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Disturbance Rejection and Set-point Following of Periodic Signals Using Predictive Control with Constraints

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Abstract: This paper proposes a continuous-time model predictive control design for disturbance rejection and set-point following of periodic signals. By assuming input disturbance in the form of sinusoid, the periodic frequency is embedded into the design model. Hence, from internal model principle, the steady-state error of the model predictive control system is ensured to be zero for both disturbance rejection and set-point following. Furthermore, with the design framework of model predictive control, hard constraints on the derivative and amplitude of the control signals are imposed as part of the performance specification. Simulation studies have been used to show the efficacy of the design with or without hard constraints.

Keywords: Periodic set-point signal, periodic disturbance, predictive control, constrained control, optimization.

1. INTRODUCTION

Control system applications in mechanical systems, manufacturing systems and aerospace systems often require setpoint following of a periodic trajectory. The reference periodic signals have given frequencies and amplitudes, however, with unknown phase information. Another type of control system applications is the rejection of periodic disturbances, where typically the frequency information of the disturbance is given either through experimental data analysis or understanding of the system. In both situations, design of a control system that has the capability to produce zero steady state error is paramount.

It is well known through the internal model control principle that in order to reject a periodic disturbance or following a periodic reference signal with zero steady-state error, the generator for the disturbance or the reference is included in the stable closed-loop control system (Francis and Wonham,1976). In a standard state estimate feedback control, this is achieved either by incorporating the periodic disturbance model into the design or by estimating the sinusoidal disturbance through an observer (see for example, Goodwin et al. 2000). In this paper, the design framework of periodic control system is extended to model predictive control system where the input disturbance is assumed to be periodic, and as a consequence, the periodic disturbance model is naturally incorporated into an augmented design model.

The majority of the development of model predictive control in the past a few decades is based on discrete models (see for example, Mayne et al. 2000, Rawlings, 2000). Continuous-time model predictive control design using state space models emerged in the recently years (Gawthrop and Ronco, 2002, Wang 2001). There are a few reasons that lead to the design in the continuous-time domain. One of the key advantages is that the continuous-time design is based a continuous-time model and its implementation is less sensitive to the choice of sampling interval. Intermittent predictive control is also investigated recently (Gawthrop and Wang, 2006).

The central idea of the design is to use a set of Laguerre exponential functions to describe the control trajectory, similar in spirit to the approaches used by Wang (2001) and Gawthrop and Ronco(2002). However, because the focus is on periodic signals, in the proposed design, a set of continuous-time Laguerre functions, which are orthonormal, are used to describe the filtered control signal where the filter is the inverse of the disturbance model. By doing so, the optimal control trajectory of the predictive control is captured by the coefficients of the Laguerre polynomials and the set of known Laguerre exponential functions. The predictive control problem is converted to a real-time optimization problem that finds the optimal Laguerre coefficients subject to constraints. Because the predictive control system is designed using receding horizon control principle, the operational constraints, such as the limits on the derivative and amplitude of the control signal, are systematically imposed in the design. The results in this paper show that when the constraints become activated, the predictive control system produces optimal results. In comparison, with saturation scheme, the control performance degrades significantly.

The remainder of the paper is organised as follows. In Section 2, the continuous-time model predictive control algorithm is proposed; in Section 3, extensive simulation studies of set-point following and disturbance rejection of periodic signals, with or without constraints, are presented and discussed.

2. FORMULATION OF THE MODEL

Suppose that the plant to be controlled is an m input-q output multivariable system having a state space model:

$$\begin{aligned}\dot{x}_m(t) &= A_m x_m(t) + B_m u(t) + \Omega_m \mu(t)\\ y(t) &= C_m x_m(t)\end{aligned} \quad (1)$$

where $x_m(t)$ is the state vector of dimension n_1, while $\mu(t)$ represents input disturbance. In the past, by assuming that the input disturbance $\mu(t)$ was a source of integrated white noise, the predictive controller naturally embedded an integrator in its structure (See Clarke et al.(1987a) for the discrete case, and Wang (2001) for the continuous time case). Along similar lines, in the following, we derive predictive control systems that have the capability to reject sinusoidal input disturbance and double integrated input disturbance. As a consequence, the predictive control systems will also follow the same type of input signals with zero steady -state errors.

2.1 Periodic Input Disturbance

Assume that the input disturbance $\mu(t)$ is a sinusoidal signal with unknown amplitude and phase, however with known frequency ω_0. The input disturbance $\mu(t)$ can be described by

$$\frac{d\mu(t)^2}{dt^2} + \omega_0^2 \mu(t) = \epsilon(t) \quad (2)$$

where $\epsilon(t)$ is a band-limited continuous time white noise. It is known that the feedback control system completely compensates the effect of the sinusoidal disturbance if the controller contains the module $\frac{1}{s^2+\omega_0^2}$ (Goodwin et al. 2000). The question here is how to embed this module in the continuous time predictive control while using the orthonormal basis functions (Wang,2001) to capture the control trajectory. Define the auxiliary control signal $u_s(t)$ as the function that satisfies the following differential equation

$$\frac{d^2u(t)}{dt^2} + \omega_0^2 u(t) = u_s(t) \quad (3)$$

and the auxiliary state variable $z(t)$ as

$$\frac{d^2x_m(t)}{dt^2} + \omega_0^2 x_m(t) = z(t) \quad (4)$$

By using the auxiliary variables, the state space equation (1) is transformed into

$$\frac{dz(t)}{dt} = A_m z(t) + B_m u_s(t) + \epsilon(t) \quad (5)$$

Note that

$$\begin{aligned}\frac{d^2y(t)}{dt^2} &= C_m \frac{d^2x_m}{dt^2}\\ &= C_m \frac{d^2x_m(t)}{dt^2} + \omega_0^2 C_m x_m(t) - \omega_0^2 C_m x_m(t)\\ &= C_m z(t) - \omega_0^2 y(t)\end{aligned} \quad (6)$$

We put together an augmented state space model as

$$\begin{bmatrix} \frac{dz(t)}{dt} \\ \frac{d^2y(t)}{dt^2} \\ \frac{dy(t)}{dt} \end{bmatrix} = \begin{bmatrix} A_m & 0_1 & 0_2 \\ C_m & 0_3 & -w_0^2 I_q \\ 0_4 & I_q & 0_5 \end{bmatrix} \begin{bmatrix} z(t) \\ \frac{dy(t)}{dt} \\ y(t) \end{bmatrix} + \begin{bmatrix} B_m \\ 0_6 \\ 0_7 \end{bmatrix} u_s(t) + \begin{bmatrix} \Omega_m \\ 0_6 \\ 0_7 \end{bmatrix} \epsilon(t) \quad (7)$$

where 0_k, $k = 1, 2, 3, 4, 5, 6, 7$ are the zero matrices with appropriate dimensions.

The key task in the design of continuous time model predictive control is to model the auxiliary control signal $u_s(t)$ using a set of orthonormal basis functions. From Equation (2) it is seen that $u_s(t)$ is the inversely filtered control signal by the sinusoidal dynamics. When the closed-loop system is stable, $u_s(t)$ satisfies the property that

$$\lim_{T_p\to\infty} \int_0^{T_p} u_s(t)^2 dt < \infty \quad (8)$$

although the control signal $u(t)$ itself does not obey this rule (for example, see the simulation examples). Therefore, it is appropriate to model $u_s(t)$, instead of $u(t)$ with a set of orthonormal basis functions. Once the optimal control $u_s(t)$ is found, the control signal $u(t)$ can be constructed through Equation (3).

The special case of the sinusoidal input signal when $\omega_0 = 0$ leads to the case of double integrated input disturbance. This is evident from Equation (2). Therefore, the design model remains the same with $\omega_0 = 0$, when the input disturbance becomes a double integrated disturbance.

3. PREDICTION AND OPTIMIZATION

Suppose in a general description of dynamic systems, we have m control signals. For a given prediction horizon T_p and $0 \le \tau \le T_p$, let the derivative of the control signal be expressed as

$$u_s(\tau) = [u_s(\tau)^1 \;\; u_s(\tau)^2 \;\; \ldots u_s(\tau)^m]^T$$

and the input matrix be partitioned as

$$B = [B^1 \;\; B^2 \;\; \ldots B^m]$$

where $u_s(.)^i$ is the ith filtered control signal and B^i is the *ith* column of the B matrix. Then the *ith* control signal $u_s(\tau)^i$ $(i = 1, 2 \ldots, m)$ is described by using a set of Laguerre functions as

$$u_s^i(\tau) = L(\tau)^T \eta_i$$

where $L(t)^T = [\, l_1(t) \; l_2(t) \; \ldots \; l_N(t) \,]$

and $\eta_i = \left[\, \xi_1^i \; \xi_2^i \; \ldots \; \xi_N^i \,\right]^T$. More specifically, the set of Laguerre functions are defined explicitly by the differential equation as below, with initial condition $L(0) = \sqrt{2p}\,\underbrace{[1 \;\; 1 \ldots \;\; 1]^T}_{N}$.

$$\dot{L}(t) = A_p L(t) \quad (9)$$

where

$$A_p = \begin{bmatrix} -p & 0 & \ldots & 0 \\ -2p & -p & \ldots & 0 \\ \vdots & & & \\ -2p & \ldots & -2p & -p \end{bmatrix}$$

Here the parameter p is called scaling factor and N the number of terms used in the description of systems. The set of Laguerre functions will be different if the scaling factor p is chosen to be different. p and N can be selected for each individual input signal in the design.

With this formulation, we compute the prediction of state variables. The model used in the prediction is based on (7), where the control signal to the model is the filtered control. We assume that at the current time, t_i, the state variable $x(t_i)$ is available. If the state variable $x(t_i)$ is not available, then an observer is needed to access the state information through the measurement of input and output signals, which will be discussed later. Then at the future time τ, $\tau > 0$, the predicted state variable $x(\tau \mid t_i)$ is described by the following equation

$$x(\tau \mid t_i) = e^{A\tau}x(t_i) + \int_0^\tau e^{A(\tau-\gamma)}Bu_s(\gamma)d\gamma \quad (10)$$

By substituting the description of the control $u_s(\gamma)$ using the Laguerre functions, the predicted future state at time τ is parameterised by η as

$$x(\tau \mid t_i) = e^{A\tau}x(t_i) + \int_0^\tau e^{A(\tau-\gamma)} \left[B^1L(\gamma)^T \ B^2L(\gamma)^T \ \ldots \ B^mL(\gamma)^T \right] d\gamma\eta$$

$$= e^{A\tau}x(t_i) + \left[I_{int}(\tau)^1 \ I_{int}(\tau)^2 \ \ldots \ I_{int}(\tau)^m \right] \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_m \end{bmatrix} \quad (11)$$

where $I_{int}(\tau)^i$ is the analytical solution of the ith integral equation Wang [2001] given by the algebraic equation

$$AI_{int}(\tau)^i - I_{int}(\tau)^iA_p^T = -B^iL(\tau)^T + e^{A\tau}B^iL(0)^T \quad (12)$$

where $L(\tau)^T$, $L(0)^T$ and A_p are the Laguerre function vector, initial vector and its state matrix; B_i is the ith column of the input matrix B. Since the state matrix of the Laguerre functions A_p is a lower triangular matrix, Equation (12) is solved in a closed-form through a set of linear equations (See Wang 2001).

Let C denote the matrix with the dimension $(2\times q)\times(n_1 + 2\times q)$ defined by

$$C = [\, o_{n1} \ I_{2q} \,]$$

where o_{n1} is a zero matrix with the dimension $(2\times q)\times n_1$ and I_{2q} is the identity matrix with dimension $2q \times 2q$. By using the matrix C, the prediction of plant output and its derivative prediction can be represented by

$$\begin{bmatrix} \dot{y}(\tau \mid t_i) \\ y(\tau \mid t_i) \end{bmatrix} = Cx(\tau \mid t_i) = Ce^{A\tau}x(t_i) + \phi(\tau)^T\eta \quad (13)$$

$$\phi(\tau) = (C \left[I_{int}(\tau)^1 \ I_{int}(\tau)^2 \ \ldots \ I_{int}(\tau)^r \right])^T$$

Note that the predicted plant output and its derivative are expressed in terms of the coefficient vector η.

Suppose that at time t_i, the future setpoint signals and their derivatives are given as

$$r(t_i) = [\, r_1(t_i) \ r_2(t_i) \ \ldots \ r_q(t_i \,]^T$$

$$\dot{r}(t_i) = [\, \dot{r}_1(t_i) \ \dot{r}_2(t_i) \ \ldots \ \dot{r}_q(t_i) \,]^T$$

We assume that $r(t_i)$ and $\dot{r}(t_i)$ remain constant within one optimization window. Namely $r(t_i + \tau) = r(t_i)$ and $\dot{r}(t_i + \tau) = \dot{r}(t_i)$ for $0 \le \tau \le T_p$. The desired trajectories include periodic signals, ramp signal and step signals. We emphasis that setpoint tracking of these signals requires desired derivative information in addition to the information about the signals themselves. Because of the receding horizon control principle used in the design of predictive control, the setpoint information can be readily varied from one optimization window to another.

In order to achieve perfect setpoint following of the periodic signals, the design objective of model predictive control is to find the control law that will drive the predicted plant output $y(\tau \mid t_i)$ and the predicted derivative of the plant output as close as possible, in a least squares sense, to the future setpoint $r(t_i)$ and the future derivative $\dot{r}(t_i)$. To this end, we define the error signal

$$e(\tau \mid t_i) = \begin{bmatrix} \dot{r}(t_i) - \dot{y}(\tau \mid t_i) \\ r(t_i) - y(\tau \mid t_i) \end{bmatrix}$$

The cost function to reflect on the design objective is chosen to be

$$J = \int_0^{T_p} e(\tau \mid t_i)^TQe(\tau \mid t_i)d\tau + \int_0^{T_p} u_s(\tau)^TRu_s(\tau)d\tau \quad (14)$$

where Q and R are symmetric matrices with $Q > 0$ and $R \ge 0$. By taking advantage of the orthonormal property of the Laguerre functions, the cost function J is then equivalently given by

$$J = \int_0^{T_p} e(\tau \mid t_i)^TQe(\tau \mid t_i)d\tau + \eta^T\bar{R}\eta \quad (15)$$

where $\bar{R} = diag\{R^i\}$ and $R^i = \lambda_i I_{N_i\times N_i}$ (a unit matrix with dimension N_i).

Note that

$$e(\tau \mid t_i) = \begin{bmatrix} \dot{r}(t_i) \\ r(t_i) \end{bmatrix} - Ce^{A\tau}x(t_i) - \phi(\tau)^T\eta$$

By defining

$$w(\tau \mid t_i) = \begin{bmatrix} \dot{r}(t_i) \\ r(t_i) \end{bmatrix} - Ce^{A\tau}x(t_i) \quad (16)$$

then the quadratic cost function (15) can be written in the following standard form:

$$J = \eta^T\{\int_0^{T_p} \phi(\tau)Q\phi(\tau)^Td\tau + \bar{R}\}\eta - 2\eta^T \int_0^{T_p} \phi(\tau)Qw(\tau \mid t_i)d\tau + \int_0^{T_p} w(\tau \mid t_i)^TQw(\tau \mid t_i)d\tau \quad (17)$$

which can be written explicitly in terms of setpoint signal $r(t_i)$, its derivative $\dot{r}(t_i)$ and the state variable $x(t_i)$:

$$J = \eta^T\Pi\eta - 2\eta^T\{\Psi_1 \begin{bmatrix} \dot{r}(t_i) \\ r(t_i) \end{bmatrix} - \Psi_2x(t_i)\} + \int_0^{T_p} w(\tau \mid t_i)^TQw(\tau \mid t_i)d\tau \quad (18)$$

where

$$\Pi = \int_0^{T_p} \phi(\tau)Q\phi(\tau)^Td\tau + \bar{R}$$

$$\Psi_1 = \int_0^{T_p} \phi(\tau) Q d\tau; \Psi_2 = \int_0^{T_p} \phi(\tau) Q C e^{A\tau} d\tau$$

The minimum of (18), without hard constraints on the variables, is then given by the simple least squares solution:

$$\eta = \Pi^{-1}\{\Psi_1 \begin{bmatrix} \dot{r}(t_i) \\ r(t_i) \end{bmatrix} - \Psi_2 x(t_i)\} \tag{19}$$

With the optimal parameter vector, η, the optimal control $u_s(\tau)$, $0 \leq \tau \leq T_p$, can be reconstructed using the Laguerre functions as

$$\begin{aligned} u_s(\tau) &= \left[u_s^1(\tau) \; u_s^2(\tau) \; \ldots \; u_s^m \right]^T \\ &= \begin{bmatrix} L(\tau)^T & o_L & \ldots & o_L \\ o_L & L(\tau)^T & \ldots & o_L \\ \vdots & & & \\ o_L & o_L & \ldots & L(\tau)^T \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_m \end{bmatrix} \end{aligned} \tag{20}$$

where o_L is a zero vector with dimension $1 \times N$.

By applying the principle of receding horizon control (i.e. the control action will use only the information $u_s(\tau)$ at $\tau = 0$), the optimal control $u_s(t)$ for the unconstrained problem at time t_i is

$$u_s(t_i) = \begin{bmatrix} L(0)^T & o_L & \ldots & o_L \\ o_L & L(0)^T & \ldots & o_L \\ \vdots & & & \\ o_L & o_L & \ldots & L(0)^T \end{bmatrix} \Pi^{-1}\{\Psi_1 \begin{bmatrix} \dot{r}(t_i) \\ r(t_i) \end{bmatrix} - \Psi_2 x(t_i)\} \tag{21}$$

The predictive controller gain matrix K, from (21) which is associated with state variable $x(t_i)$, is calculated using

$$K = \begin{bmatrix} L(0)^T & o_L & \ldots & o_L \\ o_L & L(0)^T & \ldots & o_L \\ \vdots & & & \\ o_L & o_L & \ldots & L(0)^T \end{bmatrix} \Pi^{-1}\{\Psi_1 \begin{bmatrix} \dot{r}(t_i) \\ r(t_i) \end{bmatrix} - \Psi_2\} \tag{22}$$

Hence, the closed-loop system matrix is

$$A_{cl} = A - BK \tag{23}$$

from which we can assess the closed-loop performance of the predictive control system when constraints are not imposed.

Note that the information about the optimal $u_s(t)$ at time t_i needs to be converted to the actual control signal $u(t)$ at t_i for control implementation. In order to achieve this conversion, the following differential equation that relates $u_s(t)$ to $u(t)$ is presented.

$$\begin{bmatrix} \frac{d^2u(t)}{dt^2} \\ \frac{du(t)}{dt} \end{bmatrix} = \begin{bmatrix} o_m & -w_0^2 I_m \\ I_m & o_m \end{bmatrix} \begin{bmatrix} \frac{du(t)}{dt} \\ u(t) \end{bmatrix} + \begin{bmatrix} I_m \\ o_m \end{bmatrix} u_s(t) \tag{24}$$

where o_m is a zero matrix with dimension $m \times m$ and I_m is an identity matrix with dimension $m \times m$. With approximation to the differential equation (24), assuming that the sampling interval is Δt, we obtain the optimal control at t_i:

$$\begin{aligned} \begin{bmatrix} \dot{u}(t_i) \\ u(t_i) \end{bmatrix} &= \begin{bmatrix} o_m & -w_0^2 I_m \\ I_m & o_m \end{bmatrix} \begin{bmatrix} \dot{u}(t_{i-1}) \\ u(t_{i-1}) \end{bmatrix} \Delta t \\ &+ \begin{bmatrix} I_m \\ o_m \end{bmatrix} u_s(t_i)\Delta t + \begin{bmatrix} \dot{u}(t_{i-1}) \\ u(t_{i-1}) \end{bmatrix} \end{aligned} \tag{25}$$

where the backward difference approximation, $\frac{df(t)}{dt} \approx \frac{f(t)-f(t-\Delta t)}{\Delta t}$, is used. The actual control $u(t_i)$ is computed using the optimal signal $u_s(t_i)$ and the previous states of the control, $\dot{u}(t_{i-1})$ and $u(t_{i-1})$. Note that (25) is expressed in a so-called 'velocity form', meaning that the steady state information of the control and its derivative is not required in the implementation and the control is computed iteratively.

The main strength of model predictive control lies in its ability to incorporate hard constraints in the design with on-line optimisation. The hard constraints on the derivative of the control and the control itself are formulated as follows. For notational simplicity, we consider a single input signal $u(t)$ and consider putting constraints on the first sample of the optimal signals (i.e. $\tau = 0$). From (25), we express $\dot{u}(t_i)$ and $u(t_i)$ as functions of the parameter vector η ($u_s(t_i) = L(0)^T \eta$):

$$\dot{u}(t_i) = \overbrace{-w_0^2 u(t_{i-1})\Delta t + \dot{u}(t_{i-1})}^{c_1} + L(0)^T \eta \Delta t \tag{26}$$

$$\begin{aligned} u(t_i) &= \dot{u}(t_i)\Delta t + u(t_{i-1}) \\ &= \overbrace{-w_0^2 u(t_{i-1})\Delta t^2 + \dot{u}(t_{i-1})\Delta t + u(t_{i-1})}^{c_2} \\ &\quad + L(0)^T \eta \Delta t^2 \end{aligned} \tag{27}$$

Note that the elements c_1 and c_2 under the $\overbrace{\cdot}$ are independent of the parameter vector η.

Assuming that the hard constraints on $\dot{u}(t)$ are $\dot{u}_{min} \leq \dot{u}(t) \leq \dot{u}_{max}$, the constraints on the derivative of the control are expressed as

$$L(0)^T \Delta t \eta \leq \dot{u}_{max} - c_1 \tag{28}$$

$$-L(0)^T \Delta t \eta \leq -\dot{u}_{min} + c_1 \tag{29}$$

Similarly, assuming that $u_{min} \leq u(t) \leq u_{max}$ then

$$L(0)^T \Delta t^2 \eta \leq u_{max} - c_2 \tag{30}$$

$$-L(0)^T \Delta t^2 \eta \leq -u_{min} + c_2 \tag{31}$$

Collecting (28)-(31), as an illustration, we arrive at a set of linear inequalities that reflect the hard constraints imposed in the design:

$$\begin{bmatrix} L(0)^T \Delta t \\ -L(0)^T \Delta t \\ L(0)^T \Delta t^2 \\ -L(0)^T \Delta t^2 \end{bmatrix} \eta \leq \begin{bmatrix} \dot{u}_{max} - c_1 \\ -\dot{u}_{min} + c_1 \\ u_{max} - c_2 \\ -u_{min} + c_2 \end{bmatrix} \tag{32}$$

Now the predictive control problem with hard constraints imposed in the design becomes the problem to find the optimal solution of the quadratic cost function (18) subject to the linear inequality constraints (32). This is a standard constrained quadratic minimization problem, and the optimal solution can be found using a quadratic programming algorithm.

4. SIMULATION RESULTS

The system used in the simulation study is a power electronic device with the state-space model given as

$$\dot{x}_m(t) = A_p x_m(t) + B_p u(t)$$
$$y(t) = C_p x_m(t) \quad (33)$$

where

$$A_p = \begin{bmatrix} 0 & -2.6667 \times 10^3 \\ 80 & -66.6667 \end{bmatrix}; B_p = \begin{bmatrix} 2 \times 10^5 \\ -25 \end{bmatrix}$$
$$C_p = [\,0 \;\; 1\,]$$

This system is severely underdamped, which has a pair of complex eigenvalues as $\lambda_1 = -33.3333 + i460.6758$, $\lambda_2 = -33.3333 - i460.6758$.

4.1 Set-point following with constraints

Assume that the set-point signal is a ramp signal $\omega_0 = 0$ to start, followed by a sinusoidal input signal with frequency $\omega_0 = 2.8$, and we assume that the signals have a smooth transition from one to the other. The design objective is that the output follows the reference signal as close as possible subject to the constraints on the amplitude of the control:

$$-0.015 \leq u(t) \leq 0.015$$

For a comparison, we simulate three cases, without constraints, predictive control with embedded constraints and state feedback control with saturation.

We include these two frequencies in the design of model predictive control to obtain two sets of control parameter matrices for the quadratic cost function (18). The observer is also designed using the two frequencies. The control law automatically switches when the set-point signal changes its frequency.
The parameters for the continuous-time predictive control are chosen as $p = 63$, $N = 3$, $T_p = 0.4762$, $Q = I$ and $R = 0.000$. p is chosen around the twice of the real part of the open-loop poles.

The control law for the ramp signal is determined by minimizing the cost function (18). For $\omega_0 = 0$,

$$\Pi = \begin{bmatrix} 5.7985 & -5.1993 & 4.6071 \\ -5.1993 & 4.6799 & -4.1606 \\ 4.6071 & -4.1606 & 3.7155 \end{bmatrix}$$

$$\Psi_1 = \begin{bmatrix} 0.4567 & 1.4159 \\ -0.4089 & -1.2275 \\ 0.3618 & 1.0525 \end{bmatrix}$$

$$\Psi_2 = \begin{bmatrix} 0.0002 & 0.0000 & 0.4567 & 1.4159 \\ -0.0002 & -0.0000 & -0.4089 & -1.2275 \\ 0.0001 & 0.0000 & 0.3618 & 1.0525 \end{bmatrix}$$

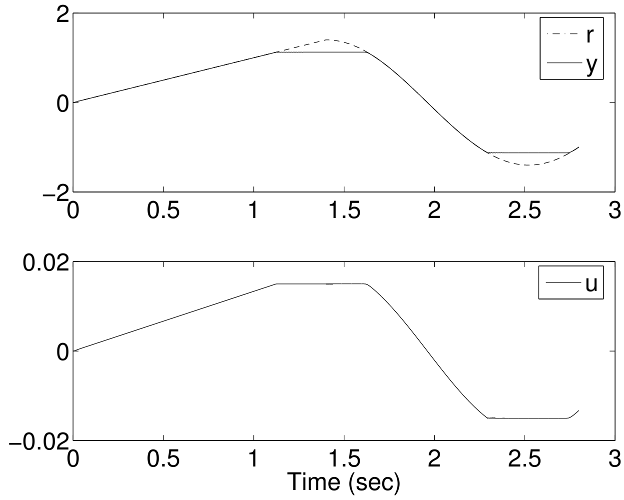
For $\omega_0 = 2.8$

$$\Pi = \begin{bmatrix} 4.1445 & -3.7968 & 3.4261 \\ -3.7968 & 3.4985 & -3.1728 \\ 3.4261 & -3.1728 & 2.8957 \end{bmatrix}$$

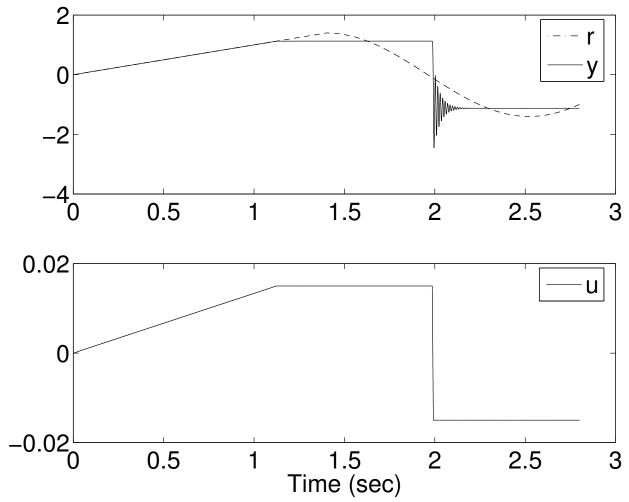
$$\Psi_1 = \begin{bmatrix} 0.3227 & 0.7467 \\ -0.2950 & -0.6362 \\ 0.2655 & 0.5339 \end{bmatrix}$$

$$\Psi_2 = \begin{bmatrix} 0.0001 & 0.0000 & 0.3227 & 0.7467 \\ -0.0001 & -0.0000 & -0.2950 & -0.6362 \\ 0.0001 & 0.0000 & 0.2655 & 0.5339 \end{bmatrix}$$

As we can see, the predictive control laws are different for different ω_0. The observers are designed using pole-placement method, where the desired closed-loop poles are selected as $-189.0000 - 189.5000 - 190.0000 - 190.5000$, roughly located at $-3 \times p$. The closed-loop system is simulated using a sampling interval of 0.00005sec. Without constraints, the closed-loop output response closely follows the set-point signal. In fact the absolute maximum tracking error occurred at the transition point between the two setpoint signals, and is 0.049. When the constraints



(a) Tracking of the reference with constraints



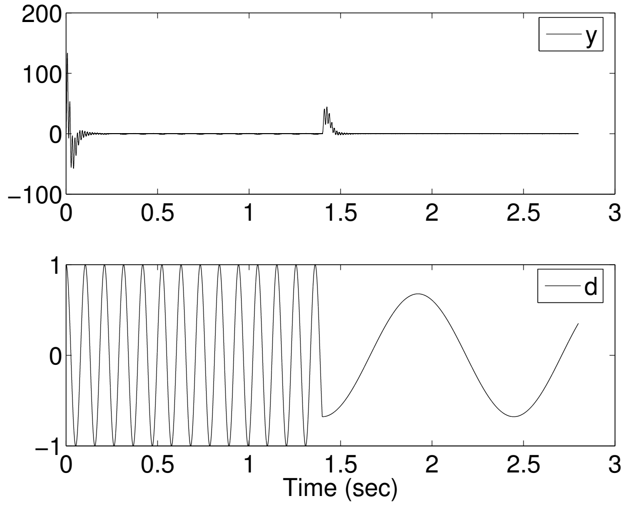
(b) Tracking of the reference with saturation

Fig. 1. Comparison of predictive control with constraints and with saturation

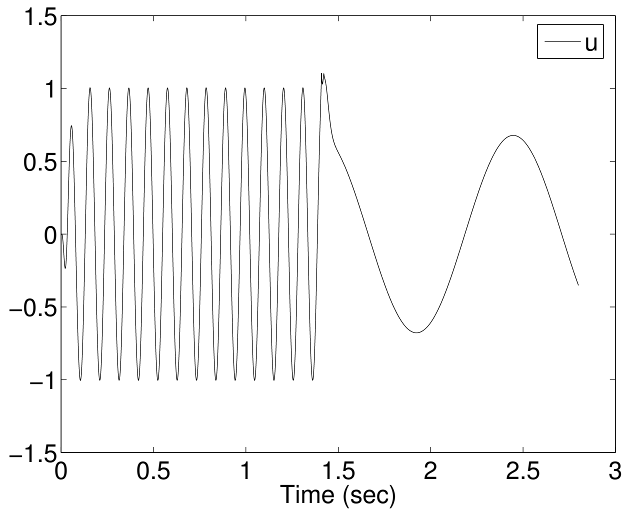
are imposed in the design, the predictive control finds the optimal control signal subject to the constraints (see Figure 1a). For comparison purpose, we simulated the case when a saturation of the control amplitude is used, instead of using the predictive control scheme. Figure 1b shows the simulation results. It is seen that the tracking performance degrades when the control signal amplitude is constrained at 0.015, and control system is not able to recover after the saturation is introduced.

4.2 Disturbance Rejection with Constraints

We consider rejection of input disturbance. This time, we introduce the disturbance as two sinusoidal signals with $\omega_0 = 60$ and $\omega_0 = 6$. The disturbance signal is added to the control signal, and a white noise with 0.1 standard deviation is added to the output to simulate the existing measurement noise. The design parameters for the predictive control system are identical to the ones used in the set-point following case, except that the frequency parameters are different. The constraints on the control amplitude is specified as $-1.1 \leq u(t) \leq 1.1$. Figure 2 shows the simulation results with constraints.



(a) Output and Disturbance



(b) Control signal

Fig. 2. Predictive control with constraints. Input disturbance rejection case in the presence of measurement noise

5. CONCLUSIONS

This paper proposed an approach to disturbance rejection and setpoint following of periodic signals in the framework of predictive control with constraints. The predictive control system is designed with embedded periodic component in the augmented model and a set of continuous-time Laguerre functions is used to describe the inversely filtered control signal. A consequence, the control system tracks (or rejects) sinusoidal signals with zero steady state errors. Simulation results show that the predictive control system produce optimal results with constraints.

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