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Ship Motion Prediction for Maritime Flight Operations

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Abstract: This paper presents a novel and feasible prediction procedure for ship motion in the presence of uncertain tendency of ship motion dynamic variations and stochastic sea state disturbances. An appropriate model aiming to feature the characteristics of the dynamic relationship between an observer and a ship deck is constructed, from which an initial algorithm is implemented. The optimal system order based on Bayes Information Criterion (BIC) is derived, resulting in the development of an accurate adaptive multi-step predictor for estimation of ship motion dynamics. Simulation results demonstrate that the proposed prediction approach substantially reduces the model complexity and exhibits excellent prediction performance, making it suitable for integration into ship-helicopter approaches and landing guidance systems.

Keywords: System order determination; Ship motion prediction; Wave spectrum.

1. INTRODUCTION

The effective prediction of ship motion dynamics is crucial to a wide range of maritime operations which can take place in a variety of situations (Zhao et al. 2004). One potential utilization is to provide necessary decision-making information for a helicopter pilot to plan an optimal descent trajectory and subsequently employ a corresponding control strategy for safe landing operations. However, the uncertainty and randomness of environmental disturbances in high sea states greatly complicate attempts to obtain satisfactory prediction results. One of the main challenges is to propose a precise and elaborate estimation model in the presence of uncertain stochastic processes (e.g. wind, sea wave), unknown ship motion behaviour characteristics, and random unmodeled dynamic disturbances. The accumulation of landing position prediction error due to variations of relative motion between a helicopter and a ship deck exacerbates the difficulty of designing an accurate predictor. Furthermore, in situations where an automated landing must be made urgently and without warning (e.g. unexpected weather, mechanical failure), a safe landing necessitates the incorporation of an efficient and rapidly converging estimation algorithm into the Flight Control System.

The main challenge for ship motion estimation is to develop an appropriate prediction model, resulting from complicated wave-excitation dynamics caused by the local stochastic sea states such as barometric pressure, wind speed and wave heights (Benstead et al. 2005). It is reported that ship motion dynamics are not too remarkably influenced by the local sea state as a result of the narrowband feature of its power spectrum around the central frequency (Ra et al. 2006), and it is common to represent sea wave dynamics as a superposition of sinusoidal forms covering a wide range of wave frequencies by abnegating high-frequency components (Chung et al. 1990; Ra et al. 2006). However, in many cases, such sinusoidal superpositions are obtained from experimental results under particular conditions. Therefore, these conclusions are subject to question as to whether they can be valid for the other cases. Clearly, the prediction results can be significantly improved when the real system parameters are accurately approached.

Recently, Ma et al. (2006) suggested an Auto-Regressive fitting model. This method lacks long-term prediction capability. Ship motion prediction using state-space approach has been subject to extensive investigation in a considerable number of papers, and significant efforts, including theoretical analysis and experimental research, have been made to deal with different practical problems in ship motion prediction. Lainiotis et al. (1992) focused on deriving a state-space model based on a sufficient knowledge of ship motion dynamics, which suffers from the dependency on available information. Ra et al. (2006) regarded the ship motion as a particular sinusoidal form, and obtained a recursive robust least squares frequency estimator by assuming that the ship motion frequency changed slowly. An initial prediction algorithm using Minor Component Analysis developed by Zhao et al. (2004) requires substantial computation effort for updating identifying coefficients, which compromises its practicality in real time prediction.

This paper focuses on improving prediction performance of ship motion dynamics in the maritime environment disturbed by a series of uncertain stochastic processes. Assuming white noise distribution of sea excitations, we concentrate in detail on the derivation of a proper model revealing intrinsic dynamic features of ship motion and a corresponding prediction algorithm. Firstly, the current state observation is treated as a linear function of its previous states and system input up to the time of interest without knowing their respective orders. Then a novel identification information criterion is proposed to obtain optimal output and input orders using the Bayes Information Criterion. Our order selection principle considers error accumulation, fitting complexity and prediction capability, attempting to increase prediction horizon by achieving a tradeoff among the three main factors. The suggested criterion avoids excessive dependency on a prior knowledge of the initial system order information. Next, an approximate prediction model is presented in which system order is specified using the new criterion. Finally, the model coefficients identified from the Recursive Least-Square (RLS) method are employed to predict the ship motion dynamics. Simulation results demonstrate that the suggested algorithm can efficiently predict the ship motion dynamics with acceptable accuracy.

2. DETERMINATION OF OPTIMAL SYSTEM ORDER AND COEFFICIENTS

The proposed methodology is inspired by phase-lead networks after the investigation of the dynamic relationship between the true data and the predicted data in Fig. 1. Here, the true and the predicted data are considered as input $u(t)$ and output $y(t)$, respectively. A phase-lead network constructed properly, with a large phase lead, means a reasonable prediction (solid) can be obtained as early as possible. The expectant predictor with phase lead feature has the transfer function in the form of

$$\frac{Y(z)}{U(z)} = \frac{b_{(n,0)} + b_{(n,1)}z^{-1} + \cdots + b_{(n,n-1)}z^{-(n-1)}}{1 + \bar{a}_{(m,1)}z^{-1} + \cdots + \bar{a}_{(m,m)}z^{-m}}. \quad (1)$$

According to (1), we describe the relationship between the current and previous ship dynamics by the following model:

$$y(t) = A(q^{-1})y(t) + B(q^{-1})u(t) + e(t). \quad (2)$$

$$A(q^{-1}) \triangleq \sum_{i=1}^{m} a_{(m,i)}q^{-i}, m \in N. \quad (3)$$

$$B(q^{-1}) \triangleq \sum_{j=0}^{n-1} b_{(n,j)}q^{-j}, n \in N, n < m. \quad (4)$$

Here $a_{(m,i)} = -\bar{a}_{(m,i)}$,i=1,..., m, and $b_{(n,j)}$,j=0,..., $n-1$ denote system coefficients, m and n are the output order and input order of our model, respectively. For ship motion dynamics, the main stochastic disturbance comes from sea wave excitation. The wave spectra are usually obtained from empirical observations at a particular maritime environment. This provides a clue for analysis of wave features. Numerous wave spectra analysis methods (e.g., D. G. Lainiotis et al. (1992) and I. M. Weiss et al. (1977)) suggest that the sea wave excitation can be treated as white noise. Hence, we consider the ship motion dynamics as a stochastic process with a statistical distribution $N(0, \sigma_e^2)$.

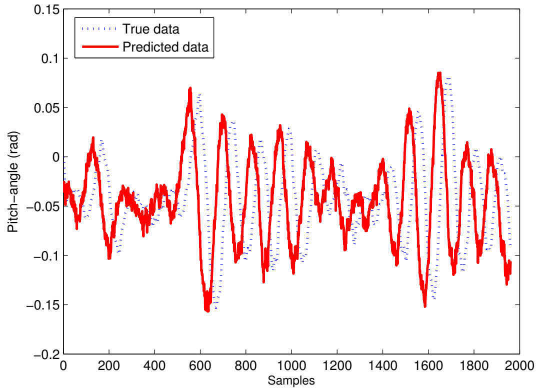

Fig. 1. Relationship between true and predicted data

Without loss of generality, it is assumed that ordered pairs (m, n) lie within the following bounds:

$$m \in V_1 = \{m | 1 \le m \le m_{max}, m \in N\}, \quad (5)$$

$$n \in V_2 = \{n | 1 \le n \le n_{max}, n \in N\}, \quad (6)$$

where m_{max} and n_{max} are upper bounds on the output order and input order, respectively. For the purpose of determining an optimal output order m^* and an input order n^*, reasonable bounds on the system order (m_{max}, n_{max}) should be assigned in advance. Smaller upper bounds on the system order will lead to a simplistic model unable to represent ship motion dynamics accurately. Hence, upper bounds on the system order should be large enough to guarantee an acceptable accuracy of the model. Meanwhile, the selection of upper bounds (m_{max}, n_{max}) has a significant influence on the complexity of the system model, i.e., excessively large upper bounds would increase the complexity of the model and aggravate computational burden. Therefore, an appropriate model without loss of prediction accuracy is preferable. Based on empirical results, a feasible selection scheme is to select (m_{max}, n_{max}) such that:

$$m_{max} = O(\sqrt{T}), \quad (7)$$

$$n_{max} = O(\frac{\sqrt{T}}{2}), \quad (8)$$

here, T denotes the number of the measured data. (7) and (8) constrain the searching scope for the optimal system order selection by avoiding either a too simplistic model or excessive computational burden .

By introducing of the stochastic regressive vector

$$\varphi^T(t) = [y(t-1), \ldots, y(t-m), u(t), \ldots, u(t-n+1)] \quad (9)$$

and the following notation

$$\theta^T(m, n, t) = [a_{m,1}(t), \ldots, a_{m,m}(t), b_{n,0}(t), \ldots, b_{n,n-1}(t)] \quad (10)$$

we can write from (2)

$$y(t) = \theta^T(m, n, t)\varphi(t) + e(t). \quad (11)$$

$\theta^T(m, n, t)$ can be effectively estimated via the RLS algorithm provided $\{y(t)\}$ is available. Using the quadratic criteria function (Ljung et al. 1987).

$$J(\theta) = \sum_{j=1}^{t} [y(j) - \theta^T(m,n,j)\varphi(j)]^2. \quad (12)$$

leads to the following estimates for the coefficients:

$$\hat{\theta}(m,n,t) = [\sum_{j=1}^{t} \varphi(m,n,j)\varphi^T(m,n,j)]^{-1} \cdot [\sum_{j=1}^{t} \varphi(m,n,j)y(j)]. \quad (13)$$

The latter can be computed recursively by

$$\hat{\theta}(m,n,t+1) = \hat{\theta}(m,n,t) + M(m,n,t+1) \cdot [y(t+1) - \varphi^T(t+1)\hat{\theta}(m,n,t)], \quad (14)$$

$$M(m,n,t+1) = P(m,n,t)\varphi(t+1) \cdot [1 + \varphi^T(t+1)P(m,n,t)\varphi(t+1)]^{-1}, \quad (15)$$

$$P(m,n,t+1) = P(m,n,t) - M(m,n,t+1)\varphi^T(t+1)P(m,n,t), \quad (16)$$

$$\hat{\theta}(m,n,0) = 0,\ P(m,n,0) = \alpha I,\ \alpha = 10000. \quad (17)$$

Define the prediction error as

$$\xi(m,n,t+1) = y(t+1) - \varphi^T(m,n,t+1)\hat{\theta}(m,n,t) \quad (18)$$

and compute the maximum likelihood estimate of the error covariance until time T

$$\hat{\sigma}^2(m,n,T) = \frac{1}{T-m-n} \sum_{m+n+1}^{T} \xi^2(m,n,t). \quad (19)$$

$\hat{\sigma}^2(m,n,T)$ will be used subsequently for optimal order determination.

Some available methods to specify system order are the AIC (Akaike, 1974), the BIC (Schwarz, 1978) the Predictive Least Squares Principle (PLS) (Hemerly et al. 1989), and the Feedback Control System Information Criterion (CIC) (Chen et al. 1990):

$$AIC(m,n,T) = \log \hat{\sigma}^2(m,n,T) + \frac{2(m+n)}{T}, \quad (20)$$

$$BIC(m,n,T) = \log \hat{\sigma}^2(m,n,T) + \frac{(m+n)\log T}{T}, \quad (21)$$

$$PLS(m,n,T) = \hat{\sigma}^2(m,n,T), \quad (22)$$

$$CIC(m,n,T) = \sum_{m+n+1}^{T} \xi^2(m,n,t) + (m+n)(\log T)^2. \quad (23)$$

For our model, the AIC is not recommended since the consistency feature of the AIC cannot be guaranteed (Kashyap et al. 1980). For the CIC, if the magnitude of the error accumulated is much smaller compared with the second term, the variation tendency of the CIC would be obliterated as the second term plays a decisive role, which leads to failure to determine optimal system order. Such phenomena arise when model coefficients are determined very accurately by the RLS at the initial computation stage, thus preventing finding optimal system order. Meanwhile, the CIC also suffers from sufficient information on initial system order, which is almost inaccessible in ship motion prediciton.

It follows from the strong consistency of the BIC that the unique system order can be obtained when the BIC value reaches minimum. Our model requires the joint determination of m and n. For every given input order $n \leq n_{max}$, the BIC value changes convexly. Thus, the minimum BIC value corresponds to optimal output order for a given input order, which results in the difficulty of selecting the desired system order in the global sense. In our case, selection of the optimal pairs (m^*, n^*) should include a tradeoff among prediction ability, accumulated prediction error, and model complexity.

In ship motion estimation problem, our main concern is the prediction capability. Meanwhile, the accumulated prediction error and identification model complexity should be considered.

The following three important aspects should be analysed:

1) How can ordered pairs $(m,n), m \in V_1, n \in V_2$ be determined to maximize the prediction horizon?

2) How to reduce the model complexity to reduce the computational burden?

3) How can the prediction error accumulated be contained within the acceptable range?

Regarding the first question, a tradeoff should be achieved between the seemingly incompatible aspects. When recursive prediction models are considered, prediction capability should come first. Our main purpose is to increase prediction horizon, as large as possble, with acceptable prediction error. The proposed selection principle begins with computing the candidate output order series

$$m_i^* = arg\{min(BIC(j,i,T))\},\ j = 1,\ldots,m_{max} \text{ for every } i = 1,\ldots,n_{max}, \quad (24)$$

then it selects the largest output order m^* in the candidate output order series

$$m^* = max\{m_i^*\}. \quad (25)$$

For the m^*, there usually exist several input order $n_1, n_2, \ldots, n_r, n_r \leq n_{max}$. One possible method is to select optimal input order n^* such that

$$n^* = arg\{min(\frac{m^* + n_k}{m^*})\}, k = 1,2,\ldots,r. \quad (26)$$

Equation (26) seeks to reduce the model complexity in consideration of long-term prediction requirement, i.e., the model with the smallest system order while achieving satisfactory prediction ability is obtained.

After optimal system order is determined, we would like to check the accumulated prediction error. The simulation results in Section 4 and 5 demonstrate that the accumulated prediction error calculated from our algorithm is acceptable and indicates that the proposed procedure is suitable for ship motion estimation.

3. SHIP MOTION PREDICTION ALGORITHM

After the optimal output order m^*, input order n^* and corresponding coefficients of the model are calculated from the RLS, the efforts will be focused on the prediction of ship motion dynamics.

Suppose the prediction step is L . Rewrite our model as follows

$$[1 - A(q^{-1})]y(t) = B(q^{-1})u(t) + e(t), \tag{27}$$

and notice the following identity (Wittenmark 1974) still holds in our case

$$F(q^{-1})[1 - A(q^{-1})] + q^{-L}G(q^{-1}) = 1. \tag{28}$$

Here

$$F(q^{-1}) = \sum_{i=0}^{L-1} f_i q^{-i},\ f_i = \sum_{j=0}^{i-1} f_j a_{(m^*,i-j)},$$
$$f_0 = 1,\ i = 1, \ldots, L-1, \tag{29}$$

$$G(q^{-1}) = \sum_{i=0}^{m^*-1} g_i q^{-i},\ g_i = \sum_{j=0}^{L-1} f_j a_{(m^*,i+L-j)},$$
$$i = 0, \ldots, m^* - 1. \tag{30}$$

It follows from (27)-(30) that (Wittenmark 1974)

$$y(t+L) = F(q^{-1})e(t+L) + G(q^{-1})\xi(t) + G(q^{-1})\hat{y}(t|t-L)$$
$$+B(q^{-1})F(q^{-1})u(t+L), \tag{31}$$

here $\hat{y}(t|t-L)$ is the estimated value of $y(t)$ based on the measured data up to time $t-L$.

Since

$$y(t+L) = \xi(t+L) + [1 - A(q^{-1})]F(q^{-1})\hat{y}(t+L|t),$$
$$+q^{-L}G(q^{-1})\hat{y}(t+L|t). \tag{32}$$

Combing (31) and (32), we obtain

$$\xi(t+L) = G(q^{-1})\xi(t) - [1 - A(q^{-1})]F(q^{-1})\hat{y}(t+L|t)$$
$$+B(q^{-1})F(q^{-1})u(t+L) + F(q^{-1})e(t+L). \tag{33}$$

The prediction error covariance

$$V = E\{\xi(m,n,t)^2\} \tag{34}$$

is minimized if the predictor is chosen as

$$\hat{y}(t+L|t) = A(q^{-1})\hat{y}(t+L|t) + \frac{G(q^{-1})\xi(t)}{F(q^{-1})}$$
$$+B(q^{-1})u(t+L). \tag{35}$$

In our case, the inputs are assumed to be the measured data, then (35) can be modified to

$$\hat{y}(t+L|t) = A(q^{-1})\hat{y}(t+L|t) + \frac{G(q^{-1})\xi(t)}{F(q^{-1})}$$
$$+B(q^{-1})y(t) \tag{36}$$

If we wish to predict further, (36) can be transformed into the following formula

$$\hat{y}(t+L|t) = A(q^{-1})\hat{y}(t+L|t) + B(q^{-1})y(t) \tag{37}$$

4. SIMULATION RESULTS

The performance of the proposed predictor is demonstrated in this section. The ship motion data were generated from the FREYDYN 8.0 software package for an 8,500-ton LPA class amphibious platform. For maritime flight operations, aircrafts are prone to jounce on the ship deck over high sea states when the deck pitches substantially, which necessitates an accurate predictor to assist in a safe landing trajectory design in case of emergency. Hence, we focused on the prediction of pitch motion here. The pitch motion data were sampled at every 0.25s at sea state 3 which had a typical wave height of 1m.

The data were divided into two segments: the first group of N_T points were used for training and another of N_P points as a test. We chose N_T and N_P large enough in the sense that N_T points could capture pitch motion feature and N_P could be utilized for testing. We chose $N_T = 500, 1000, 1500, 2000$ for training, and every time $N_P = N_T - L$ points with combination of white noise to check the prediction results. Numerous simulations were carried out, and the process of determining system order is shown in Fig. 2 for N_T=1000. The predicted and the true pitch motion data versus time with N_P=980 are plotted in Fig. 3 (20-step-ahead), and with N_P=970 in Fig. 4 (30-step-ahead). The solid lines correspond to the true motion data, and the dashed lines are the predicted.

It is seen that the prediction results produced by the proposed algorithm matches pretty well with the true pitch motion data, and the lead phase margin is 107.85 degree in Fig. 3, and 81.34 degree in Fig. 4. With the increase in prediction points, the prediction error for posterior points is not necessarily worse than previous ones.

5. COMPARATIVE STUDIES

To test the validity of our method, we compared the our algorithm with other conventional predictors. A brief description of those predictors is listed below.

5.1 Order-predefined predictor

This comparison aimed to check the performance of the proposed order determination method. From the classical control viewpoint, it is usually preferred to choose a phase-lead network with small system orders, and here a second-order predictor was adopted (Matt Garratt et al. 2007)

$$\frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}. \tag{38}$$

5.2 AutoRegressive model predictor (AR)

Based on the previous measured data, the forecasts of an AR process with system order p can be obtained (Hamilton 1994) by iterating on

$$\hat{y}(t+j|t) = a_1\hat{y}(t+j-1|t) + \cdots + a_p\hat{y}(t+j-p|t) \tag{39}$$

for $j = 1, \ldots, L$. The key to prediction is to define the system order p . To avoid the inconsistency of the AIC, the BIC is used. Several predictors are required with the first one producing a one-step-ahead prediction, the second one producing a two-step-ahead, so on and so forth.

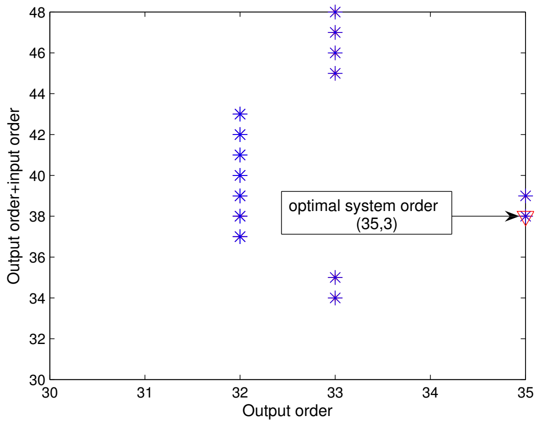

Fig. 2. Order selection for NT=1000

5.3 Performance comparison among three predictors

In this investigation, we used N_T points to obtain system order, and another group of N_P points to check prediction results. Besides, a zero-mean Gaussian random noise was added to pitch data in order to represent the sea wave dynamics. The peak amplitude percentage rate of the white noise to the measured data is 10%. The mean squared prediction error Φ was employed to measure the overall performance:

$$\Phi = \frac{1}{N_P} \sum_{i=T+1}^{T+N_P} [y(i) - \hat{y}(i)]^2. \tag{40}$$

The maximum prediction error for N_P points was evaluated by

$$\Psi = max|y(i) - \hat{y}(i)|, \tag{41}$$

where $y(i)$ and $\hat{y}(i)$ were the true and the predicted data.

To check the variations of Φ, we employed the index $20 \log_{10} \frac{\sqrt{\Phi}}{|y_{max}|}$. As is shown is Fig. 5, the index remains less than -20dB until 25 steps, i.e., the prediction error within 10% of the true data can be obtained up to 25 steps ahead. This is assumed to be acceptable in the considered application.

Table 1 summarizes the experimental results on the Φ and Ψ of three predictors, each taking four groups of N_P points and predicting 20 and 40 steps ahead, respectively. For 20-step-ahead prediction, the proposed algorithm gives consistently acceptable performance even when N_P is much larger, whereas the order-predefined predictor and AR predictors produce greater Φ. The order-predefined predictor and AR predictors suffer from much inaccuracy when we predict 40 steps. For 40-step-prediction, our algorithm predicts with acceptable Φ while producing larger Ψ, which indicates the new method sacrifices Ψ to compensate for overall performance. Fortunately, there is a limited number of such points, and general trends of motion can be captured. Since our algorithm focuses on prediction capability, it cannot always achieve the smallest accumulated prediction error. However, the Φ is within a relatively acceptable range.

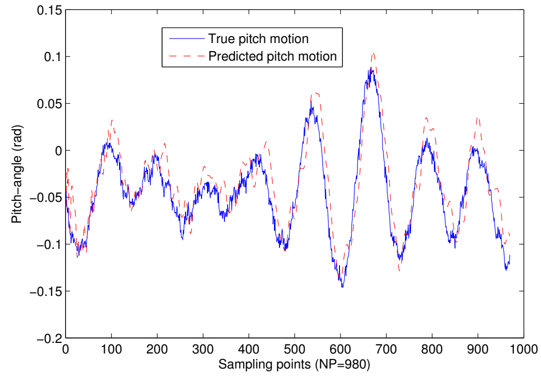

Fig. 3. Pitch motion prediction (20-step-ahead)

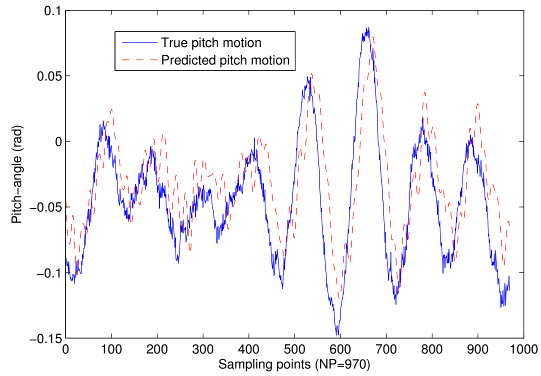

Fig. 4. Pitch motion prediction (30-step-ahead)

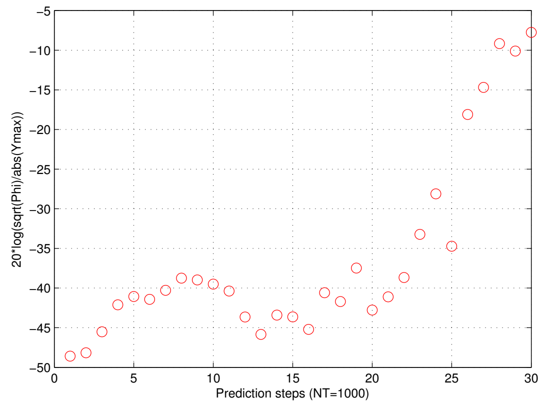

Fig. 5. Accumulated prediction error for different prediction steps

6. CONCLUSION

In this paper we concentrate on building an estimation model for ship motion dynamics. A feasible principle is addressed to solve the problem of system order selection. Based on determination of optimal system order and associated coefficients, a multi-step self-tuning predictor is employed for prediction. Simulation results demonstrate that the proposed prediction approach exhibits satisfactory performance. Furthermore, the proposed procedure facilitates the accurate prediction of ship motion dynamics for use in ship-helicopter flight operations. Future work will be aimed at increasing prediction precision when more prediction steps are expected in high sea states.

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Table 1: Performance comparison for three predictors

Prediction methods	NP	System Orders (m, n)	20-step-ahead		40-step-ahead	
			Φ	Ψ	Φ	Ψ
The proposed predictor	500	(18,2)	7.8368 *E-5	0.06831	0.0012	0.1487
	1000	(35,3)	1.3523 *E-6	0.06614	6.6912 *E-4	0.3652
	1500	(36,1)	4.2983 *E-4	0.06548	1.3672 *E-4	0.1449
	2000	(39,2)	4.5836 *E-4	0.06897	6.5211 *E-4	0.1339
Order-predefined predictor	500	(2,3)	3.2252 *E-5	0.09133	0.00136	0.1484
	1000	(2,3)	5.5225 *E-5	0.07173	0.00659	0.1215
	1500	(2,3)	8.0312 *E-4	0.0767	1.0652 *E-4	0.1365
	2000	(2,3)	0.00214	0.08329	0.0023	0.1460
AR predictor	500	5	5.4278 *E-4	0.0714	0.0012	0.1432
	1000	7	1.7516 *E-4	0.0645	0.0154	0.1298
	1500	8	8.3272 *E-4	0.0698	3.0945 *E-4	0.1315
	2000	6	0.0026	0.0722	0.0021	01456