\boldsymbol{m}_0$ appropriately chosen, not only controllable part of symmetric affine system is asymptotically stabilized but also uncontrollable part can be made to converge to the desired value. Applying the PI control with $\boldsymbol{m}_0$, we can control the symmetric affine system without transforming into the "chained form". The effectiveness of the method was confirmed by the simulation results for various plants.

1. INTRODUCTION

The nonholonomic system, in most cases, is described as a nonlinear affine system. In particular, mechanical systems with nonholonomic speed constraints are represented with a symmetric affine system without a drift term.

It is well known that the symmetric affine system cannot be asymptotically stabilized by the continuous time-invariant state feedback control, even if it is controllable (refer to Brockett's Theorem Brockett [1983]). Accordingly, discontinuous switching feedback control (Khennouf and Wit [1995], Mita [2000], Ikeda et al. [2000]) and/or time-varying feedback control (Pomet [1992], Sordalen and Egeland [1995], Kiyota and Sampei [1999], Samson [1995]) has been proposed. However, most of them are restricted to the so-called "chained form" which is a canonical system of symmetric affine system. For example, Khennouf and Wit [1995] utilized a structure of the chained form skillfully and designed a two-stage switching scheme using an invariant manifold. It is difficult to extend to a system besides the chained form, however.

Generally speaking, methods based on the transformation into the chained form are complex, individual and skillful. Further it yields a problem of singular point caused by the transformation. So it is expected to develop a control method without such transformation.

Ikeda et al. [2000], Mita [2000] proposed Variable Constraint Control (VCC) which could be applied for the symmetric affine system without transforming into the chained form. Note that it is also of two-stage scheme based on the use of invariant manifold.

In Shimizu and Tamura [2004] we proposed applying DGDC(Direct Gradient Descent Control) (Otuka and Shimizu [2001], Shimizu and Otsuka [1999]) to realize VCC, where each stage of VCC was executed by the DGDC for an individual performance function in each stage. Our method then achieved stabilization with the switching DGDC of a two-stage type.

Furthermore, in Shimizu and Tamura [2007], we applied the DGDC for an original symmetric affine system without making any coordinate transformation, and showed that asymptotical stabilization could be achieved by setting an initial condition of a dynamic controller optimally.

In this paper we consider a setpoint servo problem for symmetric affine system. We propose applying PI control to an original plant without making any coordinate transformation. We show that asymptotical stabilization can be achieved by setting the manual reset quantity of PI control or on-line by adjusting an initial condition of I operation. Optimization of the manual reset quantity is made on-line by use of the Nelder-Mead method (Nelder and Mead [1965]). Further the best setting of the manual reset quantity is not necessarily made one time. We can perform asymptotical stabilization by repeating it on all such occasions as PI control is done at several stages.

The proposed method has a merit of utilizing a simple scheme as the PI control and a parameter as the manual reset quantity easily adjusted on-line.

Lastly, we confirmed the effectiveness of the proposed method by the simulation results for two-wheeled vehicle, four-wheeled vehicle, flying robot, etc.

2. PI CONTROL FOR SYMMETRIC AFFINE SYSTEM

Nonholonomic systems, in most cases, are represented by the following nonlinear state equation

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$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + G(\boldsymbol{x})\boldsymbol{u}$$

where $\boldsymbol{x} \in R^n$, $\boldsymbol{u} \in R^r$ are the state vector and the input vector, respectively. It is called the nonlinear affine system, and the term $\boldsymbol{f}(\boldsymbol{x})$ is called the drift term because it is composed of no input. In particular the nonlinear affine system without drift

$$\dot{\boldsymbol{x}} = G(\boldsymbol{x})\boldsymbol{u}, \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{1}$$

is called the symmetric affine system, where $G(\boldsymbol{x})$ is of column full rank. Examples are moving vehicles, a flying robot, etc..

The difficulty of symmetric affine system is that inadequate control makes the system converge to a point $\boldsymbol{x}^s \neq \boldsymbol{0}$ such that $G(\boldsymbol{x}^s)\boldsymbol{u}^s = \boldsymbol{0}$. Since the symmetric affine system does not satisfy the condition of Brockett's theorem (Brockett [1983]), one cannot asymptotically stabilize it by the continuously differentiable state feedback control law $\boldsymbol{u} = \boldsymbol{\alpha}(\boldsymbol{x})$.

Let us consider PI control for system (1)

$$\boldsymbol{u} = K_P(\boldsymbol{x}^* - \boldsymbol{x}) + K_I \int_0^t (\boldsymbol{x}^* - \boldsymbol{x}) d\tau + \boldsymbol{m}_0 \tag{2}$$

where $\boldsymbol{x}^*$ is the desired value, $K_P \in R^{r\times n}$ *and* $K_I \in R^{r\times n}$ denote the proportional gain matrix and integral one, respectively. $\boldsymbol{m}_0$ denotes the so-called manual reset quantity. The PI control (2) can be equivalently represented as

$$\dot{\boldsymbol{z}} = \boldsymbol{x}^* - \boldsymbol{x}, \quad \boldsymbol{z}(0) = \boldsymbol{0} \tag{3}$$

$$\boldsymbol{u} = K_P(\boldsymbol{x}^* - \boldsymbol{x}) + K_I \boldsymbol{z} + \boldsymbol{m}_0 \tag{4}$$

(1), (3), (4) are expressed as

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} O & O \\ -I & O \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} + \begin{bmatrix} G(\boldsymbol{x}) \\ O \end{bmatrix} \boldsymbol{u} + \begin{bmatrix} O \\ \boldsymbol{x}^* \end{bmatrix}, \quad \begin{bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{z}(0) \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{0} \end{bmatrix} \tag{5}$$

$$\boldsymbol{u} = K_P(\boldsymbol{x}^* - \boldsymbol{x}) + K_I \boldsymbol{z} + \boldsymbol{m}_0 \tag{6}$$

When it holds that $\boldsymbol{x}_e = \boldsymbol{x}^*$ at the equilibrium $(\boldsymbol{x}_e, \boldsymbol{z}_e)$ of (5), (6), the corresponding $\boldsymbol{z}$ and $\boldsymbol{u}$ must satisfy $G(\boldsymbol{x}^*)\{K_I\boldsymbol{z}_e + \boldsymbol{m}_0\} = \boldsymbol{0}$. As $G(\boldsymbol{x}^*)$ is of column full rank, however, we have $K_I\boldsymbol{z}_e + \boldsymbol{m}_0 = \boldsymbol{0}$. Therefore, we obtain the input corresponding to $(\boldsymbol{x}_e, \boldsymbol{z}_e)$ as $\overline{\boldsymbol{u}} = K_P(\boldsymbol{x}^* - \boldsymbol{x}_e) + K_I\boldsymbol{z}_e + \boldsymbol{m}_0 = \boldsymbol{0}$.

Now linearizing (5) at $(\boldsymbol{x}_e, \boldsymbol{z}_e, \overline{\boldsymbol{u}}) = (\boldsymbol{x}^*, \boldsymbol{z}_e, \boldsymbol{0})$ yields

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} O & O \\ -I & O \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} + \begin{bmatrix} G(\boldsymbol{x}^*) \\ O \end{bmatrix} \boldsymbol{u} := \widehat{A} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} + \widehat{B}\boldsymbol{u} \tag{7}$$

regardless of $\boldsymbol{z}_e$ where for simplicity the linearlized variables $\delta\boldsymbol{x}, \delta\boldsymbol{z}, \delta\boldsymbol{u}$ are rewritten as $\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{u}$ again. This system possesses uncontrollable eigenvalues on the imaginary axis (zero eigenvalues), because a rank of the controllability matrix M_c becomes $\mathrm{rank} M_c = 2r < 2n$ as $r < n$. Accordingly, by the similar transformation

$$\begin{bmatrix} \overline{\boldsymbol{x}}^S \\ \overline{\boldsymbol{x}}^C \end{bmatrix} = T \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} \tag{8}$$

the linearized system (7) can be transformed into a canonical system being block-diagonalized to a controllable part and uncontrollable one:

$$\begin{bmatrix} \dot{\overline{\boldsymbol{x}}}^S \\ \dot{\overline{\boldsymbol{x}}}^C \end{bmatrix} = \begin{bmatrix} \overline{A}^S & O \\ O & \overline{A}^C \end{bmatrix} \begin{bmatrix} \overline{\boldsymbol{x}}^S \\ \overline{\boldsymbol{x}}^C \end{bmatrix} + \begin{bmatrix} \overline{B}^S \\ O \end{bmatrix} \boldsymbol{u} \tag{9}$$

where

$$\begin{bmatrix} \overline{A}^S & O \\ O & \overline{A}^C \end{bmatrix} = T\widehat{A}T^{-1}, \quad \begin{bmatrix} \overline{B}^S \\ O \end{bmatrix} = T\widehat{B}$$

Here, $\overline{\boldsymbol{x}}^S \in R^{2r}$ and $\overline{\boldsymbol{x}}^C \in R^{2n-2r}$ are the controllable state vector and the uncontrollable one, respectively. Evidently, eigenvalues of A^C are all 0 and uncontrollable modes of the linearized system (7). But the uncontrollable state variables are stable in the sense of Lyapunov (of not diverging).

Now, since $\{\overline{A}^S, \overline{B}^S\}$ is controllable, (9) becomes

$$\begin{bmatrix} \dot{\overline{\boldsymbol{x}}}^S \\ \dot{\overline{\boldsymbol{x}}}^C \end{bmatrix} = \begin{bmatrix} \overline{A}^S - \overline{B}^S\overline{K}^S & O \\ O & \overline{A}^C \end{bmatrix} \begin{bmatrix} \overline{\boldsymbol{x}}^S \\ \overline{\boldsymbol{x}}^C \end{bmatrix} \tag{10}$$

by the state feedback

$$\boldsymbol{u} = -\begin{bmatrix} \overline{K}^S & O \end{bmatrix} \begin{bmatrix} \overline{\boldsymbol{x}}^S \\ \overline{\boldsymbol{x}}^C \end{bmatrix} = -\overline{K}^S\overline{\boldsymbol{x}}^S \tag{11}$$

This implies that the controllable part $\overline{\boldsymbol{x}}^S$ of the extended system (9) can be asymptotically stabilized.

$\overline{K}^S$ may be obtained by either solving Algebric Riccati Equation for optimal regulator or using any eigenvalue assignment method.

Next, in order to obtain PI parameter matrices which asymptotically stabilize the controllable variables in the linearized system, we consider to obtain such K_P, K_I that PI control law for system (7) (the linearized one of PI control law (4) for system (1),(3))

$$\boldsymbol{u} = -K_P\boldsymbol{x} + K_I\boldsymbol{z} = [\,-K_P \;\; K_I\,] \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} \tag{12}$$

coincides with the linear state feedback law (11).

Since (11) can be expressed as

$$\boldsymbol{u} = -\begin{bmatrix} \overline{K}^S & O \end{bmatrix} \begin{bmatrix} \overline{\boldsymbol{x}}^S \\ \overline{\boldsymbol{x}}^C \end{bmatrix} = -\begin{bmatrix} \overline{K}^S & O \end{bmatrix} T \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} \tag{13}$$

it is sufficient for (12) and (11) to be equal that

$$[\,-K_P \;\; K_I\,] = -\begin{bmatrix} \overline{K}^S & O \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \tag{14}$$

holds. Namely, K_P *and* K_I are obtained as

$$K_P = \overline{K}^S T_{11}, \quad K_I = -\overline{K}^S T_{12} \tag{15}$$

If we apply the PI control (3),(4) with such K_P, K_I asymptotically stabilizing the controllable part to the original nonlinear system (1), then the controllable variables are locally asymptotically stabilized around $(\boldsymbol{x}^*, \boldsymbol{z}_e)$ and the uncontrollable ones converge to a stationary value, an offset remains though, provided that if the dynamics on the center manifold is stable, as mentioned below.

Now substitute (4) into (5) to get

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} O & O \\ -I & O \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} + \begin{bmatrix} G(\boldsymbol{x}) \\ O \end{bmatrix} \times \left\{ [\, -K_P \;\; K_I \,] \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} + \boldsymbol{m}_0 + K_P \boldsymbol{x}^* \right\} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{x}^* \end{bmatrix}, \quad \begin{bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{z}(0) \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{0} \end{bmatrix} \tag{16}$$

By putting (16) as

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{z}} \end{bmatrix} = F\left(\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix}; K_P, K_I; \boldsymbol{m}_0 \right), \quad \begin{bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{z}(0) \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{0} \end{bmatrix} \tag{17}$$

this equation (17) is transformed into

$$\begin{bmatrix} \dot{\overline{\boldsymbol{x}}}^S \\ \dot{\overline{\boldsymbol{x}}}^C \end{bmatrix} = TF\left(T^{-1} \begin{bmatrix} \overline{\boldsymbol{x}}^S \\ \overline{\boldsymbol{z}}^C \end{bmatrix}; K_P, K_I; \boldsymbol{m}_0 \right), \quad \begin{bmatrix} \overline{\boldsymbol{x}}^S(0) \\ \overline{\boldsymbol{x}}^C(0) \end{bmatrix} = \begin{bmatrix} \overline{\boldsymbol{x}}_0^S \\ \overline{\boldsymbol{x}}_0^C \end{bmatrix} \tag{18}$$

by the similar transformation (8).

Generally speaking, in case where the linearlized system possesses 0 eigenvalues, its stability is generally discussed by using center manifold theory. The following is well known about the center manifold.

"All trajectories starting from a neighborhood of the origin are attracted exponentially to the center manifold $S^c(0)$, and stability of an equilibrium point can be checked only from the dynamics on the center manifold. That is, if an equilibrium point on $S^c(0)$ is stable, asymptotically stable, unstable on $S^c(0)$, then the equilibrium point is stable, asymptotically stable, unstable in the whole region, respectively. "

Put the center manifold mapping of system (18) as $\overline{\boldsymbol{x}}^S = \boldsymbol{\pi}(\overline{\boldsymbol{x}}^C)$. Then the dynamics on the center manifold is expressed as

$$\dot{\overline{\boldsymbol{x}}}^C = F^C(\boldsymbol{\pi}(\overline{\boldsymbol{x}}^C), \overline{\boldsymbol{x}}^C) \tag{19}$$

where F^S and F^C are a part of TF corresponding to $\overline{\boldsymbol{x}}^S$ and $\overline{\boldsymbol{x}}^C$, respectively. Since the dynamics of $\overline{\boldsymbol{x}}^S$ is exponentially stable, from the mentioned above, the stability of (18) is guaranteed if (19) is stable (Aeleys [1985], Shimizu and Sato [2005]).

Meanwhile, the desired equilibrium of $\begin{bmatrix} \overline{\boldsymbol{x}}^S \\ \overline{\boldsymbol{x}}^C \end{bmatrix}$ can be calculated from the relation

$$\begin{bmatrix} \overline{\boldsymbol{x}}^{S*} \\ \overline{\boldsymbol{x}}^{C*} \end{bmatrix} = T \begin{bmatrix} \boldsymbol{x}^* \\ \boldsymbol{z}_e \end{bmatrix}$$

Since a solution $\begin{bmatrix} \overline{\boldsymbol{x}}^S(t) \\ \overline{\boldsymbol{x}}^C(t) \end{bmatrix}$ of (18) depends on $\boldsymbol{m}_0$, we have to choose $\boldsymbol{m}_0$ adequately in order to make $\overline{\boldsymbol{x}}^C(t)$ converge to $\overline{\boldsymbol{x}}^{C*}$ from any $\overline{\boldsymbol{x}}(0) = \begin{bmatrix} \overline{\boldsymbol{x}}_0^S \\ \overline{\boldsymbol{x}}_0^C \end{bmatrix}$ (regardless of value of $\overline{\boldsymbol{x}}(0)$), when $\overline{\boldsymbol{x}}^S(t)$ converges to $\overline{\boldsymbol{x}}^{S*}$.

Considering the above mentioned in regard to the original system before the transformation, we must choose $\boldsymbol{m}_0$ adequately in order to let $\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix}$ (both the controllable variables being asymptotically stable and uncontrollable variables but being Lyapunov stable) converge to $\begin{bmatrix} \boldsymbol{x}^* \\ \boldsymbol{z}_e \end{bmatrix}$. Hence we assume the following:

(Assumption 1) System (1) is reachable and the manual reset quantity $\boldsymbol{m}_0$ in (2) has sufficient freedom such that PI control can make $\boldsymbol{x}(t)$ come close to $\boldsymbol{x}^*$ as $t \to \infty$.

Accordingly, in order to let $\boldsymbol{x}(t)$ converge to the desired value $\boldsymbol{x}^*$, we have to search such $\boldsymbol{m}_0$ by minimizing the norm $\| \boldsymbol{x}^* - \boldsymbol{x}(\infty) \|$ in executing (1),(3),(4).

Without letting $t \to \infty$ actually, however, we can find it by solving the following problem with sufficiently large t_1:

$$\min_{\boldsymbol{m}_0} \sum_{j=1}^{n} w_j (x_j^* - x_j(t_1))^2, \quad w_j > 0 \tag{20a}$$

$$\text{subj. to } \dot{\boldsymbol{x}}(t) = G(\boldsymbol{x}(t))\boldsymbol{u}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{20b}$$

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{x}^* - \boldsymbol{x}(t), \;\; \boldsymbol{z}(0) = \boldsymbol{0} \tag{20c}$$

$$\boldsymbol{u}(t) = K_P(\boldsymbol{x}^* - \boldsymbol{x}(t)) + K_I \boldsymbol{z}(t) + \boldsymbol{m}_0 \tag{20d}$$

where $w_j, \; j = 1, 2, \cdots, n$ are weight coefficients. Basically the optimal $\boldsymbol{m}_0$ can be found one time.

Optimal $\boldsymbol{m}_0$ is not necessarily found at one stage. If $x_j(\infty), \; j \in J$ has converged with an off set to the undesired equilibrium $x_j^e, \; j \in J$ by the PI control with $\boldsymbol{m}_0$ not reaching an optimum, we need search the manual reset quantity $\boldsymbol{m}_0$ of the second stage under setting

$$x_j(t_1) \approx \begin{cases} x_j^e, & j \in J \\ 0, & j \notin J \end{cases}$$

as the initial state. Note that J denotes the set of subscripts of the state variables which have converged to the undesired equilibrium. Accordingly, we solve:

$$\min_{\boldsymbol{m}_0} \sum_{j \in J} w_j (x_j^* - x_j(t_1'))^2, \quad w_j > 0 \tag{21a}$$

$$\text{subj. to } \dot{\boldsymbol{x}}(t) = G(\boldsymbol{x}(t))\boldsymbol{u}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}(t_1) \tag{21b}$$

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{x}^* - \boldsymbol{x}(t), \;\; \boldsymbol{z}(0) = \boldsymbol{0} \tag{21c}$$

$$\boldsymbol{u}(t) = K_P(\boldsymbol{x}^* - \boldsymbol{x}(t)) + K_I \boldsymbol{z}(t) + \boldsymbol{m}_0 \tag{21d}$$

where t_1' is large enough.

Even when optimization of $\boldsymbol{m}_0$ has not been made exactly , it is possible to let all state variables converge to $\boldsymbol{x}^*$ gradually by repeating such a process several times. At least, we can let them converge to a point in the neighborhood of $\boldsymbol{x}^*$ within permissible accuracy in practice. (We confirmed this fact by simulation for several plants, no proof though.)

Such the best setting or updating $\boldsymbol{m}_0$ gives the same effect as switching the state feedback control law.

We can apply the Nelder-Mead method (Nelder and Mead [1965])to solve problem (20) or (21) on-line.

[Nelder-Mead's Method] This is an improved algorithm of Simplex method, which is a kind of optimization technique without using gradients. A man of business finds it useful because of the simplicity. It is very effective for problems with a relatively small number of decision variables. Since it brings an approximate solution within permissible accuracy in the finite number of iteration steps, it may be said very convenience when one cannot calculate gradients of an objective function.

Meanwhile, the PI control (2) can be equivalently represented as

$$\dot{\boldsymbol{z}} = (\boldsymbol{x}^* - \boldsymbol{x}), \quad \boldsymbol{z}(0) = \boldsymbol{z}_0 \tag{22}$$

$$\boldsymbol{u} = K_P(\boldsymbol{x}^* - \boldsymbol{x}) + K_I \boldsymbol{z} \tag{23}$$

From (22) we get $\boldsymbol{z} = \int_0^t (\boldsymbol{x}^* - \boldsymbol{x})dt + \boldsymbol{z}_0$. Substitute it into (23) to get

$$\boldsymbol{u} = K_P(\boldsymbol{x}^* - \boldsymbol{x}) + K_I \int_0^t (\boldsymbol{x}^* - \boldsymbol{x})dt + K_I \boldsymbol{z}_0 \tag{24}$$

Hence the PI control (22), (23) is equivalent to the PI control having the manual reset quantity $\boldsymbol{m}_0 = K_I \boldsymbol{z}_0$.

(1), (22), (23) can be expressed as

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} O & O \\ -I & O \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} + \begin{bmatrix} G(\boldsymbol{x}) \\ O \end{bmatrix} \boldsymbol{u} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{x}^* \end{bmatrix}, \quad \begin{bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{z}(0) \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{z}_0 \end{bmatrix} \tag{25}$$

$$\boldsymbol{u} = K_P(\boldsymbol{x}^* - \boldsymbol{x}) + K_I \boldsymbol{z} \tag{26}$$

After this is analogous to (12) $\sim$ (15) and we get

$$K_P = \overline{K}^S T_{11}, \quad K_I = -\overline{K}^S T_{12}.$$

Further, we have

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} O & O \\ -I & O \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} + \begin{bmatrix} G(\boldsymbol{x}) \\ O \end{bmatrix} \left\{ [\, -K_P \ \ K_I \,] \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} + K_P \boldsymbol{x}^* \right\} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{x}^* \end{bmatrix}, \quad \begin{bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{z}(0) \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{z}_0 \end{bmatrix} \tag{27}$$

which corresponds to (16). Put this as

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{z}} \end{bmatrix} := F\left(\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix}; K_P, K_I \right), \quad \begin{bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{z}(0) \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{z}_0 \end{bmatrix} \tag{28}$$

Then this is transformed into

$$\begin{bmatrix} \dot{\overline{\boldsymbol{x}}}^S \\ \dot{\overline{\boldsymbol{x}}}^C \end{bmatrix} := TF\left(T^{-1} \begin{bmatrix} \overline{\boldsymbol{x}}^S \\ \overline{\boldsymbol{x}}^C \end{bmatrix}; K_P, K_I \right), \quad \begin{bmatrix} \overline{\boldsymbol{x}}^S(0) \\ \overline{\boldsymbol{x}}^C(0) \end{bmatrix} = \begin{bmatrix} \overline{\boldsymbol{x}}^S_0 \\ \overline{\boldsymbol{x}}^C_0 \end{bmatrix} \tag{29}$$

by the similar transformation (8). Since a solution $\begin{bmatrix} \overline{\boldsymbol{x}}^S(t) \\ \overline{\boldsymbol{x}}^C(t) \end{bmatrix}$ of (29) is a function of $\overline{\boldsymbol{x}}(0) = \begin{bmatrix} \overline{\boldsymbol{x}}^S_0 \\ \overline{\boldsymbol{x}}^C_0 \end{bmatrix}$, we have to choose $\overline{\boldsymbol{x}}(0)$ adequately in order to make $\overline{\boldsymbol{x}}^C(t)$ converge to $\overline{\boldsymbol{x}}^{C*}$ regardless of value of $\overline{\boldsymbol{x}}(0)$, when $\overline{\boldsymbol{x}}^S(t)$ converges to $\overline{\boldsymbol{x}}^{S*}$. This implies choosing $\boldsymbol{z}_0$ adequately for any $\boldsymbol{x}_0$, since $\overline{\boldsymbol{x}}(0) = T \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{z}_0 \end{bmatrix}$.

Consider the above mentioned in regard to the original system. Then we must choose $\begin{bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{z}(0) \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{z}_0 \end{bmatrix}$ adequately in order to let $\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix}$ converge to $\begin{bmatrix} \boldsymbol{x}^* \\ \boldsymbol{z}_e \end{bmatrix}$. It can be achieved by choosing an appropriate $\boldsymbol{z}(0) = \boldsymbol{z}_0$ as a function of $\boldsymbol{x}(0) = \boldsymbol{x}_0$, i.e., $\boldsymbol{z}_0(\boldsymbol{x}_0)$ for arbitrarily given $\boldsymbol{x}_0$.

The Nelder-Mead method can apply to search an optimal $\boldsymbol{z}_0$ as well as optimizing $\boldsymbol{m}_0$ previously.

3. PI CONTROL OF 4-WHEELED VEHICLE

We consider a four-wheeled vehicle shown in **Fig.1**. This

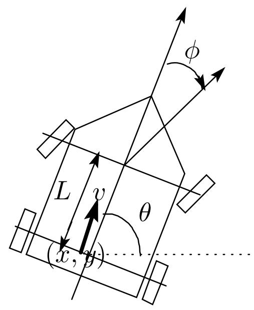

Fig.1. Four-Wheeled Vehicle

vehicle has a distance L from a middle point of a rear wheel shaft to that of a front wheel shaft.

Let four generalized coordinates be the plane position of body (x, y), the attitude angle of body θ and the steering angle of front wheels ϕ, and let control inputs be moving velocity $u_1 = v$ and steering angular velocity $u_2 = \dot{\phi}$. Then this system can be modeled as follows (Mita [2000]).

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \cos x_3 & 0 \\ \sin x_3 & 0 \\ \frac{1}{L}\tan x_4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = G(\boldsymbol{x})\boldsymbol{u} \tag{30}$$

where $\boldsymbol{x} = (x, y, \theta, \phi)^T \in R^4$, and $L = 1.5$.

The control input is manipulated by the PI control (2). We consider a setpoint servo problem with the desired value $\boldsymbol{x}^* = (x_1^*, x_2^*, x_3^*, x_4^*)^T$.

First we calculate PI parameter matrices asymptotically stabilizing the controllable part of the linearized system arround $\boldsymbol{x}^*$. For that purpose linearize the extended system (5) at $\boldsymbol{x}^*$ to get

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} O & O \\ -I_4 & O \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} + \begin{bmatrix} G(\boldsymbol{x}^*) \\ O \end{bmatrix} \boldsymbol{u} \tag{31}$$

where

$$G(\boldsymbol{x}^*) = \begin{bmatrix} \cos x_3^* & 0 \\ \sin x_3^* & 0 \\ (1/L)\tan x_4^* & 0 \\ 0 & 1 \end{bmatrix}$$

Since a rank of the controllable matrix M_c is $rankM_c = 4$, an order of uncontrollable part is 8-4=4.

By setting the similar transformation as

$$T = \begin{bmatrix} 1/\cos x_3^* & 0\ 0\ 0 & 0 & 0\ 0\ 0 \\ 0 & 0\ 0\ 1 & 0 & 0\ 0\ 0 \\ 0 & 0\ 0\ 0 & -1/\cos x_3^* & 0\ 0\ 0 \\ 0 & 0\ 0\ 0 & 0 & 0\ 0\ -1 \\ -\sin x_3^*/\cos x_3^* & 1\ 0\ 0 & 0 & 0\ 0\ 0 \\ -\tan x_4^*/L\cos x_3^* & 0\ 1\ 0 & 0 & 0\ 0\ 0 \\ 0 & 0\ 0\ 0 & -\sin x_3^*/\cos x_3^* & 1\ 0\ 0 \\ 0 & 0\ 0\ 0 & -\tan x_4^*/L\cos x_3^* & 0\ 1\ 0 \end{bmatrix},$$

we obtain the transformed system (9) with

$$\overline{A}^S = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 1&0&0&0 \\ 0&1&0&0 \end{bmatrix}, \overline{B}^S = \begin{bmatrix} 1&0 \\ 0&1 \\ 0&0 \\ 0&0 \end{bmatrix}, \overline{A}^C = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ -1&0&0&0 \\ 0&-1&0&0 \end{bmatrix}$$

The state feedback gain $\overline{K}^S$ asymptotically stabilizing the controllable part $\overline{A}^S - \overline{B}^S\overline{K}^S$ is obtained by using the Riccati equation as follows.

$$\overline{K}^S = \begin{bmatrix} 1.7321 & 0.0 & 1.0 & 0.0 \\ 0.0 & 1.7321 & 0.0 & 1.0 \end{bmatrix}$$

, from which eigenvalues is calculated as $\sigma(\overline{A}^S - \overline{B}^S\overline{K}^S) = \{-0.8660 \pm 0.5000i, -0.8660 \pm 0.5000i\}$. Accordingly, from (15) we get $K_P = \begin{bmatrix} 1.7321/\cos(x_3^*) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.7321 \end{bmatrix}$, $K_I = \begin{bmatrix} 1/\cos(x_3^*) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Finally, we solve problem (20) by Nelder-Mead's method to obtain $\boldsymbol{m}_0$ which asymptotically stabilize $\boldsymbol{x}^*$.

In Nelder-Mead's method, we set weights in performance function (20a) as $w_1 = 1, w_2 = 1,\ w_3 = 1$, a reflection coefficient α, an expansion coefficient γ, a contraction coefficient β as $\alpha = 1.0,\ \gamma = 1.5,\ \beta = 0.5$, respectively.

Example 1 is a regulation problem with $\boldsymbol{x}^* = \boldsymbol{0}$. An initial state was taken as $\boldsymbol{x}(0) = (5,\ -5,\ \pi/6,\ \pi/4)^T$. The best $\boldsymbol{m}_0$ was obtained as $\boldsymbol{m}_0 = (-10.25,\ -1.754)^T$ with 66 iterations starting from initial simplexes

$$\boldsymbol{m}_0^1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \boldsymbol{m}_0^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ \boldsymbol{m}_0^3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The simulation results are shown in Fig. 2.

Example 2 is a setpoint servo problem with $\boldsymbol{x}^* = (6,\ 5,\ \pi/4,\ 0)^T$. An initial state was taken as $\boldsymbol{x}(0) = \boldsymbol{0}$. The best $\boldsymbol{m}_0$ was obtained as $\boldsymbol{m}_0 = (-31.00,\ 0.5216)^T$ with 70 iterations starting from initial simplexes

$$\boldsymbol{m}_0^1 = \begin{bmatrix} -3 \\ -3 \end{bmatrix},\ \boldsymbol{m}_0^2 = \begin{bmatrix} 3 \\ 0 \end{bmatrix},\ \boldsymbol{m}_0^3 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

The simulation results are shown in Fig. 3.

Example 3 is also a setpoint servo problem with $\boldsymbol{x}^* = (-5,\ -10,\ \pi/4,\ \pi/4)^T$ and $\boldsymbol{x}(0) = \boldsymbol{0}$. The best $\boldsymbol{m}_0$ was obtained as $\boldsymbol{m}_0 = (5.758,\ -3.233)^T$ with 42 iterations starting from initial simplexes

$$\boldsymbol{m}_0^1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix},\ \boldsymbol{m}_0^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ \boldsymbol{m}_0^3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The simulation results are shown in Fig. 4.

4. CONCLUSIONS

We showed that the PI control worked effectively for the symmetric affine system. In particular the manual reset quantity was used effectively for asymptotical stabilization. Note that since our method need not transform the symmetric affine system into the chained form. It can avoid a difficulty of singular point avoidance at the time of inverse transformation.

The proposed method looks like a kind of path planning, because whole trajectory has been calculated when one searches the manual reset quantity $\boldsymbol{m}_0$. However, we consider it is not path planning. In fact the control input is actually manipulated by the PI controller with the best $\boldsymbol{m}_0$. We consider $\boldsymbol{m}_0$ as one parameter in the PI control.

Note that the effective $\boldsymbol{m}_0$ is very sensitive to $\boldsymbol{x}(\boldsymbol{0})$, i.e., a small variation in $\boldsymbol{x}(\boldsymbol{0})$ may lead to a large off set. Thus the optimum $\boldsymbol{m}_0$ must be searched for a precisely measured $\boldsymbol{x}(\boldsymbol{0})$ at each driving.

On the other hand, as mentioned before, an optimum $\boldsymbol{m}_0$ is not unique for a fixed $\boldsymbol{x}(\boldsymbol{0})$, in fact there exist many effective $\boldsymbol{m}_0$, and furthermore the optimum $\boldsymbol{m}_0$ is not necessarily searched at one stage. If the PI control made the plant converge to the undesired equilibrium with the first $\boldsymbol{m}_0$ then repeat the same process at the second stage, considering that equilibrium point as an initial state $\boldsymbol{x}(\boldsymbol{0})$.

Since computational time of Nelder-Mead's method is very short, our method can be practically implemented on-line.

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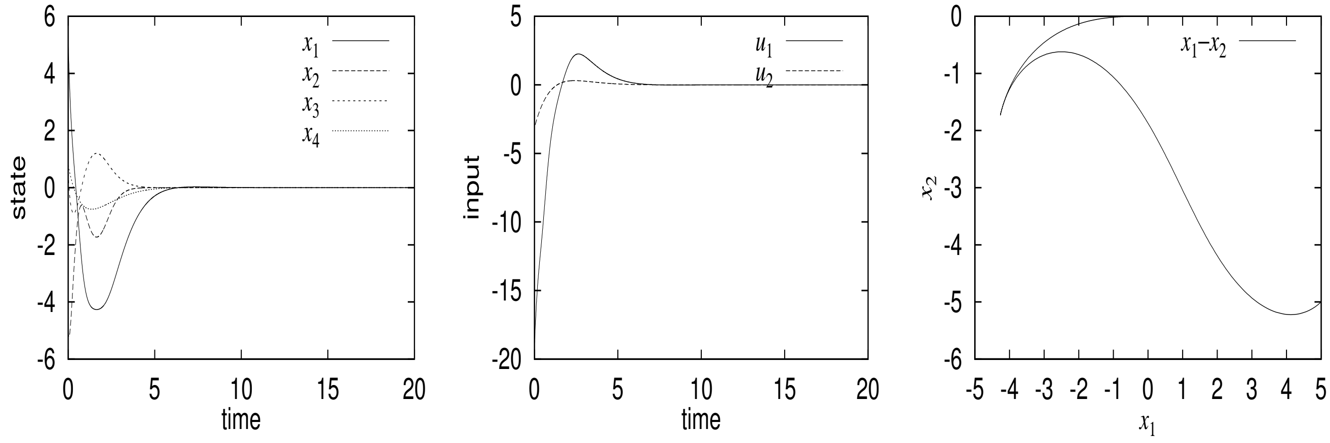

Fig.2 Example 1 (state, input, x-y plane) $(\boldsymbol{x}(0) = (5,\ -5,\ \pi/6,\ \pi/4)^T,\ \boldsymbol{x}^* = \mathbf{0})$

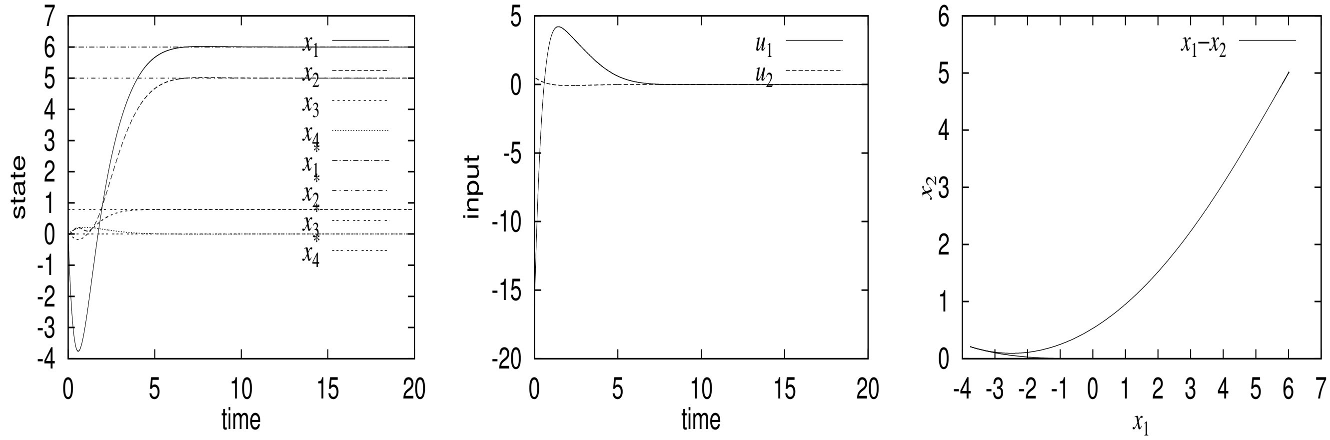

Fig.3 Example 2 (state, input, x-y plane) $(\boldsymbol{x}(0) = \mathbf{0},\ \boldsymbol{x}^* = (6,\ 5,\ \pi/4,\ 0)^T)$

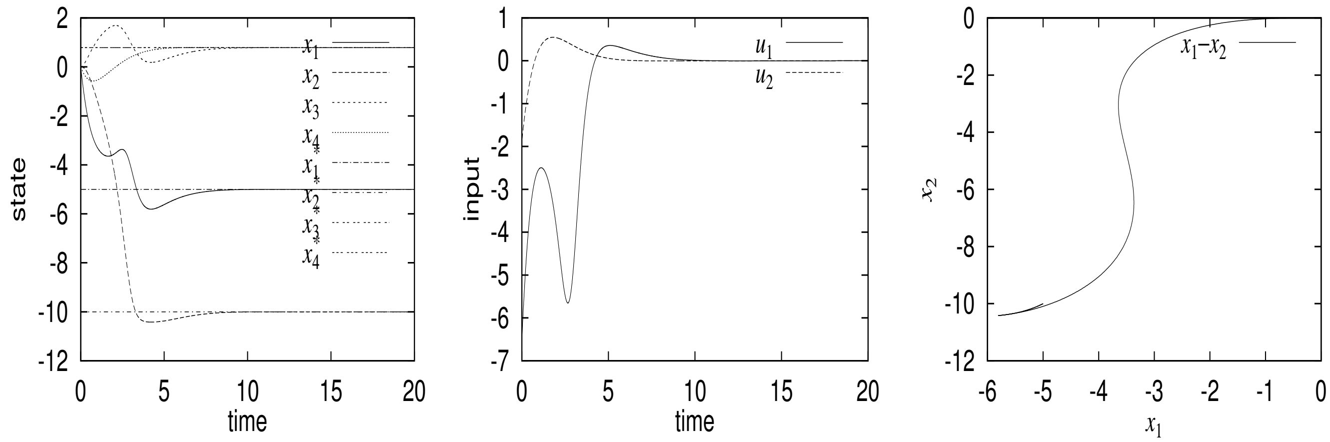

Fig.4 Example 3 (state, input, x-y plane) $(\boldsymbol{x}(0) = \mathbf{0},\ \boldsymbol{x}^* = (-5,\ -10,\ \pi/4,\ \pi/4)^T)$