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A modelling methodology for natural dam-river network systems

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Abstract: An approach for modelling a natural dam-river network system is proposed in this paper. Generally, the relationships among the variables of a natural dam-river network system are complex and difficult to be described. In this paper, we present some simple first principle relationships among the water levels measured at a limited number of points of the network system such that a model is achieved, and data-driven. The model is identified and validated with the real time operational data. An example is given and the result shows the feasibility of the modelling methodology. It is expected that the proposed approach can be used in the operation of natural dam-river or river network systems.

1. INTRODUCTION

Due to the scarcity of water resources, modelling and control of water resources in open channel flows have attracted a lot of studies in recent years. The common natural examples of open channels are water flows in rivers and streams while manmade examples are in irrigation canals and sewer lines. Generally, the channel bed and channel geomorphological features of a manmade channel are regular and the material types of the channel are uniform while the channel bed and channel geomorphological features of a natural channel are irregular and the material types of channel vary in different space locations.

Except for flood control, the ongoing subject is mainly on the dynamics of manmade channels. This is because the dynamics are regular and the dynamical parameters are regular in manmade channels, enabling the possibility of the model-based study. A recent survey of the models can be seen in Zhuan and Xia (2007), where the models are classified into physical principle models and data-driven models. There are two principles used in water flow dynamics. One principle is the so-called Saint Venant equations (Chow (1954)). From Saint Venant equations, a class of models are derived (discretized in Balogun et al. (1988); Garcia et al. (1992); Georges (1994), and linearized in Litrico and Georges (1997) and Baume and Sau (1997)). The other principle is water volume or mass balance principle (Corriga et al. (1979); Schuurmans et al. (1995) and Schuurmans et al. (1999)), with which some volume (mass) balance models are presented. The parameters in data-driven models are identified from real time data. Such models include black-box models in Elfawal-Mansour et al. (2000), grey-box models in Weyer (2001) and Maxwell and Warnick (2006), high order transfer function models in Sawadogo et al. (1998) and neural network models in Toudeft and Gallinari (1996). The difference between black-box models and the grey-box models is that grey-box models partially satisfy volume/mass balance principle.

For natural channels, although the dynamics can be described by Saint Venant equations, the equations are not as useful as in manmade channels because the parameters in Saint Venant equations vary with respect to the different space and time coordinates. The variation of the parameters leads to difficulties in studying the flow dynamics and therefore the control strategies of the flow dynamics with a mathematical model. The control objective of open channels is to transport the water resources from one area to other area(s) such that water flows and water levels meet the users' demands, ecology demand, safety requirements (flood control), navigation control, pollution control, and decreasing water waste.

For a natural dam-river network system, the existing control strategies are scheduled on the basis of the experimental operation and information in a limited local area (points) for a limited period without accurate prediction. For example, in a flood-preventing hydraulic structure–Enclosing Dike, a dike surrounding an area with gates/pumping stations connecting the outside and inside of the dike, is operated in a very simple way. When the water level at a point of inside area is detected to be higher than the security water level, the water in inside area will be immediately pumped out through the pumping station. When the water level is so low that the environment of inside area is scarce of water, the gate will be immediately opened to introduce water from outside. All those operations are based on the accumulated experiences and based on the observation of local present states without considering the whole system and predicting the development trends. Such a kind of decision-making of control strategies cannot realize global optimal control for a large and complicated river network system. The study subject in this paper is such a kind of large and complicated natural river network system. An investigation will be conducted to obtain a mathematical model for such a network system and it is expected that the model can facilitate the design of control strategies.

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A simple modelling methodology is proposed in this paper for a natural dam-river network system. Such a dam-river system is very complex and at present there is no simple mathematical model being applied to the system operation. A model established with the proposed approach is simple because it only includes water level measurements at some points and control variables. The model structure is proposed based on the simplification of some hydraulic principles. The parameters in the model can be identified with the experimental data.

The structure of this paper is: The modelling methodology is proposed in section 2 while in section 3 an application example is given to show the feasibility of the modelling methodology. The conclusions are in section 4.

2. MODELLING METHODOLOGY

The variables in modelling a dam-river network considered in this paper are only the water levels at different points. For simplicity, the control variables here are the gate openings. The relationship between the water levels can be classified into three types: gate connection, channel connection and non-channel connection. Gate connection indicates that there is one and only one gate structure between the two points with water levels (upstream and downstream of a gate) measured. Channel connection means there exists a visible channel with water flow connecting the two points with water levels measured. When the connection between two points does not belong to gate connection or channel connection, it is called non-channel connection.

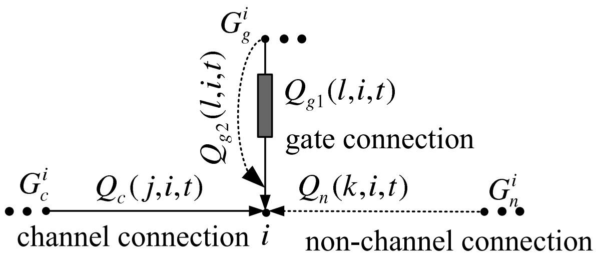

Fig. 1. Connection types

In this paper, the flow rates in channels are practically assumed not to be measured and to vary in a small range at least in a short time period.

The flow through porus medium (aquifer) in non-channel connection is generally expressed by Darcy's Law formulated based on experiments (refer to Todd and Mays (2005)). The calculation of flow through porus medium is usually in relation to flow nets, which is very complex. According to Darcy's law, the flow rate through porus medium is proportional to the head loss (generally the difference between water levels) and hydraulic conductivity or intrinsic permeability, and inversely proportional to the length of the flow path (distance). In this paper, such a flow rate $Q_n(i,j,t)$ to point j from the point i is just approximated by

$$Q_n(i,j,t) = \alpha_n(i,j)y(i,t-t_d(i,j)) + \beta_n(i,j)y(j,t) \quad (1)$$

where $y(i,t-t_d(i,j))$ is water level at point i at the time $t-t_d(i,j)$, $y(j,t)$ is the water level at point j at the time t, $t_d(i,j)$ is the time delay and $\alpha_n(i,j)$ and $\beta_n(i,j)$ are coefficients. This equation can also be approximated by a higher order series, but it is simply approximated by a first order function in this paper.

For gate connection, the relationship between the water levels inside and outside of the gate depends on two items. The first item is the flow through the gate when the gate is open. The second is the flow through porus medium.

When the gate is open, the flow through the gate depends on the water levels on both sides of the gate, the gate opening and the gate type. In Eurén and Weyer (2007), the flow rate $Q_g(i,j,t)$ for an undershot gate is approximated by

$$Q_{g1}(i,j,t) = c\cdot\delta d(t)\cdot\mathrm{sign}(y(i,t)-y(j,t))\sqrt{|y(i,t)-y(j,t)|} \quad (2)$$

where c is a constant determined by the gate characteristics, $\delta d(t)$ is the gate opening and $y(i,t)$ and $y(j,t)$ are the water levels at points i and j (the water levels on both sides of the gate).

The flow rate through porus medium in gate connection is in the same form as the equation (1), *i.e.*,

$$Q_{g2}(i,j,t) = \alpha_g(i,j)y(i,t-t_d(i,j)) + \beta_g(i,j)y(j,t). \quad (3)$$

So for a gate connection, the flow rate is described by

$$Q_g(i,j,t) = Q_{g1}(i,j,t) + Q_{g2}(i,j,t). \quad (4)$$

The flow rate of channel connection can be calculated by Saint Venant equations, a group of partial differential equations. With the assumption that the flow speed varies slowly, the flow rate $Q_c(i,j,t)$ to point j at time t in the channel connecting to point i is approximated by

$$Q_c(i,j,t) = \alpha_c(i,j)y(i,t-t_d(i,j)) + \beta_c(i,j)y(j,t) \quad (5)$$

with $y(i,t)$ and $y(j,t)$ water levels at points i,j at time t, $t_d(i,j)$ time delay and $\alpha_c(i,j)$ and $\beta_c(i,j)$ coefficients.

Another assumption in this paper is that the water volume storage in the neighbour of point i is proportional to the water level $y(i,t)$. With this assumption, according to mass balance principle, $y(i,t)$ could be predicted as follows.

$$\gamma(i)\frac{dy(i,t)}{dt} = \sum_{l\in G_g^i} Q_{g1}(l,i,t) + \sum_{l\in G_g^i} Q_{g2}(l,i,t) + \sum_{j\in G_c^i} Q_c(j,i,t) + \sum_{k\in G_n^i} Q_n(k,i,t) + e(t),$$

where G_c^i, G_n^i and G_g^i are the sets in which the elements with respect to point i, respectively belong to channel connection, non-channel connection and gate connection, $\gamma(i)$ is a coefficient and $e(t)$ is the disturbance or unmodelled flow, such as rainfall and irrigation flow.

Comparing the equations (1), (3) and (5), they are in the same form. Let $G^i = G_g^i \cup G_c^i \cup G_n^i$. With the above equations combined, the equation (6) holds,

$$\frac{dy(i,t)}{dt} = \sum_{k\in G^i} (\alpha(k,i)y(k,t-t_d(k,i)) + \beta(k,i)y(i,t)) + \sum_{l\in G_g^i} \nu(i)Q_{g1}(l,i,t) + e(t), \quad (6)$$

where α, β, and ν are coefficients and can be identified with real data.

The model (6) can be discretized with various discretization methods, resulting in different kinds of discrete models. In this paper, for simplicity, we choose an AutoRegressive model with eXogenous input (ARX model) in the form

$$\begin{aligned} A(q)y(t) &= B(q)u(t) + P(q)w(t) + Q(q)e(t), \\ &= [B(q)\ P(q)][u(t)\ w(t)]^T + e_s(t), \end{aligned} \tag{7}$$

with $e(t)$ unknown disturbance, $y(t) = y(i,t)$ ith output, $u(t) = Q_{g1}(t)$ control input, $w(t) = (y(k,t))_{k \in G^i}$ uncontrollable input, $e_s(t)$ the unknown disturbance in a new form, q^{-1} delay operator and t discrete time series.

To date, there is no accurate model to describe the dynamics of natural dam-river network systems. Even the introduction of Saint Venant equations is based on some assumptions and furthermore, the parameters in Saint Venant equations are variable in natural channels and difficult to be determined. The model (7) is based on some practical assumptions and the coefficients are determined by system identification with realtime data. For the slow-varying coefficients, they can be updated with the most recent operational data. The model (7) is very simple compared with Saint Venant equations or Darcy law.

The more the measurement points there are, the more accurate the model is. However, when the number of the measurement points is limited, we can schedule the measurement points at the places where there are obvious geomorphological variations to improve the accuracy of model (7), *i.e.,* the geomorphological features can be considered in scheduling the measurement points to improve the model accuracy. For example, a measurement point can be scheduled at the place where the cross section changes abruptly, to improve the accuracy of the model.

Another remark is that (7) is obtained from the simple model (6) by employing a discretization scheme. This discretization scheme could be for instance analogous to the finite element scheme in solving the Saint Venant equations. This may result in high orders in terms of q in model (7), therefore with higher accuracy than (6).

3. APPLICATION OF MODELLING METHODOLOGY

An example is given in this section to show the feasibility of the proposed modelling methodology. The example, namely Enclosing Dike, is a typical kind of hydraulic structure, composed of enclosing dike, sluice gates and pumping stations, which is constructed beside a river connected to the sea in South China to dampen the influence of the large tidal variations and to allow the consolidation of upstream urban infrastructure.

Sixiang Enclosing Dike, as shown in Fig. 2, located in Nanhai city, Guangdong Province, protects the city from flooding and ensures the environment balance. It is composed of 17 sluice gates, ten pumping stations and a long enclosing dike. The main aim is to drain away flooded water when the water level in Enclosing Dike excesses the security level, and to draw water from the river when Enclosing Dike is in need of water. In the past, the sluice gates and pumping stations were operated by hand with the experiences of the operators. With the automatic operation facilities equipped and optical fiber cabling net-

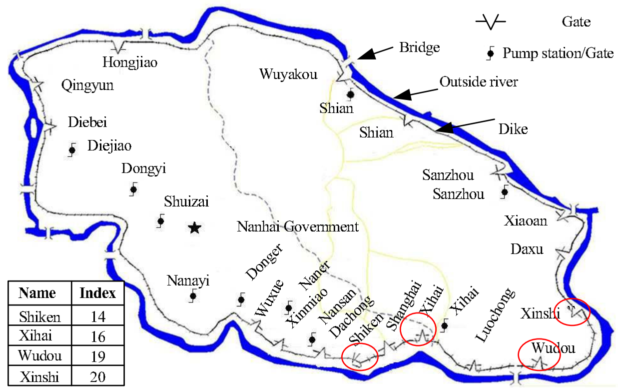

Fig. 2. Sixiang Enclosing Dike

work built in recent years, all information is collected and transmitted to a supervising and controlling center. The information includes running states of the facilities, such as gate openings, running states of the pumps, and water levels at some points inside and outside of Enclosing Dike, and even the video images of the supervised points. Thus the operators can know the situations of all pumping stations and sluice gates, and furthermore decide and send operational commands to the stations and gates. With the project applied, the level of Enclosing Dike automatic control is advanced and the reliability is enhanced. It is possible to realize the optimal control in the global viewpoint of Enclosing Dike. The present control is still local and the performance depends on the experiences of the operators. The difficulty of the application of global automatic control lies in the absence of a model describing the whole dynamics of Enclosing Dike.

In the supervising and controlling center, data are collected from pumping stations and undershot sluice gates (hydraulic structures), including water levels inside and outside of the hydraulic structures, the pump status (on/off) and gate openings. At present, gate openings only have two states: open and close, where open state means that the plate bottom is over the water surface and close state means there is no water flowing through the gate.

The measured variables in the above-mentioned project are water levels $y_{nj}(i)$ and $y_{wj}(i)$ inside and outside of the hydraulic structures (i is the index of the hydraulic structure). The control variables are the gate openings $\delta d(i)$ and pump state $\delta d(i)$.

The available operational data of Enclosing Dike were limited to four sluice gates (indexes of the hydraulic structures are 14, 16, 19 and 20) and one pumping station (its index is 17, it is connected to structure 16 and the two structures are almost in the same place). The data for the other hydraulic structures were not continuously measured or recorded. During the period of data sampling, there was no record about the operation of the pumps. Because the project is used to show the applicability of the modelling approach in Section II, the above mentioned data only with four points in Enclosing Dike are enough.

The data have the following features:

- The time length of the data is about 17 days.
- The measured water levels are Yellow Sea's altitudes.
- The data were collected with asynchronous samples.

- The time intervals of the data are about 5 minutes.
- Some data were missed in the record.
- With the installation of the sensors and calculation formula considered, there are different offsets for various water levels.

Based on the available data, the model as (8) is expected to be established for Enclosing Dike. In (8), Q_{g1} are in the form of (2), and the coefficients $A(i)$, $B(i)$ and $P(i)$ will be identified with the operational data.

$$A(i,q)y_{nj}(i,t) = B(i,q)u(i,t) + P(i,q)w(i,t) + e_i(t), \quad (8)$$

where $u(i,t) = Q_{g1}(i,t)$, $i = 14, 16, 19, 20$,

$$w(14,t) = [y_{wj}(14,t), y_{nj}(16,t), y_{nj}(19,t), y_{nj}(20,t)]^T,$$
$$w(16,t) = [y_{wj}(16,t), y_{nj}(14,t), y_{nj}(19,t), y_{nj}(20,t)]^T,$$
$$w(19,t) = [y_{wj}(19,t), y_{nj}(14,t), y_{nj}(16,t), y_{nj}(20,t)]^T,$$
$$w(20,t) = [y_{wj}(20,t), y_{nj}(14,t), y_{nj}(16,t), y_{nj}(19,t)]^T.$$

The water levels outside of the dike are mainly influenced by the tidal process and flow rate from upstream of the river. They can be estimated by using the approach proposed in this paper in a wider system incorporating the tidal prediction. In this paper, the water levels outside of the dike are assumed to be known.

In the above model, Q_{g1} is thought as the control variable. In modelling Enclosing Dike, pump flow rates of pumping stations can also be expressed in the the form as control variables and thus we can study the control problem of Enclosing Dike.

3.1 Data preprocessing

The sampled data were processed prior to using them for identification. The preprocess includes virtual synchronization, offset correction and missing data estimation.

The sampled data were not synchronous because they were independently collected at different points and it was inconvenient to synchronize the sampling time for them. However, in the model (8), the data are required to be synchronous. So, the data are virtual synchronized in the way of linear interpolation.

The offsets in the data for different variables are various. In our model, it is difficult to know the offsets of each measurement. The measurement sensors were installed individually and calibrated with respect to the Yellow Sea's altitude according to the workers' experiences. Each measurement has a different offset. So it is better to employ a second-order model to decrease the effect of offsets.

The missing data were corrected by linear interpolation. If the time interval of the continuous missing data is short, such a method will not lead to a large error in the model.

3.2 Model identification and validation

The time length of the experiment data is about 17 days (4700 × 5 minutes), in which half of the length (2500 sampling points) is used for the model identification, and remaining data for the model validation.

The model structure is a second order ARX model ($A(q), B(q)$ and $P(q)$ are polynomials of q^{-1} of two consecutive orders). The disturbance is not considered for it is difficult to know its distribution class. A second order ARX model is employed and expected to decrease the influence of the offsets of measurements and the disturbance in a limited period.

The next step is to identify the time delays in (8). The delays in the variables, gate flows and water levels outside of the gate, are 1. The other time delays are estimated based on comparison of ARX models with different delays in the range of [1 100]. The result is shown in Table 1.

Table 1. Time delays identified from the data

Output \ Input	14	16	19	20
14	-	14	67	97
16	72	-	19	38
19	98	35	-	8
20	95	32	14	-

The parameters in an ARX model (7) are estimated with least squares method, and the results are shown as follows.

If $y(t)$ is the water level $y_{nj}(14,t)$, then the parameters are

$$\begin{aligned} A(14,q) &= 1 - 1.456q^{-1} + 0.459q^{-2}, \\ B(14,q) &= 0.073q^{-1} - 0.014q^{-2}, \\ P(14,q) &= \begin{bmatrix} -0.004q^{-1} + 0.010q^{-2} \\ -0.039q^{-14} + 0.041q^{-15} \\ -0.085q^{-67} + 0.085q^{-68} \\ -0.007q^{-97} + 0.009q^{-98} \end{bmatrix}. \end{aligned} \quad (9)$$

If $y(t)$ is the water level $y_{nj}(16,t)$, then the parameters are

$$\begin{aligned} A(16,q) &= 1 - 1.342q^{-1} + 0.346q^{-2}, \\ B(16,q) &= 0.256q^{-1} - 0.061q^{-2}, \\ P(16,q) &= \begin{bmatrix} -0.039q^{-1} + 0.0484q^{-2} \\ -0.075q^{-72} + 0.081q^{-73} \\ -0.002q^{-19} - 0.016q^{-20} \\ -0.011q^{-38} - 0.004q^{-39} \end{bmatrix}. \end{aligned} \quad (10)$$

If $y(t)$ is the water level $y_{nj}(19,t)$, then the parameters are

$$\begin{aligned} A(19,q) &= 1 - 1.015q^{-1} + 0.028q^{-2}, \\ B(19,q) &= 0.327q^{-1} - 0.031q^{-2}, \\ P(19,q) &= \begin{bmatrix} -0.006q^{-1} + 0.009q^{-2} \\ 0.049q^{-98} - 0.046q^{-99} \\ 0.113q^{-35} - 0.121q^{-36} \\ 0.170q^{-8} - 0.170q^{-9} \end{bmatrix}. \end{aligned} \quad (11)$$

If $y(t)$ is the water level $y_{nj}(20,t)$, then the parameters are

$$\begin{aligned} A(20,q) &= 1 - 1.661q^{-1} + 0.668q^{-2}, \\ B(20,q) &= 0.260q^{-1} - 0.163q^{-2}, \\ P(20,q) &= \begin{bmatrix} -0.054q^{-1} + 0.059q^{-2} \\ -0.068q^{-95} + 0.072q^{-96} \\ 0.016q^{-32} - 0.028q^{-33} \\ 0.020q^{-14} - 0.029q^{-15} \end{bmatrix}. \end{aligned} \quad (12)$$

It is noticed that the control input $u(i)$ is zero during most of the experimental period, which may result in difficulty in identifying $A(q)$. The coefficient $A(q)$ may be identified with the subset of the data when $u(i)$ is not equal to zero. In this paper, $A(q)$ is identified together with $B(q)$ and

$P(q)$ because $u(i)$ in the identification data is assumed to be *persistently exciting* (Ljung (1999)).

When the identified model is used for 24-step (two-hour) ahead prediction, the result for $y_{nj}(14, t)$ is shown in Fig. 3.

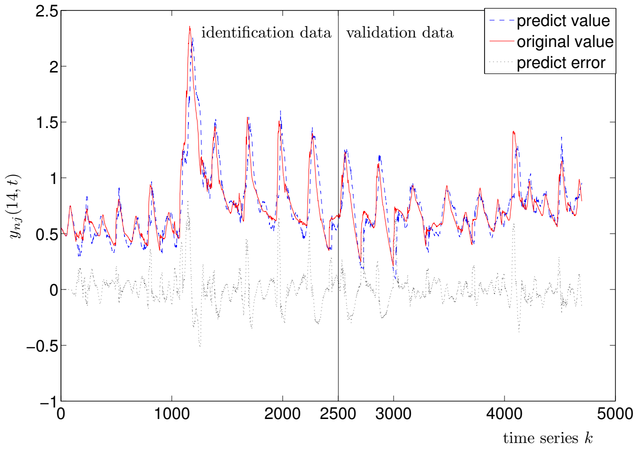

Fig. 3. Validation result for water level $y_{nj}(14, t)$

3.3 Analysis

From Table 1 and Fig. 2, it can be seen that the length of time delay is consistent with the distance between the measurement points. The longer the distance is, the longer the time delay is. This is because in the studied Enclosing Dike, the connection types of the measurement points (excluding the water levels outside of Enclosing Dike) are non-channel connections, whose time delay is influenced by the hydraulic conductivity and distance. In the regional area of Enclosing Dike, hydraulic conductivities are almost the same. So the time delays mainly reflect the distance difference, which is confirmed by the identification result.

It can be seen from Fig. 4 that the water levels inside and outside of the gate are almost equal when the gate is open. This is easy to be understood. When there is a difference between the water levels, a flow will occur to compensate for the difference. In the model, the flow rate through the gate is very small when the gate is open if it is calculated through the difference of the water levels at a specific time. Actually, the flow rate refers to the flow during a sampling period, so in the model, such a flow is calculated based on the difference between inside water level at present and outside water level at next sampling time. With the offsets of the measurements considered, it is calculated based on the difference between the inside (or outside) water levels at present and next sampling time. The contribution to inside water level variation is from the outside water level when gate is open. Such a phenomenon is reflected on the parameter value of the flow rate through gate, which is usually larger than other parameters in the model, as can be seen from equations (9)–(12). For a more accurate model, it can be simply assumed that when the gate is open, the inside water level is equal to the outside water level and is not related to other variables. When the gate is closed, the relationships with other variables are identified with experimental data.

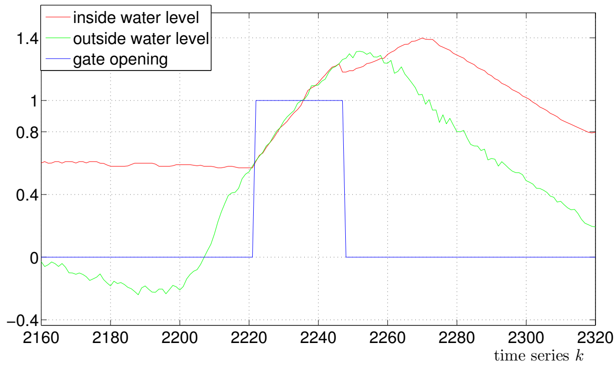

Fig. 4. Water levels variation with gate opening

When a gate is closed, the inside water level is influenced much by the disturbances or unmodelled variables, for example, unmodelled water resources, rainfall, irrigation demands, evaporation. The fast variation of unmodelled variables leads to the large error of the prediction. So, in further study, more variables, such as the rainfall, evaporation rate, water levels in other points of the area and irrigation flows, could be included in the model.

The statistics of predict errors are shown in Table 2 for identification period and Table 3 for prediction period. Comparing Table 2 with Table 3, it can be seen that the prediction errors are not much worse in prediction period than those in identification period, which means that the predict errors are mainly from the unmodelled variables. With more variables included in the model, the errors will decrease. This shows the feasibility of the modelling methodology.

Table 2. Predict error in identification period

	min	max	mean	std
$y_{nj}(14,t)$	-0.5216	0.8047	0.0130	0.1637
$y_{nj}(16,t)$	-0.2301	2.4509	0.7914	0.4076
$y_{nj}(19,t)$	-0.2701	2.4171	0.5088	0.3943
$y_{nj}(20,t)$	-0.4527	2.5366	0.5730	0.4583

Table 3. Predict error in prediction period

	min	max	mean	std
$y_{nj}(14,t)$	-0.3832	0.6324	-0.0035	0.1366
$y_{nj}(16,t)$	0.0489	1.8466	0.7894	0.3014
$y_{nj}(19,t)$	-0.1309	1.0925	0.4181	0.2422
$y_{nj}(20,t)$	-0.2582	1.2037	0.5135	0.3143

From Table 3, it is also seen that the errors with two hours ahead prediction are acceptable in the decision-making of the management and operation of Enclosing Dike. When a certain security redundancy of the water level is considered, the operation can be planned based on the model (8) and the prediction error will be tolerant. With the decrease of the prediction period and increase of measurement points, the accuracy will be improved.

4. CONCLUSION

A very simple modelling methodology is proposed in this paper for modelling natural dam-river network systems. In the model, only water level measurements are included.

Considering the complexity of a natural dam-river network system, this modelling methodology is very simple. The proposed model structure is proposed based on some hydraulic principles. With the experimental data, the parameters in the model can be identified.

An application example is given to show the feasibility of the modelling methodology. Some application issues in the example are considered, such as data synchronization, data offsets and missing data. With the analysis of the model identification and validation results, the physical significance of time delays is discussed with the hydraulic characteristics. Some improvement measures of the model are proposed based on the analysis results.

The accuracy of such a model in the application of prediction is discussed. The model shown in this paper is good enough for two-hour ahead prediction. This means, the decision of the management and operation can be made based on two-hour ahead prediction. Comparing with the present descision-making mechanism (based on present states), it is an improvement. Moreover, the existing decision-making is based on local present measurements, while with the proposed model, such a decision-making can be based on the measurements in the network system. An optimal operation is possible from the point of view of the network system.

Further study based on a specific dam-river network system will be proceeded with application issues considered. The other direction is the application of a model based on the proposed modelling methodology in the operation.

The model (7) facilitates the study on the control problem. With a model, we can formulate different kinds of control problem for a dam-river network system. For example, when there are several points considered in a study, there are coupled equations in the form of (7) composing a model for the network systems. Then based on the series of models, we may formulate the control problem in a global viewpoint. For example, an optimal water level regulation can be formulated as follows,

Optimal water level regulation: To find a controller as

$$u(t) = \mathcal{L}(y(t), w(t)), \tag{13}$$

such that the performance function

$$f = \int (y^T \Lambda y + u^T \Pi u) dt, \tag{14}$$

is minimized with Λ, Π as weighting matrices.

Of course, many other formulations can be proposed based on the mathematical model (7). These control problems will be dealt with elsewhere.

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