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A Practical Loop Shaping Design Procedure with Classical Control Criteria and Its Application to Hard Disk Drives ⋆

Keitaro Ohno * Shinji Hara * Naoshi Yamahira *
Takayuki Kawabe ** Tsugito Maruyama **

* *Graduate School of Information Science and Technology, The University of Tokyo, Tokyo, Japan (e-mail:keitaro.ohno@nifty.com).*
** *Storage Laboratories, Fujitsu Laboratories Limited, Kanagawa, Japan (e-mail: kwb@jp.fujitsu.com)*

Abstract: This paper is concerned with integration of classical control criteria into the $\mathcal{H}_\infty$ robust control design with the use of the $\mathcal{H}_\infty$ Loop Shaping Design Procedure (LSDP) by McFarlane and Glover. Classical control criteria such as gain crossover frequency and phase margin still play important roles to designing and evaluating feedback control systems in practical applications including Hard Disk Drives (HDDs) where empirical knowledge has great importance. The systematic use of the robust control design has been tried in industry. However, it has yet to be fully adopted simply because of the familiarity with the classical control criteria. In this paper, we will propose a way to design digital robust control systems with the use of the LSDP in which we can specify the classical control criteria. Application to HDDs will be demonstrated with a set of simulations to validate the proposed method.

Keywords: Robust control, $\mathcal{H}_\infty$ Loop shaping design, Phase margin, Gain crossover frequency, Hard Disk Drives.

1. INTRODUCTION

Standard $\mathcal{H}_\infty$-based robust control design methods have received a lot of focuses for a last couple of decades because of its systematic design capability of balancing contradictory objectives of robustness and control performance. However, in practice, such methods have yet to be fully adopted in industry. The reason is that, in industry, classical control criteria, including the gain crossover frequency, the phase margin and the gain margin of the open-loop frequency responses, still play a central role in practical design. The $\mathcal{H}_\infty$-based methods proposed so far have not provided good and concrete perspectives on the relations between the advanced control criteria and the classical control ones.

McFarlane and Glover [1][2] proposed the $\mathcal{H}_\infty$ loop shaping design procedure (LSDP), which incorporates open-loop shaping by a set of compensators to obtain performance/robust stability tradeoffs. The procedure is basically straightforward. Designers are expected to follow a design/re-design procedure on an open-loop frequency characteristics basis by handling a couple of compensators for loop shaping purposes. An appropriate weighting function setting will automatically yield a robust controller.

One of the advantages of the LSDP is its familiarity with classical loop shaping control theory, where the empirical sense are easily made available. Yet, concrete classical design criteria such as phase margin cannot be directly specified in the design procedure. This motivates us to integrate the classical criteria into the LSDP. Moreover, concrete ideas on how to specify the weighting functions are still lacking. One simple reason is that the LSDP itself can treat wide range of control problems that results in the difficulty of establishing a general systematic weighting function setting procedure. This motivates us to establish such a concrete procedure by imposing a limitation to a class of target control systems.

In this paper, following our previous researches [3][4], we propose a way to integrate the classical control criteria into the advanced control design method with the use of the LSDP. In Section 2, we propose a class of target control systems to be considered to establish a general and systematic design procedure. In Section 3, we describe the basic idea of the proposed design procedure, in which we review the LSDP and provide an outline of the proposed design procedure. Section 4 provides a concrete procedure for robust control design with classical control criteria. Section 5 demonstrates our proposed design procedure to validate it by applying it to a Hard Disk Drive (HDD). A set of simulations will be carried out on a benchmark problem for HDD proposed by a Japanese industrial/academic group. Section 6 is a summary.

2. A CLASS OF TARGET FEEDBACK SYSTEMS

In order to establish a concrete procedure of weighting function setting, we propose to impose some reasonable limitation to the feedback system to be considered. The class of such systems is summarized as a control block

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diagram in Fig. 1, where $\mathcal{S}$ is a sampler, $\mathcal{H}$ is a hold, $K[z]$ is a digital controller, $N[z]$ is a notch filter to notch out a set of resonant modes, and $\tilde{P}(s)$ is a detailed plant that can be written as,

$$\tilde{P} := \sum_{i=0}^{N} \frac{A_i}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}, \quad (1)$$

where ζ_i, ω_i, and A_i are respectively a damping factor, resonant frequency, and residue of the i-th resonant mode. Notice that the first mode may be a rigid body model by setting $\omega_0 = \zeta_0 = 0$.

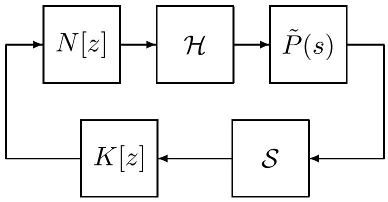

Fig. 1. A typical control block diagram of digital control systems.

The proposed class of systems is reasonable for flexible mechanical systems, where almost all the plants including HDDs lie in this class and some notch filters are usually utilized to compensate for the resonant modes.

3. BASIC IDEA OF PROPOSED PROCEDURE

3.1 Review of the Loop Shaping Design Procedure

Let us first review the LSDP proposed in [1][2]. Figure 2 shows a typical control system considered in LSDP, where P is a target plant, W is a compensator for design, $P_s := PW$ is a shaped plant, K_∞ is an optimal controller calculated for P_s, and $K := K_\infty W$ is a resultant controller. A design procedure begins with the loop shaping by W to yield P_s by considering the desired frequency characteristics, e.g., roll-off at high frequency range and disturbance attenuation at low frequency range. This may be performed by a classical control design perspective.

Optimal controller K_∞ that guarantees closed-loop robustness is then calculated via minimization of the following $\mathcal{H}_\infty$ cost function over K_∞ to yield an optimal γ_{opt},

$$\gamma_{opt} := \begin{matrix}\min \\ K_\infty\end{matrix} \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I - P_s K_\infty)^{-1} \begin{bmatrix} P_s & I \end{bmatrix} \right\|_\infty . \quad (2)$$

It should be pointed out that the following properties of LSDP are known.

Property 1: $\sigma(PW\gamma_{opt}) \geq \sigma(PK) \geq \sigma(PW/\gamma_{opt})$ is roughly assured, where γ_{opt} is always grater or equal to 1 [1][2].

Property 2: Phase margin (P_M) as a classical control design criteria has a property of $P_M \geq 2\arcsin\left(\frac{1}{\gamma_{opt}}\right)$ [7].

Property 3: $\gamma_{opt} \leq \sqrt{4+2\sqrt{2}}$ and $\gamma_{opt} \leq 2\sqrt{3} + \sqrt{6}$ are strictly assured for a class of second and third order models, respectively [3][4].

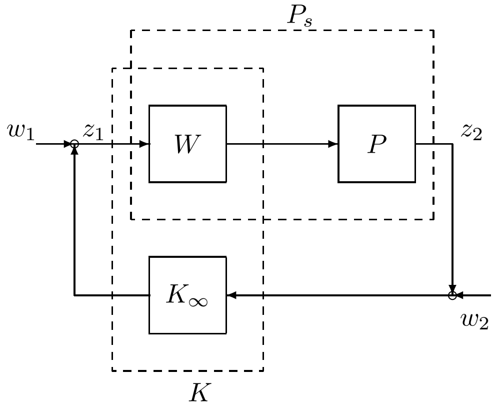

Fig. 2. A typical control block diagram considered in LSDP.

A clear advantage of the LSDP is its ability to straightforwardly obtain a robust controller by open-loop frequency shaping through the appropriate selection of W. Now, the question is how to reasonablly and systematically select W using classical control perspectives to satisfy given design specifications.

3.2 Integration of classical control criteria into LSDP

The basic idea of the proposal consists of (i) a way to configure the weighting function, (ii) a way of specifying a servo bandwidth, and (iii) a way of specifying a target phase margin.

Proposal 1: Configure the weighting function

As we impose the limitation on the class of target systems as in Section 2, the objectives of the weighting function are also limited to the following properties: (A) Suppressing DC disturbace by an integral action, (B) Ensuring sufficient roll-off at high frequency region for robust stability, (C) Suppressing narrow band disturbance at specific frequency region, e.g., disturbance component around 1kHz in Fig. 7, (D) Specifying a servo bandwith as in Proposal 1, (E) Specifying a phase margin as in Proposal 2. Thus, we propose the following configuration as the weighting function.

$$W := kW_{PI}W_{RO}W_{FT}W_{PR}, \quad (3)$$

where k is a constant design parameter for (D), and W_{PI}, W_{RO}, W_{FT}, W_{PR} are respectively correspond to (A), (B), (C) and (E). In other word, roughly speaking, each of the weighting function specifies the sensitivity function and complimentaly sensitivity function as Fig. 3. Notice that, the gain of W is selected by k, meaning that the gain of each function does not necessarily have to be cared.

Proposal 2: Specify the servo bandwidth

The servo bandwidth is one of the most important criteria in classical control design. From Property 1 of Section 3, the open-loop gain crossover frequency, ω_c, is roughly assured to be around the gain crossover frequency of P_s or PW. Thus we can specify the servo bandwidth by PW through adjusting the gain k of W.

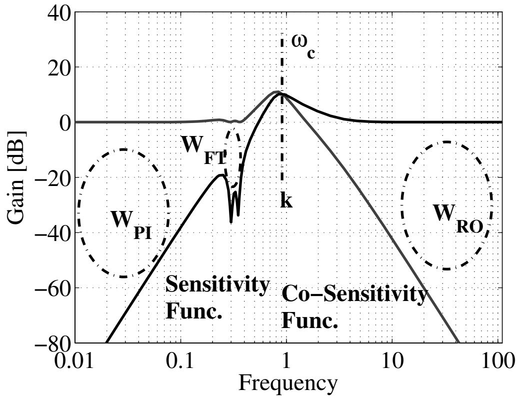

Fig. 3. Function of each component of W

Proposal 3: Specify the phase margin

The phase margin is also important to gain insight about the closed-loop characteristics including the sensitivity function. Now let us review the Property 3. The upper bound of γ_{opt} for the second and third order systems come from the worst case of $1/s^2$ and $1/s^3$, respectively, whose phase delay are 180 and 270 degrees.

Let us define the phase delay of P_s at ω_c as θ, and the order of P_s as n. Then, from the Propoerty 3, we can rewrite γ_{opt} as a function of n as

$$\gamma_{opt} = \sqrt{1+\alpha^{2(n-1)}}, \alpha = 1+\sqrt{2}, \tag{4}$$

where n is 2 or 3. γ_{opt} can be considered to be a monotonically increasing function with respect to θ, and it is reasonable to characterize those relation from (4) by simply assuming $n = 2\theta/\pi$, where the $n = 2$ system has π-rad/s phase delay and $n = 3$ system has $3\pi/2$-rad/s phase delay, which results in

$$\gamma_{opt} = \sqrt{1+\alpha^{2(2\theta/\pi-1)}}. \tag{5}$$

On the other hand, from the Property 2, the uppper bound of P_M can be expressed as

$$P_M = 2\arcsin\left(\frac{1}{\gamma_{opt}}\right). \tag{6}$$

Consequently, replacing γ_{opt} in (6) by (5) and solving it for θ yields

$$\theta = \frac{\pi}{4}\frac{\ln(\gamma_t^2-1)}{\ln(1+\sqrt{2})} + \frac{\pi}{2}, \gamma_t := \frac{1}{\sin(P_M\frac{\pi}{360})}. \tag{7}$$

Specifying the target phase margin P_M provides a target phase delay θ at the gain crossover frequency of PW, which can be manipulated by W_{PR} of W.

3.3 Outline of the proposed design procedure

The proposed design procedure basically consists of three parts. First part, which corresponds to Step 1 and 2 in the following section, is to obtain an equivalent nominal plant model of a target control system to incorporate it in the LSDP, where we consider a limited class of systems as in Section 2. Second part, which corresponds to Step 3 through 7, is a selection of a weighting function with a concrete idea on how to incorporate the classical control criteria as is described in Subsection 3.2. Last part, which corresponds to Step 8 and after, is to calculate K_∞, reconfigure it with W to have $K = K_\infty W$, and discretize it to have $K[z]$ to implement on the digital control systems.

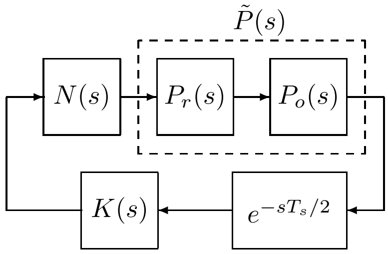

Fig. 4. Conversion into a continuous-time equivalent system.

4. PRACTICAL DESIGN PROCEDURE

4.1 Nominal plant selection

A typical control block diagram shown in Fig 1 cannot be straightforwardly handled by the LSDP and should first be converted into the system as in Fig 2.

Step 1: Obtain a continuous-time equivalent system

First, discrete-time signals in Fig 1 should all be transformed into continuous-time signals. This can be performed by simply approximating the effect of sampler and hold as some time delay, $\exp(-sT_s/2)$, as shown in Fig. 4, where T_s is a sampling time. Notice that, in this figure, as a preparation for the following design step, detailed model of the plant, $\tilde{P}$, is divided into two part, $P_r(s)$ and $P_o(s)$, namely, $\tilde{P} = P_r P_o$, which are respectively a set of resonant modes and a rigid mode.

Step 2: Obtain a resonant modes compensated model

Secondly, it is desirable not to have resonant modes near the desired gain crossover frequency, especially when multiple gain crossover points appear because of those resonant modes [5]. In such a case, we propose to pre-compensate for the plant by a notch filter N as indicated in Fig. 4. N is designed a priori with the conventional classical control technique such that $|N(j\omega)P_r(j\omega)| \simeq 1$ for $\omega \leq q\omega_c$, where ω_c is the gain cross-over frequency and $q \geq 1$ is a design parameter that is practically ranging from 1 to 5.

The notch-compensated resonant mode NP_r is then approximated as an all-pass filter $P_{ap} = \{\forall\omega : |P_{ap}(j\omega)| = 1\}$ for simplicity such that $N(j\omega)P_r(j\omega) - P_{ap}(j\omega) = 0, \forall\omega < q\omega_c$ is assured. $|N(j\omega)P_r(j\omega) - P_{ap}(j\omega)| \simeq 0$ can be designed through N. Thus P_{ap} only cares about the phase drop. One simple candidate of P_{ap} is a linearized model of the time delay by the use of the Padé approximation [6], and the phase drop of NP_r is approximated by the order and the time delay of the P_{ap}.

On the other hand, the physical time delay caused by a sampler and hold can also be approximated as a transfer function P_d by Padé approximation in the same manner as in [6].

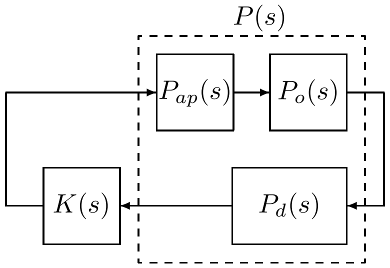

Fig. 5. Equivalent plant system P for design.

Now we have an equivalent control system for design as shown in Fig. 5, where $P \simeq P_{ap}P_oP_d$.

4.2 Weigh selection

Step 3: Specify W_{PI} for (A) in Proposal 3.

Proportional and integral filter is proposed as W_{PI} which is simple but practically sufficient transfer function for (A) in Proposal 3.

$$W_{PI} = \left(1 + \frac{\omega_p}{s}\right). \quad (8)$$

Basically it is sufficient to define the characteristic frequency $\omega_p = 2\pi f_p$ to fit the exogenous disturbance shown in Fig. 7. It may be used as a tuning parameter to adjust the integral gain.

Step 4: Specify W_{RO} for (B) in Proposal 3

Low pass filter of the following form is proposed as W_{RO} which is simple but sufficient transfer function for (B) in Proposal 3.

$$W_{RO} = \frac{s + \omega_l}{s + \omega_h}, \quad (9)$$

where $\omega_l = 2\pi f_l$ and $\omega_h = 2\pi f_h$ are cutoff frequencies. As implied, f_l may be set a little higher than the target crossover frequency, and f_h may be set high enough frequency to have sufficient roll-off characteristics.

Step 5: Specify W_{FT} for (C) in Proposal 3

A series of peak filters of the following form is proposed as W_{FT} which is a simple but practical transfer function for (C) in Proposal 3.

$$W_{FT} = \Pi_{i=1}^{m} \frac{s^2 + 2\zeta_i\eta_i\omega_i s + \omega_i^2}{s^2 + 2\eta_i\omega_i s + \omega_i^2}, \quad (10)$$

where m is a number of disturbance components to be treated, and ζ_i, η_i, ω_i are all design parameters which are basically adjusted for W_{FT} to fit to the target disturbance.

Step 6: Specify k for (D) in Proposal 3

Set $W_{PR} = 1$ for now and adjust the target gain crossover frequency $\omega_c = 2\pi f_c$ by k as stated in Proposal 1.

Step 7: Specify W_{PR} for (E) in Proposal 3

Phase lead-lag filter of the following form is proposed as W_{PR} which is simple but practical transfer function for (E) in Proposal 3.

$$W_{PR} = \frac{s + \omega_c(1 - \beta)}{s + \omega_c(1 + \beta)} \quad (11)$$

where ω_c is the target gain crossover frequency, and β is a parameter to specify the target phase margin as stated in Proposal 2. Namely, β is to adjust the phase delay θ of shaped plant P_s.

Step 8: Calculation of LSDP controller and its discretization

Final design step is to calculate the LSDP controller K_∞ to get $K = K_\infty W$ by following the procedure reviewed in Subsection 3.1. It is followed by some model reduction because the order of the resultant controller ($\mathcal{O}(K_\infty) = \mathcal{O}(PW)$) is oftenly too high to implement. It is also followed by some discretization in order to implemente it on the digital control systems as in Fig. 1.

5. APPLICATION TO HARD DISK DRIVES

We demonstrate the use of the proposed design procedure on HDDs to improve the positioning accuracy.

5.1 A Brief Description of Hard Disk Drive

Typical HDD characteristics lie in the class of systems proposed in Section 2, where a typical frequency response of the actuator, a typical estimated exogenous disturbance, and a typical control block diagram are respectively shown in Fig. 6, Fig. 7, and Fig. 1 [1].

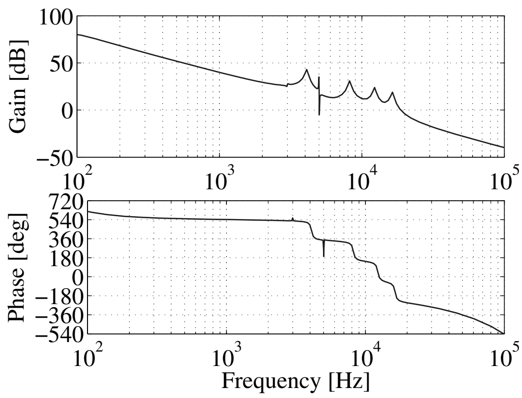

Fig. 6. A typical frequency response of a VCM actuator [8].

5.2 A control design with proposed LSDP

Let us now design a control system for a benchmark model proposed in [8], which can be downloaded through internet. The primary objective is to improve the positioning accuracy by at least 10% compared to the conventional controller that can also be downloaded as a PID controller from [8].

In order to achieve such a positioning accuracy, we will design a controller with gain crossover frequency f_c = 1500-Hz and phase margin P_M = 30-degree.

First, by following the procedure from Step 1 to Step 2 in Section 4, we have a plant model P for design as shown in Fig. 8, where the all-pass filter P_{ap} is designed to fit the

[1] The multi-rate hold is oftenly utilized as the hold function $\mathcal{H}$ to enhance the positioning performance in HDD control. It means that the control update frequency is higher than the sampling frequency in this case.

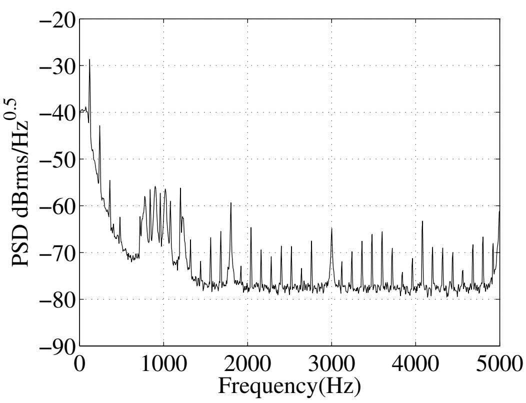

Fig. 7. A typical estimated exogenous disturbance of a HDD [8].

notch compensated model NP_r as illustrated in Fig. 9. Note that, although the P_{ap} does not look like a good approximation of NP_r as in Fig. 9, the important point is to approximate the phase characteristics for $\omega \leq \omega_c$, since the proposed method focuses on the phase at ω_c. For practical use, because the proposed method does not exactly assure the phase margin satisfy the target one, we propose to approximate the phase characteristics for frequency range of $\omega \leq q\omega_c$, where in this design case we set $q = 2$.

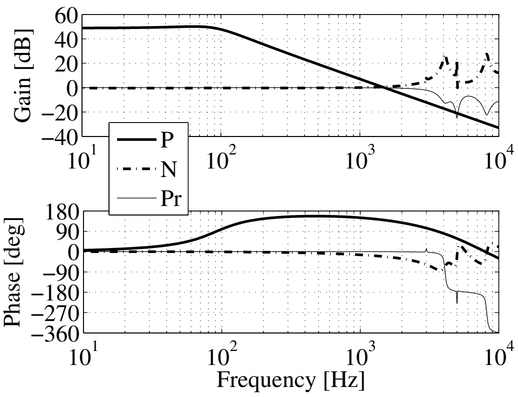

Fig. 8. Resonant modes Pr, notch filter N, and modified plant model P for design

Following the step 3, we select $f_p = 800$ for W_{PI} to best represent the exogenous disturbance shown in Fig.7. Regarding the step 4, we select $f_l = 1 \times 10^4$ and $f_h = 2 \times 10^4$ for W_{RO} since the Nyquist frequency of the target system is 1.32×10^4. Regarding the step 5, from Fig. 7, we can see the broadband disturbances at the frequency of about ω_i =800, 900 1050, 1250, 1800, 3000 and 5000-Hz. Each one can be modeled as a peak filter with $\zeta_i = 0.3$ and $\eta_i = 0.1$. These identified parameters directly specify W_{FT}. Then, after adjusting the gain by k by following the step 6, we specify W_{PR} for the step 7. Now, from the target phase margin and (7), $\theta = 0.9292$ or 233.2 degree. Thus, β is selected to 0.95 to satisfy the condition of the step 7. Theose weighting functions are shown in Fig. 10 and the shaped plant PW is shown in Fig. 11. Now, we can see that the f_c is set to 1500-Hz and θ is 224.5-degree (γ_t=3.864) from Fig. 11.

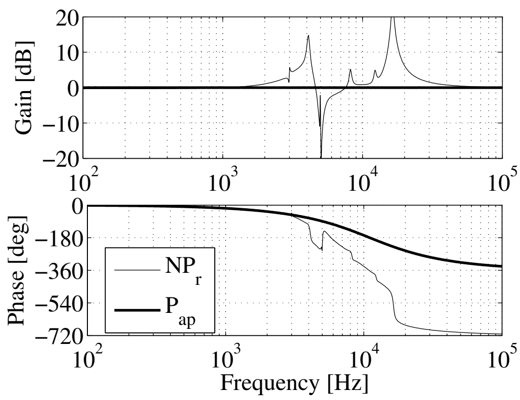

Fig. 9. Notch-compensated model NP_r and its approximation by an all-pass filter P_{ap}.

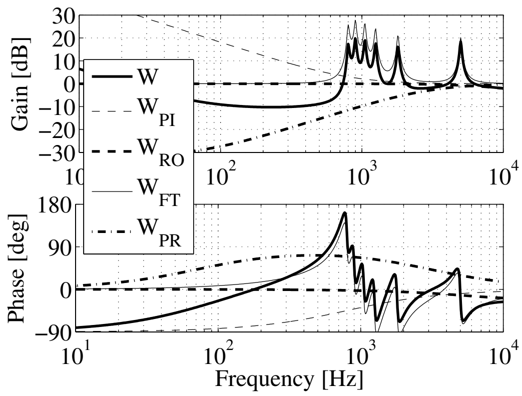

Fig. 10. Weighting function W and its components.

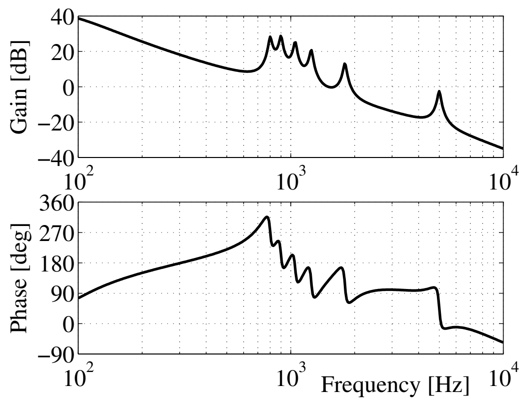

Fig. 11. Shaped plant PW.

5.3 Simulation results

The resultant controller has γ_{opt}=4.18, f_c=1452-Hz and P_M=35.3-degree[2], where we can see all those criteria are roughtly assured.

The open-loop freqency response is shown in Fig. 12. At around 700 Hz, we can see some notch-like characteristics. This is generated through the $\mathcal{H}_\infty$ norm minimization pro-

[2] Resultant phase margin is greater than the target phase margin. This may be caused by W_FT that has peak filter near the gain crossover frequency.

cess to recover the phase to establish a robast stability. The corresponding sensitivity function is depicted in Fig. 13. As an effect of W_{FT}, the sensitivity gain at around 1 kHz is reduced to suppress the narrow band disturbance.

Note that, in the standard classical control design, it is difficult to introduce such a peak filter at this frequency region since we have to care both the gain and phase at the same time to establish the robust stability. One of the advantages of the approach used here is that we can simply introduce any filter as a weighting function from the classical control perspective and the rest of the concerns is automatically taken care of by the $\mathcal{H}_\infty$ framework.

The positioning error is reduced from 0.078 to 0.068-$\mu m - 3\sigma$rms, which corresponds to 10% reduction compared to the conventional PID system.

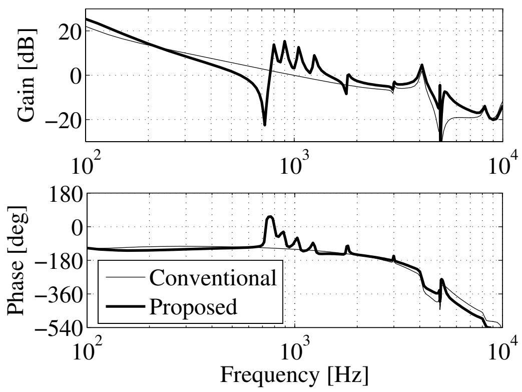

Fig. 12. Openloop frequency responses with proposed and conventional controller.

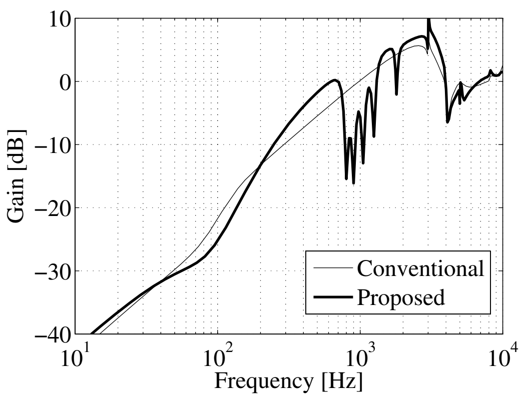

Fig. 13. Sensitivity functions for proposed/conventional control systems.

6. CONCLUSION

We have proposed a practical robust control design procedure with the use of the $\mathcal{H}_\infty$ LSDP, in which the classical control criteria is integrated. By imposing a reasonable limits on the class of target control system, we have established a detailed general systematic procedure, where the design criteria including gain crossover frequency and phase margin are shown to be design specification. We have carried out a set of simulations on a HDD as a

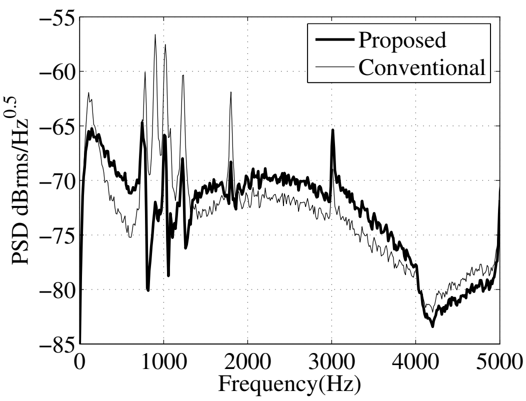

Fig. 14. Power spectral density of the position error signal for proposed/conventional control systems.

design example to validate our proposed procedure, and concluded that it can easily yield controller that outperformances conventional controllers by 10% in our design case in terms of the positioning accuracy.

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