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Optimal Control of Fuel Processing System Using Generalized Linear Quadratic Gaussian and Loop Transfer Recovery Method

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Abstract: This paper originally proposes an optimal control system which consists of both feedforward and state-feedback controllers using a generalized linear quadratic Gaussian and loop transfer recovery (GLQG/LTR) method. The control objective is focused on the regulatory performances of output vector in response to a desired stack current command in face of load variation. The proposed method provides another degree-of-freedom in optimal controller design and makes the compensated system have a prescribed degree of stability. Finally, the numerical simulations of a compensated fuel processing system reveal that the proposed method achieves better performance and robustness properties in both time-domain and frequency-domain responses than those obtained by the traditional LQ Method.

1. INTRODUCTION

A Linear Quadratic Gaussian and Loop Transfer Recovery (LQG/LTR) process, originally proposed by Doyle and Stein (1981), provides a prominent "loop shaping" concept in a two-step design procedure for the corresponding principal gains of return ratio (Stein and Athan, 1987; Maciejowski, 1989). In general, a control problem can be considered as a tracking issue by taking both reference command tracking and output vector regulation into consideration simultaneously. The control objectives are focused on the performance and stability specifications, such as reference command tracking, noises rejection, and robustness characteristics. For a multivariable control system, such requirements can be naturally transformed into the frequency-domain requirements in term of the singular values of sensitivity function and complementary sensitivity (co-sensitivity) function in a closed-loop control system. The sensitivity function is related to the return ratio which is evaluated by breaking at either the input or output point of compensated plant. Pukrushpan *et al*. (2003; 2005; 2006) have used a well-developed linear Quadratic (LQ) optimization technique to design an observer-based state-feedback controller for a fuel processing system (FPS) with a catalytic partial oxidation (CPO) reactor. However, the controlled CPO-based FPS is non-minimum phase and this effect does not take into consideration in the process of control system design. These motivate us to develop an optimal two-degree-of-freedom control system by a generalized LQG/LTR (GLQG/LTR) procedure for the problems of load tracking and output vector of a general system.

Fuel cell system (FCS) is potentially intended for stationary and mobile power generations with low greenhouse emissions and high electrochemical efficiency. The role of a FPS is to convert fossil and/or renewable fuel sources into suitable fuels, especially hydrogen-rich synthesis gas (H_2-rich syngas), for the electrochemical conversion in the FCS. Of all primary fossil fuels, natural gas is the cleanest fuel resource and the most environment-friendly one in terms of its products of combustion. Although it is a non-renewable fuel resource, natural gas is naturally preferred as the first candidate of available fuels because of its wide availability (Dicks, 1996), high-efficiency H_2 reforming (Ahmed and Krumpelt, 2001; Brown, 2001), environmental friendliness, as well as sufficient infrastructure for refuelling, distribution, and storage. Therefore, natural gas will play an important role in the ever-increasing energy consumption in the upcoming future. Among all of the fuel reforming processes, CPO is the most suitable one for mobile applications with rapid start-up, good tracking ability of load variation, and compactness. Nevertheless, CPO reaction suffers from lower hydrogen concentration and reforming efficiency than the other reforming process.

The main contribution of this paper is to originally derive a two-degree-of-freedom optimal control structure using the proposed GLQG/LTR methodology for both minimum and non-minimum phase systems. Both reference command and desired output trajectory tracking are simultaneously taken into consideration. Such a design procedure is used to redesign the CPO-based natural-gas-fuelled FPS in the work of Pukrushpan *et al.* (2005). The numerical results of simulation demonstrate both performance and robustness properties of compensated system are obviously improved in the time-domain response and frequency-domain analysis as well.

2. PROBLEM DEFINITION AND METHODOLOGY

2.1 Problem Definition

Let the dynamic equations of a multivariable control system shown in Fig. 1 be as follows

$$\dot{x}(t) = Ax(t) + Bu(t) + B_r r_c(t) + \Gamma w(t) \tag{1}$$

and

$$y(t) = Cx(t) + v(t) \tag{2}$$

where $x(t) \in \Re^{n}$, $u(t) \in \Re^{m}$, $r_c(t) \in \Re^{r}$, and $y(t) \in \Re^{q}$ are the state, input, reference command, and output vectors. $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $B_r \in \Re^{n\times r}$, $\Gamma \in \Re^{n\times p}$, and $C \in \Re^{q\times n}$ are the state, input of control, input of reference command,

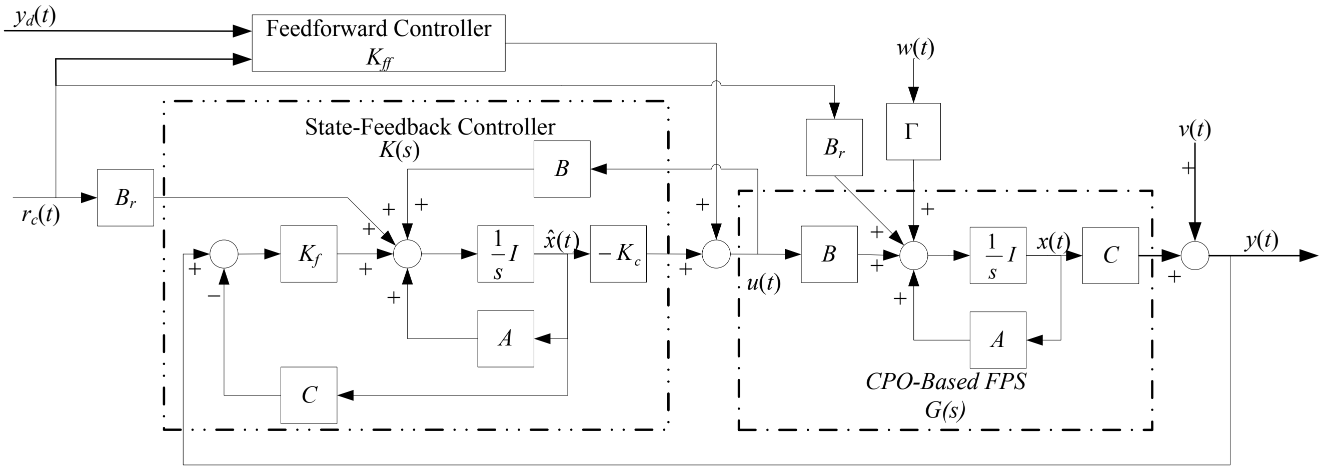

Fig. 1. Two-degree-of-freedom GLQG/LTR control structure.

input of disturbance, and output matrices, respectively. System disturbance $w(t)$ and measurement noise $v(t)$ are, respectively, p- and q-dimensional uncorrelated Gaussian white noise processes with zero-mean, and their covariances are given by

$$E\{w(t)w^T(\tau)\} = W(t)\delta(t-\tau) \quad (3)$$

$$E\{v(t)v^T(\tau)\} = V(t)\delta(t-\tau) \quad (4)$$

and

$$E\{v(t)w^T(\tau)\} = 0 \quad (5)$$

where $E\{\cdot\}$ is an expectation function operator, $W(t)$ and $V(t)$ are the covariance matrices of system disturbance and measurement noise, respectively. The problem is to derive an optimal control law minimizing the following LQ performance index

$$J = E\{0.5\int_0^{t_f} \exp(2\alpha t)[e^T(t)Qe(t) + u^T(t)Ru(t)]dt\} \quad (6)$$

where $e(t)$ is a tracking error between output response and desired output vector, Q is a q×q positive semi-definite weighting matrix, R is an m×m positive definite control weighting matrix, and α is a nonnegative constant which can provide a prescribed degree of stability in the proposed regulation problem.

2.2 Methodology Formulation

According to separation principle, a Kalman filter is first applied to provide an optimal estimated state vector and shape the principal gains of return ratio at the output of controlled plant. Secondly both feedforward and state-feedback controllers subject to the LQ performance index is designed in the LTR process.

A. Kalman Filter Design for Target Loop Transfer Function

The first step is to design a Kalman filter to provide an optimal estimate $\hat{x}(t)$ of $x(t)$, which minimizes the mean of estimated error $x(t)-\hat{x}(t)$. For a minimum-phase plant (A_m, B_m, C_m), the Kalman filter can be derived by the following state estimation equation

$$\dot{\hat{x}}(t) = A_m\hat{x}(t) + B_m u(t) + K_f\left[y(t) - C_m\hat{x}(t)\right] \quad (7)$$

where K_{mf} is the gain matrix of a Kalman filter calculated by

$$K_{mf} = P_{mf}C_m^T V^{-1} \quad (8)$$

and where P_{mf} is the covariance of $x(t)-\hat{x}(t)$ defined as

$$P_{mf} = E\left\{[x(t)-\hat{x}(t)]^T[x(t)-\hat{x}(t)]\right\} \quad (9)$$

P_{mf} can be obtained by the following Filter Algebraic Riccati Equation (FARE)

$$A_m P_{mf} + P_{mf}A_m^T + \Gamma W \Gamma^T - P_{mf}C_m^T V^{-1} C_m P_{mf} = 0 \quad (10)$$

It is well known that the right-hand plane (RHP) zeros of a non-minimum phase plant may be collected into a stable all-pass filter. The similar factorization is used to describe the RHP zeros in terms of structured uncertainty. Suppose the original plant is non-minimum phase, i.e. there is at least one zero in the RHP. Given a non-minimum phase system (A, B, C) with l RHP zeros, all the zeros can be factored in the form of multiplicative input uncertainty described as

$$G(s) = G_m(s)[1+\Delta(s)] \quad (11)$$

where $\Delta(s) = -2z/(s+z)$ is a structured uncertainty, and $z = \sum z_i$ is the sum of all the RHP zeros. By this way the non-minimum phase system can be expressed as a minimum-phase plant (A_m, B_m, C_m) with RPH zeroes in the form of multiplicative uncertainty. Suppose that K_f and K_{mf} are the gain matrices of Kalman filter design for (A, B, C) and (A_m, B_m, C_m), respectively. It has been well- proofed that $K_f = K_{mf}$ ([16,17]. Thus the gain matrix of Kalman filter for a general plant is given by

$$K_f = P_f C^T V^{-1} \quad (12)$$

where P_f can be obtained by the following Filter Algebraic Riccati Equation (FARE)

$$AP_f + P_f A^T + \Gamma W \Gamma^T - P_f C^T V^{-1} C P_f = 0 \quad (13)$$

Since the disturbances would couple into the system through the inputs rather than directly on the states, Γ is chosen as the input matrix B, i.e., $\Gamma = B$. The gain matrix of Kalman filter for a general system can be determined by manipulating the covariance matrices W and V.

B. Loop Transfer Recovery at Plant Output

An optimal control law for both feedforward and state-feedback controllers as shown in Fig. 1, can be derived as follows. Firstly, the corresponding Hamilton function is defined as

$$H = 0.5\exp(2\alpha t)\left(y_d^T Q y_d - 2y_d^T QC\hat{x} + \hat{x}^T C^T QC\hat{x} + u^T Ru\right) + P^T\left(A\hat{x} + Bu + B_r r_c\right) \quad (14)$$

An optimal control law can be derived by satisfying the following Euler-Lagrange equations. The optimal control can be obtained

$$u = -\exp(-2\alpha t)R^{-1}B^T P \quad (15)$$

Assume that $P = P_C\hat{x} + g$. After some manipulations, one has

$$\dot{P}_C + P_C A + A^T P_C - \exp(-2\alpha t)P_C BR^{-1}B^T P_C + \exp(2\alpha t)C^T QC = 0 \quad (16)$$

and

$$\dot{g} + A^T g - \exp(-2\alpha t)P_C BR^{-1}B^T g + \exp(2\alpha t)P_c B_r r_c - \exp(2\alpha t)C^T Q y_d = 0 \quad (17)$$

According to a sub-optimal control law, the optimal control law with feedforward and state-feedback controllers can be obtained

$$u(t) = -K_c\hat{x}(t) - R^{-1}B^T g(t) \quad (18)$$

where K_c is a optimal control gain matrix and is derived as

$$K_c = R^{-1}B^T P_c \quad (19)$$

and where P_c is a positive definite symmetric matrix, which is defined by the following Controller Algebraic Riccati equation (CARE)

$$P_c(A + \alpha I) + (A + \alpha I)^T P_c - P_c BR^{-1}B^T P_c + C^T QC = 0 \quad (20)$$

Therefore, the closed-loop dynamic equation of compensated system can be arranged as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \\ \dot{g}(t) \end{bmatrix} = \begin{bmatrix} A & -BK_c & 0 \\ K_f C & A - BK_c - K_f C & 0 \\ 0 & 0 & -A^T + P_C BR_{-1}B^T \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \\ g(t) \end{bmatrix} + \begin{bmatrix} 0 & B_r & \Gamma & 0 \\ 0 & B_r & 0 & K_f \\ C^T Q & P_C B_r & 0 & 0 \end{bmatrix} \begin{bmatrix} y_d(t) \\ r_c(t) \\ w(t) \\ v(t) \end{bmatrix} \quad (21)$$

and

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix}\begin{bmatrix} x(t) & \hat{x}(t) \end{bmatrix}^T + \begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}\begin{bmatrix} y_d(t) & r_c(t) & w(t) & v(t) \end{bmatrix}^T \quad (22)$$

Since the transfer function of an observer-based state feedback controller is

$$K(s) = -K_c(sI - A + BK_c + K_f C)^{-1}K_f \quad (23)$$

the resulting return ratio evaluated at the input of compensated plant is

$$G(s)K(s) = -C(sI - A)^{-1}BK_c(sI - A + BK_c + K_f C)^{-1}K_f \quad (24)$$

and the associated sensitivity function and complementary sensitivity function for the compensated plant are

$$S_{GK}(s) = [I + G(s)K(s)]^{-1} \quad (25)$$

and

$$T_{GK}(s) = S(s)G(s)K(s) \quad (26)$$

In a traditional LQG/LTR approach, the weighting matrices Q and R are tuneable parameters that can be manipulated to get better performance and robustness properties. In general, we can synthesize these optimal controllers by setting $Q = I$ and $R = \rho I$. It is well proven that $\lim_{\rho \to 0} G(s)K(s,\rho) = G_t(s)$ in the LTR procedure. We manipulate these variables to recover the principal gains of return ratio $G(s)K(s)$ at the output of compensated plant to the ones of target loop transfer function $G_t(s)$ as close as possible.

3. NUMERICAL SIMULATION

The linearized model of the CPO-based FPS in the work of Pukrushpan *et al.* (2005) can be reformulated in the form of state-space realization.

$$\dot{x} = Ax(t) + Bu(t) + B_r r_c(t) \quad (27)$$

and

$$y = Cx(t) \quad (28)$$

with

$$A = \begin{bmatrix} -3.333 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 212.63 & -124.5 & 0 & 112.69 & 112.69 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -32.43 & 32.304 & 32.304 & 0 & 0 & 0 & 0 & 0 \\ 0 & 221.97 & 0 & -254.9 & -253.2 & 0 & 0 & 32.526 & 0 & 0 \\ 0 & 0 & 331.8 & -341 & -344 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0748 & -3.53 & -0.074 & 0 & 1\times10^{-6} & 0 & 0 \\ 0 & 0 & 0 & 1.214 & 1.8309 & 0 & -0.358 & -3.304 & 0 & 2.0354 \\ 0 & 0 & 0 & 5.3994 & 5.6043 & 0.0188 & 0 & -13.61 & 0 & 8.1642 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.5582 & 13.911 & -1.468 & -25.3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33.586 & -156 \end{bmatrix} \quad (29)$$

$$B = \begin{bmatrix} 0.12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1834 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad (30)$$

$$B_r = \begin{bmatrix} 0.0265 & 0 & 0.0504 & 0 & 0 & 0 & 0 & 0 & -0.328 & -0.024 \end{bmatrix}^T \quad (31)$$

and

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.994 & -0.088 \end{bmatrix} \quad (32)$$

where the state, control input, reference command input, and output vectors of dynamic equations are defined as

$$x(t) = \begin{bmatrix} r_{\mathrm{BLO}} & p_{\mathrm{HEX}} & p_{\mathrm{DES}} & p_{\mathrm{MIX}}^{\mathrm{air}} & p_{\mathrm{MIX}}^{\mathrm{CH_4}} & T_{\mathrm{CPO}} & p_{\mathrm{WPO}}^{\mathrm{H_2}} & p_{\mathrm{WPO}} & p_{\mathrm{AN}}^{\mathrm{H_2}} & p_{\mathrm{AN}} \end{bmatrix}^T \quad (33)$$

$$u(t) = \begin{bmatrix} u_{\mathrm{BLO}} & u_{\mathrm{VAL}} \end{bmatrix}^T \quad (34)$$

$$r_c(t) = \begin{bmatrix} T_{\mathrm{CPO},r} & \gamma_{\mathrm{AN}}^{\mathrm{H_2}} & I_{st} \end{bmatrix}^T \quad (35)$$

and

$$y(t) = \begin{bmatrix} T_{\mathrm{CPO}} & \gamma_{\mathrm{AN}}^{\mathrm{H_2}} \end{bmatrix}^T \quad (36)$$

The desired steady-state output vector is selected that the operating temperature in the CPO reactor is T_{CPO}=972°K and the mole fraction of hydrogen in the anode is $\gamma_{\mathrm{AN}}^{\mathrm{H_2}} = 0.088$. The control objective is to regulate both T_{CPO} and $\gamma_{\mathrm{AN}}^{\mathrm{H_2}}$ in face of the variation of stack current I_{ST}. For mobile fuel cell system applications, an optimal control law in the form of feedforward and state-feedback controllers is designed to meet the requirements that a desired stack current I_{ST} is commanded to make vehicles in a preferred maneuvering direction rapidly, and both temperature of CPO reactor and molar fraction of hydrogen are simultaneously regulated at the desired working conditions.

3.1 Kalman Filter Design for Target Loop Transfer Function

The principal gains of return ratio $C(sI-A)^{-1}B$ for the nominal plant are shown in Fig. 2, where $\overline{\sigma}(\cdot)$ and $\underline{\sigma}(\cdot)$ are maximum and minimum principal gain functions. The controlled plant being a type-0 system, it is necessary to augment the model by inserting integral action before each input to eliminate steady-state errors. From a practical consideration, the poles of augmented model are placed at -0.01, and then the model of augmented plant is written as

$$A_a = \begin{bmatrix} A & \Gamma I_m \\ 0_{m\times m} & -0.01 I_m \end{bmatrix} \tag{37}$$

$$B_a = \begin{bmatrix} B & 0_{m\times m} \end{bmatrix}^T \tag{38}$$

$$C_a = \begin{bmatrix} C & 0_{m\times m} \end{bmatrix} \tag{39}$$

and

$$\Gamma_a = \begin{bmatrix} \Gamma\cdot 0_{m\times m} & I_m \end{bmatrix}^T \tag{40}$$

With Γ, *W*, and *V* being specified as mentioned above, we can adjust ρ_1 to design the Kalman filter and meet the frequency requirements. As ρ_1=0.2, the Kalman filter gain matrix is obtained as

$$K_f = \begin{bmatrix} 4.9741 & -0.1399 & 0.0207 & 0.4072 & 0.0605 & 0.3173 & -2.6891 & 3.0295 & -0.5571 & 0.0977 & 1.9532 & -1.0560 \\ -0.1409 & 0.8298 & 0.0360 & 0.3953 & 0.0369 & 0.3752 & 0.1813 & 0.1863 & 0.0585 & 0.1675 & 1.0454 & 1.9226 \end{bmatrix}^T \tag{41}$$

The principal gains of target loop transfer function $G_t(s)$, sensitivity function $S_f(s)$, and co-sensitivity function $T_f(s)$ at the plant output are shown in Figs. 3-4. As comparing with Fig. 4, integration action is clearly obvious in each channel and the minimum principal gains have been increased by almost exactly 20 dB at lower frequencies shown in Fig. 3.

3.2 Optimal Control Law Design in LTR Process

The parameters *Q*, *R*, and *α* are manipulated to shape the principal gains of return ratio, sensitivity function, and co-sensitivity function to have better recoverable quality in the LTR process. After some iterations, the parameters are manipulated as $Q=I$, $R=10^{-4}\times I$, and $\alpha=0.03$. The gain matrix of state-feedback controller is obtained as

$$K_c = \begin{bmatrix} 85.3053 & 35.9057 & -3.4315 & 33.7868 & 307.2167 & -60.1125 & -9.3214 & 20.4594 & 26.4546 & 42.1590 & 3.4626 & 3.6491 \\ -51.4848 & 60.5491 & -7.7289 & -15.3702 & -91.8720 & 78.8152 & 10.4117 & -9.6272 & 41.6871 & 30.4922 & -13.1081 & -10.8164 \end{bmatrix} \tag{42}$$

The principal gains of target loop transfer function $G(s)K(s)$, sensitivity function $S_{GK}(s)$, and co-sensitivity function $T_{GK}(s)$ at the plant input are shown in Figs. 5 and 6, respectively. For the comparison purpose, the results obtained by Pukrushpan's LQ method are also depicted in Figs. 5-6. Fig. 5 shows with the proposed method the separation between maximum and minimum principal gains are obviously lessened at lower frequencies and the condition number $\overline{\sigma}(GK)/\underline{\sigma}(GK)$ is decreased at all interested frequencies. This result unveil that the proposed method is much robust in face of plant's uncertainty. The principal gains of co-sensitivity function at higher frequencies are also declined about 10dB as shown in Fig. 6. These contributions make the compensated system have better rejection ability in the presence of high-frequency measurement noise.

It should be noted that there is a tradeoff between the recoverable quality of LTR and the performance of time-domain response. Furthermore, the time-domain simulations of compensated fuel processing system in response to an additional 50A command of stack current at the instant of 600 second for both proposed LQG/LTR and Pukrushpan's LQ methods are simultaneously shown in Figs. 7-11. Figs. 8 and 9 obviously reveal the better regulation ability of both CPO reactor temperature and hydrogen molar fraction by the proposed method. The penalty is both blower and fuel value inputs have relatively peak amplitudes in transient response as shown in Figs. 10-11. However, it is still satisfactory to meet the input limitation of 0-100. In addition, the root mean square of both blower and fuel value inputs are also listed in Table 1. The proposed method does not increase the power consumption of inputs at all. To evaluate the robustness of compensated system, the covariance responses of output vectors in the face of system disturbance covariance *W*=1 and measurement noise covariance *V*=1 are listed in Table 2. The proposed GLQG/LTR method significantly lessens the deviation of operating condition from normality and increases the robustness and performance properties of compensated system in the present of white noises.

4. CONCULSIONS

From the previous derivation and simulation, the proposed GLQG/LTR is an effective and efficient method from a frequency specification standpoint. The control system design of integrated feedforward and state-feedback control structure using the proposed method can obviously improve time– and frequency-domain responses. The proposed GLQG/LTR method for a two-degree-of-freedom controller can not only have better tracking ability of output vector in response to reference command but offer better robustness of noise rejection.

REFERENCES

Ahmed, S. and M. Krumpelt (2001). Hydrogen from Hydrocarbon Fuels for Fuel Cells. *International Journal Hydrogen Energy*, Vol. 26, No. 4, pp. 291-301.

Brown, L.F. (2001). A Comparative Study of Fuels for On-board Hydrogen Production for Fuel-Cell-Powered Automobiles. *International Journal Hydrogen Energy*, Vol. 26, No. 4, pp. 381-397.

Dicks, A.L. (1996). Hydrogen Generation from Natural Gas for the Fuel Cell Systems of Tomorrow. *Journal of Power Sources*, Vol. 61, No. 1-2, pp. 113-124.

Doyle, J. C. and G. Stein (1981). Multivariable Feedback Design: Concepts for a Classical / Modern Synthesis," *IEEE Transaction on Automatic Control*, Vol. AC-26, No. 1, pp. 4-16.

Maciejowski, J. M. (1989). *Multivariable Feedback Design*, Addition-Wesley Publishing Co., Chap. 5, pp. 222-264.

Pukrushpan, J. T., A. G. Stefanopoulou, S. Varigonda, L. M.

Pedersen, S. Ghosh, and H. Peng (2003). Control of Natural Gas Catalytic Partial Oxidation for Hydrogen Generation in Fuel Cell Applications", *Proceeding of the American Control Conference*, Vol. 3, pp. 2030-2036.

Pukrushpan, J. T., A. G. Stefanopoulou, S. Varigonda, L. M. Pedersen, S. Ghosh, and H. Peng (2005) Control of Natural Gas Catalytic Partial Oxidation for Hydrogen Generation in Fuel Cell Applications. *IEEE Transactions on Control System Technology*, Vol. 13, No. 1, pp. 3-14.

Pukrushpan, J., A. Stefanopoulou, S. Varigonda, J. Eborn, and C. Haugsteretter (2006) Control-Oriented Model of Fuel Processor for Hydrogen Generation in Fuel Cell Applications. *Control Engineering Practice*, Vol. 14, No. 3, pp. 277-293.

Stein G. and M. Athan (1987) The LQG/LTR Procedure for Multivariable Feedback Control Design. *IEEE Transaction on Automatic Control*, Vol. AC-32, No. 2, pp. 105-114.

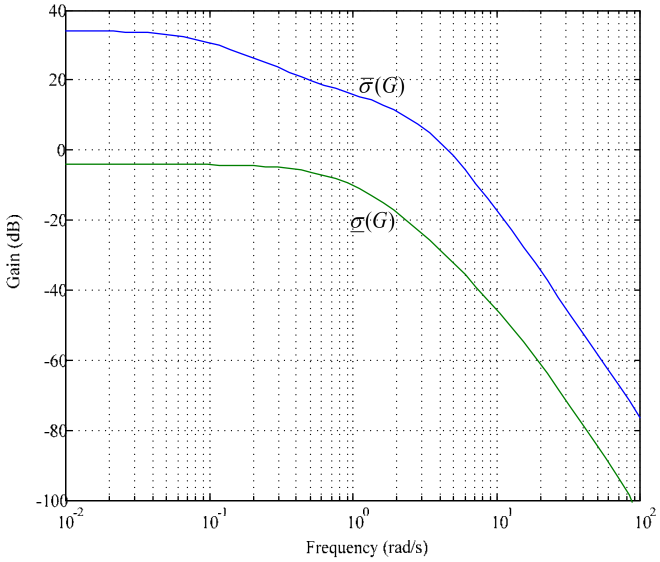

Fig. 2. Principal gains of return ratio $C(sI-A)^{-1}B$.

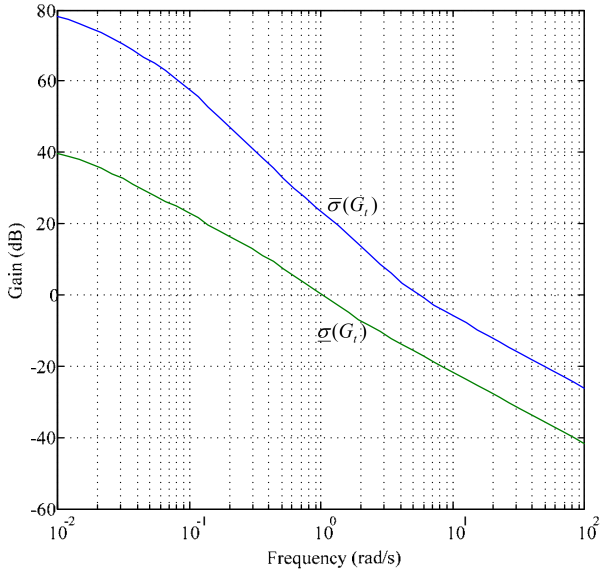

Fig. 3. Principal gains of target loop transfer function $G_t(s)$.

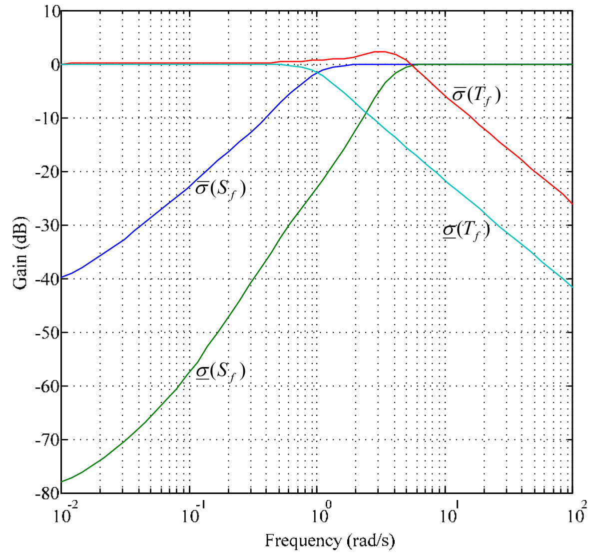

Fig. 4 Principal gains of $S_f(s)$ and $T_f(s)$.

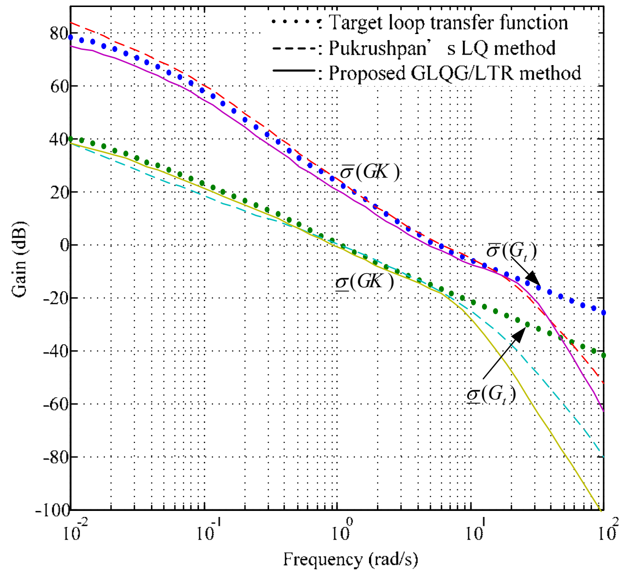

Fig. 5. Principal gains of return ratio $G(s)K(s)$.

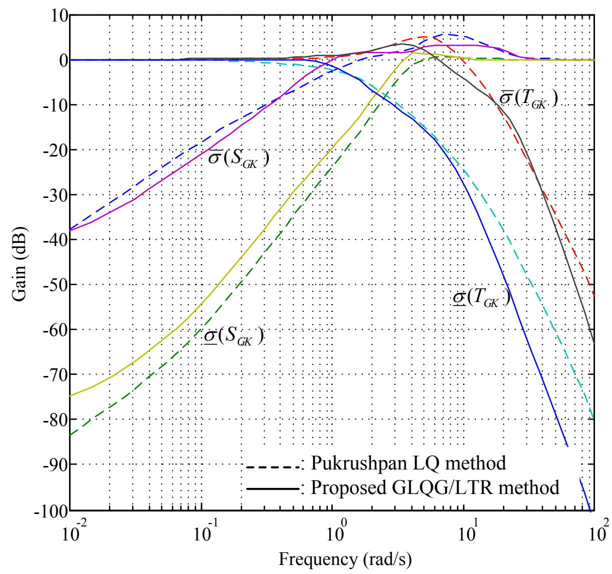

Fig. 6. Principal gains of $S_{GK}(s)$ and $T_{GK}(s)$.

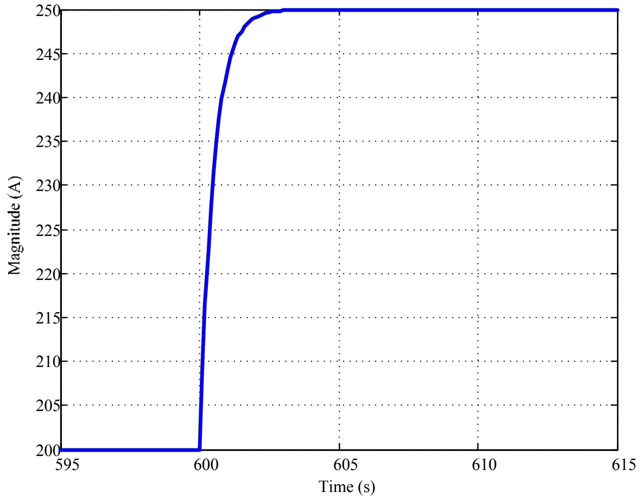

Fig. 7. Desired stack current increases by 50A at 600 second.

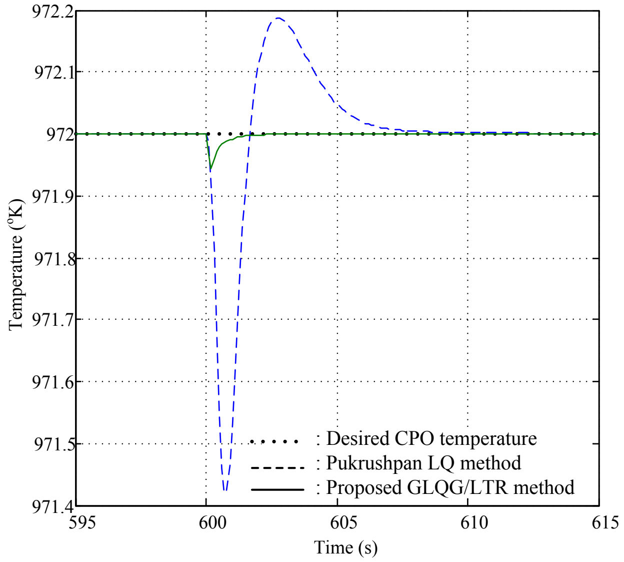

Fig. 8. Temperature response of CPO reactor.

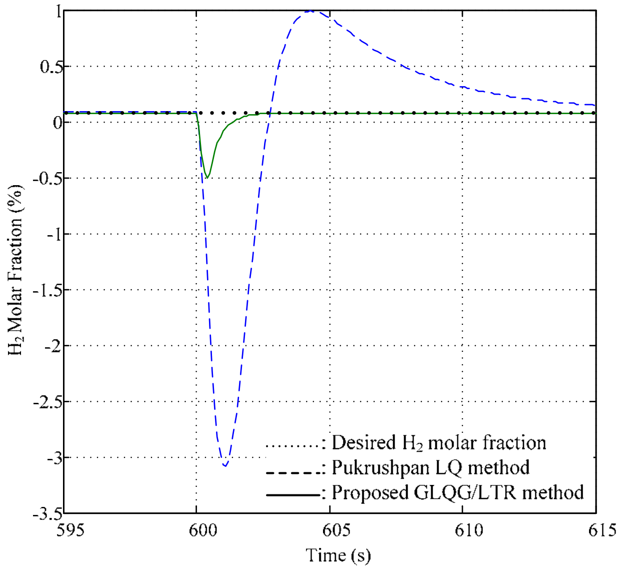

Fig. 9. Hydrogen molar fraction response of anode.

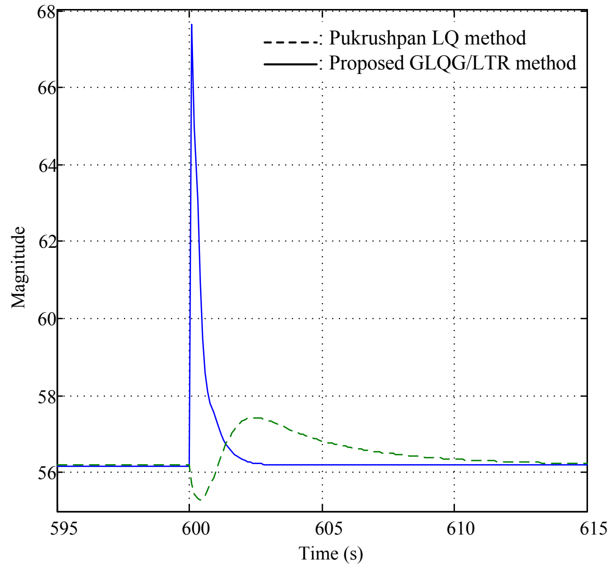

Fig. 10. Blower control input for commanded stack current.

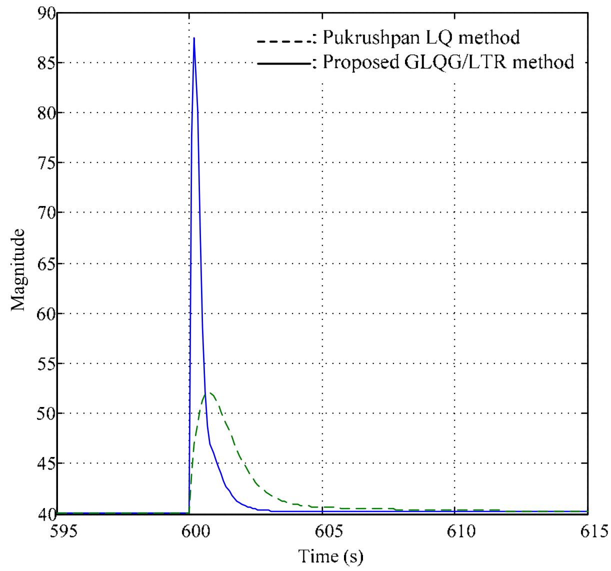

Fig. 11. Fuel value input response for commanded stack current.

Tabel 1. Root mean square of input vector

Items	GLQG/LTR method	Pukrushpan's LQ method
rms(u_{BLO})	55.7545	56.5822
rms(u_{VAL})	40.9715	41.5668

Tabel 2. Covariances of output responses with white noises

Items	GLQG/LTR method	Pukrushpan's LQ method
$E(e_{T_{\mathrm{CPO}}} e_{T_{\mathrm{CPO}}}^T)$	43.4260	115.2935
$E(e_{\gamma_{\mathrm{AN}}^{\mathrm{H_2}}} e_{\gamma_{\mathrm{AN}}^{\mathrm{H_2}}}^T)$	1.8042	5.3066