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Disturbance Attenuation in Hamiltonian Systems via Direct Discrete-Time Design

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Abstract: The discrete-time disturbance attenuation problem for a class of Hamiltonian systems is considered. In order to give a sufficient condition for the solution of the considered problem, firstly an appropriate discrete gradient is proposed, which enables the derivation of the discrete time version of the given Hamiltonian systems. The disturbance attenuation problem characterised by means of L_2 gain is redefined in the discrete-time setting. The proposed direct discrete-time design method is used to solve the disturbance attenuation problem for the double pendulum system in simulations.

I. INTRODUCTION

In last decades, there have been researches on the subject of the modelling and control of complex nonlinear systems, especially where electrical and mechanical sub-systems have to be considered together. The port-controlled Hamiltonian (PCH) approach has been introduced for modelling and control of this type of nonlinear systems. Since the total energy of the system, namely the Hamiltonian function of the system, which is naturally utilized in PCH approach, could be used as a Lyapunov function for the system, this approach in the design of a control rule would be relatively simpler than other approaches (A. van der Schaft, 2000, R. Ortega *et al*., 2002, R. Ortega *et al*. 2004).

On the other hand, the disturbance attenuation problem and the design of controllers under parametric and/or structural uncertainties are important issues in practical applications. In the literature, the H_∞ approach has been used to solve the disturbance attenuation problem and to provide robust control for nonlinear systems. Specially, the problem of the local disturbance attenuation for continuous nonlinear systems has been studied comprehensively, and (Isidori and Astolfi, 1992, Isidori and Kang, 1995, Shen and Tamura, 1995, Lu *et al,* 2000) can be referred on this subject. While the disturbance attenuation problem characterised by means of the so-called L_2 gain of a general non-linear system is required to solve the Hamilton-Jacobs-Issac (HJI) partial differential inequality, the same problem for Hamiltonian systems can be reduced to solve an algebraic HJI. For this reason, in (Xi and Cheng, 2000, Mei *et al,* 2005), some nonlinear H_∞ control problems for Hamiltonian systems have been defined and some sufficient conditions have been presented to solve the proposed problems.

However, it is well known that nowadays computer-controlled systems using industrial processors are preferred in engineering practice because of the simplicity of their implementation. Therefore, it has been increasingly important to develop modelling and control techniques for discrete-time nonlinear systems.

In literature, there are several studies on discrete time nonlinear systems, which can be classified, roughly, in two groups. While one group deals with the concepts of the losslessness, the feedback equivalence and the global stabilization of discrete-time non-linear systems, (Byrnes and Lin,1994, Lin and Byrnes, 1995a, Sengör, 1995, Göknar and Sengör,1998, Navarro-López, 2002a, Navarro-López *et al*., 2002b), the other group works deriving the discrete-time counterpart of the H_∞ control techniques which are developed using the exact model of the system (Lin and Byrnes, 1995b,1996). It should be noted that; some results on the stabilization problem for sampled-data non-linear systems using their approximate discrete-time models has been presented in (Nesic and Teel, 2004), and a direct discrete-time PBC (Passivitiy Based Control) control method by using an approximate discrete-time Hamiltonian model has been developed by Astolfi and Laila (2005,2006a,2006b), recently.

In this study, the discrete-time counterpart of disturbance attenuation problem for a class of Hamiltonian systems is investigated and a sufficient condition for the solution of the problem is given. To fulfil, firstly a discrete-time equation, which corresponds to the given Hamiltonian system, is derived using an appropriate discrete gradient. Afterwards, using this equation, the disturbance attenuation problem characterised by means of L_2 gain is defined and the results are presented as a theorem, which provides a sufficient condition for the existence of solution. The proposed direct discrete-time design method is utilized to solve the disturbance attenuation problem of the double pendulum system by simulations.

II. PRELIMINARIES

In this section, we will restate the fundamental definitions and the bounded real lemma used in the literature (Lin and Byrnes, 1995, 1996) to construct the H_∞ control problem for discrete-time nonlinear systems which are affine in the input. Consider the discrete-time nonlinear system defined by,

$$\begin{aligned} x_{k+1} &= f(x_k) + g_1(x_k)w_k + g_2(x_k)u_k \\ z_k &= h_1(x_k) + k_{11}(x_k)w_k + k_{21}(x_k)u_k \\ y_k &= h_2(x_k) + k_{12}(x_k)w_k \end{aligned} \tag{1a}$$

where $x_k \in R^n$ is the state vector, y_k is the system output, z_k is the penalty signal and $w_k \in R^{m_1}, u_k \in R^{m_2}$ are disturbance and regulation inputs respectively. Here, an admissible static feedback control rule $u = c(x_k)$ would bring about a closed-loop system in the form of

$$\begin{aligned} x_{k+1} &= f_{cl}(x_k, w_k) \\ z_k &= h_{cl}(x_k, w_k) \end{aligned} \tag{1b}$$

Definition 1: (Lin and Byrnes, 1995, 1996)

A discrete-time nonlinear system in the form of (1b) is said to be dissipative with *supply rate* $\frac{1}{2}(\gamma^2\|w_k\|^2 - \|z_k\|^2)$ if there exist a nonnegative function $V : R^n \to R$ with $V(0) = 0$, which is called *storage function*, such that $\forall w_k \in R^{m_1}$ and $\forall k = 0,1,2,....$ the inequality

$$V(k+1) - V(k) \le \frac{1}{2}(\gamma^2\|w_k\|^2 - \|z_k\|^2)$$

is satisfied. □

Definition 2: (Lin and Byrnes, 1995, 1996)

Let γ be a positive real number. A nonlinear system of the form (1b) is said to have an L_2 gain less than or equal to γ from input w_k to output z_k if,

$$\sum_{k=0}^{N}\|z_k\|^2 \le \gamma^2 \sum_{k=0}^{N}\|w_k\|^2$$

holds for $\forall N \in Z_+$ and $\forall w_k \in l_2([0,N], R^{m_1})$ □

Lemma (Bounded real lemma): (Lin and Byrnes, 1995, 1996)

Suppose that a discrete-time nonlinear system of the form (1b) has $x=0$ as a locally asymptotically stable equilibrium. Then the system has an L_2 gain less than or equal to γ if it is dissipative with storage function V, with respect to the supply rate $\frac{1}{2}(\gamma^2\|w_k\|^2 - \|z_k\|^2)$. Conversely, suppose that system (1b) has an L_2 gain less than or equal to γ and (1b) is reachable from $x=0$. Then the system (1b) is dissipative with storage function V, with respect to the supply rate $\frac{1}{2}(\gamma^2\|w_k\|^2 - \|z_k\|^2)$. □

III. MAIN RESULT

As mentioned before, our aim is to construct the discrete counterpart of the disturbance attenuation problem of Hamiltonian systems which are described by,

$$\begin{aligned} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= (J - R(q,p))\nabla H(q,p) + G_1(q)w + G_2(q)u \\ y(t) &= G_1^T(q)\nabla H(q,p) \end{aligned} \tag{2}$$

where $(q,p) \in X \subset R^{2nx2n}$ an 2n-dimensional manifold, $y, w \in R^{m_1}$ are disturbance input and system output, respectively, $u \in R^{m_2}$ is the regulation input, J is a standard skew-symmetric structure matrix and $R(q,p) \in R^{2nx2n}$ is nonnegative symmetric matrix given as,

$$J = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix},\ R(q,p) = \begin{bmatrix} 0 & 0 \\ 0 & R_1(q,p) \end{bmatrix},$$

$$R_1(q,p) \in R^{nxn} \ge 0 \text{ and } G_1(q) = \begin{bmatrix} 0 \\ g_1(q) \end{bmatrix}, G_2(q) = \begin{bmatrix} 0 \\ g_2(q) \end{bmatrix}$$

where $g_1(q) \in R^{nxm_1}$, $g_2(q) \in R^{nxm_2}$ are disturbance force matrix and input force matrix, respectively. The notation $\nabla_{(\bullet)}H$ is used to denote the gradient vector of a scalar function $H(q,p)$ with respect to $(.)$. Furthermore, $H : R^{2n} \to R$ is the Hamiltonian function of the system or the energy function of the system in the following form,

$$H(q,p) = \frac{1}{2}p^T M(q)^{-1} p + P(q)$$

in which $M(q) = M(q)^T \ge 0$ is the generalized inertia matrix. If $M(q) = M \in R^{nxn}$ i.e. a constant matrix, the system is called a *separable Hamiltonian system* and if $m_2=n$ the system is said to be *full actuated*. Throughout the paper we will assume that the system is full actuated.

In order to obtain the discrete-time version of disturbance attenuation problem for the Hamiltonian systems, we need to use a term corresponding to the discrete version of the gradient term in (2), so the definition of discrete gradient given in (Gonzalez and Simo, 1996) and restated below will be considered.

Definition 3: Let $H(x)$, be a differentiable scalar function in x then $\overline{\nabla} H(x(k), x(k+1))$ is *a discrete gradient* of H if it is continuous in x and,

$$\overline{\nabla} H(x(k), x(k+1))^T \left[x(k+1) - x(k)\right] = H(x(k+1)) - H(x(k) \tag{3}$$
$$\overline{\nabla} H(x(k), x(k)) = \nabla H(x(k))$$

It should be noted that the main results of this study, which will be presented as a theorem, would be derived under the assumption such that there exists a discrete gradient, i.e., $\overline{\nabla} H(x(k), x(k+1))$, $x = \{q, p\} \in R^{2nx2n}$ which satisfies the conditions in Definition 3, exactly. A detailed discussion will be given later for the case where the conditions in Definition 3 are not precisely satisfied.

Approximating to derivatives of state variables of (2) by Forward Euler,

$$\dot{q} = \frac{q_{k+1} - q_k}{T}, \quad \dot{p} = \frac{p_{k+1} - p_k}{T}$$

and replacing the gradient term in (2) by the discrete gradient $\overline{\nabla} H(x(k), x(k+1))$, the discrete-time description of the system given in (2) can be obtained as follows;

$$\begin{bmatrix} q_{k+1} - q_k \\ p_{k+1} - p_k \end{bmatrix} = T\,(J - R(q_k, p_k))\overline{\nabla} H + T\, G_1(q_k) w + T\, G_2(q_k)\ u \tag{4}$$
$$y_k = G_1^T(q_k)\overline{\nabla} H$$

where T is sampling period. Note that, system (4) is a discrete approximation to the linear-gradient system with input of the form (2), (McLachlan, 1998).

In order to construct the feedback control rule $u_k = c(q_k, p_k)$ that keeps the system (4) asymptotically stable and minimizes the effect of the disturbance input w to penalty signal output z by means of L_2 gain, the penalty signal is defined as in (5),

$$z(q_k, p_k) = \begin{bmatrix} y_k \\ d_k(q_k, p_k) u_k \end{bmatrix} \tag{5}$$

and the disturbance rejection performance is described as following

$$\sum_{k=0}^{N} \{y_k^T y_k + u_k^T D(q_k, p_k) u_k\} \le \gamma^2 \sum_{k=0}^{N} \left\| w_k \right\|^2$$

where, $D(q_k, p_k) = d^T(q_k, p_k) d(q_k, p_k)$ is a given weighting matrix with a full rank matrix $d(q_k, p_k)$ and γ corresponds the disturbance attenuation level.

We can now give the following theorem on the solution of disturbance attenuation problem for the Hamiltonian system described by (4-5).

Theorem1: Consider the discrete-time port-controlled Hamiltonian system given in (4-5), and suppose the system satisfies the following conditions,

I. The weighting matrix $d(q_k, p_k)$ has full column rank.

II. The equilibrium point $x^* = 0$ is a strict local minimum of $H(x)$.

III. There exist a discrete gradient, i.e., namely $\overline{\nabla} H(x(k), x(k+1))$, $x = \{q, p\} \in R^{2nx2n}$ which satisfies the conditions in Definition 3, exactly.

If there exists a positive real number α satisfying the following Hamilton-Jacobs-Issac (HJI) inequality for a prescribed γ ,

$$\begin{aligned} &\alpha^2 \gamma^{-2} T^2 g_1(q_k) g_1^T(q) - \alpha^2 T^2 g_2(q_k)\{d_k^T d_k\}^{-1} g_2^T(q_k) \\ &- 2\alpha T R_1(q_k, p_k) + g_1(q_k) g_1^T(q_k) \le 0 \end{aligned} \tag{6}$$

then feedback control rule $u_k = c(q_k, p_k)$ that minimizes the effect of the disturbance input w to penalty signal output z defined in (5) by means of L_2 gain is given by:

$$u_k = -\alpha\, T\, \{d_k^T d_k\}^{-1} g_2{}^T(q_k) \overline{\nabla}_p H \tag{7}$$

Proof: Suppose there exists a α satisfying HJ1 given in (6), when we multiply (4) from left by $\overline{\nabla}^T H$ and use the first condition of the discrete gradient definition, the relation in below can be obtained as,

$$\begin{aligned} H(k+1) - H(k) &= \begin{bmatrix} \overline{\nabla}_q H \\ \overline{\nabla}_p H \end{bmatrix}^T \begin{bmatrix} q_{k+1} - q_k \\ p_{k+1} - p_k \end{bmatrix} \\ &= -\overline{\nabla}_p^T H\, T\, R_1(q_k, p_k) \overline{\nabla}_p H + \overline{\nabla}_p^T H\, T\, g_1(q_k) w_k \\ &\quad + \overline{\nabla}_p^T H\, T\, g_2(q_k) u_k \end{aligned} \tag{8}$$

since the matrix J is skew-symmetric so $\overline{\nabla}^T H\, J\, \overline{\nabla} H = 0$. The strictly positive function $V(k)$ may be defined by,

$$V(k) = \alpha H(k)$$

then, the following relation can be written;

$$\begin{aligned} V(k+1) - V(k) &= -\alpha \overline{\nabla}_p^T H\, T\, R_1(q_k, p_k) \overline{\nabla}_p H \\ &+ \alpha \overline{\nabla}_p^T H\, T\, g_1(q_k) w_k + \alpha \overline{\nabla}_p^T H\, T\, g_2(q_k) u_k \end{aligned} \tag{9}$$

Once the inequality (6) is written considering the term R_1 is extracted from HJI and this inequality is evaluated together with (9), the following inequality can be written,

$$V(k+1)-V(k) \le -\frac{\alpha^2\gamma^{-2}T^2}{2}\bar{\nabla}_p^T H\, g_1(q_k)g_1^T(q_k)\bar{\nabla}_p H$$
$$+\frac{\alpha^2 T^2}{2}\bar{\nabla}_p^T H\, g_2(q_k)\{d_k^T d_k\}^{-1} g_2^T(q_k)\bar{\nabla}_p H$$
$$-\frac{1}{2}\bar{\nabla}_p^T H\, g_1(q_k)g_1^T(q_k)\bar{\nabla}_p H + \alpha\bar{\nabla}_p^T H\, T g_1(q_k)w_k$$
$$+\alpha T\nabla_p^T H g_2(q_k)u_k$$

and after some algebraic manipulations, the following relation is obtained,

$$V(k+1)-V(k) \le -\frac{1}{2}\left\|\frac{\alpha T}{\gamma}g_1^T(q_k)\bar{\nabla}_p H - \gamma w_k\right\|^2$$
$$+\frac{1}{2}\left\|\alpha\ T\{d_k^T d_k\}^{-\frac{1}{2}} g_2^T(q_k)\times\bar{\nabla}_p H + \{d_k^T d_k\}^{-\frac{1}{2}} u_k\right\|^2$$
$$+\frac{\gamma^2}{2}\|w_k\|^2 - \frac{1}{2}\{y_k^T y_k + u_k^T d_k^T d_k u_k\}$$

On the right hand-side of this inequality, the first term is always negative, and the second term becomes zero when the control rule (7) is substituted in this term. Therefore, this inequality gives the dissipation inequality,

$$V(k+1)-V(k) \le \frac{\gamma^2}{2}\|w_k\|^2 - \frac{1}{2}\{y_k^T y_k + u_k^T d_k^T d_k u_k\}$$

or

$$V(k+1)-V(k) \le \frac{\gamma^2}{2}\|w_k\|^2 - \frac{1}{2}\|z_k\|^2$$

This means that, if control (7) is applied to system (4-5) then the robust L_2 disturbance attenuation objective is achieved for a prescribed γ, so the proof has been completed. □

The following remark is given to discuss the case when the discrete gradient used in Theorem 1 does not precisely satisfy the conditions (3) given in Definition 3.

Remark: Suppose that the discrete gradient $\bar{\nabla}H(x(k),x(k+1))$, $x=\{q,p\}\in R^{2nx2n}$ used in Theorem 1 satisfies the second condition in Definition 3 exactly but it does not satisfy the first condition precisely. In this case, we can write the following energy difference relation in terms of discrete energy function $\hat{H}(x_k)$,

$$\bar{\nabla}^T H\left[J-R(x_k)\right]\bar{\nabla}H = -\bar{\nabla}^T H\, R(x_k)\bar{\nabla}H = \frac{\hat{H}(x_{k+1})-\hat{H}(x_k)}{T}$$

If this energy difference could be written in terms of the energy function of continuous system $H(x)$ in the following form,

$$\frac{\hat{H}(x_{k+1})-\hat{H}(x_k)}{T} = \frac{H(x_{k+1})-H(x_k)}{T} + \varepsilon(x_{k+1},x_k)$$

then we could say that, there exists the $\hat{R}(x_k)$ such that the following equation holds,

$$\bar{\nabla}^T H\left[J-\hat{R}(x_k)\right]\bar{\nabla}H = \frac{H(x_{k+1})-H(x_k)}{T}$$ □

As a consequence, it can be stated that the above discussion should be taken into account while the condition HJ1 in Theorem 1 is used for the design of control rule given in (7), especially when slow sampling is used. Obviously, for $T\to 0$ this extra term tends to zero, i.e. $\varepsilon(x_{k+1},x_k)\to 0$.

IV. DISCRETE GRADIENT

In the sequel, we will use the following definition for the discrete gradient

Definition 4: Consider the energy function $H(q,p)$ of (2) which is $H(q,p)=K(q,p)+P(q)=\frac{1}{2}p^T M^{-1}(q)p+P(q)$

and its gradient given in the form of

$$\nabla H(q,p)=\begin{bmatrix} P_{gr}(q) & S(q,p)\\ 0 & M^{-1}(q)\end{bmatrix}\begin{bmatrix} q\\ p\end{bmatrix} = Q(q,p)\begin{bmatrix} q\\ p\end{bmatrix}$$

$$S(q,p)=\begin{bmatrix}\left[p^T\dfrac{\partial M^{-1}(q)}{\partial q_1}\right]\\ \left[p^T\dfrac{\partial M^{-1}(q)}{\partial q_2}\right]\\ \vdots\\ \left[p^T\dfrac{\partial M^{-1}(q)}{\partial q_n}\right]\end{bmatrix}$$

in which the matrix $P_{gr}(q)$ is described as,

$$\nabla P(q) = P_{gr}(q)q$$

then the discrete gradient of $H(q,p)$ is defined as,

$$\bar{\nabla}H = Q(\bar{q}_{k,k+1},\bar{p}_{k,k+1})\begin{bmatrix}\bar{q}_{k,k+1}\\ \bar{p}_{k,k+1}\end{bmatrix} \quad (10)$$

where

$$\bar{q}_{k,k+1}=\frac{q_{k+1}+q_k}{2},\quad \bar{p}_{k,k+1}=\frac{p_{k+1}+p_k}{2} \quad (11)$$

□

It can be easily seen that, for separable Hamiltonian systems, the discrete gradient defined above satisfies both of the conditions given in Definition 3, exactly, but it does not satisfy the first condition precisely, for non-separable case.

It is convenient to isolate the relations $\overline{\nabla}_q H$ and $\overline{\nabla}_p H$ from $\overline{\nabla} H$ as follows,

$$\overline{\nabla} H = \begin{bmatrix} P_{qr}(\bar{q}_{k,k+1}, \bar{p}_{k,k+1}) & S(\bar{q}_{k,k+1}, \bar{p}_{k,k+1}) \\ 0 & M^{-1}(\bar{q}_{k,k+1}, \bar{p}_{k,k+1}) \end{bmatrix} \times \begin{bmatrix} \bar{q}_{k,k+1} \\ \bar{p}_{k,k+1} \end{bmatrix}$$

$$\overline{\nabla}_q H = \overline{\nabla}_q K + \overline{\nabla}_q P = P_{qr}(\bar{q}_{k,k+1}, \bar{p}_{k,k+1})\bar{q}_{k,k+1} + S(\bar{q}_{k,k+1}, \bar{p}_{k,k+1})\bar{p}_{k,k+1}$$

$$\overline{\nabla}_p H = \overline{\nabla}_p K = M^{-1}(\bar{q}_{k,k+1}, \bar{p}_{k,k+1})\bar{p}_{k,k+1} \qquad (12)$$

The following approximation, which is inspired from the first order hold mechanism, is used for the calculation of q_{k+1} and p_{k+1} to avoid the non-causality.

$$p_{k+1} = p_k + (p_k - p_{k-1}) = 2p_k - p_{k-1}$$
$$q_{k+1} = q_k + (q_k - q_{k-1}) = 2q_k - q_{k-1}$$

When these relations are used in (11-12), the below relations are obtained,

$$\bar{p}_{k,k+1} = \frac{3p_k - p_{k-1}}{2}, \quad \bar{q}_{k,k+1} = \frac{3q_k - q_{k-1}}{2}$$

$$\overline{\nabla}_p H = M^{-1}\left(\frac{3q_k - q_{k-1}}{2}, \frac{3p_k - p_{k-1}}{2}\right)\left(\frac{3p_k - p_{k-1}}{2}\right)$$

The last expression will be used for the discrete gradient term $\overline{\nabla}_p H$ in the realization of the control rule (7), in the example.

One has to consider that the computational complexity of the control rule using this discrete gradient expression is nearly the same as the computational complexity of the emulation controller; this property might provide an important advantage especially in industrial applications.

V. EXAMPLE and SIMULATION RESULTS

In order to analyse the performance of the proposed method, we have considered the double pendulum system given as,

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = (J - R(q,p))\nabla H(q,p) + \begin{bmatrix} 0 \\ g_1(q) \end{bmatrix} w + \begin{bmatrix} 0 \\ g_2(q) \end{bmatrix} u$$
$$y(t) = \begin{bmatrix} 0 & g_1^T(q) \end{bmatrix} \nabla H(q,p)$$

$$g_1(q) = I_2,\ g_2(q) = I_2,\ R = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 I_2 \end{bmatrix}$$

with Hamiltonian function $H(q,p) = \frac{1}{2} p^T M(q)^{-1} p + P(q)$ in which,

$$M = \begin{bmatrix} l_1^2(m_1 + m_2) & m_2 l_1 l_2 \cos(q_1 - q_2) \\ m_2 l_1 l_2 \cos(q_1 - q_2) & l_2^2 m_2) \end{bmatrix}$$

$$P(q) = -\left[m_2 g l_2 \cos q_2 + (m_1 + m_2) g l_1 \cos q_1\right]$$

where $m_1 = m_2 = 1kg$, $l_1 = 0.2m$, $l_2 = 0.3m$ and g=0.98ms.

To construct the control rule in (7), the weighting coefficient in the penalty signal is chosen as $d_k = I_2$ and $T = 0.04$, then it is found as $\alpha = 28$ satisfying HJI inequality given in (6). Finally, the control rule has been obtained as,

$$u_k = -1.12\overline{\nabla}_p H$$

The disturbance considered in simulation is taken as $w = 3.5$ during the time 1-3 seconds. The time response of the uncontrolled system under the disturbance is shown in Figure1.

The Figure 2 and 3 illustrate the simulation results of the system-time response under the direct discrete-time control, the continuous-time control and emulator controller, altogether.

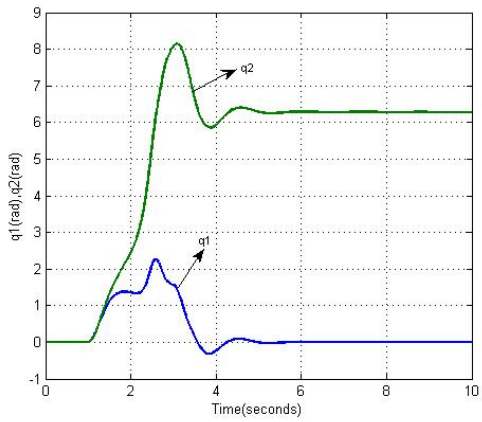

Figure 1. The time response of the uncontrolled double pendulum system under the disturbance

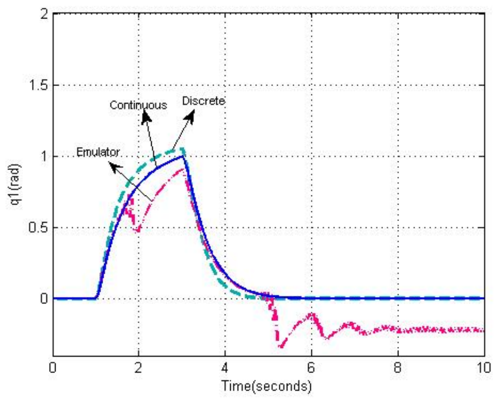

Figure 2. The time response q_1 of the controlled double pendulum system under the disturbance

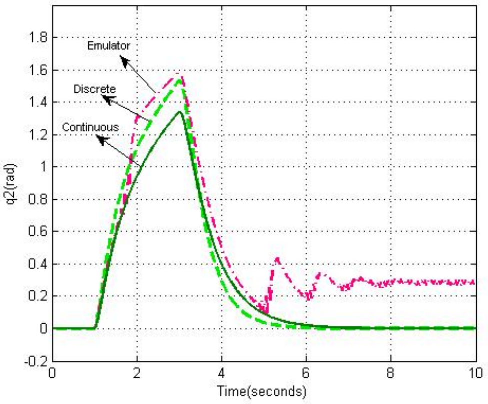

Figure 3. The time response q_2 of the controlled double pendulum system under the disturbance

These results have demonstrated that the controller obtained using the method developed in this paper has better performance than the emulator controller for sampled data Hamiltonian systems.

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