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Optimal input design for online identification: a coupled observer-MPC approach

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Abstract: This paper presents a parametric sensitivity based controller for on line optimal model parameter identification using constrained closed loop control tools and an observer. In optimal input design problem, analytical solution exists for few particular cases based on a relatively simple model. The approach proposed here may be used for a process based on a continuous model in the time domain, with two assumptions on the observability and the general structure of the model. The new proposed approach is to solve a model predictive control problem coupled with an on line process parameter estimation at each time using an observer. A dynamic parametric sensitivity model (derived from the process model) is also used on line to get the parametric sensitivity that has to be optimized. Both optimal input and estimated model parameter are therefore obtained on line. The case study presented here is a powder coating curing process where the main thermal parameter to identify influences the powder curing. First simulation results show here the efficiency of the approach in the control software (MPC@CB) developed under Matlab.

Keywords: design and control, identification for control, model predictive optimization-based control, nonlinear observer and filter design.

1. INTRODUCTION

Identification is a methodology which aims to approximate the process model parameters Ljung (1999). In this paper, one seek to discuss about one essential question in term of optimal input design for identification: is it possible to design online a certain input such that the model parameter θ may be identified on line and such that the input signal u maximizes the parametric output sensitivity $\frac{\partial y_p}{\partial \theta}$ (where y_p is the measured process output) ?

The problem of optimal design for identification has received attention in the literature Gevers (2005). Two papers Keesman and Stigter (2002) Stigter and Keesman (2001) include the analytical solution of optimal input design with respect to one specific parameter θ of the model. In this case, the Fisher Information Matrix reduces to a scalar value which may be optimized using a singular control law. It can be applied on a singular arc in the state space, and it may be solved by a bang-bang control. The singular control law is derived by solving a set of algebraic equations, generated through repeated differentiation of the Pontryagin optimality condition $\frac{dH}{du} = 0$ on the compact time interval $[t_1, t_2]$, where H is the familiar Hamiltonian associated with the process model and $u(t)$ is a goal function L. S. Pontryagin and Mishchenko (1986). In addition, if the cost criterion is based on the trace of the Fischer Information Matrix, i.e. the A-criterion, the input design problem may also be solved analytically: it

involves a maximization of the output sensitivities of a number of parameters for which an optimal input signal needs to be found. Recently, Stigter and Keesman (2004) have found recursive algorithm solutions for an optimal input design for a specific fed batch bioreactor. In this case an adaptive receding horizon optimal control problem, involving the so-called E-criterion, is solved on line using the current estimation of the parameter θ at each sample instant ($t_k, k = 0, \ldots, N - N_c$), where N marks the end of the experiment and N_c the control horizon. The optimal feed rate $F_{in}^*(t_k)$ thus obtained is applied and the measure $y(t_{k+1})$ that becomes available is subsequently used in a recursive prediction error algorithm in order to find an improved estimate of the parameter estimate $\hat{\theta}(t_k)$. Meanwhile, such analytical methods are not easy to use for complex model based systems. This is often the case, like in chemical engineering.

The goal of this paper is to present a general framework to optimally estimate on line the real value of the parameter θ using the input-output measures and the process model. This approach may be used for a process modeled by a continuous dynamic model in the time domain (belonging to a particular family of models), like a nonlinear partial differential equation (PDE) based model. An ordinary differential equation (ODE) based model may also be used. In this approach, the parametric sensitivity model is explicitly derived from the process model and is used by the controller. Since the optimal control law may not be easily obtained, one has to rely on numerical optimization

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methods, involving intensive numerical calculations: we develop here a control strategy combining a process model, the coupled parametric sensitivity model, an observer and a model predictive control (MPC) strategy. The idea is to maximize on line, over a receding horizon, the parametric sensitivity of the process output under input-output constraints. Since the sampling time may be small (a few seconds), a linearization of the models is also used to decrease the on line computational burden. With such optimization, the optimal input u is found on line while the real value of the parameter θ is also estimated on line.

MPC has found large applications Qin and Badgwell (2003), particularly in the chemical processing industry. The idea of this strategy is to solve on line an open-loop based optimization problem over a finite prediction horizon. This procedure is repeated at each sample time with the update of process measurements. In this paper, the MPC is based on two models in cascade: the process model and the associated parametric sensitivity model.

Furthermore, the use of an observer is often needed for the implementation of the control law. Numerous studies were interested to design of different kind of observers, for example: high gain observer J.P. Gauthier and Othman (1992), Kalman filters etc. In this approach, an observer is used to estimate on line the model parameter, which value is also fed into the controller to improve the choice of u.

The case study presented here deals with the optimal input design problem in the field of a powder coating curing process control K. Abid and Laurent (2007). Powder coating curing process presents numerous advantages although the applications of the powder coatings remain unchanged: architecture, furniture, domestic appliances L. Vechot and Lieto (2006). Few studies correlates the optimization of the cure process and the identification of the different parameters of the process, whereas one of the uncertain parameter has here a large impact over control results.

This paper is organized as follows: In Section 2, some preliminaries are presented. The idea of optimal parametric sensitivity control is detailed in Section 3 leading to a nonlinear observer based model predictive control problem. In Section 4 the mathematical model of the case studied (a powder coating curing process) is described. A brief detail about the MPC@ CB software used here is given. Finally the simulation results are presented in Section 5.

2. PRELIMINARIES

Let the model structure be defined by the general dynamic system

$$\begin{cases} \dfrac{dx(t)}{dt} = f(x(t), \theta) + g(u(t), \theta) \\ y(t) = C.x(t) \end{cases} \tag{1}$$

where $x(t)$ is the n-dimensional state vector, $u(t)$ is the scalar input, $y(t)$ is the scalar output of the system. In order to simplify our work and the notation, θ is the scalar parameter of the model to be identified. f, g and h are nonlinear functions of suitable sizes.

An estimator is a dynamic system obtained from the nominal model by adding a correction term which is proportional to some output deviation. The advantage of this specific algorithm is that, under some properties of the model structure, it can be used to accurately estimate the states not measured and certain unknown parameters used in the model. It is assumed that the augmented state $\tilde{x} = \begin{bmatrix} x & \theta \end{bmatrix}^T$ may be directly observed. Therefore, an observer may be constructed from the model (1) to estimate on line $\hat{\tilde{x}} = \begin{bmatrix} \hat{x} & \hat{\theta} \end{bmatrix}^T$. Let us also assume that this observer may be written as a state affine system:

$$\begin{cases} \dfrac{d\hat{\tilde{x}}(t)}{dt} = \tilde{f}(u(t)).\hat{\tilde{x}}(t) + \tilde{g}(u(t)) \\ \qquad - S^{-1}\tilde{C}^T.(\tilde{C}.\hat{\tilde{x}}(t)) - y(t)) \\ \dfrac{dS(t)}{dt} = -\mu.S - \tilde{f}(u(t))^T.S - S.\tilde{f}(u(t)) - \tilde{C}^T.\tilde{C} \end{cases} \tag{2}$$

where $\mu > 0$. More details about this observer may be found in G. Bornard and Celle (1998); Hammouri and Leon (1990).

In parametric models, the sensitivity of the output y with respect to a parameter θ (i.e, $y_\theta = \frac{\partial y}{\partial \theta}$) determines how the parameter may be estimated from the input/output data. If y_θ is "small", then the instrumentation is not well chosen or the input sequence $u(t)$ may not excite sufficiently the parametric sensitivity. Here, the aim is to find a control law that maximizes y_θ, based on both the process model and its parametric sensitivity model. The associated parametric sensitivity model is derived from the model (1) and is written as follows:

$$\begin{cases} \dfrac{dx_\theta(t)}{dt} = \dfrac{\partial f}{\partial x}(x(t), \theta).x_\theta(t) + \dfrac{\partial f}{\partial \theta}(x(t), \theta) \\ y_\theta(t) = \dfrac{\partial h}{\partial x}(x(t)).x_\theta(t) \end{cases} \tag{3}$$

where $x_\theta \equiv \frac{\partial x}{\partial \theta}$ and $y_\theta \equiv \frac{\partial y}{\partial \theta}$.
The idea of this strategy is to find the control law based on the model (1) coupled with the sensitivity function (3) and the observer (2).

3. OPTIMAL CONTROL PROBLEM

The proposed approach is to solve on line the coupled input design and identification problem as a global control problem where the unknown parameter is estimated at each time using an observer. The control strategy used here is the MPC, or receding horizon control (RHC). It consists in solving an explicit optimization problem formulated into the future. It is based on a cost function that represents the optimal control problem: process yield, energy efficiency, cost of energy and raw materials, product prices ... The main advantage of the MPC technique is that the constraints (due to physical limits, safety, environmental regulation, ...) may be explicitly specified into this formulation. In this structure, a model aims to predict the future behavior of the process and the best one is chosen by correct tuning of the manipulated variables. This procedure is repeated at each sampling time with the update on the process measurements. In recent applications, K. Abid and Laurent (2007) and N. Daraoui and Hammouri (2007) have shown how the current special PDE model based MPC framework developed in P. Dufour and Laurent (2003) may be used for the control of such PDE systems, in spite of the relatively large size and nonlinearity of the model state computed during the optimization, modeling errors and

uncertainties. In this approach, control problems with a short sampling period (a few seconds) may be handled. In this particular approach, the control problem is a general optimization problem over a receding horizon N_p where the cost function J to be minimized reflects any control problem (trajectory tracking, processing time minimization, energy consumption minimization, sensitivity maximization,...):

$$J(u) = \sum_{j=k+1}^{j=k+N_p} a(y_{ref}(j), y_p(k), y_m(j), u(j)) \quad (4)$$

where k (resp. j) is the actual (resp. future) discrete time index, u is the input, y_{ref} describe the specified behavior for the considered process output y_p, y_m is the continuous model output. The cost function J has to be minimized under:

- Input constraints on the magnitude and velocity of the manipulated variable:

$$\begin{cases} u_{min} \le u(k) \le u_{max}, \forall k > 0 \\ \Delta u_{min} \le u(k) - u(k-1) \le \Delta u_{max}, \forall k > 1 \end{cases} \quad (5)$$

- Output constraints related to operating conditions, safety, quality, ...:

$$g_i(y_p(k), u(k)) \le 0, \forall k > 0, \forall i \in I^n = (1, ..., n) \quad (6)$$

The cost function of this optimization problem is transformed into:

$$J_{tot}(p(u)) = J(p(u)) + J_{ext}(p(u)) \quad (7)$$

where the unconstrained optimization argument is p: it is obtained from a simple hyperbolic transformation of the magnitude and velocity constraints (5) specified for the manipulated variable u P. Dufour and Laurent (2003). Output constraints (6) are explicitly formulated as a penalty term Fletcher (1987) used in 7:

$$J_{ext}(p(u)) = \sum_{j=k+1}^{j=k+N_p} (\sum_{i \in I^n} \omega_i max^2(0, g_i(y_p(j), p(u(j))))) (8)$$

where w_i is an adaptive positive defined weight tuned according to Fletcher (1987).

Any unconstrained optimization algorithm may be used to minimize the penalized cost function (7): widely known and used for its robustness and convergence properties, the Levenberg-Marquardt's algorithm is used where the optimization argument p is determined at each sample time k using the process measurement, the model prediction and the cost function J_{tot}.

In this work, our interest is to maximize the parametric sensitivity of the process output with respect to the parameter ($\frac{\partial y_p}{\partial theta}$) with an optimal tuning of the parameter p, such that θ becomes optimally identified. Based on the parametric sensitivity model, the cost function J to minimize can therefore be expressed in the following form:

$$J(p(u)) = \sum_{j=k+1}^{j=k+N_p} \frac{1}{\left\| \epsilon + \frac{\partial y_p}{\partial \hat{\theta}}(k) \right\|^2} \quad (9)$$

where ϵ is a constant tuned such that J is always defined. From a practical point of view, one of the numerical

problems of the MPC is to reduce the computational time needed to solve the unconstrained optimization problem during the short sampling period (a few seconds). Based on a MPC approach developed and used for on line control problems P. Dufour and Laurent (2003), we use a linearization method of the nonlinear model (1) and the nonlinear parametric sensitivity model (3) chosen and computed off-line. Finally, the off-line solved nonlinear and nonlinear parametric sensitivity models, and the on line solved time varying linearized and time varying linearized parametric sensitivity models replace the on line initial nonlinear and nonlinear parametric sensitivity models in the internal model control (IMC) approach. For this new approach coupling a model based estimator, the process model, the coupled sensitivity model, the linearization and a predictive controller, the final control structure is presented in Fig. 1. Therefore, the current estimation $\hat{\theta}$ of the parameter is used to find u, which is then used gain to update the estimation $\hat{\theta}$. The procedure to apply at each time instant k is to:

(1) Measure the current measurement $y_p(k)$
(2) Estimate $\hat{\theta}(k)$ through the observer,
(3) Compute $\frac{\partial y_m}{\partial \theta_m}(k)$ over the time horizon $[k+1, .., k+N_p]$ and minimize the penalized cost function (7)
(4) Apply the optimal input $u(k)$ thus obtained on the interval $[k, k+1[$

4. CASE STUDY

4.1 A Dynamical Model for Powder Coating Curing Process

The mathematical partial differential equation model of a powder coating curing process is based on the knowledge of the fundamental mechanisms inside the powder during the infrared flow curing. It is governed by two nonlinear phenomena: the coupled thermal transfer and the curing.

The thermal model is based on the Fourier law of heat conduction and Figure 2 shows the boundary conditions applied at the top surface of the powder and at the bottom of the metallic substrate. The thermal balance uses both the temperature variable varying in the thickness of the powder coated metal sample, and the degree of cure conversion (which ranges from 0^+ at the beginning to 1 at the end). The terms C_i used in the model are positive known constant values. Inside the powder, it leads to the following equation:

$$\frac{\partial T_p(z,t)}{\partial t} = C_1 \frac{\partial^2 T_p(z,t)}{\partial z^2} - C_2 e^{(\frac{-C_3}{T_p(z,t)})} x^m (1-x)^n, \quad (10)$$
$$\forall z \in]0, e_p[, \forall t > 0$$

where T_p is the temperature across the powder film, which thickness is e_p and x is the degree of cure. The thermal balance inside the substrate leads to the following equation for the temperature T_s inside the substrate, which thickness is e_s:

$$\frac{\partial T_s(z,t)}{\partial t} = C_4 \frac{\partial^2 T_s(z,t)}{\partial z^2} \quad \forall z \in]e_p, e_p + e_s[, \forall t > 0 \quad (11)$$
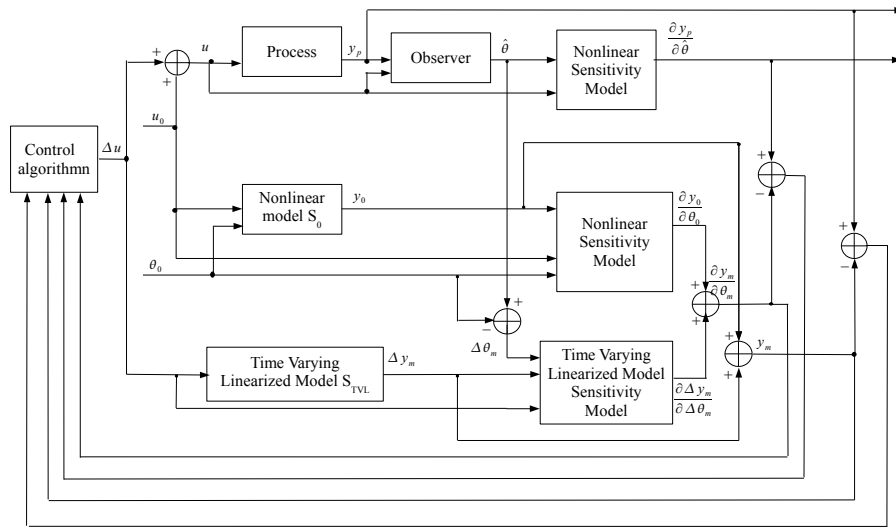
The boundary conditions are:

Fig. 1. Linearized IMC-MPC observer based structure for coupled input design and parameter estimation.

$$-C_5\frac{\partial T_p(t,z)}{\partial z} = \alpha_p\phi_{ir}(t) - C_6(T_p^4(z,t) - T_{ext}^4) \\ -C_7(T_p(z,t) - T_{ext}) \quad at \quad z = 0, \forall t \quad (12)$$

where $\phi_{ir}(t)$ is the manipulated variable u and α_p is the absorption coefficient that has to be optimally estimated.

$$\frac{\partial T_p(z,t)}{\partial z} = C_8\frac{\partial T_s(z,t)}{\partial z} \quad at \quad z = e_p, \forall t > 0 \quad (13)$$

expresses the continuity of the thermal flow at the interface of the powder and the substrate, and:

$$-C_9\frac{\partial T_s(z,t)}{\partial z} = -C_{10}(T_s^4(z,t) - T_{ext}^4) \\ -C_{11}(T_s(z,t) - T_{ext}) \quad at \quad z = e_p + e_s, \forall t > 0 \quad (14)$$

The initial condition is:

$$T_p(z,t) = T_s(z,t) = T_{ext}\forall z \in [0, e_p + e_s], t = 0 \quad (15)$$

Concerning the degree of cure $x(z,t)$ of the powder, the polymerization reaction is characterized by the Sestak-Berggren law:

$$\frac{\partial x(z,t)}{\partial t} = C_{12}e^{(\frac{-C_3}{T_s(z,t)})}x^m(1-x)^n\forall z \in [0, e_p], \forall t > 0 \quad (16)$$

The initial condition is:

$$x(z,t) = 0^+\forall z \in [0, e_p], t = 0 \quad (17)$$

The thermo physical properties of the substrate have been found in the literature. The thermo physical properties of the paint were provided by our paint supplier. More details about the modeling may be found in I. Bombard and Lieto (2006).

In this case, one is interested to find the optimal control $\phi_{ir}(t)$ which maximizes the sensitivity of the measure $T_s(e_p + e_s, t)$ with respect to parameter α_p, such that α_p is optimally estimated. Indeed, a study allowed concluding that the parameter that influences the most the kinetic of the coating is the absorption coefficient of painting
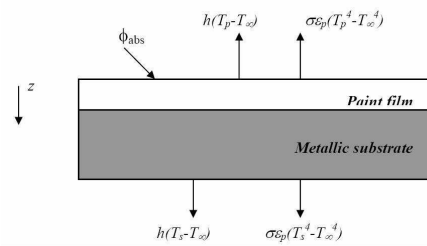

Fig. 2. Schematic drawing of the "substrat+powder" sample.

α_p, which is difficult to identify, since it depends on the temperature of the infra-red lamps.

In order to implement the control law, the original PDE model is solved as an algebraic differential equation (ADE) model. There are several methods making it possible to obtain the solution of the model by a numerical solution. Among these methods, it is necessary to note: the finite element method, the method of finite volume and the method the finite differences. The latter is used in our study.

4.2 Control Software: Main Features of MPC@CB

The codes of the MPC@CB Software [2] have been written with Matlab. It allows to realize the MPC under constraints of any continuous process. The advantages of these codes is first the easy use for any continuous SISO process (Single Input Single Output), through the user files (where model equations have to be specified), synchronized by few main standards files (where the use has to make few (or no) changes). The model may features any number of states variable, it may be linear or not linear, time variant or time invariant, based on ODE and/or PDEs. Another original feature of the software is the straightforward resolution

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of various model based control problems through different choices:

- MPC for a trajectory tracking problem, with or without the output constraint. The user may specify any reference trajectory.
- MPC to solve an operating time problem, with or without the output constraint.
- MPC to solve any user defined constrained optimal control problem.
- In order to study the robustness of the control law with simulations, it is easy to introduce, for any parameter, different values in the model (used in the controller) and in the simulated process.
- Introduce a software sensor for the estimate of the variables not measured (currently under development for this process).

The other originality is the method used to develop the codes: it is very easy to introduce new parts in the code, such as: user defined problem, handle SIMO, MISO or MIMO model (M stands for multiple, S stands for single) and apply the software for a real time application or simulation.

5. SIMULATION RESULTS

As mentioned previously, our aim is to find the optimal control $\phi_{ir}(t)$ which maximizes the output sensitivity $\partial T_s(e_p + e_s, t)/\partial \hat{\alpha}_p(t)$. For this reason, the MPC@CB software is tuned to solve 3 different control problems to compare the following approaches:

- open loop control with a step at the input satisfying the input constraints,
- closed loop control defined in this paper, with input constraints,
- closed loop control defined in this paper, with input constraints and an output constraint.

For the input constraints, the constraints (5) for the manipulated variable are:

$$\begin{cases} 0 \ W.m^{-2} \leq \phi_{ir}(k) \leq 23000 \ W.m^{-2} \\ -2000 \ W.m^{-2} \leq \phi_{ir}(k) - \phi_{ir}(k-1) \leq 2000 \ W.m^{-2} \end{cases} (18)$$

and the process output constraint (6) that allows to avoid overheating that deteriorates the powder during the curing is:

$$T_s(e_p + e_s, k) \leq \ 380 \ \ K \qquad (19)$$

In each cases, the unknown parameter α_p is estimated after 30 s (Fig. 3). In case of closed loop control with input constraints, the parametric sensitivity is more maximized than is the open loop case (Fig. 4) and the temperature in process output takes very large values (Fig. 5). In the meantime, the estimation of α_p is more continuous. In the third case, the output constraint (19) is always satisfied (Fig. 5) by a correct tuning of the input (Fig. 6): the parameter is in this case estimated on line while the process is under closed loop control and in a safe use specified by the constraints of the user. In both closed loop cases, the trajectory of the estimation are very closed since the observer converges before the saturation of the output constraint (19).
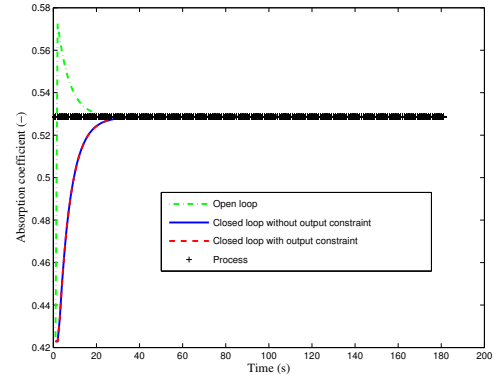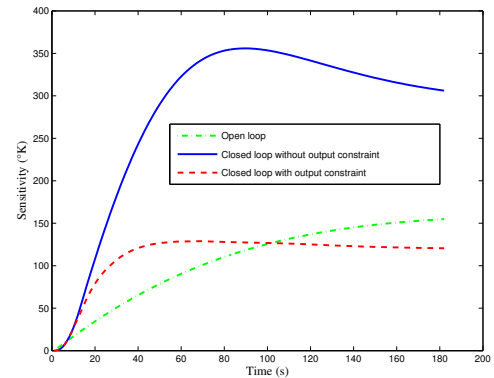

Fig. 3. Parameter estimation.

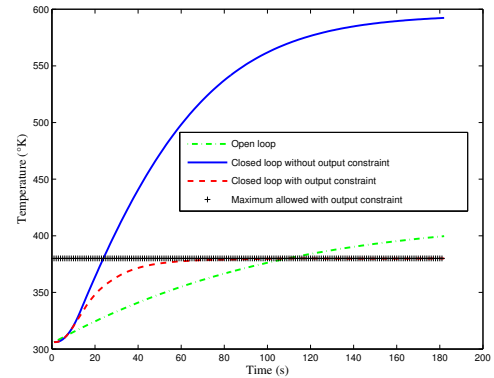Fig. 4. Sensitivity of the process output with respect to the estimated parameter.
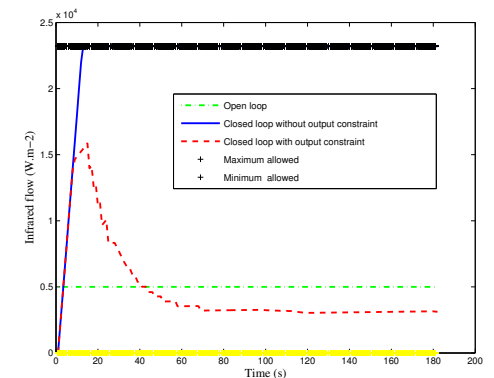

Fig. 5. Process output.

Fig. 6. Process input.

6. CONCLUSION

A new approach coupling a particular continuous time-domain process model, the parametric sensitivity model (derived from the process model), a model based observer and a model based predictive controller was used on-line for optimal identification. It consisted in solving on-line an explicit constrained optimization problem over a receding horizon to maximize the considered parametric sensitivity. In the MPC, the process model and its associated parametric sensitivity model are used. In order to decrease the computational burden, a linearization of the models has been done. During the prediction, the current estimation of the unknown parameter is used to find the optimal control action. The obtained results provide an optimal parametric sensitivity in output, helping for a process model parameter identification. The optimal control problem is solved using a control software (MPC@CB) developed under MATLAB.

In the future, this optimal identification strategy will also be used to find a compromise between dual effects, in terms of its ability to both control the system in closed loop for the use under normal operating specifications, and excite it enough to be able to obtain an optimal identification. This approach will also be extend for other process models, and for a parameter vector case. Noise sensitivity and robustness with respect to the initial value used in the observer will also be studied.

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