Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

PI Tracking Control with Mixed H_2 and H_∞ Performance of Descriptor Time Delay System for Output PDFs Based on B-Spline Neural Networks

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Abstract: This paper presents a robust PI tracking control strategy with mixed H_2 and H_∞ performance for general non-Gaussian systems based on the square root B-spline model for the probability density functions (PDFs).The main objective is to design a generalized proportional-integral (PI) control strategy such that the PDF can follow a target one with the enhanced robustness. Different form the previous models, a descriptor time delay system model based on square root B-spline approximation is first established. To enhance the robust performance for the tracking problem, the mixed H_2 and H_∞ performance is applied instead of the only H_∞ performance. The novel mixed H_2 and H_∞ tracking problem is formulated as a optimization problem. Instead of the non-convex design algorithms, the improved linear-matrix-inequality (LMI) based convex algorithms are also proved for controller design. Furthermore, simulations on particle distribution control problems are given to demonstrate the efficiency of the proposed approach and encouraging results have obtained.
Keyword: probability density function; non-Gaussian stochastic systems; PI controller; B-spline expansion; mixed H_2 and H_∞ performance

1. INTRODUCTION

Stochastic tracking control has been received much attention for Gaussian systems over the past decades. However, for stochastic systems with non-Gaussian variables, the classical approaches may not be able to cover the requirement of closed loop control, where only the output mean and covariance are controlled. Recently, probability density function (PDF) control (or stochastic distribution control) methods has been proposed for general stochastic systems with non-Gaussian variables, where the control objective focused on the shape control of output PDF rather than its mean and variance (Wang, 2000). In order to provide realizable PDF control methods, recently B-spline expansions (Wang, 1999) have been introduced for the output PDF modeling and controller design problem in both theoretical studies and practical applications (Wang, 1999,2000; Guo & Wang, 2003,2004;Yue,Leprand,A.J.A &Wang, 2005). It is shown that linear B-spline NN models result in positive constraints and square root B-spline NN models lead to episode constrains.In (InYue,Leprand,A.J.A & Wang, 2005) , stochastic distribution control on descriptor systems has been discussed. However, it is shown that only numeral optimization algorithms were given. Especially, the positive constrain cannot be eliminated by using the descriptor time delay systems.

Descriptor systems describe a broad class of systems which are not only of theoretical interest but also have great practical significance. Models consist of differential equations and additional algebraic equations. In the past two decades, there are quite a few studies related to the robust PI tracking control of stochastic descriptor systems (Guo & Wang, 2003,2005). To meet the requirement of the weighting tracking, the H_∞ tracking control theory is used to formulate the disturbance attenuation. It is noted that the results also have the independent significance in robust PI tracking control fields.

In this paper, a new design framework is established for the NN-based approaches of PDF tracking theory. The square root B-spline NN models are adopted for the output PDF approximations. Since descriptor system and time delay are frequently encountered in many practical processes, it is shown that the weighting errors dynamics can be modeled via a descriptor time delay system without any constrains based on the generalized PI control model and the characteristics of the output PDFs. Consequently, PDF tracking can be transformed into the weighting tracking subject for stochastic system. Furthermore, the generalized PI tracking control and the mixed H_2 and H_∞ robust control will be presented for the weighting tracking control problem for descriptor time delay system which is established to describe the relationship between the weighting errors and control input. Here, the objective is to find the gains of the PI controllers such that

- the closed-loop descriptor time delay system is admissible;
- the weight dynamics can follow the desired set of weight;
- the constraint can be guaranteed;
- the disturbance can be attenuated for the tracking error.

In the following, if not otherwise stated, matrices are supposed to have compatible dimensions. The identity and zero matrices are denoted by I and 0, respectively, with appropriate dimensions. For a symmetric matrix M, the notations $M > (\geq)0$ are used to denote that is positive definite (positive semi-definite). The case $M < (\leq)0$ for follows similarly. For a vector $v(t)$, it is denoted that $\|v(t)\|_\infty := sup_{t\geq 0}\sqrt{v(t)^T v(t)}$.

The rest of this paper is organized as follows. In Section 2, the model of problem is formulated. Firstly , we approximate the known PDFs through the square root B-spline expansion technique. Establishing the weighting dynamical model with the exogenous using identification processes or neural network modeling processes ,and with such a model, the corresponding PI control schemes to descriptor time delay systems are studied in Section 3. In the meantime, the PI tracking controller design method is presented based on LMIs. Finally, simulations are given to demonstrate the feasibility of the results in Section 4 and the main results are concluded in Section 5.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 Model Representation

For some general stochastic systems, the control objective turns to the shape control of the conditional output PDFs, rather than the output mean and variance. In order to simplify the modeling and control methods, square root B-spline expansions have been adopted to model the measured output PDFs so that the PDF control problem can be reduced to the classical control problem for dynamical weights error.

it is supposed that $u(t) \in R^m$ is the control input and $z(t) \in [a, b]$ is the system output. Then, the probability of output $z(t)$ lying inside $[a, \sigma]$ can be described as

$$P(a \leq z(t) < \sigma, u(t)) = \int_a^\sigma \gamma(y, u(t))dy$$

where $\gamma(y, u(t))$ is the PDF of the stochastic variable $z(t)$ under control input $u(t)$.

To avoid the complex computation involved in partial differential equations and provide crisp control strategies, the linear B-spline approximation has been presented (Wang, 2000), where $\gamma(y, u(t))$ can be represented by

$$\sqrt{\gamma(z, u(t))} = B(y)V(t) \tag{1}$$

where

$$B(y) = [b_1(y)\cdots\ b_{n-1}(y)\ \ b_n(y)]$$
$$V(t) = [v_1\ \cdots\ v_{n-1}\ \ v_n]^T \tag{2}$$

$b_i(y)(i = 1, \cdots, n-1, n)$ are pre-specified basis function defined on $y \in [a, b]$, $v_i(u(t) := v_i(t)(i = 1, \cdots, n-1, n)$ are the weights of the such an expansion.

Corresponding to (1), a given desired PDF $g(y)$ can also be expressed by

$$\sqrt{g(y)} = B(y)Vg \tag{3}$$

where Vg is the desired weight vector corresponding to the same group of $b_i(y)(i = 1, \cdots, n-1, n)$.The purpose of the controller design is to find $u(t)$ so that $\gamma(y, u(t))$ can follow $g(y)$. The error between the output PDF and the target one can be formulated as

$$e(y, t) = \sqrt{g(y)} - \sqrt{\gamma(y, u(t))} = B(y)E(t) \tag{4}$$

which is a function of both $y \in [a, b]$ and the time instant t, where the weight error vector is defined as

$$E(t) = Vg - V(t)$$

where $E(t) = [e_1(t)\cdots\ e_{n-1}(t)\ \ e_n(t)]^T = [E_1(t)\ \ e_n(t)]^T$.

After the basis functions are determined, it is noted that only $n-1$ weight errors are independent due to constraint

$$\int_a^b e(y, t)dy = \int_a^b B(y)E(t)dy = \sum_{i=1}^n e_i(t)\int_a^b b_i(y)dy = 0 \tag{5}$$

In this case, equation (5) can be rewritten as

$$e_n(t) = -\sum_{i=1}^{n-1}\frac{b_i}{b_n}e_i(t) = A_1E_1(t) \tag{6}$$

where it can be supposed that $\int_a^b b_n(y)dy := b_n \neq 0$. And , equation (6) is the constraints resulting form the B-spline expansions for the output PDFs.

Remark1. Based on the continuity theory of functions, it is noted that$e(y, t) \to 0$, if and only if $E(t) \to 0$. Thus, the output PDFs tracking problem can be transformed to the weights tracking problem.

3. ROBUST PI CONTROLLER DESIGN

3.1 Descriptor Time Delay Model Based on Weighting Error Model

We have transformed the output PDFs tracking problem into the weights tracking problem and presented the weight error constraints (6) based on the square root B-spline expansions. The next step is establish the dynamic model between the control input and the weight errors, this procedure has been widely used in PDF control and entropy optimization problem. This procedure can be carried out by the corresponding identification processes in (Wang, 2000),or a neural network modeling process in (Guo & Wang, 2004). In this paper, we adopt the following time delay model with exogenous disturbances

$$\dot{E}_1(t) = A_0E_1(t) + A_{d0}E_1(t-d) + B_{01}w(t) + B_{02}u(t) \tag{7}$$

where $E_1(t) = [e_1\ \cdots\ e_{n-1}]^T$ is the independent weight errors denoted in (5). $u(t) \in R^q$ and $w(t) \in R^m$ represent the control and the exogenous input , respectively. Constant square matrices A_0 andA_{d0}, and constant matrices B_{01} and B_02 have compatible dimension.

Based on weighting error system (7) and the constraint (6), we introduce a new state variable

$$x(t) := [E_1^T(t)\ \ (\int_0^t E_1(\tau)d\tau)^T\ \ e_n(t)]^T$$

Then the weighting error model can be transformed into an equivalent descriptor form(Xu & Lam, 2006; Kim, 2001)

$$\Sigma_f : \begin{cases} E\dot{x}(t) = Ax(t) + A_dx(t-d) + B_1w(t) + B_2u(t) \\ z(t) = Cx(t) + C_dx(t-d) + Dw(t) \\ z_0(t) = C_0x(t) + C_{0d}x(t-d) \\ x(t) = \phi(t), t \in [-d, 0] \end{cases} \tag{8}$$

where

$$E = \begin{bmatrix} I_{n-1} & 0 & 0 \\ 0 & I_{n-1} & 0 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} A_0 & 0 & 0 \\ I_{n-1} & 0 & 0 \\ A_1 & 0 & -1 \end{bmatrix}$$

$$A_d = \begin{bmatrix} A_{d0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_i = \begin{bmatrix} B_{0i} \\ 0 \\ 0 \end{bmatrix}, i = 1, 2$$

In(8) $\phi(t)$ is a continuous vector function. $d(t)$ is the time delay satisfying $\dot{d}(t) \leq \beta < 1$. $x(t-d)$ represent the delayed state. In addition, $z_0(t) \in R^{p_0}$ and $z(t) \in R^p$ represent the reference outputs involved in the H_2 and H_∞ performance measure respectively.

Remark2. Based on the continuity theory of functions, it is noted that $e(y,t) \rightarrow 0$, if and only if $x(t) \rightarrow 0$. As a result, after establishing dynamic models (1) and (8) which combines the output PDFs with the control input through the weight error vector, a new robust tracking performance problem is investigated for the descriptor systems with the exogenous disturbances .

Remark3. Compared with tracking control to Gaussian systems, the output PDFs tracking control can consider non-Gaussian variables.Moreover, compared with other tracking control to the output PDFs, the constraints(Wang, 2000; Guo & Wang, 2003,2004) resulting form the B-spline expansions for the output PDFs can be guaranteed through PI tracking control for the descriptor time delay system.

3.2 PI Controller Structure

At this stage, the considered PDF control problem can be formulated into the tracking problem for the above weight errors system, and the control objective is to find such that the tracking performance, disturbance attenuation performance, state constraints and admissible are guaranteed simultaneously.

Proportional-integral (PI) and Proportional-integral-diffe- rential(PID) control has been widely applied in engineering (Guo & Wang, 2004,2005;Lin, Wang & Lee, 2004). Some recent developments for PI or PID control methodologies in state space form can been in (Lin, Wang & Lee, 2004). However, the above results cannot be applied to the considered PDF tracking problem with the state weighting constraints. New design methods are required to deal with the corresponding generalized PI controller under the state.

The classical PI control is unavailable for PDF tracking since$e(y,t)$ cannot be used for feedback. As a result, we adopt the following generalized PI control structure

$$u(t) = K_P E_1(t) + K_I \int_0^t E_1(\tau)d\tau \tag{9}$$

where K_P and K_I are control gains to be determined.

With such an augmented descriptor time delay system(8), tracking problem can be further reduced to a admissible framework because the PI controller described by (9) can be formulated by

$$u(t) = Kx(t), K = [K_P \;\; K_I \;\; 0] \tag{10}$$

The control objective is to find robust PI controllers (9)such that the closed-loop systems is asymptotically stable which can achieve the output PDFs tracking control and the disturbance can be restrained for the descriptor time delay system(8).

3.3 Mixed H_2 and H_∞ Optimization Performance for Descriptor time delay System

It has been shown that tracking control can be solved via H_∞ setting.However, few tracking results have been provided for the descriptor time delay systems. In this part, a new mixed H_2 and H_∞ optimization formulation will be applied to the above weighting tracking problem. The H_2 and H_∞ optimization measure can be described as follows respectively(Wang ,Zhang & Feng, 2006).

definition 1. The H_2 **performance measure** for Σ_f is defined as $J_2 := \|z_0(t)\|_2^2$.

As stated above, in most existing standard H_∞ control for delay system, $\phi(t) = 0$ is assumed for $t \in [-d, 0]$. In order to apply the H_∞ performance measure for the non-zero initial condition, the following generalized measure is presented for Σ_f.

definition 2. The H_∞ **performance measure** for Σ_f is defined as

$$J_\infty(\gamma, P_0, S_0) := (\|z_0(t)\|_2^2 - \gamma^2\|w\|_2^2) - \delta(P_0, S_0) \tag{11}$$

where

$$\delta(P_0, S_0) := \phi^T(0)P_0\phi(0) + \int_{-d}^0 \phi^T(\tau)S_0\phi(\tau)d\tau \tag{12}$$

$\gamma > 0$ is a scalar and $P_0 > 0$, $S_0 > 0$are weighting matrices.

Denote $sym(A) := A + A^T$ and $MM^T := \int_{-d(0)}^0 \phi^T(\tau)\phi(\tau)d\tau$. For finding an robust PI controller(10) ,we give the mixed H_2 and H_∞ performance of descriptor time delay system as the following Theorem.

Theorem 1. For Σ_f with scalars $\gamma > 0$, if the following problem

$$\delta_0 := min\{\phi^T(0)P\phi(0) + \sum_{i=1}^N tr(M^T S M)\}$$

subject to

$$E^T P = P^T E \geq 0 \tag{13}$$

$$\Phi := \begin{bmatrix} sym(A^T P) + S & P^T A_d & P^T B_1 & C^T & C_0^T \\ A_d^T P & -(1-\beta)S & 0 & C_d^T & C_{0d}^T \\ B_1^T P & 0 & -\gamma^2 I & D^T & 0 \\ C & C_d & D & -I & 0 \\ C_0 & C_{0d} & 0 & 0 & -I \end{bmatrix} < 0 \tag{14}$$

is feasible with respect to P and $S > 0$, then Σ_f is admissible and satisfies

$$J_\infty(\gamma, P, S) < 0, J_2 \leq \delta(P, S) \leq \delta_0 \tag{15}$$

where $J_\infty(\gamma, P, S)$and $\delta(P, S)$ are denoted by (11)and (12) respectively.

Proof. For Σ_f, there exist two invertible matrices $U \in R^{(2n-1)\times(2n-1)}$ and $V \in R^{(2n-1)\times(2n-1)}$ such that

$$\bar{E} := UEV = \begin{bmatrix} I(2n-2) & 0 \\ 0 & 0 \end{bmatrix}, \bar{A} := UAV = \begin{bmatrix} A_{11} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\bar{A_d} := UA_dV = \begin{bmatrix} A_{d11} & 0 \\ 0 & 0 \end{bmatrix} \tag{16}$$

$$\bar{P} := U^{-T}PV = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, \bar{S} := V^T S V = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} \tag{17}$$

where,

$$U = \begin{bmatrix} I_{2n-1} & 0 \\ 0 & 1 \end{bmatrix}, V = \begin{bmatrix} I_{2n-1} & 0 \\ V_{21} & -1 \end{bmatrix}$$

$$V_{21} = [A_1 \;\; 0], A_{11} = \begin{bmatrix} A_0 & 0 \\ I_{n-1} & 0 \end{bmatrix}, A_{d11} = \begin{bmatrix} A_{0d} & 0 \\ 0 & 0 \end{bmatrix}$$

Therefore, Σ_f is regular and impulse-free.

Then, from (14) and (15), and the notations in (16) and (17), we have

$$\bar{E}^T\bar{P} = \bar{P}^T\bar{E} \geq 0 \tag{18}$$

$$\begin{bmatrix} sym(\bar{A}^T\bar{P}) + \bar{S} & \bar{P}^T\bar{A}_d \\ \bar{A}_d^T\bar{P} & -(1-\beta)\bar{S} \end{bmatrix} < 0 \tag{19}$$

Noting the expression of $\bar{E}$ in (16) and using (18), we can deduce that$P_{11} = P_{11}^T \geq 0$ and P_{12}. Therefore, $\bar{P}$ reduces to

$$\bar{P} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{20}$$

Now, substituting (16),(17) and (20) into (19), we obtain

$$\begin{bmatrix} \Psi_1 & P_{21}^T S_{12} & P_{11}^T A_{d11} & 0 \\ P_{21} + S_{12}^T & P_{22} + P_{22}^T + S_{22} & 0 & 0 \\ A_{d11}^T P_{11} & 0 & -(1-\beta)S_{11} & -(1-\beta)S_{12} \\ 0 & 0 & -(1-\beta)S_{12}^T & -(1-\beta)S_{22} \end{bmatrix} < 0 \tag{21}$$

where $\Psi_1 := sym(A_{11}^T P_{11}) + S_{11}$

Form (21), we can get

$$\Phi_{11} = \begin{bmatrix} sym(A_{11}^T P_{11}) + S_{11} & P_{11}^T A_{d11} \\ A_{d11}^T P_{11} & -(1-\beta)S_{11} \end{bmatrix} < 0 \tag{22}$$

Now, let $V^{-1}x(t) = [x_1(t)^T \ \ x_2(t)^T]$, $x_1(t) \in R^{2n-1}$, $x_2(t) \in R$. Using the expressions in (16) and (17), the singular delay system in (8) can be rewritten as

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{d11}x_1(t-d) \tag{23}$$

$$0 = x_2(t) \tag{24}$$

Then, it is easy to see that the stability of the singular delay system in (8) is equivalent to that of the system in (23) and (24).Obviously,system (24) is stable.

In view of this, next we shall prove that the system in (23) is stable. Define a Lyapunov function candidate as

$$V_1(x_1(t), t) = (x_1(t))^T P_{11}x_1(t) + \int_{t-d(t)}^{t} (x_1^T(\tau)S_{11}x_1(\tau)d\tau \tag{25}$$

For $P_{11} = P_{11}^T \geq 0$, $S_{11} > 0$,we can get $V_1(x_1(t), t) > 0$.

$$\begin{aligned} \dot{V}_1(x_1(t), t) &= \dot{x}_1(t)^T P_{11}x_1(t) + x_1(t)^T P_{11}^T \dot{x}_1(t) + x_1(t)^T S_{11}x(t) \\ &\quad -(1-\dot{d}(t))x_1^T(t-d)S_{11}x_1(t-d) \\ &\leq x_1(t)^T (A_{11}^T P_{11} + P_{11}^T A_{11} + x_1^T(t-d)A_{d11}x_1(t) \\ &= \varsigma_1^T(t)\Phi_{11}\varsigma_1(t) \end{aligned}$$

where $\varsigma_1^T(t) := [x_1^T(t) \ \ x_1^T(t-d)]$.

Because of $\Phi_{11} < 0$ and for any $\varsigma_1(x) \neq 0$, we obtain

$$\dot{V}_1(x_1(t), t) \leq \varsigma_1^T(t)\Phi_{11}\varsigma_1(t) < 0$$

Then, system (23) is stable.Therefore, Σ_f is regular, impulse-free and stable.It follows that Σ_f is admissible in the absence of the exogenous input.

Next we consider the performance to be optimized for Σ_f. Define a Lyapunov function candidate as

$$V(x(t), t) = (Ex(t))^T Px(t) + \int_{t-d(t)}^{t} (x^T(\tau)Sx(\tau)d\tau \tag{26}$$

By using schur complement formula on the last column and row, (14) is equivalent to

$$\Phi' < 0 \tag{27}$$

where

$$\Phi' := \begin{bmatrix} sym(A^T P) + S + C_0^T C_0 & * & * & * \\ A_d^T P + C_{0d}^T C_0 & -(1-\beta)S + C_{0d}^T C_{0d} & * & * \\ B^T P & 0 & -\gamma^2 I & * \\ C & C_d & D & -I \end{bmatrix}$$

Form the first, second and the fifth columns and rows of the left matrix in inequality (27) it can be seen that

$$\Phi_1 := \begin{bmatrix} sym(A^T P) + S + C_0^T C_0 & P^T A_d + C_0^T C_{0d} \\ A_d^T P + C_{0d}^T C_0 & -(1-\beta)S + C_{0d}^T C_{0d} \end{bmatrix} < 0 \tag{28}$$

Also, it is noted that

$$\begin{bmatrix} -\gamma^2 I & D^T \\ D & -I \end{bmatrix} < 0$$

also holds based on (27) as a static condition.

Along the solution of Σ_f in the absence of $w(t)$ and $u(t)$ based on (13), it can be shown that

$$\begin{aligned} \dot{V}(x(t), t) &= \dot{x}(t)^T E^T Px(t) + x(t)^T P^T E\dot{x}(t) \\ &\quad +x(t)^T Sx(t) - (1-\dot{d}(t))x^T(t-d)Sx(t-d) \\ &\leq x(t)^T (A^T P + P^T A + S)x(t) + X^T(t-d)A_d Px(t) \\ &\quad +x(t)^T P^T A_d x(t-d) - (1-\beta)x^T(t-d)Sx(t-d) \\ &= \varsigma^T(t)(\Phi_1 - \Gamma^T\Gamma)\varsigma(t) \end{aligned}$$

where $\Gamma := [C_0 \ \ C_{0d}]$ and $\varsigma^T(t) := [x^T(t) \ \ x^T(t-d)]$.

When $\Phi_1 < 0$ holds, it is seen that for any $\varsigma(t) \neq 0$, we have $\dot{V}(x(t), t) \leq \varsigma^T(t)(\Phi_1 - \Gamma^T\Gamma)\varsigma(t) < -\varsigma^T(t)(\Gamma^T\Gamma)\varsigma(t) = -z_0^T z_0 \leq 0$.

Denote two auxiliary functions as follows

$$J_0 := z_0^T z_0 + \dot{V}(x(t), t) \tag{29}$$

$$J_1(t) := z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(x(t), t). \tag{30}$$

Following the definition of the H_2 performance measure for the time delay systems, we only consider the case in the absence of $w(t)$. By using of the above formulation of $\dot{V}(x(t), t)$, it can be further verified that $J_0 \leq \varsigma^T(t)\Phi_1\varsigma(t)$, where Φ_1 is denoted in (28).

In the presence of $w(t)$ and in the absence of $u(t)$, taking the derivation of the function $V(x(t), t)$ along the solution of Σ_f yields that

$$\begin{aligned} \dot{V}(x(t), t) &\leq x(t)^T (sym(A^T P) + S)x(t) + sym(x(t)^T P^T B_1 w(t)) \\ &\quad +sym(x(t)^T P^T A_d x(t-d)) - (1-\beta)x^T(t-d)Sx(t-d) \end{aligned}$$

with which it can be verified that $\theta^T(t)\Phi_2\theta(t)$, where

$$\Phi_2 := \begin{bmatrix} \Psi & P^T A_d + C^T C_d & P^T B_1 + C^T D \\ A_d^T P + C_d^T C & -(1-\beta)S + C_d^T C_d & C_d^T D \\ B_1^T P + D^T C & D^T C_d & -\gamma^2 I + D^T D \end{bmatrix} \tag{31}$$

where $\Psi := sym(A^T P) + S + C^T C$, $\theta^T(t) := [x^T(t) \ \ x^T(t-d) \ \ w^T(t)]$.

Applying the Schur complement formula again to $\Phi' < 0$ (where Φ' is denoted in (27))yields $\Phi'' < 0$where

$$\Phi'' := \begin{bmatrix} \Psi + C_0^T C_0 & P^T A_d + C^T C_d + C_0^T C_{0d} & P^T B_1 + C^T D \\ * & -(1-\beta)S + C_d^T C_d + C_{0d}^T C_{0d} & C_d^T D \\ * & * & -\gamma^2 I + D^T D \end{bmatrix}$$

which implies that $\Phi_2 < 0$ since $\Phi'' - \Theta^T\Theta = \Phi_2$, where $\Theta := [C_0 \ \ C_{0d} \ \ 0]$. Thus, it can be claimed that both $J_0 < 0$ and $J_1 < 0$ can be guaranteed under the condition $\Phi < 0$.

Because of (26), it is shown that

$$V(x(T)) = (Ex(T))^T Px(T) + \int_{T-d(T)}^{T} (x^T(\tau)Sx(\tau)d\tau \geq 0$$

Integrating J_0 and J_1 form the initial time to ∞ implies that

$$0 \geq \int_0^\infty J_0(\tau)d\tau \geq \|z_0\|_2^2 + \lim_{T\to\infty} V(x(T)) - \delta(P, S) \geq \|z_0\|_2^2 - \delta(P, S) \tag{32}$$

$J_2 \leq \delta(P, S)$ and from (32) we get $J_\infty(\gamma, P, S) < 0$. Q.E.D

3.4 PI Tracking Control

Now,consider the state feedback controller$u(t) = Kx(t)$ for the augmented descriptor system with PI control structure for the originate system as $u(t) = K_P E_1(t) + K_I \int_0^t E_1(\tau)d\tau$, and substituting $u(t) = Kx(t)$ into system (8) , the corresponding closed-loop descriptor time delay system can be described by

$$\begin{cases} E\dot{x}(t) = A_k x(t) + A_d x(t-d) + B_1 w(t) \\ z(t) = Cx(t) + C_d x(t-d) + Dw(t) \\ z_0(t) = C_0 x(t) + C_{0d} x(t-d) \\ x(t) = \phi(t), t \in [-d, 0] \end{cases}, A_k = A + B_2 K \quad (33)$$

Remark4: Given a constant $\gamma > 0$. For the system (8), the state feedback (10) is said to be an PI tracking controller if system (33) is admissible and satisfies $J_\infty(\gamma, P, S) < 0, J_2 \le \delta(P, S) \le \delta_0$.The objects of this paper are to find the necessary and sufficient conditions for the existence of a new mixed H_2 and H_∞ PI tracking controller in terms of LMIs. Then, we have the following PI tracking control result which resolves the output PDFs tracking problem with disturbance attenuation performance.

Theorem 2. Given constants $\gamma > 0$. For the system (33) ,the following statements are equivalent:

(1)There exists a mixed H_2 and H_∞ PI tracking controller described by (10) ;

(2) There exist matrix $W \in R^{q\times 2n-1}$ and nonsingular matrix $X \in R^{2n-1\times 2n-1}$ described by

$$X = \begin{bmatrix} X_1 & 0 \\ X_{21} & X_2 \end{bmatrix} \quad (34)$$

which satisfy $X_{21} \in R^{1\times(2n-2)}$, $X_2 \in R \neq 0$, nonsingular matrix $X_1 \in R^{(2n-2)\times(2n-2)}$ and

$$X_1 < 0 \quad (35)$$

$$\begin{bmatrix} sym(AX + B_2W) + T & A_dX & B_1 & (CX)^T & (C_0X)^T \\ (A_dX)^T & -(1-\beta)T & 0 & (C_0X)^T & (C_{0d}X)^T \\ B^T & 0 & -\gamma^2 I & D^T & 0 \\ CX & C_dX & D & -I & 0 \\ C_0X & C_{0d}X & 0 & 0 & -I \end{bmatrix} < 0 \quad (36)$$

is feasible with respect to P and $S > 0$, then system (33) is admissible and satisfies

$$J_\infty(\gamma, P, S) < 0, J_2 \le \delta(P, S) \le \delta_0$$

where $J_\infty(\gamma, P, S)$ and $\delta(P, S)$are denoted by (11)and (12) respectively. Thus, Construct $K = WX^{-1} = [K_P \ K_I \ 0]$, then we get the mixed H_2 and H_∞ PI tracking control law $u = WX^{-1}x$.

Proof. Similar to the prove of Theorem 1 , closed-loop system (33) is regular and impulse-free. Define $X = P^{-1}$ and $W = KX$, By pre-multiplying $diag\{P^T \ P^T \ I \ I \ I\}$ and post-multiplying $diag\{P \ P \ I \ I \ I\}$ to (36) ,we can get

$$\begin{bmatrix} \Pi & P^T A_d & P^T B_1 & C^T & C_0^T \\ A_d^T P & -(1-\beta)P^T TP & 0 & C_0^T & C_{0d}^T \\ B^T P & 0 & -\gamma^2 I & D^T & 0 \\ C & C_d & D & -I & 0 \\ C_0 & C_{0d} & 0 & 0 & -I \end{bmatrix} < 0$$

where $\Pi = sym((A + B_2K)^T P) + P^T TP$ Form $T = P^{-T} SP^{-1} > 0$ and $A_k = A + B_2K$, we can get the following matrix inequation as

$$\begin{bmatrix} sym((A_k)^T P) + S & P^T A_d & P^T B_1 & C^T & C_0^T \\ A_d^T P & -(1-\beta)S & 0 & C_0^T & C_{0d}^T \\ B^T P & 0 & -\gamma^2 I & D^T & 0 \\ C & C_d & D & -I & 0 \\ C_0 & C_{0d} & 0 & 0 & -I \end{bmatrix} < 0 \quad (37)$$

$X = \begin{bmatrix} X_1 & 0 \\ X_{21} & X_2 \end{bmatrix}$, $X_{21} \in R^{1\times(2n-2)}$, $X_2 \in R \neq 0$, nonsingular matrix $X_1 \in R^{(2n-2)\times(2n-2)}$ and $X_1 < 0$,therefore,

$$\begin{bmatrix} I_{2n-2} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 & X_{12} \\ X_{21} & X_2 \end{bmatrix} = \begin{bmatrix} X_1 & X_{12} \\ X_{21} & X_2 \end{bmatrix}^\tau \begin{bmatrix} I_{2n-2} & 0 \\ 0 & 0 \end{bmatrix} \ge 0$$

$$EX = (EX)^T \quad (38)$$

By pre-multiplying P^T and post-multiplying P to (38) , we obtain (13).

According to Theorem 1,we can get close-loop system (33)is admissible, and satisfies $J_\infty(\gamma, P, S) < 0$,$J_2 \le \delta(P, S) \le \delta_0$ Q.E.D.

4. SIMULATION

It is noted that for an integrated stochastic distribution control problem, the modeling process for the output PDFs is required except the robust PI tracking controller design for the weighting error system. In (Wang, 2000), it has been shown how the neural network modeling for the retention system in paper making systems has been established. This modeling procedure can be achieved for many practical processes (Wang, 2000). However, it is noted that such a procedure for the conditional PDFs of the system output need the corresponding data processing or image processing techniques. Due to limitation of the space, in this section, we only consider a similar example as in (Guo & Wang, 2003) and will give a comparison between the proposed algorithm with the previous ones.

In the particle distribution control problems, the shape of output PDF usually has two or three peaks(Wang, 2000). For a stochastic system with non-Gaussian process, it is supposed that the output PDF can be formulated to be (1) with

$$V(t) = [v_1(t) \ v_2(t) \ v_3(t)]^T, V(0) = [1.1283 \ 2.26 \ 0.385]^T$$

$$b_i = \begin{cases} |\sin 2\pi y|, & y \in [0.5(i-1), \ 0.5i] \\ 0, & y \in [0.5(j-1), \ 0.5j], \ i \neq j \end{cases}$$

The desired weight vector value is set to be

$$V_g = [\frac{\pi}{5} \ \frac{2\pi}{5} \ \frac{3\pi}{5}]^T \quad (39)$$

corresponding to the desired PDF.

In this context, the dynamical relations with respective to $x(t)$ and $u(t)$ is described by (8) with the selections

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & -1 \end{bmatrix}, B_1 = [2 \ 0 \ 0]^T, B_2 = [6 \ 0 \ 0]^T, C = [1 \ 0 \ 0],$$

In simulations, for $\gamma = 1.0$, $d(t) = 0.2$ and $A_{d0} = 0.5$, solving (36) together with (34) and (35) yields

$$K_P = -1.0669, K_I = -0.0705 \quad (40)$$

Correspondingly, robust PI tracking controller can be obtained by using Theorem 1. When the H_∞ PI control law is applied, the closed-loop system responses for the dynamical weighting

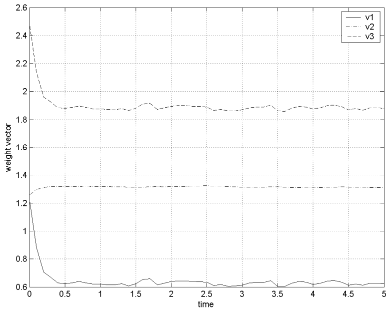

Fig. 1. Response of the dynamical weight vector

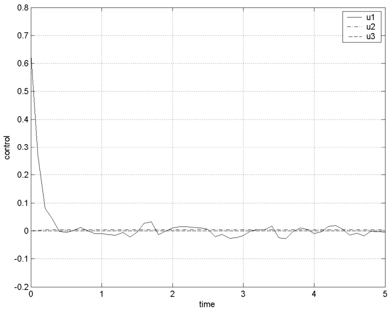

Fig. 2. Gains of the PI tracking Control input of the dynamical system

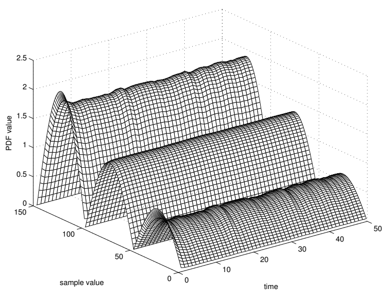

Fig. 3. 3-D-mesh plot of the output function with the robust PDF controller

are shown in Fig.1.The control gains are shown in Fig.2.The practical PDFs for the descriptor uncertain weighting error system and under the proposed robust control strategy is shown in Fig.3. It is demonstrated that the satisfactory tracking performance and robustness are achieved.

5. CONCLUSION

This paper considers the robust tracking problem for the output PDFs of non-Gaussian processes by using the mixed H_2 and H_∞ PI tracking controllers. B-spline NN expansions and descriptor weighting error time delay systems are applied to formulate the tracking problem. Different from the previous related works, descriptor weighting error time delay systems and exogenous disturbances are considered to enhance the robustness, and the constraints of the weighting error vectors are guaranteed by the PI tracking control law. Feasible controller design procedures are provided to guarantee the closed loop admissible and the tracking convergence. Different from the existing results on PDF control, the control strategy proposed in this paper has a simple fixed structure and can guarantee both admissible and robustness of the closed loop system. Simulations are provided to show the effectiveness and advantages of the proposed approach.

6. ACKNOWLEDGMENT

This paper is supported by the Hwaying Education and Culture Foundation, NSFC (60474050) and NCET program. The authors would like to thank Professor L Guo for his assistance for its first version.

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