Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Gain-Scheduled Smith PID Controllers for LPV Systems with Time Varying Delay: Application to an Open-flow Canal

Yolanda Bolea, Vicenç Puig, Joaquim Blesa

Automatic Control Department, Technical University of Catalonia (UPC)
Pau Gargallo 5, 08028 Barcelona; (e-mail: yolanda.bolea@upc.edu)

Abstract: In this paper, a new approach to design gain scheduled robust linear parameter varying (LPV) PID controllers with pole placement constraints (through LMI regions) is proposed for LPV systems with second order structure and time-varying delay. The controller structure includes a Smith predictor, real time estimated parameters that schedule the controller (including the known part of the delay) and unstructured dynamic uncertainty which covers the unknown portion of the delay. Finally, the proposed control technique is validated in a real case study based on real a single reach canal: the Lunax Gallery at Gascogne (France).

1. INTRODUCTION

Although the control community has developed new and, in many aspects, more powerful control techniques during the last three or four decades, the PID controller is still used in many of the real world control applications. The reason is that controllers designed with the aid of modern control techniques are usually of high order, difficult to implement and virtually impossible to re-tune on line. Furthermore, if implementation issues have been overlooked, they can produce extremely fragile controllers (small perturbations of the coefficients of the controller destabilise the closed-loop control system). Besides, there are a great number of techniques to tune the PID gains that have been proposed in the literature for linear time invariant (LTI) systems. Since the 60's, the empirical or classical gain-scheduling (GS) control has been used for controlling non-linear and time-varying systems. But, this control methodology achieves closed loop stability, without guarantees, for slowly varying parameters. In order to overcome this deficiency, linear parameter-varying gain-scheduling (LPV GS) controllers are introduced to allow arbitrarily smooth or discontinuous variations of plant dynamics. The LPV GS method guarantees closed loop stability based on the concept of quadratic stability (QS) (Becker&Packard,1994) for all real parameter trajectories inside a given region. This methodology allows multi-objective criteria (H_∞, H_2, pole-placement) as well (Apkarian *et al.*, 1995a,b). Under additional hypotheses, such kind of synthesis problems can be transformed to a convex optimisation problem involving linear matrix inequalities (LMI's). This results in a well behaved and computationally tractable problem. For analysis, when the LMI conditions depend on the system parameter vector in a multi-affine way, it suffices to verify these conditions only at the vertices of the parameter polytope

The main contribution of this paper is to design a Linear Parameter Varying PID Smith Predictor controller (LPV PID+SP) for second order plus time delay linear parameter varying systems, taking into account robust stability, performance, closed loop pole constraints and essentially the time varying nature of the plant to be controlled: a single reach open-flow canal. This is a system with such dynamical behavior, being the case study used in this paper. The varying parameters are measured (estimated) in real time and used to schedule PID parameters. A "*delay scheduling*" Smith predictor scheme is used to compensate most of the estimated delay. However, there is still a remaining delay due to the inaccuracy in its estimation that it will be represented as unstructured dynamic uncertainty in a robust control framework. For a general LPV system case, the design of a LPV PID controller should be formulated as an output feedback control that usually derives in solving a non-convex optimization problem based on BMI's (Mattei, 2003). But, because of the special structure of the plant model considered (second order plus delay), the basic idea in our approach is to tune the PID controller reformulating it as a convex state-feedback problem.

The structure of the paper is the following: In *Section 2*, the background on LPV theory and on the design of a PID control problem as a state feedback is presented. The formulation, synthesis and implementation of the PID control of a second order plus delay LPV system in LPV framework using a "delay scheduling" Smith predictor (Smith LPV PID) are presented in *Sections 3*. In *Section 4*, to validate the proposed methodology this is applied to a real case study based on the control of a single reach canal (Lunax Gallery). In *Section 5*, final conclusions are drawn.

2. BACKGRAOUND ON LPV CONTROL

2.1 A LPV General Framework

Given an LPV system described by state-space equations of the form

$$\begin{aligned}\dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t) + B_w(\theta)w(t)\\ z(t) &= C_z(\theta)x(t) + D_{zu}(\theta)u(t) + D_{zw}(\theta)w(t)\\ q(t) &= C_q(\theta)x(t) + D_{qu}(\theta)u(t) + D_{qw}(\theta)w(t)\end{aligned} \tag{1}$$

where $x \in \Re^{n}$ is the state vector, $u \in \Re^{m_1}$ and $w \in \Re^{m_2}$ are the control and disturbance input vectors, respectively, $z \in \Re^{p1}$ and $q \in \Re^{p_2}$ are the measured and controlled output vectors, respectively. $A(\cdot)$, $B(\cdot)$, $B_w(\cdot)$, $C_z(\cdot)$, $C_q(\cdot)$, $D_{qu}(\cdot)$, $D_{zu}(\cdot)$, $D_{zw}(\cdot)$, $D_{qw}(\cdot)$ are continuous matrix valued functions of the time varying parameter vector $\theta(t) \in \Theta \subset \Re^{l}$, Θ being a polytope with r vertices. We assume the time varying parameters $\theta(t)$ can be measured (or estimated in the case of quasi-LPV models) in real time as in (Apkarian *et al.*, 1995b) (Becker and Packard, 1994). Performance is defined as requiring a bounded output $q(t)$ for any bounded external signal $w(t)$, both measured by their energy integral. The synthesis technique for LPV systems is based on the following results:

Theorem 1. *(Quadratic H_∞ Performance) (see Apkarian et al., 1995b). The LPV system given by Eq.(1) is QS and has quadratic H_∞ performance if there exists a positive definite matrix X>0 such that*

$$B^{0}_{[A(\theta),B(\theta),C(\theta),D(\theta)]}(X,\gamma) := \begin{bmatrix} A^T(\theta)X + XA(\theta) & XB(\theta) & C^T(\theta) \\ B^T(\theta)X & -\gamma I & D^T(\theta) \\ C(\theta) & D(\theta) & -\gamma I \end{bmatrix} < 0$$

for all admissible values of the parameter θ.

Remark 1. According to the self-scheduled H_∞ control synthesis problem for LPV systems developed by (Apkarian et al., 1995a), a control design which guarantees the Quadratic H_∞ performance for the closed-loop system, should fulfill the following necessary and sufficient conditions:

(i) $D_{qu}(\theta) = 0$ or equivalently $D_{qu_i} = 0$ for i=1,2,..,r.

(ii) $B(\theta), C_q(\theta), D_{zu}(\theta), D_{qw}(\theta)$ are parameter independent or equivalently $B_i = B, C_{q_i} = C, D_{zu_i} = D_{zu}, D_{qw_i} = D_{qw}$ for i=1,2,..,r.

(iii) The pairs $(A(\theta), B)$ and $(A(\theta), C_q)$ are quadratically stabilizable and detectable over Θ, respectively.

Theorem 2 (*Quadratic D stability*) (see *Chilali et al.*, 1999). *Consider the LPV system $\dot{x} = A(\theta)x$ with parameter θ , when θ is a fixed value ("frozen" time). Its pole location in the LMI-Region[1] D at each time t ("frozen" time) can be described by:* M_D= $[\ \alpha_{kl} X + \beta_{kl} A(\theta)X + \beta_{lk} XA(\theta)^T\ ]_{1\le k,l\le m}$, *where X is a positive definite matrix, and $M_D[A(\theta),X]$ and $f_D(z)$ can be related by the following substitution, $\left[X, A(\theta)X, XA(\theta)^T \leftrightarrow (1, z, \bar{z})\right]$. Then, the matrix $A(\theta)$ is quadratic D stable if and only if there exists a symmetric positive definite matrix X such that $M_D[A(\theta),X]<0$ for all admissible values of the parameter θ.*

Based on the fact that a finite set of LMI can be solved in the multi-affine case when the parameters vary in a polytope, a computationally feasible solution to the problem exists, first formulated in (Becker and Packard, 1994), as follows.

Theorem 3. (Vertex Property) (see Apkarian *et al.*, 1995b). Consider a polytopic linear parameter-varying plant as in Eq. (1), where

$$\begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix} \in \Theta := Co\left\{ \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix},\quad i = 1,\dots,r \right\}$$

and assume A,B,C,D are affine functions of θ, then the following items are equivalent:

i. The system is quadratic D–stable with Quadratic H_∞ performance γ.

ii. There exists a positive definite matrix X>0, which satisfies the following LMI's:

$$M_D(A_i, X) < 0$$
$$B^{0}_{[A_i,B_i,C_i,D_i]}(X,\gamma) < 0,\quad i = 1,2,\dots,r$$

If *Theorem 3* is fulfilled, *Theorem 1* and *2* only should be verified on the vertices of the parameter polytope Θ. This implies that the number of inequalities needed to test the analysis conditions of these theorems can be reduced to a finite one, which makes such an approach appealing.

2.2 PID LPV Control as State Feedback Control Problem

For a general LPV system case, the design of a LPV PID controller, $K(s,\theta)$, should be formulated as an output feedback control that usually derives in solving a non-convex optimisation problem based on BMI's. However if the system to be controlled has the following structure,

$$G(s,\theta) = \frac{b_0(\theta)}{s^2 + a_1(\theta)s + a_0(\theta)} \tag{2}$$

where $a_0(\theta)$, $a_1(\theta)$ and $b_0(\theta)$ are varying-parameters, a convex state feedback problem can be formulated (see (Ge *et al.*,2002) (Zheng *et al.*, 2002) for details), leading to the following state space description:

$$\begin{aligned} \dot{x} &= A(\theta)x + B(\theta)u + B_r r, \\ u &= -K(\theta)x + K_P(\theta)r + K_D(\theta)\dot{r}, \\ y &= C\,x, \end{aligned} \tag{3}$$

where y is the system output, $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ the state with variables defined by $x_1 = y, x_2 = \dot{x}_1$, $x_3 = -\int e dt, e = r - y$, r the reference input, and

$$A(\theta) = \begin{bmatrix} 0 & 1 & 0 \\ -a_0(\theta) & -a_1(\theta) & 0 \\ 1 & 0 & 0 \end{bmatrix}, B(\theta) = \begin{bmatrix} 0 \\ b_0(\theta) \\ 0 \end{bmatrix}, B_r(\theta) = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},\ K(\theta) = \begin{bmatrix} K_P(\theta) & K_D(\theta) & K_I(\theta) \end{bmatrix}. \tag{4}$$

In this state-space model, the PID controller design becomes a static state feedback controller, and the static feedback gain $K(\theta)$ simply contains all the PID controller parameters. Note also that there are three varying parameters in (2) and (4) (Ge *et al.*, 2002).

[1] A subset D of the complex plane is called an LMI-Region if there exists a symmetric matrix $\alpha = [\alpha_{kl}] \in \Re^{m\times m}$ and a matrix $\beta = [\beta_{kl}] \in \Re^{m\times m}$ such that: $D = \{z \in C : f_D(z) < 0\}$, $f_D(z) := \alpha + z\beta + \bar{z}\beta^T = [\alpha_{kl} + \beta_{kl} z + \beta_{kl}\bar{z}]_{1\le k,l\le m}$

3. LPV GAIN_SCHEDULING CONTROL METHODOLOGY

3.1 Smith LPV PID Control Problem Set-up

Let us consider the following LPV system

$$G(s,\theta) = D(s,\theta)e^{-s\tau(\theta)} \tag{5}$$

with $D(s,\theta)$ equal to (2), whose parameters are fixed functions of some vector of varying parameters $\theta(t)$ that can be measured on-line as in the case of general LPV systems presented in *Section 2.1*. The parameter range Θ is a box defined by $[b_{0\min} \quad b_{0\max}]$ for the gain $b(\theta)$, and $[a_{0\min} \quad a_{0\max}]$ $[a_{1\min} \quad a_{1\max}]$, for denominator coefficients $a_0(\theta)$, $a_1(\theta)$ and $[\tau_{min} \quad \tau_{max}]$ for the time delay $\tau(\theta)$. Our objective is to design a gain-scheduling PID controller using LPV theory for the plant model described by (5) which is an usual representation of many industrial and environmental processes. By including the parameter measurements/estimations, this controller adjusts to the variations in the plant dynamics in order to maintain stability and high performance along all trajectories $\theta(t)$. In other words, the controller is 'self-scheduled', that is automatically gain-scheduled with respect to $\theta(t)$. The variable delay in (5) can be handled in two different ways: (1) As an LTI dynamic uncertainty covered conveniently by a weight W_Δ as in (Skogestad *et al.*, 1997) (2) As a time-varying parameter which updates a Smith Predictor. The first approach could be conservative, and unnecessarily decrease the overall performance. On the other hand, the second approach could provide a far better performance, but it does not take into account the measurement error of the time-varying delay $\tau(\theta)$. In this paper, it is proposed to combine both approaches by assuming that a real time estimation $\hat{\tau}(\theta)$ of the delay is available, which will be used to update a Smith Predictor (Fig. 1). The difference between the actual and the estimated delay is considered as global dynamic uncertainty as in (Skogestad *et al.*, 1997) (Sánchez-Peña *et al.*, 1998) (Morari & Zafiriou, 1989) and is used in the design and robustness conditions. Therefore, we assume that the time delay dynamics has a time varying nature although its estimation error dynamics is time invariant, with a constant bound. The latter can be explained as follows: sensors are usually modelled as time invariant systems, with a bounded error provided by the manufacturer, as we have assumed here. The dependence of the delay with the operating point can be determined by physical modelling (Bolea *et al.*, 2004) or identification and is measured (estimated) in real time. Proceeding in such a way, most of the delay is compensated and the remaining portion, denoted as

$$\Delta\tau(\theta) = \tau(\theta) - \hat{\tau}(\theta) \tag{6}$$

can be covered by LTI unstructured uncertainty. This measurement error is always smaller that the actual delay, therefore the uncertainty is less conservative, which in turn has a lower impact on performance. This uncertainty is handled here as multiplicative output uncertainty and the following weight "covers" the delay measurement error frequency response as tightly as possible (see Chapter 11, (Sánchez-Peña *et al.*, 1998).):

$$W_\Delta(s,\Delta\tau) = \frac{2.05\Delta\tau_{\max}s}{\Delta\tau_{\max}s+1} \quad with \quad \Delta\tau(\theta) \le \Delta\tau_{max}. \tag{7}$$

Although the delay is time varying, by assuming that the delay measurement error is time invariant, the same robust stability analysis of the Smith predictor can be performed, following the approach proposed in (Sánchez-Peña *et al.*, 1998) (Morari & Zafiriou, 1989) for the LTI case. This is due to the fact that the remaining system, after the cancellation of delay with the use of its estimation, can be considered as finite dimensional LTI, according to this assumption. Therefore, the delay scheduled Smith Predictor eliminates the infinite dimensional as well as the time varying nature of the delay, reducing it to a LTI dynamic uncertainty. This is one of the main contribution of this work as compared to previous approaches (Ge *et al.*, 2002).

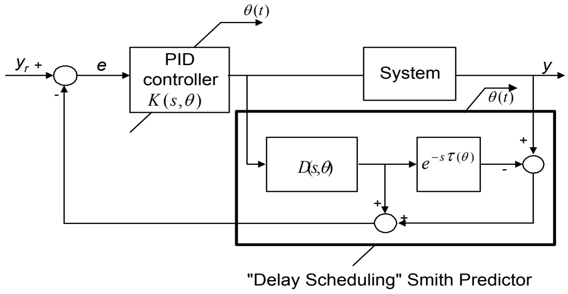

Fig.1. "Delay scheduled" Smith Predictor scheme.

3.2 Statement of the Smith LPV PID Controller

The LPV PID controller design of the system described by (5) will now be formulated as a state feedback problem and will be embedded in a self-scheduled LPV control problem as developed by (Apkarian, 1995a) (Becker&Packard, 1994) briefly summarised in *Section 2.1*. The control design specifications that will be considered are a mixture of performance and robustness objectives arranged as a MSP (Skogestad *et al.*, 1997) (see Fig.2), as follows:

$$\left\| \left[W_e S \quad W_u KS \quad W_\Delta T \right] \right\|_\infty^T < \gamma \le 1 \tag{8}$$

Here S is the sensitivity and T is the complementary sensitivity functions. These transfer functions represent weighted tracking error (or disturbance rejection), weighted control action and robust stability, respectively. In order to limit the control energy and bandwidth of the controller, a weight W_u is included in the design. Such weight is a transfer function with a crossover frequency approximately equal to that of the desired closed-loop bandwidth. The weight for the complementary sensitivity, W_Δ, captures the uncertainty of the plant model (in this case coming from the delay measurement error) and also limits the closed loop bandwidth. Typically, a disturbance in the system output is a low frequency signal, and therefore it will be successfully rejected if the minimum value of S is achieved over the same frequency band. This is performed by selecting a weight W_e, with a bandwidth equal to that of the disturbance in the controller design specifications. Robustness is presented as an H_∞ bound and is related with the dynamic uncertainty coming from the real time delay estimation error.

Performance is a combination of weighted error and control action minimization measured in terms of the energy integrals of the input and output signals involved. A PID controller is a good approximation of a robust high order controller at low frequencies, especially because of the inclusion of the integral action. Then, the resulting PID controller is expected to preserve the disturbance rejection performance of a high-order controller. Furthermore, the time response is tuned via a selected closed loop pole placement LMI region (Chilali *et al.*, 1999). This control design problem will be solved using the notion of QS and closed loop pole placement applied to a MSP, considering the delay measurement error as multiplicative dynamic uncertainty (see *Section 3.1*). A MSP can always be formulated as a Linear Fractional Transformation (LFT), and solved recasting two previous theoretical results (see *Section 2.1*): 1) Quadratic H_∞ performance. 2) Robust and Quadratic D–Stability.

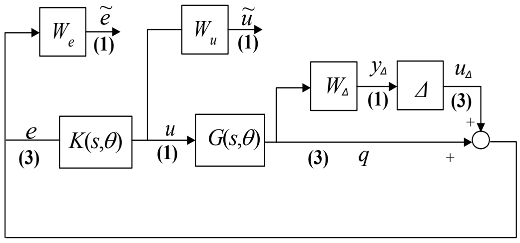

Fig.2. Proposed LPV feedback system scheme (MSP scheme).

The problem statement is as follows:

Problem 1. *Given the system in Eq. (1), find a gain-scheduling PID controller using an augmented LPV plant, guaranteeing QS and an H_∞ norm bound less than a positive number γ on the w-z input-output channel $\forall\theta\in\Theta$, and pole placement requirements applied to the MSP in Eq. (8).*

The control design scheme proposed for Problem 1, which combines measured (estimated) LPV parameters and unstructured output uncertainties is presented in Fig.2, and it is represented as a LFT. In such a LFT representation, the second order LPV model in (2) and (5) is represented by (1).

To achieve a PID controller as a state feedback, it is necessary to consider the following issues:

1. The performance and control effort weight functions need to be constants ($W_e = D_e$, $W_u = D_u$), so that the order of the augmented model is the same as the one in *Section 2.2*, and a PID controller can be designed.
2. In order not to increase the augmented model's order, the uncertainty weight in (8) is modified as follows:
$$\widetilde{W}_\Delta(s, \Delta\tau) = 2.05\Delta\tau_{max}s\,, \tag{9}$$
so that $W_\Delta = \widetilde{W}_\Delta G_f$.

Then, using a Smith Predictor scheme (Fig. 1) and the uncertainty weight introduced in (9) bounding the delay measurement error in (6), the following LFT LPV system representation is achieved:

$$\begin{aligned}\dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t) + B_{u_\Delta}(\theta)u_\Delta(t)\\ z(t) &= C_z(\theta)x(t) + D_{zu}(\theta)u(t) + D_{zu_\Delta}(\theta)u_\Delta(t) \qquad (10)\\ q(t) &= C_q(\theta)x(t) + D_{qu}(\theta)u(t) + D_{qu_\Delta}(\theta)u_\Delta(t)\end{aligned}$$

with $x = [x_1 \;\; x_2 \;\; x_3]^T = [y \;\; x_\Delta \;\; x_I]^T = [y \;\; \dot{x} \;\; x_I]^T$, $\underline{u} = [u_\Delta \;\; u]^T$, $z = [y_\Delta \;\; \tilde{u} \;\; \tilde{e}]^T$, and

$$A(\theta) = \begin{bmatrix} 0 & 1 & 0 \\ -a_0(\theta) & -a_1(\theta) & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad B(\theta) = \begin{bmatrix} 0 & b_0(\theta) & 0 \end{bmatrix}^T,$$

$$B_{u_\Delta}(\theta) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T,\; C_z(\theta) = \begin{bmatrix} D_\Delta & C_\Delta & 0 \\ 0 & 0 & 0 \\ -D_e & 0 & 0 \end{bmatrix},$$

$$C_q(\theta) = \begin{bmatrix} -1 & 0 & 0 \end{bmatrix},\, D_{zu}(\theta) = \begin{bmatrix} 0 & D_u & 0 \end{bmatrix}^T,$$

$$D_{zu_\Delta} = \begin{bmatrix} 0 & 0 & -D_e \end{bmatrix}^T,\; D_{qu_\Delta} = -1,\; D_{qu} = 0. \tag{11}$$

3.3 Implementation of the Smith LPV PID Controller

Since the gain, $b_0(\theta)$ of the system in (5)-(6) varies with parameter θ, to fulfill hypothesis (ii) associated to *Remark 1* in *Section 2.1*, the time varying gain of the system can be compensated in the following way. First, the LPV gain-scheduling PID controller $K(s,\theta) = K[s,a_0(\theta),a_1(\theta)]$] is designed taking into account only the variation of the parameters $a_0(\theta)$ and $a_1(\theta)$, and assuming that the parameter $b_0(\theta)$ has a nominal value b_{0nom}. Finally, keeping the same inner loop through equation

$$\widetilde{K}[s, a_1(\theta), a_1(\theta), b_0(\theta)] = K[s, a_1(\theta), a_1(\theta)]\frac{b_0(\theta)}{b_{0\,nom}} \tag{12}$$

the variation of parameter $b_0(\theta)$ is considered in the design of the controller. Due to the fact that the time varying parameters enter affinely in the augmented model equations (see (10) and (11)), the parameter region is polytopic and since condition (ii) is fulfilled through the transformation introduced by (12), the model of the LPV system can be represented by:

$$\begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix} = \sum_{i=1}^{r} \lambda_i \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}.$$

The delay $\tau(\theta)$ has already been considered as a scheduled (time varying) parameter in the Smith Predictor implementation, and the delay estimation error bounded by a multiplicative uncertainty in the design process, as explained in *Section 3.1*. Next compute a static time varying state feedback controller, which satisfies QS and the quadratic H_∞ performance specifications. Such a controller can be transformed by the equivalence introduced in *Section 2.2*, in a PID controller as in (3). This controller schedules the parameters $a_0(\theta)$, $a_1(\theta)$ and by means of the transformation in (12), the scheduling of parameter $b_0(\theta)$ is added. This controller guarantees QS and Quadratic H_∞ Performance, as well as ("frozen") closed loop pole location inside the desired LMI region. Since the plant is polytopic, the controller $K(s,\theta) = K(\theta)$ is designed as a polytopic model and implemented according to:

$$K(\theta) \in Co\{K(v_1), K(v_2),..., K(v_r)\} := \{\sum_{i=1}^{r} \lambda_i K_i \,;\, K_i = K(v_i)\}$$

where: $\sum_{i=1}^{r} \lambda_i(\theta) = 1$, $\lambda_i(\theta) > 0$. (13)

This technique is known as a convex decomposition technique, and *Co* is the function that generates the convex hull of the polytope vertices. The polytopic coordinates are calculated by fast algorithms in such a way that each vertex v_i, i=1,...,r has coordinates:

$$\lambda_i = \prod_{j=1}^{l} \tilde{\vartheta}_j^i \qquad \text{with}$$

$$\tilde{\vartheta}_j^i = \begin{cases} \vartheta_j^i & \text{if } \underline{\vartheta}_j^i \text{ is a coordinate of } v_i \\ 1-\vartheta_j^i & \text{if } \overline{\vartheta}_j^i \text{ is a coordinate of } v_i \end{cases} \qquad \text{where}$$

$$\tilde{\vartheta}_j^i = \frac{(\overline{\theta}_j^i - \theta_j^i)}{(\overline{\theta}_j^i - \underline{\theta}_j^i)}, \; j=1,...,l, \tag{14}$$

and $(\underline{\theta}_j^i, \overline{\theta}_j^i)$ represent the upper and lower bounds of θ_j^i, and l fulfils that $r=2^l$.

Finally, the closed-loop system is $\dot{x}_{cl} = A_{cl}(\theta)x_{cl} + B_w w$, with matrices $A_{cl}(\theta)$ and $C_{cl}(\theta)$ that depend on the parameter vector θ described as follows:

$$A_{cl} = \left\{ \sum_{i=1}^{r} \lambda_i(\theta)\left(A_i + B_w K_i\right) \right\}, B_{cl} = \left\{ \sum_{i=1}^{r} \lambda_i(\theta)\left(C + D_{zw} K_i\right) \right\}.$$

4. APPLICATION TO A SINGLE REACH CANAL

4.1 Lunax Gallery

The Lunax gallery is located at Gascogne in southwestern region of France. The dam-gallery is used to supply with water the river Gesse. As depicted in Fig.3, the plant consists in a single reach canal, the dam gate controls the upstream flow. The control objective aims at regulating the downstream flow even if the measurement point is far from the gate, at the output of the gallery. The geometry of the gallery is circular. Geometrical data are: d (diameter) = 1.8m, l (length) = 946.65m, i (slope) =0.0026rad, Strickler coefficient = 65.

4.2 LPV Model Motivation

The Saint-Venant equations can be simplified by considering the two following assumptions: lateral discharges are null and inertia terms are negligible compared to one representing the energy lost by friction. The approximated equations, known as diffusive wave equation, can be linearized around a reference flow Q_r. According to (Litrico et al., 1999), from the diffusive wave equation and the moment matching method, the following transfer function, known as Hayami model, linking the upstream flow Q_{ups} and downstream flow Q_{dns} for a single reach canal of length X can be derived

$$G(s) = \frac{Q_{dns}(s)}{Q_{ups}(s)} = \frac{e^{-s\tau(\theta)}}{1 + k_1(\theta)s + k_2(\theta)s^2} \tag{15}$$

where the scheduling parameter $\theta = Q_r = Q_{ups}$. In the Lunax Gallery, the time varying parameters of the model (15) can be bounded taking into account that operating range of the scheduling variable is $\theta \in [0, 5]$: $k_1(\theta) \in [199, 245]$, $k_1(\theta) \in [8396, 17355]$ and the time delay $\tau(\theta) \in [73, 252]$ (s).

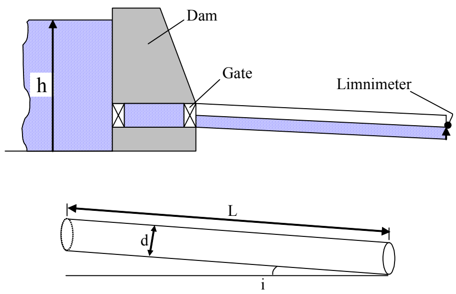

Fig. 3. Lunax gallery.

4.3 Controller Design and Simulation Results[2]

Comparing (15) with (2), it can be obtained $b_0(\theta) = \frac{1}{k_2(\theta)}$, $a_0(\theta) = \frac{1}{k_2(\theta)}$ and $a_1(\theta) = \frac{k_1(\theta)}{k_2(\theta)}$. Then, the control methodology presented in *Section 3* can be applied. Additionally, the delay estimation error is considered to be bounded $\Delta\hat{\tau}(\theta) \in [0.1, 5]$. The error in the time delay is taken into account in the control design as an LTI unstructured multiplicative uncertainty $W_\Delta(s, \Delta\hat{\tau}) = 10.25s$. Once the main time varying delay has been compensated by the Smith Predictor (Fig.1) and the remaining delay error considered as the weight $W_\Delta(s, \Delta\hat{\tau})$ of a multiplicative dynamic uncertainty (see *Section 3.1*), a PID controller is designed as a state feedback. This controller should guarantee closed loop stability and the following (step response) performance specifications: 1) tracking error of 0.1, 2) control signal within [0, 5] and 3) closed-loop damping of $\xi \geq 0.5$ and settling time in $t_{ss} \approx 230$(s), for any arbitrarily fast parameter variation. The tracking error and the bounded control signal are represented by performance weights $W_e = 1$ and $W_u = 0.4$, respectively. Furthermore, to achieve this desired transient behaviour and prevent controller fast dynamics, a pole clustering constraint is added. To this end, a LMI region $S(h_1, h_2, \alpha)$ is defined as a combination of three subregions: 1) A conic sector with apex at $x = 0$ and angle $\alpha = 3\pi/4$, which captures the closed-loop damping constraint $\xi \geq 0.5$. 2) Left half plane that guarantees the maximum settling time (h_1 = -0.016). 3) Left half plane that guarantees the minimum settling time (h_2 = -0.0018). The obtained closed loop responses using the LPV PID+SP design are shown in Fig.4.

[2] The real canal behaviour is accurately reproduced by Saint-Venant's equations using a simulator developed by the group of "Modelling and Control of Hydraulic Systems" at the UPC.

Fig. 5 shows the flow released upstream by the gate while Fig. 6 presents the evolution of the model parameters. It can be observed that the performance specifications are achieved for the whole admissible operating range. On the other hand, if a robust LTI H_∞ PID controller with a standard LTI Smith Predictor designed for the worst case set of parameters is used, the time response is slower than the one obtained by its LPV counterpart and do not satisfy the performance specifications for the whole admissible operating range.

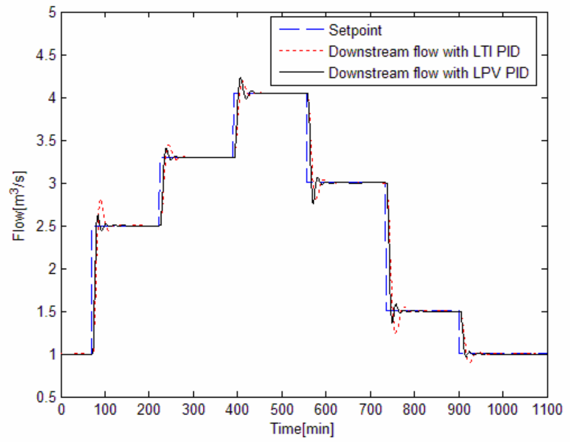

Fig. 4 Closed-loop response for different operating points.

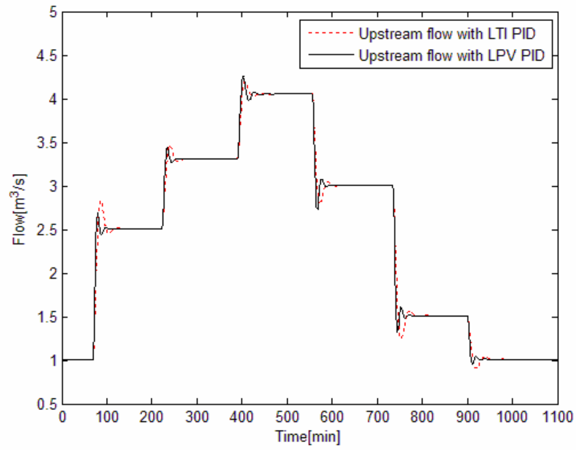

Fig. 5 Upstream flow (control action) corresponding to operating points changes presented in Fig. 4.

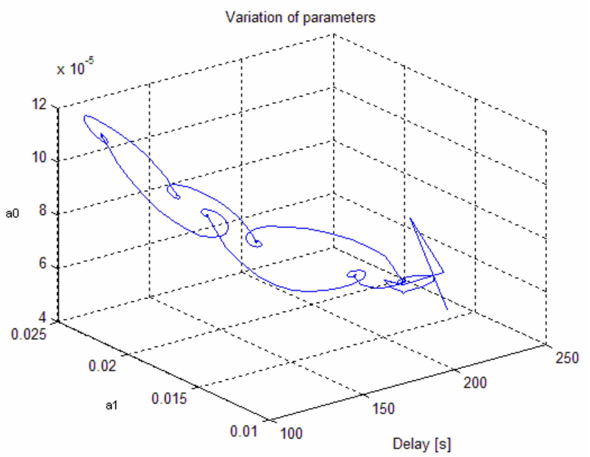

Fig.6 Evolution of the model parameters corresponding to the operating point changes presented in Fig. 4.

5. CONCLUSIONS

The main contribution of this paper is the development of a new approach to design a gain-scheduled Smith PID controller for LPV second order systems plus delay solving a MSP problem with closed loop pole placement constraints. The time varying delay is handled by a "delay-scheduling" Smith predictor and the estimated delay error is treated as an unstructured dynamic uncertainty. Thanks to the second order system structure, the PID controller design can be viewed as an state-feedback controller whose design can be transformed to a convex optimisation problem involving LMI's. This approach has successfully been applied real case study based on the control of a single reach canal (Lunax Gallery).

ACKNOWLEDGMENTS

The authors wish to thank the support received by the Research Commission of the Generalitat of Catalunya (Grup SAC ref. 2005SGR00537) and by CICYT (ref. DPI-2005-05415) of Spanish Ministry of Education.

REFERENCES

P. Apkarian, P. Gahinet, A convex characterization of gain-scheduled H_∞ controllers, IEEE Trans. on Automatic Control 40 (1995a) 853-864.

P. Apkarian, P. Gahinet, G. Becker, Self-Scheduled H∞ control of linear parameter-varying systems: A design example, Automatica 31(9) (1995b) 1251-1261.

G. Becker, A. Packard, Robust performance of linear parametrically varying systems using parametrically-dependent linear feedback, System and Control Letters 23 (1994) 205-213.

F. Bianchi, R. Mantz, Control PID LPV Gain Scheduled Robusto, XIX Congreso Argentino de Control Automático, AADECA (2004).

Y. Bolea, V. Puig, J. Blesa, M. Gómez, J. Rodellar, An LPV model for canal control, Proc. of the 10th IEEE International Conference on Methods and Models in Automation and Robotics (MMAR), Miedzyzdroje, Poland, (2004).

M. Chilali, P. Gahinet, P. Apkarian, Robust Pole Placement in LMI Regions, IEEE Trans. Automatic Control 44(12) (1999) 2257-2270.

V. T. Chow, Open-channel hydraulics, McGraw-Hill, New York, 1959.

M. Ge, M.-S. Chiu, Q.-G. Wang, Robust PID controller design via LMI approach, Journal of Process Control 12 (2002) 3-13.

Litrico, X. and D. Georges, "Robust continuos-time and discrete-time flow control of a dam-river system: (I) Modelling", Appl. Mathematical Modelling, vol.23, no.11, pp.809-827, 1999a.

M. Mattei, Robust multivariable PID control for linear parameter varying systems, Automatica 37 (2001) 1997-2003.

M. Morari, E. Zafiriou, Robust Process Control, Upper Saddle River, NJ: Prentice-Hall, 1989.

R.S. Sánchez Peña, M. Sznaier, Robust Systems Theory and Application, John Wiley & Sons, 1998.

S. Skogestad, I. Postlethwaite, Multivariable feedback control. Analysis and Design, John Wiley & Sons, 1997.