

# Fault Diagnosis with the Use of the Knowledge about Symptoms Delay Intervals

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**Abstract**: The paper refers the issue of utilisation of knowledge about symptoms delays in fault isolation. Firstly, formal conditions for fault isolability in the case of reasoning based on binary diagnostics matrix and the intervals of the symptoms delays are formulated and its influence of fault isolability is discussed. Finally, the new isolation algorithm utilising such knowledge is proposed. The presented approach is illustrated with the example of fault isolability analysis for three tank system.

## 1. INTRODUCTION

In most of the diagnostic strategies the mapping of the diagnostic signal space (evaluated residual values) onto the fault space is necessary for fault isolation. Different forms of this representation are known (Gertler, 1998; Chen and Patton, 1999; Korbicz, et al., 2004; Patton et al., 2000). Most of them have static nature. However, the diagnosed processes are dynamical systems. Therefore, some time elapses from the moment of fault occurrence to the moment when one can obtain measurable symptoms. In general, this time period is different for each fault and each residual which detects that fault. Only after some period of time all symptoms are observed. Just a few approaches reference this problem (Combastel *et al.*, 2003; Vanden Daele *et al.*, 1997; Kościelny and Zakroczymski, 2001).

The wrong diagnosis could be generated if one didn't take the dynamics of symptoms into consideration. The knowledge about symptoms delays, i.e. its sequence, sometimes enables to isolate normally unisolable faults and, in many cases, to shorten the diagnosing time (Kościelny and Zakroczymski, 2001).

The accurate determination of the symptoms delays is usually impossible. Few approaches use the simplified notations of that knowledge, e.g. the lower and the upper bounds on the symptoms delay intervals for particular faults (Vanden Daele *et al.*, 1997; Kościelny and Zakroczymski, 2001) or maximal possible symptoms forming delay for particular diagnostic tests (Kościelny, 1995; Korbicz, et al., 2004).

In this paper the authors analyse the fault distinguishability in the case of the diagnosing based on binary diagnostic matrix and the symptoms delay intervals. The conditions for unconditional fault unisolability and isolability as well as conditional fault isolability are given. The possible increase of fault distinguishability in such a case is shown. Finally, the modified isolation algorithm utilising the knowledge about the symptoms delays previously described in (Korbicz *et al.*, 2004) is formulated.

## 2. ESTIMATION OF SYMPTOMS FORMING DELAYS

The delays of symptoms forming depend on the dynamic characteristic of the process, fault type (abrupt, incipient), its time development characteristic, the applied method and detection algorithm parameters. It is possible to calculate analytically these times based on the dynamic description (e.g. transmittance) of the controlled part of the process and the transient response of fault appearance.

The mathematical process description can be achieved based on the equations describing the physical effects taking place in the process. In this case, it is necessary to treat all the possible faults as separate inputs in the system of equations. After the linearization at the operating point and applying Laplace transform one achieves linear model in the form (Gertler, 1998; Korbicz, et al., 2004; Patton et al., 2000):

$$\mathbf{y}(s) = \mathbf{G}(s)\mathbf{u}(s) + \mathbf{G}^{\mathrm{F}}(s)\mathbf{f}(s) .$$
 (1)

Each constituent equation has the following form:

$$\mathbf{y}_{j}(\mathbf{s}) = \mathbf{G}_{j}(\mathbf{s})\mathbf{u}(\mathbf{s}) + \mathbf{G}_{j}^{\mathsf{F}}(\mathbf{s})\mathbf{f}(\mathbf{s}) , \qquad (2)$$

while  $G_j(s)$  (j=1,...,J – number of residuals) denotes inputoutput transmittance:

$$G_{p,i}(s) = y_{i}(s)/u_{p}(s); p = 1,...,P,$$
 (3)

and  $G^{F}_{j}(s)$  denotes the transmittance for each fault-output couple:

$$G_{k,j}^{F}(s) = y_{j}(s) / f_{k}(s); \quad k = 1,...,K.$$
 (4)

In the case of faults absence the following dependence if fulfilled:  $y_j(s) - G_j(s)u(s) = G_j^F(s)f(s) = 0$ . The residuals are calculated based on the following equation called calculation form:

$$\mathbf{r}_{j}(\mathbf{s}) = \mathbf{y}_{j}(\mathbf{s}) - \mathbf{G}_{j}(\mathbf{s})\mathbf{u}(\mathbf{s}) .$$
(5)

The equation (6) reflects the general relation between the particular residual and faults. It is called an internal form of a residual (Gertler, 1998):

$$\mathbf{r}_{j}(s) = \mathbf{G}_{j}^{F}(s)\mathbf{f}(s) = \sum_{k=1}^{K} \mathbf{G}_{j,k}^{F}(s)\mathbf{f}_{K}(s).$$
(6)

If  $r_j$  is sensitive for fault  $f_k$  and no other faults are present  $(f_m=0)$  then one achieves:

$$\mathbf{r}_{j}(\mathbf{s})\big|_{\mathbf{f}_{k}} = \mathbf{G}_{k,j}^{F}(\mathbf{s})\mathbf{f}_{k}(\mathbf{s}), \mathbf{f}_{m} = 0: m = 1, 2, ..., \mathbf{K}, \ m \neq k.$$
(7)

For so defined residual its time development function is defined by the following relation:

$$\mathbf{r}_{j}(\mathbf{t}) = \mathbf{L}^{-1} \Big( \mathbf{G}_{k,j}^{\mathrm{F}}(\mathbf{s}) \mathbf{f}_{k}(\mathbf{s}) \Big).$$
(8)

The analytical calculation of the symptoms forming delays is difficult in practice because it requires the modelling of fault influence on measurable outputs. The fault development function as well as residual threshold value must be assumed arbitrary. Because of modelling errors the precision of analytical estimation of symptom times is poor.

In practice, based on the knowledge about the process and detection algorithms, it is possible to estimate symptom times by giving their minimum and maximum values (Kościelny and Zakroczymski 2001; Korbicz, et al., 2004). Let us use the following notation:  $\theta^{1}_{k,j}$  – minimal time period from k<sup>th</sup> fault occurrence to j<sup>th</sup> symptom appearance,  $\theta^{2}_{k,j}$  - maximal time period from k<sup>th</sup> fault occurrence to j<sup>th</sup> symptom appearance. The actual symptom time belongs to interval  $< \theta^{1}_{k,j}, \theta^{2}_{k,j} >$ .

#### 3. FORMAL DESCRIPTION OF DIAGNOSED SYSTEM

Most often, for the isolation of faults from the set  $F = \{f_k : k = 1, 2, ..., K\}$  the set  $S = \{s_j : j = 1, 2, ..., J\}$  of binary diagnostics signals is used. The diagnostic signals are calculated as a result of threshold evaluation of absolute residual values (*A* – threshold value):

$$\left|\mathbf{r}_{j}\right| \le \mathbf{A} \Longrightarrow \mathbf{s}_{j} = 0 \text{ or } \left|\mathbf{r}_{j}\right| > \mathbf{A} \Longrightarrow \mathbf{s}_{j} = 1.$$
(9)

The occurrence of the diagnostic signal value of '1' is a fault symptom. It testifies about the existence of one of the faults from the subset  $F(r_j)$  of faults for which the j<sup>th</sup> residual is sensitive for:

$$F(r_{j}) = F(s_{j}) = \{f_{k} : r_{j} = \psi(f_{k})\}$$
(10)

where  $\psi(f_k)$  is a some function of  $f_k$  (or maybe also of other faults) which can be determined from (6).

Based on the residual equations in the internal form (6) one can define the subsets  $F(r_j)$  and, finally, the binary diagnostics matrix defined on the Cartesian product of the sets of faults and diagnostic signals:

$$\mathbf{Q}_{\mathrm{FS}} \subset \mathbf{F} \times \mathbf{S} \,. \tag{11}$$

The elements of the relation (11) take the shape of:

$$q(f_k, s_j) \in \{0, 1\}.$$

$$(12)$$

The simplest diagnostics system is defined by the following triplet:

$$SD = \langle F, S, Q_{FS} \rangle$$
 (13)

The minimal and maximal values of the delays of fault symptoms  $\langle \theta_{k,j}^1, \theta_{k,j}^2 \rangle$  are attributed to each pair of fault-symptom such as  $q(f_k, s_i) = 1$ .

## 4. FAULT ISOLABILITY

The analysis of achieved fault isolability for the system described by the triplet (13) and the parameters of symptoms delay intervals is presented in this section. It concerns the cases of faults that are unisolable based on binary diagnostic matrix, for which for all diagnostic signals  $s_j \in S$  the condition  $q(f_k, s_j) = q(f_m, s_j)$  is fulfilled (Gertler, 1998; Korbicz *et al.*, 2004).

**Definition 1.** Any two faults  $f_k$  and  $f_m$  are unconditionally unisolable, in respect to the intervals of symptoms delays, in the following two cases:

- they are detectable only by one, the same, diagnostics signal one must notice that the single diagnostic signal s<sub>j</sub> sensitive for faults f<sub>k</sub> and f<sub>m</sub> is not capable to distinguish these faults even if their intervals of symptoms delays are disjoint ([θ<sup>1</sup><sub>k,j</sub>, θ<sup>2</sup><sub>k,j</sub>] ∧ [θ<sup>1</sup><sub>m,j</sub>, θ<sup>2</sup><sub>m,j</sub>] = Ø) due to the lack of knowledge about the moment of fault erasing,
- the minimal and the maximal values of the symptoms delays are shifted by the same constant value τ, for all diagnostic signals:

$$\bigvee_{\substack{s_j \in S \land [q(f_k, s_j) \neq 0] \land \\ \land [q(f_m, s_j) \neq 0]}} [\theta_{m,j}^1 = \theta_{k,j}^1 + \tau] \land [\theta_{m,j}^2 = \theta_{k,j}^2 + \tau].$$
(14)

The above dependence implies that the intervals of delays have the same length for all the faults and particular diagnostic signal  $s_i$ .



Fig. 1. The graphical representation of unconditional fault unisolability

The analysis of undonditional fault isolability must be conducted for two cases. They are taken into account in the following definition.

**Definition 2.** Any two faults  $f_k$  and  $f_m$  are unconditionally isolable, in respect to the intervals of symptoms delays, in the following two cases:

• the faults are detectable by, at least two, diagnostics signals s<sub>i</sub> i s<sub>p</sub> always in different sequence:

$$= \underset{s_{j},s_{p}\in S}{=} \left(\theta_{k,j}^{2} < \theta_{k,p}^{1}\right) \land \left(\theta_{m,p}^{2} < \theta_{m,j}^{1}\right).$$

$$(15)$$



Fig. 2. Different sequence of faults detection by two diagnostics signals

• the faults are detectable by any two diagnostics signals  $s_j$  and  $s_p$  in the same sequence but the maximal value of the  $s_p$  symptom delay in respect to the symptom  $s_j$  for one of the faults (e.g.  $f_k$ ) is less than the minimal value of this delay for the second fault (e.g.  $f_m$ ), the following dependence is fulfilled:

$$(\theta_{m,p}^{1} - \theta_{m,j}^{2}) > (\theta_{k,p}^{2} - \theta_{k,j}^{1}).$$
(16)



Fig. 3. Let us assume that signal  $s_j$  detects the faults  $f_k$  and  $f_m$  earlier that signal  $s_p$  and the delays of symptom of the fault  $f_k$  are lower than the fault  $f_m$ , i.e.  $\theta_{k,j}^2 < \theta_{m,j}^1$ 

Except unconditional fault unisolability and isolability based on intervals of symptoms delays, one can define conditional isolability. Conditional isolability (or unisolability) means, that the fault can be isolable or not in respect to actual values of the symptoms delays (being situated in defined intervals) observed during diagnostic procedure. On the design stage it is not possible to decide if the faults are isolable due to the lack of knowledge about the real delays.

**Definition 3.** Two faults  $f_k$  and  $f_m$  are conditionally isolable, in respect to the intervals of symptoms delays, when these faults are not unconditionally unisolable or unconditionally isolable.

Taking into account the time of symptoms forming can increase the fault isolability comparing with diagnosing based only on binary diagnostic matrix.

## 5. FAULT ISOLATION ALGORITHM

This section presents the fault isolation algorithm that utilizes the knowledge about the diagnostic relation and the values of the minimal and maximal symptoms forming delays. It assumes single fault scenarios, however, the multiple faults issue is also addressed. The algorithm implements serial diagnostic reasoning. The following notation is used:

- DGN<sub>r</sub> final diagnosis elaborated in r<sup>th</sup> step of reasoning,
- DGN<sup>\*</sup><sub>r</sub>, DGN<sup>\*\*</sup><sub>r</sub> intermediate diagnosis.

Three main stages of reasoning algorithm can be distinguished: initialization, diagnosis specifying, and final diagnosis formulation.

**Initialisation of isolation procedure.** The isolation algorithm starts in the time  $t^1=0$  when the first symptom  $s^1_x=1$  is observed (fault detection). The following steps are conducted:

• <u>Determining the set of possible faults</u>. The primary set of possible faults is determined from diagnostic relation. It consists of all the faults, for which the diagnostic signal with the observed symptom is sensitive for:

$$(s_x^1 = 1) \Longrightarrow DGN_1^* = \{f_k : [q(f_k, s_x^1) = 1]\}$$
(17)

where:  $DGN_1^*$  denotes temporary diagnosis, elaborated under the condition of use of the first diagnostic signal  $s_x^1$ but without taking into account the intervals of symptoms delays;  $q(f_k, s_x^1)=1$  denotes that diagnostic signal  $s_x^1$ detects the fault  $f_k$  according to the diagnostic relation.

• <u>Reduction of primary set of possible faults</u>. Let us introduce the notations  $\theta_{k,x}^1$  and  $\theta_{k,x}^2$  for minimal and maximal periods from k<sup>th</sup> fault occurring to the first, detected symptoms  $s_x^1 = 1$  formulation, respectively. The faults, which occurrence should cause another symptoms to be observed before the symptom  $s_x^1$ , in respect to known intervals of symptoms delays, are eliminated from the set DGN( $s_x^1$ ):

$$DGN_{1} = \{ \mathbf{f}_{k} \in DGN_{1}^{*} : \bigvee_{s_{j} \neq s_{x}^{1}} \boldsymbol{\theta}_{k,j}^{2} < \boldsymbol{\theta}_{k,x}^{1} \}, \qquad (18)$$

while  $DGN_1$  denotes first, temporary diagnosis elaborated while taking into account the intervals of the symptoms delays.

• Determining the set of diagnostic signals useful for further fault isolation in the following form:

$$\mathbf{S}^* = \{\mathbf{s}_i : \mathbf{F}(\mathbf{s}_i) \cap \mathbf{DGN}_1 \neq \emptyset\} - \mathbf{s}_x^1 \,. \tag{19}$$

• Defining the intervals of symptoms possible consecutive forming. Due to the fact that the real time of fault occurring is unknown (only the time of the first symptom detection is registered) the time intervals of the appearing of the consecutive symptoms of the diagnostic signals from the set S\* must be recalculated in respect to the moment of the first symptom detection. Such calculations must be conducted for the faults pointed out in the diagnosis in the following way:

$$\beta_{k,j}^{1} = \begin{cases} 0 & \text{if } \theta_{k,j}^{1} - \theta_{k,x}^{2} \leq 0\\ \theta_{k,j}^{1} - \theta_{k,x}^{2} & \text{if } \theta_{k,j}^{1} - \theta_{k,x}^{2} \geq 0 \end{cases}$$
(20)

$$\boldsymbol{\beta}_{k,j}^2 = \boldsymbol{\theta}_{k,j}^2 - \boldsymbol{\theta}_{k,x}^1 \,. \tag{21}$$

The parameters  $\beta_{k,j}^2$  for the faults  $f_k \in DGN_1$  and the diagnostic signals  $s_j \in S^*$  are arranged in ascending order.

*Iterative diagnosis specifying*. The second part of the reasoning has iterative nature. The elaboration of the following diagnosis takes place:

- after the detection of each, successive fault symptom,
- each time when the maximal period of the symptom delay  $\beta_{k,i}^2$  from the ordered series of these parameters passes.

During this stage, the following steps are conducted iteratively:

 The reduction of the set of possible faults based on diagnostics relation. If the symptom s<sub>j</sub>=1 (s<sub>j</sub>∈S<sup>\*</sup>) was detected in the proper period of delays than the set of possible faults is reduced according to formula:

$$(s_{j}^{r} = 1) \land (s_{j} \in S^{*}) \Rightarrow DGN_{r}^{*} = = \{f_{k} \in DGN_{r-1} : q(f_{k}, s_{j}) = 1 \land (t \in [\beta_{k,j}^{1}, \beta_{k,j}^{2}]\}$$
(22)

Such an operation is realised for all the faults from the set  $f_k \in F(s_i)$ .

• The reduction of the set of possible faults based on the analysis of delays interval. The faults, which occurrence should cause another symptoms  $s_p = 1$  to be observed before the currently observed symptom, in respect to the

known intervals of symptoms delays, can be eliminated from the diagnosis elaborated in previous step:

$$DGN_{r}^{**} = \{f_{k} \in DGN_{r}^{*} : \beta_{k,p}^{2} < \beta_{k,j}^{1}\}$$
(23)

• The reduction of the set of possible faults after the analysis of the maximal times of the symptoms delays. The lack of symptom after predefined time period,  $t > \beta_{k,j}^2$ , allows for the reduction of the set of possible faults due to the formula:

$$(s_{j} = 0) \land (s_{j} \in S^{*}) \land (t > \beta_{k,j}^{2}) \Rightarrow$$
  
DGN<sub>r</sub> = {f<sub>k</sub> \in DGN<sub>r</sub><sup>\*\*</sup>} - f<sub>k</sub> (24)

*The end of fault isolation*. The algorithm stops when all the diagnostic signals from the set  $S^*$  are taken into account.

Taking into account the symptoms forming delays can increase faults distinguishability comparing with the diagnosis elaborated basing only on binary diagnostic matrix. In some cases it reduces the diagnosing time.

#### 6. EXAMPLE

The presented example illustrates the possibility of increasing the fault isolability due to taking into account the symptoms delays. Giving the precise calculations is not the aim of this example, only the main steps of isolation algorithm design are discussed. The structure of the system is presented on Fig. 4, while the considered faults are listed in Table 1.



Fig. 4. Diagnostic process – three tank system. Measured variables: medium flow F, levels in tanks  $L_1$ ,  $L_2$  and  $L_3$ , control signal U.

$\mathbf{f}_k$	Notation	Fault description
$\mathbf{f}_1$	ΔF	measurement path F fault
$\mathbf{f}_2$	$\Delta L_1$	measurement path L1 fault
$f_3$	$\Delta L_2$	measurement path L <sub>2</sub> fault
$\mathbf{f}_4$	$\Delta L_3$	measurement path L <sub>3</sub> fault
f <sub>5</sub>	$\Delta S_{\rm V}$	control-valve fault
f <sub>6</sub>	$\Delta P_P$	pump fault
f <sub>7</sub>	$\Delta S_1$	partial passage clogging between tanks 1-2
$f_8$	$\Delta S_2$	partial passage clogging between tanks 2-3
f9	$\Delta S_3$	partial outlet clogging
f <sub>10</sub>	<b>Q</b> <sub>1</sub>	leakage from tank 1
f <sub>11</sub>	Q <sub>2</sub>	leakage from tank 2
f <sub>12</sub>	Q <sub>3</sub>	leakage from tank 3

Table 1. Three tank system faults

Based on balance equations that take into account fault influence, by its linearization, one can achieve the set of residuals equations in the internal form:

$$\mathbf{r}_{1}(s) = -\Delta \mathbf{F}(s) + \mathbf{k}_{1} \Delta \mathbf{U}(s) + \mathbf{k}_{5} \Delta \mathbf{S}_{V}(s) - \mathbf{k}_{6} \Delta \mathbf{P}_{p}(s)$$

$$\begin{split} r_2(s) &= -\Delta L_1(s) + k_2 / (T_1s+1)\Delta F(s) + \\ &+ k_7 (T_1s+1)\Delta S_1(s) - k_8 (T_1s+1)Q_1(s) \\ r_3(s) &= -\Delta L_2(s) + k_3 (T_2s+)\Delta L_1(s) - k_9 (T_2s+1)\Delta S_1(s) + \\ &+ k_{10} (T_2s+1)\Delta S_2(s) - k_{11} (T_2s+1)Q_2(s) \\ r_4(s) &= -\Delta L_3(s) + k_4 (T_3s+1)\Delta L_2(s) \end{split}$$

$$-k_{12}(T_3s+1)\Delta S_2(s) + k_{13}(T_3s+1)\Delta S_2(s)$$
  
- k\_{14}(T\_3s+1)Q\_3(s)

$$\begin{aligned} & -k_{4}k_{9} / (T_{2}s+1) / (T_{3}s+1)\Delta L_{1} \\ & -k_{4}k_{9} / (T_{2}s+1) / (T_{3}s+1)\Delta S_{1} + \\ & -k_{4}k_{11} / (T_{2}s+1) / (T_{3}s+1)Q_{2} \\ & -(k_{12}(T_{2}s+1)+k_{10}) / (T_{2}s+1) / (T_{3}s+1)\Delta S_{2} + \\ & +k_{13} / (T_{3}s+1)\Delta S_{3} - k_{14} / (T_{3}s+1)Q_{3} \end{aligned}$$
(25)

The fifth residual is achieved by structurisation - substituting the transform of  $L_2$  signal in the balance equation for the third tank. Even for such a simple process the achieved equations are already the simplified ones, e.g. they do not take into account the linearization error or the uncertainty of transmittances parameters determination (deviations).

Binary diagnostics matrix corresponding to the residuals set (25) is presented in Table 2. The following subsets of faults are distinguishable based on this matrix:  $\{f_1\}$ ,  $\{f_2, f_7\}$ ,  $\{f_3\}$ ,  $\{f_4, f_9, f_{12}\}$ ,  $\{f_5, f_6\}$ ,  $\{f_8\}$ ,  $\{f_{10}\}$ ,  $\{f_{11}\}$ . The faults in the subsets are not isolable.

Table 3 presents exemplary intervals of delays  $\langle \theta_{k,j}^1, \theta_{k,j}^2 \rangle$ . These values can be calculated according to the methodology presented in Section 2, basing on residual equations (25). For the demonstration purpose the precise computations were omitted, the intervals of delays were related only to the order of transmittances. The delays could be also formulated by the expert.

Table 2. Binary diagnostic matrix for three tank system

S/F	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$	$f_6$	$f_7$	$f_8$	f9	$f_{10}$	f <sub>11</sub>	f <sub>12</sub>
s <sub>1</sub>	1				1	1						
<b>s</b> <sub>2</sub>	1	1					1			1		
<b>s</b> <sub>3</sub>		1	1				1	1			1	
<b>s</b> <sub>4</sub>			1	1				1	1			1
<b>S</b> 5		1		1			1	1	1		1	1

Table 3. The intervals of symptoms delays

S/F	$\mathbf{f}_1$	$f_2$	f <sub>3</sub>	$f_4$	$f_5$	f <sub>6</sub>	f <sub>7</sub>	f <sub>8</sub>	f9	f <sub>10</sub>	f <sub>11</sub>	f <sub>12</sub>
s <sub>1</sub>	1,2				1,2	1,2						
<b>s</b> <sub>2</sub>	5,7	1,2					5,7			5,7		
<b>s</b> <sub>3</sub>		5,7	1,2				5,7	5,7			5,7	
s <sub>4</sub>			5,7	1,2				5,7	5,7			5,7
<b>S</b> 5		8,10		1,2			8,10	8,10	5,7		8,10	5,7

The analysis of fault isolability due to intervals of symptoms delays is conducted only for faults not distinguishable based on binary diagnostic matrix:

- The faults  $\{f_5, f_6\}$  stay unconditionally unisolable.
- Faults {f<sub>2</sub>, f<sub>7</sub>} are unconditioanlly isolable, if there exist such diagnostic signals s<sub>j</sub>, s<sub>p</sub>, that the condition  $(\theta_{m,p}^1 - \theta_{m,j}^2) > (\theta_{k,p}^2 - \theta_{k,j}^1)$  is fullfiled. Let us assume: j=2, p=5, k=7, m=2. The minimal time interval between symptoms s<sub>j</sub>, s<sub>p</sub> of the fault f<sub>2</sub> equals:  $(\theta_{2,5}^1 - \theta_{2,2}^2) = (8 - 2) = 6$ . It is greater than the maximal interval between the same symptoms for fault f<sub>7</sub>:  $(\theta_{7,5}^2 - \theta_{7,2}^2) = (10 - 5) = 5$ . It assures isolability of this two faults.
- Faults { $f_9$ ,  $f_{12}$ } are unconditionally unisolable according to (14), while the fault  $f_4$  is conditionally isolable with { $f_9$ ,  $f_{12}$ }. It can be isolable in the cases when the interval between the symptoms equals 2. In such a case, the fault { $f_4$ } can be excluded. When the interval belongs to the range [0-1] the faults are not isolable.

The example below shows the reasoning according to the algorithm given in Section 5. Let us assume that the first observed symptoms was  $s_2^1 = 1$ . From that moment the time is calculated -  $t^1=1$ :

- The first diagnosis is elaborated according to (17):  $(s_2^1 = 1) \Rightarrow DGN^*(S_2^1) = \{f_1, f_2, f_7, f_{10}\}.$
- The primary diagnosis can be reduced based on the analysis of intervals of symptoms delays according to (18). The fault  $f_1$  can be rejected, because  $\theta_{1,1}^2 = 1 < \theta_{1,2}^1 = 5$ , i.e. for that fault the first observed symptom must be  $s_1=1$ . Finally: DGN( $S_2^1$ ) = { $f_2, f_7, f_{10}$ }.
- The set of diagnostic signals useful for further fault isolation is determined, according to (19):  $S^* = \{s_3, s_5\}$ .

• The time intervals of consecutive symptoms possible formulation is calculated according to (20) and (21). We achieved:  $\beta_{2,3}^1 = 3$ ,  $\beta_{2,3}^2 = 6$ ;  $\beta_{2,5}^1 = 6$ ,  $\beta_{2,5}^2 = 9$ ;  $\beta_{7,3}^1 = 0$ ,  $\beta_{7,3}^2 = 2$ ;  $\beta_{7,5}^1 = 1$ ,  $\beta_{7,5}^2 = 5$ . The parameters  $\beta_{k,j}^2$  are arranged in ascending order { $\beta_{7,3}^2 = 2$ ,  $\beta_{7,5}^2 = 5$ ,  $\beta_{2,3}^2 = 6$ ,  $\beta_{2,5}^2 = 9$ }.

Then the iterative diagnosis is conducted. Let us assume, that in time t=1 the symptom  $s_3^2 = 1$  occurred. The diagnosis reduction is conducted according to (22). The fault  $f_2$  is eliminated, because the condition for symptom delay is not fulfilled - t  $\in [\beta_{2,3}^1, \beta_{2,3}^2] = [3,6]$ , while, in the case of fault  $f_{10}$  the symptom  $s_3^2 = 1$  should not occur because  $q(f_{10}, s_3) = 0$ . The diagnosis DGN\* $(s_2^1, s_3^2) = \{f_7\}$  is a final one. It is elaborated in time t=1. In this case there is no need to wait for the successive symptom  $s_5^3 = 1$ . The achieved diagnosis time is minimal in this case.

## 7. SUMMARY

The basic method of increasing fault isolability is the increase of measured signals, which leads to the generation of additional residuals. The residual structurization, i.e. creation of secondary residuals based on primary ones (Gertler and Singer, 1990; Gertler 1991, 1998), is also a very effective method. The increase of the set of measured signals is not always possible and economically justified. Also, the residual structurization not always gives satisfactory results. The alternative or complementary methods of increasing fault isolability are the application of multiple-valued residual evaluation (Kościelny, 1999, 2001; Kościelny *et al.*, 2006; Korbicz *et al.*, 2004) or presented in this paper approach based on the knowledge about the delays of forming of fault symptoms.

The presented fault isolation algorithm that utilises the minima and maximal values of fault symptoms delays can lead to higher fault isolability. Additionally, it protects against false diagnosis caused by the dynamics of symptoms forming. In some cases, it also makes possible to achieve shorter times of diagnosis in comparison with the diagnostic algorithms based on binary diagnostic matrix when the final diagnosis can be elaborated after all values of diagnostic signals are steady.

However, to be able to determine the symptoms intervals one needs the residual equations in the internal form (or other precise mathematical description) or very precise expert knowledge. This is very difficult to obtain in practice. The alternative approaches were shown in (Kościelny *et al*, 2007). The first one simplifies the reasoning procedure by taking into account only the observed symptoms and rejecting the information carried out by the lack of symptoms (symptoms based reasoning). The second one takes into account the sequence of symptoms arising not the precise delay intervals (diagnosis based on the symptoms sequence). Both approaches are much more simpler and applicable for industrial applications. Additionally, the symptoms based reasoning has an limited ability to deal with multiple faults.

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#### REFERENCES

- Basseville M. (2001). On fault detectability and isolability. In: European Journal of Control, **7**, **No 6.**, 625-637.
- Chen J., Patton R.J. (1999). *Robust model based fault diagnosis for dynamic systems*. Kluver Akademic Publishers, Boston.
- Combastel C., Gentil S., Rognon J. P. (2003). *Toward a* better integration of residual generation and diagnostic decision, Preprints of: 5th IFAC Symposium SAFEPROCESS`2003, Washington D.C., USA, 9-11.VI
- Gertler J. (1991). Analitical redunduncy methods in fault detection and isolation. Proc. of: *IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes - SAFEPROCESS'91*, Baden-Baden , Vol. 1, 9-21.
- Gertler J. (1998). Fault Detection and Diagnosis in Engineering Systems. Marcel Dekker, Inc. New York -Basel - Hong Kong.
- Gertler J., Singer D. (1990). A new structural framework for parity equation based failure detection and isolation. *Automatica*, **26**, **no. 2**, 381-388.
- Frank P.M.(1990). Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy. *Automatica*, **26**, 459-474.
- Korbicz, J., J.M. Kościelny, Z. Kowalczuk and W. Cholewa (2004). Fault Diagnosis: Models, artificial intelligence methods, applications. Springer.
- Kościelny J.M. (1995). Fault Isolation in Industrial Processes by Dynamic Table of States Method. *Automatica*, **31**, no.5, 747-753,
- Kościelny J.M. (1999). Application of Fuzzy Logic Fault Isolation in a Three-Tank System. Proc. of: *14th World Congress IFAC*, Bejing, China, P-7e-05-1, 73-78.
- Kościelny J.M., Bartyś M., Rzepiejewski P., Sá da Costa J. (2006). Actuator fault distinguishability study. *Control Engineering Practice* (Elsevier) **14**, **2006**, 645-652.
- Kościelny J.M., Syfert M., Dziembowski B. (2007). The issue of symptoms arising delay during diagnostic reasoning. 5<sup>th</sup> Workshop on Advanced Control and Diagnosis, November 15-16, 2007 Grenoble, France (CD).
- Kościelny J.M., Zakroczymski K. (2001). Fault Isolation Algorithm that Takes Dynamics of Symptoms Appearances Into Account. *Bulletin of the Polish Academy of Sciences. Technical Sciences.* **49, no 2**, 323-336.
- Patton R., Frank P., Clark R. (Eds.) (2000). *Issues of fault diagnosis for dynamic systems*. Springer.
- Vanden Daele R., Peng Y., Kinnaert M. (1997). Fault diagnosis using belief functions. Proc. of: 3th IFAC Symposium on Fault Detection Supervision and Safety for Technical Processes SAFEPROCESS'97, Hul, UK, 546-551.