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High order sliding mode control of a sensorless induction motor

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Abstract: A new High Order Sliding Mode (HOSM) controller and an adaptive observer for sensorless induction motors (IM) drive are designed. The adaptive interconnected observer estimates the fluxes, the angular velocity, the load torque, and the stator resistance. The speed-flux control law is an original HOSM one: a sliding manifold is designed in order to ensure finite-time convergence of sliding variable and its high order time derivatives. The Lyapunov theory is used to prove the stability of the observer then the stability of the "observer-controller". To test the feasbility of the proposed solutions, a significant low frequency benchmark is used by considering the sensorless control problem of IM. Robustness with respect to parameters variations is proved and experimentally verified.

1. INTRODUCTION

Due to cost, fragility of mechanical speed sensors and the difficulty to install sensors in many applications, sensorless induction motor (IM) drives are becoming wide spread solutions for the next generation of commercial drives. However, the IM presents a challenging control problem: it is a complex highly coupled nonlinear system. Two of the states variables (rotor fluxes/mechanical speed) are not usually measurable. Due to heating, the rotor and stator resistances considerably vary with a significant impact on system dynamics. Moreover the load torque is generally unknown.

More often than not, for complex controls like Field Oriented Control (FOC) or Variable Structure Control (VSC), a shaft encoder is necessary. But in the high power range, the mounting of the sensor and its maintenance are difficult: vibrations produced by the high power motor damage the encoder coupling and the speed measure quality. An alternative is then to obtain an estimated speed value (sensorless). A major difficulty of the sensorless algorithm is the estimation of the state variable at low frequencies (Canudas et al. [2000]). Another difficulty is to ensure the robustness against parameter variations. For example the most critical parameter affecting performance at low speed is stator resistance (Holtz [2002]). In Canudas et al. [2000] and Ibarra et al. [2004], the authors demonstrate that the main conditions to lose the observability of IM are: the excitation voltage frequency is zero and the rotor speed is constant. Yet, in the literature, the sensorless algorithms are usually tested and evaluated at high and low speed nevertheless see Ghanes et al. [2006] and Montanari et al. [2006]. But, few studies have highlighted this problem of unobservability (Ghanes et al. [2006]). In this paper the "observer+controller" is tested on a "Sensorless control benchmark". The trajectories of this benchmark are chosen to evaluate the IM sensorless algorithm in observable and unobservable conditions. Unfortunately when the load torque is greater than 20% of the nominal load torque the "observer+controller" becomes unstable.

The concepts and principles of sliding mode (SM) control applied to electrical motors is introduced in Utkin [1993]. The success of this type of control for electric drives, is mainly due to its disturbance rejection, strong robustness and simple implementation. In literature, there is a large number of papers that use this approach for sensorless IM drives (Aurora et al. [2004], Barambones et al. [2004]). All of these papers used the standard approach of SM control. The specific problem of this standard approach is the chattering effect, *i.e* dangerous high-frequency vibrations of the controlled system. For example in Aurora et al. [2004], a time derivative of the currents have been regarded as auxiliary control signal to avoid the problem of chattering. In order to overcome this drawback, and to improve the controller performances, an approach called "High Order Sliding Mode" has been proposed in Bartolini et al. [1998] and Levant [1993]. These HOSM keep the main advantages of the standard sliding mode control, the chattering effect is attenuated and high order precision provided (Levant [1993]).

r^{th} order SM control solutions with finite-time convergence have been proposed in Plestan et al. [2008] and Laghrouche et al. [2004b]. At authors' best knowledge, these methods for sensorless IM drives have never been proposed. In the sequel, the HOSM speed-flux control is based on Plestan et al. [2008].

The contributions of the current paper are

- A HOSM speed-flux controller (Plestan et al. [2008]) in case of "sensorless benchmark",

- An estimation of the speed, the load torque and the stator resistance critical parameter at very low speed (Traoré et al. [2007]),
- A stability proof of the closed-loop system,
- Experimental tests on a significant "Sensorless Control Benchmark" described in Ghanes et al. [2006].

2. INDUCTION MOTOR MODEL

The IM model is based on the motor equation in a rotating **d** and **q** frame (Chiasson [1995])and reads as

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \quad (1)$$

where $x = [i_{sd}\ i_{sq}\ \phi_{rd}\ \phi_{rq}\ \Omega\ T_l\ R_s]^T$, $u = [u_{sd}\ u_{sq}]^T$, $y = [h_1\ h_2]^T = [i_{sd}\ i_{sq}]^T$

$$f(x) = \begin{bmatrix} ba\phi_{rd} + bp\Omega\phi_{rq} - \gamma i_{sd} + \omega_s i_{sq} \\ ba\phi_{rq} - bp\Omega\phi_{rd} - \gamma i_{sq} - \omega_s i_{sd} \\ -a\phi_{rd} + (\omega_s - p\Omega)\phi_{rq} + aM_{sr}i_{sd} \\ -a\phi_{rq} - (\omega_s - p\Omega)\phi_{rd} + aM_{sr}i_{sq} \\ m(\phi_{rd}i_{sq} - \phi_{rq}i_{sd}) - c\Omega - \frac{1}{J}T_l \\ 0 \\ 0 \end{bmatrix}, \quad g(x) = \begin{bmatrix} m_1 & 0 \\ 0 & m_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Let $i_{sd}, i_{sq}, \phi_{rd}, \phi_{rq}, u_{sd}, u_{sq}, \Omega, T_l, \omega_s$ denote respectively the stator currents, the rotor fluxes, the stator voltage inputs, the angular speed, the load torque and stator pulsation. The subscripts s and r respectively refer to stator and rotor. The parameters $a, b, c, \gamma, \sigma, m$ and m_1 are defined by
$a = R_r/L_r$, $b = M_{sr}/\sigma L_s L_r$, $c = f_v/J$, $m_1 = 1/\sigma L_s$
$\gamma = (L_r^2 R_s + M_{sr}^2 R_r)/\sigma L_s L_r^2, \sigma = 1 - (M_{sr}^2/L_s L_r), m = pM_{sr}/JL_r$.
R_s and R_r are the resistances. L_s and L_r are the self-inductances, M_{sr} is the mutual inductance between the stator and rotor windings. p is the number of pole-pair. J is the inertia of the system (motor and load) and f_v is the viscous damping coefficient.The load torque and stator resistance are supposed constant and unknown.
As only currents are measured $(y = (i_{sd}, i_{sq})^T$, an adaptive interconnected observers (Besançon et al. [2006]) for the sensorless IM is used. The IM model (1) may be seen as the interconnection between subsystems (2) and (3). Then, we suppose that each subsystem satisfies some required properties to build an observer. It is also considered that for each observer, the state of the other is available. Equation (1) reads as

$$\begin{bmatrix} \dot{i}_{sd} \\ \dot{\Omega} \\ \dot{R}_s \end{bmatrix} = \begin{bmatrix} 0 & bp\phi_{rq} & -m_1 i_{sd} \\ -m\phi_{rq} & -c & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{sd} \\ \Omega \\ R_s \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{J} \\ 0 \end{bmatrix} T_l + \begin{bmatrix} -\gamma_1 i_{sd} + ab\phi_{rd} + m_1 u_{sd} + \omega_s i_{sq} \\ m\phi_{rd} i_{sq} \\ 0 \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} \dot{i}_{sq} \\ \dot{\phi}_{rd} \\ \dot{\phi}_{rq} \end{bmatrix} = \begin{bmatrix} -\gamma_1 & -bp\Omega & ab \\ 0 & -a & -p\Omega \\ 0 & p\Omega & -a \end{bmatrix} \begin{bmatrix} i_{sq} \\ \phi_{rd} \\ \phi_{rq} \end{bmatrix} + \begin{bmatrix} -m_1 R_s i_{sq} - \omega_s i_{sd} + m_1 u_{sq} \\ \omega_s \phi_{rq} + aM_{sr} i_{sd} \\ -\omega_s \phi_{rd} + aM_{sr} i_{sq} \end{bmatrix}. \quad (3)$$

Then, subsystems (2) and (3) can be viewed as interconnected form

$$\begin{aligned} \dot{X}_1 &= A_1(X_2, y)X_1 + g_1(u, y, X_2, X_1) + \Phi T_l \\ y_1 &= C_1 X_1 \\ \dot{X}_2 &= A_2(X_1)X_2 + g_2(u, y, X_1, X_2) \\ y_2 &= C_2 X_2 \end{aligned} \quad (4)$$

where

$$A_1(\cdot) = \begin{bmatrix} 0 & bp\phi_{rq} & -m_1 i_{sd} \\ -m\phi_{rq} & -c & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ A_2(\cdot) = \begin{bmatrix} -\gamma_1 & -bp\Omega & ab \\ 0 & -a & -p\Omega \\ 0 & p\Omega & -a \end{bmatrix},$$

$$g_1(\cdot) = \begin{bmatrix} -\gamma_1 i_{sd} + ab\phi_{rd} + m_1 u_{sd} + \omega_s i_{sq} \\ m\phi_{rd} i_{sq} \\ 0 \end{bmatrix}, \Phi = \begin{bmatrix} 0 \\ -\frac{1}{J} \\ 0 \end{bmatrix}$$

$$g_2(\cdot) = \begin{bmatrix} -m_1 R_s i_{sq} - \omega_s i_{sd} + m_1 u_{sq} \\ \omega_s \phi_{rq} + aM_{sr} i_{sd} \\ -\omega_s \phi_{rd} + aM_{sr} i_{sq} \end{bmatrix}, \quad \text{and} \quad \gamma_1 = \frac{M_{sr}^2 R_r}{\sigma L_s L_r^2}.$$

$X_1 = [i_{sd}\ \Omega\ R_s]^T$, $X_2 = [i_{sq}\ \phi_{rd}\ \phi_{rq}]^T$ are the states, $u = [u_{sd}\ u_{sq}]^T$ is the input, and $y = [i_{sd}\ i_{sq}]^T$ is the output of the IM model. $C_1 = C_2 = [\ 1\ 0\ 0\ ]$.

Remark 1. For system (4), ω_s is assumed to be known. This assumption is necessary to build the interconnected observer, but is not restrictive because ω_s is provided by the control laws design (for more details, see Section 3).

The following observer is based on the interconnection between several observers satisfying some required properties, in particular the property of input persistency.

Remark 2. A regularly persistence input is an input that sufficiently excites the system in order to guarantee its observability (Besançon et al. [1998]).

In order to design an observer for system (4), one proceeds from the separate synthesis of the observer for each subsystem. By considering the two systems

$$\Sigma_1 \left\{ \begin{aligned} \dot{X}_1 &= A_1(X_2, y)X_1 + g_1(u, y, X_2, X_1) + \Phi T_l \\ y_1 &= C_1 X_1 \end{aligned} \right. \quad (5)$$

$$\Sigma_2 \left\{ \begin{aligned} \dot{X}_2 &= A_2(X_1)X_2 + g_2(u, y, X_1, X_2) \\ y_2 &= C_2 X_2 \end{aligned} \right. \quad (6)$$

where X_1 (resp. X_2) represents the states of Σ_1 (resp. Σ_2). For observers synthesis, X_1 and X_2 are assumed available. Before designing adaptive interconnected observers for subsystem (5)-(6), state the following assumptions

Assumption 1. Consider subsystems (5) and (6) for which (u, X_2) and (u, X_1) are regularly persistent inputs for Σ_1 and Σ_2 respectively.

Remark 3. It is clear that $A_1(\cdot)$ is globally Lipschitz w.r.t. X_2, $A_2(\cdot)$ is globally Lipschitz w.r.t. X_1, and $g_2(\cdot)$ is globally Lipschitz *w.r.t.* X_2, X_1 and uniformly *w.r.t.* (u, y).

Assumption 2. $g_1(\cdot)$ is globally Lipschitz *w.r.t.* X_2, X_1 and uniformly *w.r.t.* (u, y).

Then, assuming that Assumptions 1 and 2 are fulfiiled, a nominal adaptive observer for interconnected systems (4) reads as

$$\begin{aligned} \dot{Z}_1 &= A_1(Z_2, y)Z_1 + g_1(u, y, Z_2, Z_1) + KC_2^T(y_2 - \hat{y}_2) \\ &\quad + (\varpi\Lambda S_3^{-1}\Lambda^T C_1^T + \Gamma S_1^{-1} C_1^T)(y_1 - \hat{y}_1) + \Phi\hat{T}_l \\ \dot{\hat{T}}_l &= \varpi S_3^{-1}\Lambda^T C_1^T(y_1 - \hat{y}_1) + B_1(Z_2)(y_2 - \hat{y}_2) + B_2(Z_2)(y_1 - \hat{y}_1) \\ \dot{S}_1 &= -\theta_1 S_1 - A_1^T(Z_2, y)S_1 - S_1 A_1(Z_2, y) + C_1^T C_1 \\ \dot{S}_3 &= -\theta_3 S_3 + \Lambda^T C_1^T C_1 \Lambda \\ \dot{\Lambda} &= (A_1(Z_2, y) - \Gamma S_1^{-1} C_1^T C_1)\Lambda + \Phi \\ \hat{y}_1 &= C_1 Z_1 \\ \dot{Z}_2 &= A_2(Z_1)Z_2 + g_2(u, y, Z_1, Z_2) + S_2^{-1} C_2^T(y_2 - \hat{y}_2) \\ \dot{S}_2 &= -\theta_2 S_2 - A_2^T(Z_1)S_2 - S_2 A_2(Z_1) + C_2^T C_2 \\ \hat{y}_2 &= C_2 Z_2 \end{aligned} \quad (7)$$

where $Z_1 = \left[\hat{i}_{sd}\ \hat{\Omega}\ \hat{R}_s\right]^T$, $Z_2 = \left[\hat{i}_{sq}\ \hat{\phi}_{rd}\ \hat{\phi}_{rq}\right]^T$ are the estimated states. $\theta_1, \theta_2, \theta_3$ are positive constants, S_1, S_2 symmetric positive definite matrices (Besançon et al. [1998]), $S_3(0) > 0$, $B_1(Z_2) = km\hat{\phi}_{rd}$, $B_2(Z_2) = -km\hat{\phi}_{rq}$,

$$K = \begin{bmatrix} -k_{c1} & 0 & 0 \\ -k_{c2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha \end{bmatrix}$$

where k, k_{c1}, k_{c2}, α and ϖ are positive constants.

Remark 4. Note that

$$\begin{aligned} & B_1(Z_2)(y_2 - \hat{y}_2) + B_2(Z_2)(y_1 - \hat{y}_1) \\ & \equiv k[m(\hat{\phi}_{rd} i_{sq} - \hat{\phi}_{rq} i_{sd}) - m(\hat{\phi}_{rd} \hat{i}_{sq} - \hat{\phi}_{rq} \hat{i}_{sd})] \\ & \equiv k(T_e - \tilde{T}_e) \end{aligned}$$

with T_e and $\tilde{T}_e$ respectively the "measured" and "estimated" electromagnetic torques.

In order to prove observers convergence, define $\epsilon = X_1 - Z_1, \;\; \epsilon_2 = X_2 - Z_2, \;\; \epsilon_3 = T_l - \hat{T}_l$

Following the same idea as Zhang [2002], and applying the transformation $\epsilon_1 = \epsilon - \Lambda\epsilon_3$, it yields: $\dot{\epsilon}_1 = \dot{\epsilon} - \Lambda\dot{\epsilon}_3 - \dot{\Lambda}\epsilon_3$.

As Traoré et al. [2007], the estimation error dynamics under uncertainties parameters read as

$$\begin{aligned} \dot{\epsilon}_1 &= [A_1(Z_2, y) - \Gamma S_1^{-1} C_1^T C_1 + B_{21}]\epsilon_1 + g_1(u, y, X_2, X_1) \\ &\quad + \Delta g_1(u, y, X_2, X_1) - g_1(u, y, Z_2, Z_1) + (B_{12} - K')\epsilon_2 \\ &\quad + B_{22}\epsilon_3 + [A_1(X_2, y) + \Delta A_1(X_2, y) - A_1(Z_2, y)]X_1 \\ \dot{\epsilon}_2 &= [A_2(Z_1) - S_2^{-1} C_2^T C_2]\epsilon_2 + [A_2(X_1) + \Delta A_2(X_1) - A_2(Z_1)]X_2 \\ &\quad + g_2(u, y, X_1, X_2) - g_2(u, y, Z_1, Z_2) + \Delta g_2(u, y, X_1, X_2) \\ \dot{\epsilon}_3 &= -[\varpi S_3^{-1} \Lambda^T C_1^T C_1 \Lambda + B_2']\epsilon_3 - [\varpi S_3^{-1} \Lambda^T C_1^T C_1 + B_2'']\epsilon_1 - B_1'\epsilon_2 \end{aligned} \quad (8)$$

where the terms $\Delta A_1(\cdot)$, $\Delta A_2(\cdot)$, $\Delta g_1(\cdot)$ and $\Delta g_2(\cdot)$ contain the uncertainties $A_1(\cdot)$, $A_2(\cdot)$, $g_1(\cdot)$, $g_2(\cdot)$ respectively, and $B_{21} = \Lambda B_2(Z_2)C_1$, $B_{12} = \Lambda B_1(Z_2)C_2$, $B_{22} = \Lambda B_2(Z_2)C_1\Lambda$, $B_2' = B_2(Z_2)C_1\Lambda$, $B_2'' = B_2(Z_2)C_1$, $B_1' = B_1(Z_2)C_2$, $K' = KC_2^T C_2$.

Let $V_o = V_1 + V_2 + V_3$ define a candidate Lyapunov function with $V_1 = \epsilon_1^T S_1 \epsilon_1$, $V_2 = \epsilon_2^T S_2 \epsilon_2$ and $V_3 = \epsilon_3^T S_3 \epsilon_3$. Next, by taking the time derivative of V_o and replacing the suitable expressions (Traoré et al. [2007]) one gets

$$\dot{V}_o \leq -(1 - \varsigma)\delta V, \;\; \forall \|\epsilon\| \geq \frac{\mu}{\varsigma\delta}. \quad (9)$$

Then, the observer asymptotically converges to zero as $\|\epsilon\| \geq \frac{\mu}{\varsigma\delta}$.

Remark 5. The difference between the observer in Traoré et al. [2007] and the proposed observer is the estimation of the stator resistance value which is a critical parameter at low speed. This increases the robustness of the adaptive interconnected observer with respect to parameter variations of the IM.

3. HIGH ORDER SLIDING MODE CONTROL

After applied flux oriented control strategy (Blaschke [1972]), let us denote by Ω^* and ϕ^* the smooth bounded reference signals for the output variables to respectively control the speed Ω and the rotor flux modulus $\sqrt{\phi_{rd}^2 + \phi_{rq}^2}$. Following the strategy of field oriented control ($\phi_{rd} = \sqrt{\phi_{rd}^2 + \phi_{rq}^2}$, $\phi_{rq} = 0$), then the electromagnetic torque

$$T_e = \frac{pM_{sr}}{L_r}\phi_{rd} i_{sq} \quad (10)$$

is proportional to the product of two state variables ϕ_{rd} and i_{sq}. From (10), it is observed that by holding constant the magnitude of the rotor flux, there is a linear relationship between the variable i_{sq} and the speed dynamic.

Before carrying on the design of the controllers, let us first examine how to estimate the stator frequency (ω_s). For the flux oriented field $\phi_{rq} \equiv 0$, so that $\omega_s = p\Omega + a\frac{M_{sr}}{\phi_{rd}} i_{sq}$.

To avoid the uncertainties of IM parameters in the observer and achieve our goal ($\phi_{rq} \equiv 0$), we define

$$\tilde{\omega}_s = p\hat{\Omega} + a\frac{M_{sr}}{\hat{\phi}_{rd}} i_{sq} - \frac{(i_{sq} - \hat{i}_{sq})}{\beta_1 \hat{\phi}_{rd}} k_{\omega_s} \quad (11)$$

where $\tilde{\omega}_s$ is an estimate stator frequency, $\beta_1 = M_{sr}/\sigma L_s L_r$ and $k_{\omega_s} > 0$. The main objective is to control the speed and flux of induction motor by using high order SM controller.

3.1 High order sliding mode controller

Problem formulation. Consider a nonlinear system (1). For a sake of clarity, only single output-single output case is considered in the sequel. Let $\sigma_c(x, t)$ ($x \in I\!R^n$) the state variable) the sliding variable with a relative degree equal to r.

H1. The relative degree r of (1) with respect to σ_c is assumed to be constant and known, and the associated zero dynamics are stable.

The control objective is to fulfill the constraint $\sigma_c(x, t) = 0$ in finite time and to keep it exactly by some feedback.

The r^{th} order sliding mode control approach allows the finite time stabilization to zero of the sliding variable σ_c and its $r - 1$ first time derivatives by defining a suitable discontinuous control function. Then, the output σ_c satisfies equation (Levant [2005])

$$\sigma_c^{(r)} = \varphi_1(x, t) + \varphi_2(x)u \quad (12)$$

with $\varphi_2(x) = L_g L_f^{r-1}\sigma_c$, $\varphi_1(x) = L_f^r \sigma_c$. Assume that

H2. The solutions are understood in the Filippov sense (Filippov [1988]), and system trajectories are supposed to be infinitely extendible in time for any bounded Lebesgue measurable input.

H3. Functions $\varphi_1(x, t)$ and $\varphi_2(x)$ are bounded uncertain functions, and, without loss of generality, let also the sign of $\varphi_2(x)$ be strictly positive. Thus, there exist positive constants $K_m > 0$, $K_M > 0$ and $C_0 \geq 0$ such that $0 < K_m < \varphi_2(x) < K_M$ and $|\varphi_1(x, t)| \leq C_0$ for $x \in \mathcal{X} \subset I\!R^n$, $\mathcal{X}$ being a bounded open subset of $I\!R^n$ within which the boundedness of the system dynamics is ensured, and $t > 0$.

Then, the r^{th} order sliding mode control of (1) with respect to the sliding variable σ is equivalent to the finite time stabilization of

$$\dot{Z}_{c1} = A_{11}Z_{c1} + A_{12}Z_{c2} \quad (13)$$

$$\dot{Z}_{c2} = \varphi_1 + \varphi_2 u \quad (14)$$

with $Z_{c1} := [\sigma_c \; \dot{\sigma}_c \; \cdots \; \sigma_c^{(r-2)}]^T$, $Z_{c2} = \sigma_c^{(r-1)}$. $A_{11(r-1)\times(r-1)}$ and $A_{12(r-1)\times 1}$ are such that Z_{c1} dynamics read as linear Brunovsky form.

Controller synthesis. The synthesis of a high order sliding mode controller for (1) consists in two steps

- A linear continuous finite-time convergent control law is used in order to *induce linear reference trajectories* for system (13), which defines the sliding manifold on which the system evolves as early as $t = 0$.
- A discontinuous control law u is designed in order to maintain the system trajectories on the sliding manifold which ensures the establishment of a r^{th} order sliding mode.

Switching variable. Let S denote the switching variable defined as

$$\begin{aligned} S = \sigma_c^{(r-1)} &- \mathcal{F}^{(r-1)}(t) + \lambda_{r-2}[\sigma_c^{(r-2)} - \mathcal{F}^{(r-2)}(t)] \\ &+ \cdots + \lambda_0[\sigma_c(x, t) - \mathcal{F}(t)], \end{aligned} \quad (15)$$

with $\lambda_{r-2}, \cdots, \lambda_0$ defined such that $P(z) = z^{(r-1)} + \lambda_{r-2}z^{(r-2)} + \cdots + \lambda_0$ is a Hurwitz polynomial in the complex variable z. The function $\mathcal{F}(t)$ is a C^r one defined such that $S(t = 0) = 0$ and $\sigma_c^{(k)}(x(t_f), t_f) - \mathcal{F}^{(k)}(t_f) = 0 (0 \leq k \leq r - 1)$.

Then, from initial and final conditions the problem consists in finding the function $\mathcal{F}(t)$ such that

$$\begin{array}{l} \sigma_c(x(0),0)=\mathcal{F}(0), \quad \sigma_c(x(t_f),t_f)=\mathcal{F}(t_f)=0, \\ \dot{\sigma}_c(x(0),0)=\dot{\mathcal{F}}(0), \quad \dot{\sigma}_c(x(t_f),t_f)=\dot{\mathcal{F}}(t_f)=0, \\ \vdots \\ \sigma_c^{(r-1)}(x(0),0)=\mathcal{F}^{(r-1)}(0), \quad \sigma_c^{(r-1)}(x(t_f),t_f)=\mathcal{F}^{(r-1)}(t_f)=0 \end{array} \tag{16}$$

A solution for $\mathcal{F}(t)$ reads as $(1 \leq j \leq r)$ (Plestan et al. [2008])

$$\mathcal{F}(t) = K_c T e^{Ft} \sigma_c^{(r-j)}(0) \tag{17}$$

with F being a $2r \times 2r$-dimensional stable matrix (strictly negative eigenvalues), T being a $2r \times 1$- dimensional vector.

H4. There exists an integer j such that $\sigma_c^{(r-1)}(0) \neq 0$ and bounded.

Lemma 1. Given Hypothesis H4 and $t_F > 0$ bounded, there exists a Hurwitz matrix $F_{2r\times 2r}$ and a matrix $T_{2r\times 1}$ such that matrix $\mathcal{K}$ defined as

$$\begin{array}{l} \mathcal{K} = [F^{r-1}T\sigma_c^{(r-j)}(0) \; F^{r-1}e^{Ft_f}T \; F^{r-2}T\sigma_c^{(r-j)}(0) \\ \quad F^{r-2}e^{Ft_f}T \cdots \; T\sigma_c^{(r-j)}(0) \; e^{Ft_f}T] \end{array} \tag{18}$$

is invertible.

K_c is a $1 \times 2r$-dimensional gain matrix such system (16) is fulfilled. Then, one gets

$$K_c = \left[\sigma_c^{(r-1)}(0) \; 0 \; \sigma_c^{(r-2)}(0) \; 0 \; \cdots \; \sigma_c(0) \; 0\right] \cdot \mathcal{K}^{-1} \tag{19}$$

Then, S, the switching variable, reads as

$$\begin{array}{l} S = \sigma_c^{(r-1)} - K_c T F^{(r-1)} e^{Ft} \sigma_c^{(r-j)}(0) + \lambda_{r-2}[\sigma_c^{(r-2)} \\ \quad -K_c T F^{(r-2)} e^{Ft} \sigma_c^{(r-j)}(0)] + \cdots + \lambda_0[\sigma_c(x,t) - K_c T e^{Ft} \sigma_c^{(r-j)}(0)], \end{array}$$

H5. There exists a finite positive constant $\Theta > 0$ such that

$$\begin{array}{l} |K_c T F^r e^{Ft} \sigma_c^{(r-j)}(0) - \lambda_{r-2}[\sigma_c^{(r-1)} - K_c T F^{r-1} e^{Ft} \\ \sigma_c^{(r-j)}(0)] - \cdots - \lambda_0[\dot{\sigma}_c(x,t) - K_c T F e^{Ft} \sigma_c^{(r-j)}(0)]| < \Theta \end{array} \tag{20}$$

Equation $S = 0$ describes the desired dynamics which satisfy the finite time stabilization of $[\sigma_c^{(r-1)} \sigma_c^{(r-2)} \; \cdots \; \sigma_c]^T$ to zero. Then, the *switching manifold* on which system (13) is forced to slide on via a discontinuous control v, is defined as:

$$\mathcal{S} = \{x | S = 0\} \tag{21}$$

Given equation (19), one gets $S(t=0) = 0$: at the initial time, the system still evolves on the switching manifold. There is no reaching phase in opposition to previous approaches as in Laghrouche et al. [2004b].

Controller design. The attention is now focused on the design of the discontinuous control law u which forces the system trajectories of (13) to slide on $\mathcal{S}$ in order to reach the origin in finite time and then to maintain the system at the origin.

Theorem 1. (Plestan et al. [2008]). Consider system (1) with a relative degree r with respect to $\sigma_c(x,t)$. Suppose that it is minimum phase with respect to $\sigma_c(x,t)$ and that hypotheses H1, H2, H3 and H4 are fulfilled. Let r be the sliding mode order and t_f $(0 < t_f < \infty)$ the desired convergence time. Let $S \in IR^n$ define by (20) with K_c being the single solution (19) and suppose that assumption H5 is fulfilled. Then, the control input u defined by $u = -\alpha_c \, \mathrm{sign}(S)$ with

$$\alpha_c \geq \frac{C_0 + \Theta + \eta}{K_m}, \tag{22}$$

C_0, K_m defined in assumption H3, Θ defined by (20), leads to the establishment of a r^{th} order sliding mode with respect to σ_c. The convergence time is t_f.

3.2 Application to induction motor

The objective consists in designing a robust (with respect to uncertainties/disturbances) flux and speed controller. Define σ_ϕ and σ_Ω, the sliding variable, as $\sigma_\phi = \phi_{rd} - \phi^*$ and $\sigma_\Omega = \Omega - \Omega^*$. From (1), the relative degree of σ_ϕ and σ_Ω with respect to u equals 2 ($r = 2$), which implies that at least a 2^{nd} order SM controller is designed for the flux and speed. In order to avoid the "chattering" effect and to improve the robustness of the controller, according to previous design, 3^{rd} order HOSM controllers are designed for the two outputs, which means that the discontinuous term is applied to $\sigma_\phi^{(3)}$ and $\sigma_\Omega^{(3)}$ trough $\dot{u}$. Then, the chattering effect is decreasing in the control input. From (1), it yields

$$\begin{bmatrix} \phi_{rd}^{(2)} \\ \Omega^{(2)} \end{bmatrix} = \begin{bmatrix} \varphi_{\alpha_1} \\ \varphi_{\alpha_2} \end{bmatrix} + \varphi_\beta \begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix} \tag{23}$$

where

$$\begin{array}{l} \varphi_{\alpha_1} = -a\dot{\phi}_{rd} + (\omega_s - p\Omega)\dot{\phi}_{rq} + aM_{sr}(ba\phi_{rd} + bp\Omega\phi_{rq} - \gamma i_{sd} \\ +\omega_s i_{sq}) + \dfrac{aM_{sr}[(ba\phi_{rq} - bp\Omega\phi_{rd} - \gamma i_{sq} - \omega_s i_{sd})\phi_{rd} - i_{sq}\dot{\phi}_{rd}]}{\phi_{rd}^2}\phi_{rq} \\ \varphi_{\alpha_2} = m[\dot{\phi}_{rd} i_{sq} + \phi_{rd}(ba\phi_{rq} - bp\Omega\phi_{rd} - \gamma i_{sq} - \omega_s i_{sd}) \\ -\dot{\phi}_{rq} i_{sd} - \phi_{rq}(ba\phi_{rd} + bp\Omega\phi_{rq} - \gamma i_{sd} + \omega_s i_{sq})] - c\dot{\Omega} - \dfrac{\dot{T}_l}{J} \end{array}$$

$$\varphi_\beta = \begin{bmatrix} aM_{sr}m_1 & aM_{sr}m_1\dfrac{\phi_{rq}}{\phi_{rd}} \\ -mm_1\phi_{rq} & mm_1\phi_{rd} \end{bmatrix}. \tag{24}$$

As there are uncertainties on several parameters, one supposes that the previous terms read as

$$\begin{array}{rcl} \varphi_{\alpha 1} &=& \varphi_{\alpha 1}^{Nom} + \Delta\varphi_{\alpha 1} \\ \varphi_{\alpha 2} &=& \varphi_{\alpha 2}^{Nom} + \Delta\varphi_{\alpha 2} \\ \varphi_\beta &=& \varphi_\beta^{Nom} + \Delta\varphi_\beta \end{array} \tag{25}$$

such that $\varphi_{\alpha 1}^{Nom}$, $\varphi_{\alpha 2}^{Nom}$ and φ_β^{Nom} are the well-known nominal terms whereas $\Delta\varphi_{\alpha 1}$, $\Delta\varphi_{\alpha 2}$ and $\Delta\varphi_\beta$ contain all the uncertainties due to parameters variations and disturbance. The control input u reads as (note that matrix φ_β^{Nom} is invertible on the work domain $(\phi_{rd} \neq 0)$) [1]

$$\begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix} = \varphi_\beta^{Nom^{-1}} \left[-\begin{bmatrix} \varphi_{\alpha_1}^{Nom} \\ \varphi_{\alpha_2}^{Nom} \end{bmatrix} + \begin{bmatrix} \nu_{sd} \\ \nu_{sq} \end{bmatrix} \right] \tag{26}$$

From (23-26), switching variables dynamics read as

$$\begin{bmatrix} \phi_{rd}^{(2)} \\ \Omega^{(2)} \end{bmatrix} = \Psi_\alpha + \Psi_\beta \begin{bmatrix} \nu_{sd} \\ \nu_{sq} \end{bmatrix}$$

with Ψ_α and Ψ_β supposed to be uncertain bounded C^1-functions. Then, one gets

$$\begin{bmatrix} \sigma_\phi^{(3)} \\ \sigma_\Omega^{(3)} \end{bmatrix} = \underbrace{\dot{\Psi}_\alpha + \dot{\Psi}_\beta \begin{bmatrix} \nu_{sd} \\ \nu_{sq} \end{bmatrix} - \begin{bmatrix} \phi^{*(3)} \\ \Omega^{*(3)} \end{bmatrix}}_{\varphi_1} + \underbrace{\Psi_\beta}_{\varphi_2} \begin{bmatrix} \dot{\nu}_{sd} \\ \dot{\nu}_{sq} \end{bmatrix}$$

Note that previous system has the same form than system (12). As mentionned in previous subsection, the control law synthesis is made in 2 steps: the design of the switching

[1] The interest of a such feedback has been detailed in Castro et al. [2004]: it allows to minimize gain values of the control discontinuous function.

variable and the discontinuous input.

Switching vector. From (20) and Theorem 1, the switching vector reads as

- **For** $t \leq t_F$.

$$S_\phi = \sigma_\phi^{(2)} - \chi_\phi, \quad S_\Omega = \sigma_\Omega^{(2)} - \chi_\Omega$$

where

$$\begin{cases} \chi_\phi = K_\phi F^2 e^{Ft} T\sigma_\phi(0) - 2\zeta_\phi \omega_{n\phi}(\dot{\sigma}_\phi \\ \quad -K_\phi F e^{Ft} T\sigma_\phi(0)) - \omega_{n\phi}^2(\sigma_\phi - K_\phi e^{Ft} T\sigma_\phi(0)) \\ \chi_\Omega = K_\Omega F^2 e^{Ft} T\sigma_\Omega(0) - 2\zeta_\Omega \omega_{n\Omega}(\dot{\sigma}_\Omega \\ \quad -K_\Omega F e^{Ft} T\sigma_\Omega(0)) - \omega_{n\Omega}^2(\sigma_\Omega - K_\Omega e^{Ft} T\sigma_\Omega(0)) \end{cases}$$

- **For** $t > t_F$.

$$S_\phi = \sigma_\phi^{(2)} + 2\zeta_\phi \omega_{n\phi} \dot{\sigma}_\phi + \omega_{n\phi}^2 \sigma_\phi$$

$$S_\Omega = \sigma_\Omega^{(2)} + 2\zeta_\Omega \omega_{n\Omega} \dot{\sigma}_\Omega + \omega_{n\Omega}^2 \sigma_\Omega$$

with $K_\phi = \left[\sigma_\phi^{(2)}(0)\ 0\ \dot{\sigma}_\phi(0)\ 0\ \sigma_\phi(0)\ 0\right].\mathcal{K}_\phi^{-1}$,

$K_\Omega = \left[\sigma_\Omega^{(2)}(0)\ 0\ \dot{\sigma}_\Omega(0)\ 0\ \sigma_\Omega(0)\ 0\right].\mathcal{K}_\Omega^{-1}$ and

$$\mathcal{K}_\phi = \left[F^2 T\sigma_\phi(0) \quad F^2 e^{Ft_f} T \quad FT\sigma_\phi(0) \quad Fe^{Ft_f} T \quad T\sigma_\phi(0) \quad e^{Ft_f} T\right]$$

$$\mathcal{K}_\Omega = \left[F^2 T\sigma_\Omega(0) \quad F^2 e^{Ft_f} T \quad FT\sigma_\Omega(0) \quad Fe^{Ft_f} T \quad T\sigma_\Omega(0) \quad e^{Ft_f} T\right]$$

with F and T tuned from Lemma 1 (for details, see Plestan et al. [2008]).

Discontinuous input. The control discontinuous input reads as

$$\begin{bmatrix} \dot{\nu}_{sd} \\ \dot{\nu}_{sq} \end{bmatrix} = \begin{bmatrix} -\alpha_\phi.\mathrm{sign}(S_\phi) \\ -\alpha_\Omega.\mathrm{sign}(S_\Omega) \end{bmatrix} \tag{27}$$

From (27), it yields

$$\begin{bmatrix} \dot{S}_\phi \\ \dot{S}_\Omega \end{bmatrix} = \varphi_1 + \varphi_2 \cdot \dot{\nu} - \begin{bmatrix} \chi_\phi \\ \chi_\Omega \end{bmatrix} \tag{28}$$

4. STABILITY ANALYSIS OF THE OBSERVER-CONTROLLER SCHEME

Recalling that the main goal of this paper is to synthetize a robust sensorless control of induction motor, the speed and the flux are not measurable, and the load torque is considered as an unknown perturbation. It yields that it is necessary to replace speed and flux measurements, and the stator resistance in (26) by their estimated values. Then, one gets

$$\begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix} = \hat{\varphi}_\beta^{Nom^{-1}} \left[- \begin{bmatrix} \hat{\varphi}_{\alpha_1}^{Nom} \\ \hat{\varphi}_{\alpha_2}^{Nom} \end{bmatrix} + \begin{bmatrix} \nu_{sd} \\ \nu_{sq} \end{bmatrix} \right] \tag{29}$$

where $\hat{\varphi}_\beta$, $\hat{\varphi}_{\alpha_1}^{Nom}$ and $\hat{\varphi}_{\alpha_2}^{Nom}$ are respectively the *estimated* (*i.e.* using the estimated values given by observer(7)) values of φ_β, $\varphi_{\alpha_1}^{Nom}$ and $\varphi_{\alpha_2}^{Nom}$.

Remark 6. In order to avoid a singularity problem in (29), the observer is initialized with a flux initial condition different to zero, such that (29) is well-defined. This condition is a physical condition for IM (no flux implies no torque!).Moreover, the SM controller allows to guarantee that ϕ_{rd} reaches its reference ϕ^* in a finite time. Thus, before the motor is fluxed (i.e $\phi_{rd} = \phi^*$), the speed reference is kept to zero.

The sliding variables become $\sigma_{\hat{\phi}} = \hat{\phi}_{rd} - \phi^*$, $\sigma_{\hat{\Omega}} = \hat{\Omega} - \Omega^*$

Taking $\epsilon_{\phi_{rd}} = \phi_{rd} - \hat{\phi}_{rd}$ and $\epsilon_\Omega = \Omega - \hat{\Omega}$ the flux and speed estimation error, it yields

$\sigma_{\hat{\phi}} = \phi_{rd} - \phi^* - \epsilon_{\phi_{rd}} = \sigma_\phi - \epsilon_{\phi_{rd}}$, $\sigma_{\hat{\Omega}} = \Omega - \Omega^* - \epsilon_\Omega = \sigma_\Omega - \epsilon_\Omega$

Let $S_{\hat{\phi}}$ and $S_{\hat{\Omega}}$ define the new switching vector; then their dynamics reads as are:

$$\begin{aligned} \dot{S}_{\hat{\phi}} &= \dot{S}_\phi - [\epsilon_{\phi_{rd}}^{(3)} + 2\zeta_\phi \omega_{n\phi} \epsilon_{\phi_{rd}}^{(2)} + \omega_{n\phi}^2 \dot{\epsilon}_{\phi_{rd}}] \\ \dot{S}_{\hat{\Omega}} &= \dot{S}_\Omega - [\epsilon_\Omega^{(3)} + 2\zeta_\phi \omega_{n\Omega} \epsilon_\Omega^{(2)} + \omega_{n\Omega}^2 \dot{\epsilon}_\Omega] \end{aligned} \tag{30}$$

From (28) and (30), one gets:

$$\begin{bmatrix} \dot{S}_{\hat{\phi}} \\ \dot{S}_{\hat{\Omega}} \end{bmatrix} = \bar{\varphi}_1 + \bar{\varphi}_2 \cdot \dot{\nu} - \begin{bmatrix} \chi_\phi \\ \chi_\Omega \end{bmatrix} \tag{31}$$

where

$$\bar{\varphi}_1 = \varphi_1 - \begin{bmatrix} \epsilon_{\phi_{rd}}^{(3)} + 2\zeta_\phi \omega_{n\phi} \epsilon_{\phi_{rd}}^{(2)} + \omega_{n\phi}^2 \dot{\epsilon}_{\phi_{rd}} \\ \epsilon_\Omega^{(3)} + 2\zeta_\phi \omega_{n\Omega} \epsilon_\Omega^{(2)} + \omega_{n\Omega}^2 \dot{\epsilon}_\Omega \end{bmatrix}, \quad \bar{\varphi}_2 = \varphi_2$$

By using the same method as **Theorem 1**, it yields that there exist gains α_ϕ and α_Ω such that $\dot{S}_{\hat{\phi}} S_{\hat{\phi}} \leq -\eta_{\hat{\phi}} |S_{\hat{\phi}}|$ and $\dot{S}_{\hat{\Omega}} S_{\hat{\Omega}} \leq -\eta_{\hat{\Omega}} |S_{\hat{\Omega}}|$.

5. EXPERIMENTAL RESULTS

In this section, in order to show the feasability of the proposed approach, experimental results of previous control and observer are displayed. The motor parameters and identified parameters values of the set-up are

Nominal rate power	1.5kW
Nominal angular speed	1430 rpm
Number of pole pairs	2
Nominal voltage	220 V
Nominal current	6.1 A

R_s	1.47Ω	M_{sr}	$0.094H$
R_r	0.79Ω	J	$0.0077Kg.m^2$
L_s	$0.105H$	f_v	$0.0029Nm/rad/s$
L_r	$0.094H$	ϕ^*	$0.595Wb$

The observer parameters are chosen as $\alpha = 50$, $\varpi = 10$, $k = 0.16$, $k_{c1} = 250$, $k_{c2} = 0.5$, $k_{\omega_s} = 60$, $\theta_1 = 5000$, $\theta_2 = 7000$, $\theta_3 = 10e - 12$ to satisfy convergence conditions.

In order to optimize the behaviour and the performances of the motor, and due to technical reasons, two parameters tuning have been chosen: the first one has been chosen to induce the reaching of the motor flux, the second one to reject perturbation (such as load torque) and to ensure high level accuracy for the trajectory tracking. Then, the SM controller parameters are chosen such that $t_f = 0.3sec$ and

- $t \leq 5$ *sec.* $\zeta_\phi = 0.35$, $\omega_{n\phi} = 316$ rad/s, $\alpha_\phi = 6.10^4$, $\zeta_\Omega = 1.56$, $\omega_{n\Omega} = 32$ rad/s, $\alpha_\Omega = 8.10^4$,
- $t > 5$ *sec.* $\zeta_\phi = 0.35$, $\omega_{n\phi} = 447$ rad/s, $\alpha_\phi = 15.10^4$, $\zeta_\Omega = 0.7$, $\omega_{n\Omega} = 200$ rad/s, $\alpha_\Omega = 8.10^6$

For the experiment, only stator currents are measured. Rotor speed and flux amplitude are provided by the observer (7) whereas flux angle is provided by the estimator (11). Stator resistance observer is initialized as $R_{s0} = 1.9$ *ohm*. The experimental sampling time T equals $200\mu s$.

The experimental results of the nominal case with identified parameters (except stator resistance) are shown in Fig. 1. These figures show the good performance of the complete " Observer+Controller" system in trajectory tracking and perturbation rejection (load torque). In terms of trajectory tracking, we note that the estimated motor speed (Fig. 1.b) converges to the measured speed (Fig. 1.a) near and under unobservable conditions. It is the same conclusion for the estimated flux (Fig. 1.f) with respect to the reference flux (Fig. 1.e). The estimated load torque (Fig. 1.d) converges to the measured load torque (Fig. 1.c), in observable and unobservable conditions (between 7 and 9 sec). Nevertheless, it appears a small static error when the motor speed increases (between 4 and 6 sec). In terms of perturbation rejection, we have noted that the load torque is well rejected excepted at the time when it is applied (see (Fig. 1.h&j) at time 1.5s and 5s) and when it is removed (see (Fig. 1.h&j), at time 2.5s).

The robustness of the "Observer+Controller" is confirmed by the result obtained with rotor resistance variation (+50%) and (-50%) applied to the observer and controller parameters (Fig. 2 and Fig. 3). These figures display similar experimental results that for rotor resistance nominal case under observable conditions. To conclude, we can say that the increase of the rotor resistance value doesn't affect the performance of the speed trajectories tracking in observable conditions. It appears a static error when the motor is under unobservable condition (between 7 and 9 sec), see (Fig. 2.a&b and 3.a&b). Moreover, the static error increases transitory when the load torque is applied at time 1.5s and 5s see (Fig. 2.h&j and 3.h&j).

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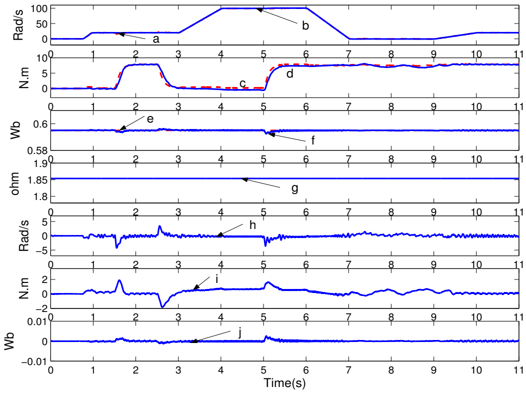

Fig. 1. Experimental result in nominal case**.

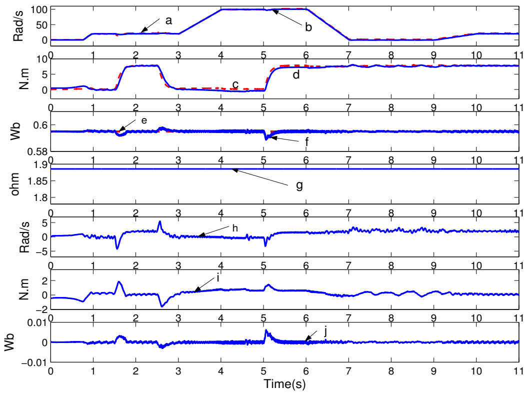

Fig. 2. Experimental result with rotor resistance variation (+50%)**.

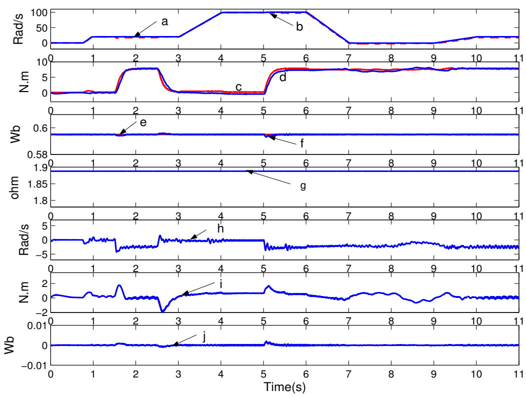

Fig. 3. Experimental result with rotor resistance variation (-50%)**.

** : **a, c: measured speed and load torque, e: reference flux, b, d, f, g: estimated speed, load torque, flux and stator resistance, h, i, j: speed, load torque and flux estimation error.**