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Aircraft Airbrakes Compensation Design Using Iterative Inversion

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Abstract: This paper deals with the synthesis of an open-loop control law of a civilian aircraft for the compensation of the pitching moment generated by the extension of airbrakes. The proposed method uses in-flight recorded data and is based on impulse response identification and inverse simulation, whose results are used to design the controller upon qualitative assumptions. Results are then given for both the inversion and the synthesis for different flight cases. The robustness of the method to measurement noise is also assessed.

Keywords: Iterative modelling and control design, Iterative inversion of model, Inverse control design, Landweber iteration, Airbrakes compensation

1. INTRODUCTION

During the development phase of an aircraft, flight control laws are often subject to change, mostly because of the inaccuracy of the models used to design them. Poorly estimated flexible modes and complex aerodynamic effects may limit the representativeness of the model.

If these uncertainties can be bounded, robust control design methods can be used but if these bounds restrain too much the robust performance, more precise models are required. The problem is that it often takes much time to retune and update the models as it needs long and specific flight test campaigns, thus delaying the retuning of the control laws.

An idea would be to develop an alternate and complementary method that uses in-flight recorded data, in order to "fine-tune" directly the control laws so as to react quickly when confronted to these uncertainties.

In this paper, we propose a method based on inverse simulation that generates optimal input signals with respect to given specifications and that synthesizes a controller, or tunes an existing one, able to reproduce these input signals. The method is then tested on the design of an open-loop compensation of the pitching moment generated by the extension of the airbrakes of civilian aircraft.

Usual approaches to inverse simulation of a system, allow to compute input signals that reproduce a desired behaviour, from either real data or specifications. One application of these methods in the aeronautical field is to assess the feasibility of given flight maneuvers, maximal maneuverability, and handling qualities as in Avanzini et al. (1998). It has also been used for the validation of simulation programs by Grünhagen (1992) and for the computation of non measurable data to match in-flight data as in Mouyon and Losser (2002) and Mouyon and Vacher (2001). Similar problems have already been dealt with in Mouyon et al. (2002) and Mouyon and Losser (2002), where the authors used inverse simulation by deconvolution to retune the gains of an autopilot, and in Sentoh and Bryson (1992) and in Boyle and Chamitoff (1999), where inverse simulation was used for the design of control laws.

Section 2 gives a formal view of the problem mentioned previously, a theoretical background for the methods used and deals with several parameterization issues. Section 3 details the application of the method to the airbrakes compensation of a civilian aircraft. Section 4 gives the results obtained on a couple of different cases and gives an insight into the robustness of the method to measurement noise. Finally, a conclusion recalls the main ideas and results of the paper and gives hints for future work.

2. INVERSE CONTROL LAW DESIGN

2.1 Formalization

Let us consider a system Σ, illustrated in Fig. 1, where $w(t)$ is an exogenous input signal, $u(t)$ the command signal, and $z(t)$ the output signal of size p.

In Sentoh and Bryson (1992), the authors survey different methods to perform inverse control and thus formalizes a general design problem which we are going to use and extend.

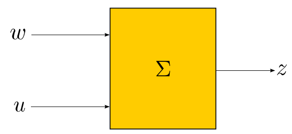

Fig. 1. System to be inverted

Design problem The first objective is to find the input control history $\Delta u^\star(t)$ so that the system Σ (see Fig. 1)

$$\dot{x} = f(x, w, \Delta u^\star), \qquad x(0) = x_0 \tag{1}$$
$$z = h(x, w, \Delta u^\star) \tag{2}$$

has the output history

$$z(t) = z^\star(t) \tag{3}$$

where $z^\star(t)$ is specified.

The aim is now to find and design a control law structure that can reproduce $\Delta u^\star(t)$, using other input and output signals of the system. We have to find a controller K (see Fig. 2)

$$\dot{x}_K = f_K(x_K, w, y), \qquad x_K(0) = x_{K0} \tag{4}$$
$$z_K = h_K(x_K, w, y) \tag{5}$$

such that:

$$z_K(t) = \Delta u^\star(t) \tag{6}$$

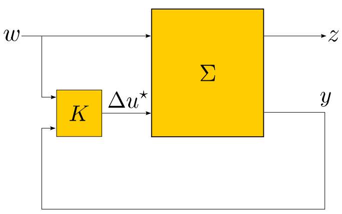

Fig. 2. Controller synthesis

2.2 Inversion by deconvolution

We want the system Σ to follow a given specification when excited by a given signal $w(t)$. For this part, we will consider that the specification is to have a desired response signal $z^\star(t)$ to $w(t)$, by using command signal $u(t)$.

Let $z_o(t)$ be the response of the system when excited by $w(t)$ and with $u(t) \equiv 0$, and $\Delta z(t) = z^\star(t) - z_o(t)$. The idea is now to compute an input signal $\Delta u^\star(t)$ that makes the output signal of Σ be $z^\star(t)$ when excited by $w(t)$.

To invert the system, we chose to use the deconvolution technique. We thus assume that the transfer function from u to z, $\Sigma_{u\to z}$, can be approximated by a finite impulse response (FIR) linear transfer $\widehat{\Sigma}_{u\to z}$, whose impulse response $h(t)$ is of length $r+1$. In order to compute the input of the FIR filter $\widehat{\Sigma}_{u\to z}$, knowing its output $z(t)$, we must identify its impulse response $h(t)$. We may also have to pre-filter the impulse signal with a known FIR filter of impulse response $f(t)$, for engineers in the aeronautical field never use impulse signals but test signals, like finite length steps. If a physically reasonable amplitude is used and the energy controlled by the step length, one may get more accurate results. The impulse response hence identified is $g(t) = h * f(t)$, where $*$ denotes the discrete convolution product, and we define the artificial signal $e(t)$ such that $u(t) = f * e(t)$.

The problem to solve can be written as follows :

Solve in $\Delta u(t)$:

$$\Delta z(t) = g * e(t), \quad \begin{array}{l}\text{knowing } g(t) = h * f(t),\\ \text{and } \Delta u(t) = f * e(t)\end{array} \tag{7}$$

The above problem can be rewritten with vectors and matrices like:

Solve in $\mathbf{u}$:

$$\mathbf{z} = G\mathbf{e}, \quad \begin{array}{l}\text{knowing } G = HF,\\ \text{and } \mathbf{u} = F\mathbf{e}\end{array}$$

This problem is now equivalent to the following least square problem :

Compute:

$$\mathbf{u} = F\mathbf{e}^\star \text{ where } \mathbf{e}^\star = \arg\min_{\mathbf{e}} \mathcal{J}_\lambda(\mathbf{e})$$
$$\text{with } \mathcal{J}_\lambda(\mathbf{e}) = \left\| \begin{pmatrix} \omega_1 \mathbf{z}_1 \\ \vdots \\ \omega_p \mathbf{z}_p \\ 0 \end{pmatrix} - \begin{pmatrix} \omega_1 G_1 \\ \vdots \\ \omega_p G_p \\ \lambda D \end{pmatrix} \mathbf{e} \right\|^2 \tag{8}$$
$$= \|\mathbf{y} - G_\lambda \mathbf{e}\|^2 \tag{9}$$

where the $\{\mathbf{z}_i\}_{i=1}^p$ are the p outputs of the system, with corresponding transfer functions G_i and associated fixed weighting ω_i, λ is the regularization parameter, and D is a regularization operator so as to make the problem well-posed as defined by Tikhonov and Arsenin (1977).

Iterative Inversion Numerous methods exist to solve such a problem, as one can found in Björck (1996). Yet, the problem we wish to solve is particular because of the matrix G, which is a lower triangular Tœplitz matrix. It would be wise to make use of this specificity. More information about Tœplitz matrices can be found in a review by Gray (2006).

Mouyon and Vacher (2001) chose the direct inverse frequency approach where the solution is computed using the fact that G is almost circulant, thus diagonalizable using the discrete Fourier transform. By this mean, the inversion of G is replaced by n scalar division, if G is a $n \times n$ matrix. Alas, due to the fact that G is not exactly circulant, side-effects of size r appear on the computed solution if r is not small enough compared to n, which is likely to happen in our application.

We chose to use an iterative method based on the work of Landweber (1951) and Hanke (1991).

Theorem 1. Let $A : A_1 \to A_2$ be a bounded operator and $\mathbf{b} \in Im(A) \oplus Im(A)^\perp$. The following sequence:

$$\begin{aligned} \mathbf{x}_0 &= 0 \\ \mathbf{x}_{k+1} &= \mathbf{x}_k + \alpha A^t (\mathbf{b} - A\mathbf{x}_k) \\ &\quad \text{with } 0 < \alpha < 2/\left\|A^t A\right\|_2 \end{aligned}$$

converges to the solution of the least squares problem $\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|^2$ and where the 2-norm of a linear operator A is given by:

$$\|A\|_2 = \sup_{\|\mathbf{x}\|=1} \|A\mathbf{x}\|$$

We can apply this theorem to criterion (9):

$$\begin{aligned} \mathbf{e}_0 &= 0 \\ \mathbf{e}_{k+1} &= \mathbf{e}_k + \alpha G_\lambda^t \left(\mathbf{y} - G_\lambda \mathbf{e}_k\right) \\ &\quad \text{with } 0 < \alpha < 2/\left\| G_\lambda^t G_\lambda \right\|_2 \end{aligned} \tag{10}$$

and by using a regularized criterion in Landweber's algorithm, we have the following lemma:

Lemma 2. Provided $\lambda > 0$ and the regularization operator D only has non-zero singular values, i.e. $D^t D$ is strictly positive definite, the algorithm (10) is guaranteed to have minimum convergence speed.

Proof: We start by recalling the expression of $G_\lambda^t G_\lambda$:

$$G_\lambda^t G_\lambda = \sum_{i=1}^{p} \omega_i^2 G_i^t G_i + \lambda^2 D^t D \tag{11}$$

As $G_i^t G_i$ matrices are symmetrical for all $i = 1 \ldots p$ and so is $D^t D$, we have, according to Weyl's theorem, the following inequalities on the eigenvalues of $G_\lambda^t G_\lambda$:

$$\begin{aligned} &\forall k = 1 \ldots n \\ \lambda_k(G_\lambda^t G_\lambda) = \sigma_k^2(G_\lambda) &\geq \sigma_k^2 \left(\sum_{i=1}^{p} \omega_i^2 G_i^t G_i \right) + \lambda^2 \underline{\sigma}^2(D) \\ &\geq \lambda^2 \underline{\sigma}^2(D) \end{aligned} \tag{12}$$

A minimum convergence speed will then be assured if λ is nonzero and if all singular values of D are nonzero [1]. □

2.3 Algorithm parameterization

Several issues can be raised from this algorithm including the tuning of the regularization parameter λ, the number of iterations necessary to compute a satisfactory approximation of the regularized solution, the computation of $\|G_\lambda^t G_\lambda\|_2$ and the computation of the various matrix-vector products when n is large.

Regularization parameter λ This problem was dealt with a method given in Mouyon and Losser (2002) and in Mouyon and Vacher (2001). The solution is based on the fact that the evolution of the optimization criterion and the norm of the solution with respect to λ, is well-known and allows an automatic tuning.

Learning rate α The exact computation of $\|G_\lambda^t G_\lambda\|_2$ would be tedious and it seems that computing an upper bound would be more appropriate. Once again, we will use the fact that the G_i are Tœplitz and lower triangular matrices:

$$\forall i = 1 \ldots p,\ G_i = \sum_{k=0}^{n-1} g_i(k) \mathcal{A}^k \tag{13}$$

where $\mathcal{A}$ is a lower nilpotent matrix such as $\mathcal{A}^n = 0$ and $\|\mathcal{A}\|_2 = 1$. This leads to :

$$\|G_i\|_2 \leq \sum_{k=0}^{n-1} |g_i(k)| \, \|\mathcal{A}\|^k \leq \sum_{k=0}^{n-1} |g_i(k)|$$

We can then deduce an upper bound for $\|G_\lambda^t G_\lambda\|$:

$$\begin{aligned} \left\|G_\lambda^t G_\lambda\right\| &\leq \lambda^2 \|D\|_2^2 + \sum_{i=1}^{p} \omega_i^2 \|G_i\|_2^2 \\ &\leq \lambda^2 \bar{\sigma}^2(D) + \sum_{i=1}^{p} \omega_i^2 \left(\sum_{k=0}^{n-1} |g_i(k)| \right)^2 \end{aligned} \tag{14}$$

We may then choose an upper bound $\widetilde{\alpha}$ for the learning rate:

$$\widetilde{\alpha} = \frac{2}{\lambda^2 \bar{\sigma}^2(D) + \sum_{i=1}^{p} \omega_i^2 \left(\sum_{k=0}^{n-1} |g_i(k)| \right)^2} \tag{15}$$

Stopping condition If we consider the convergence of the iterates $\mathbf{e}_k$ to the regularized solution $\mathbf{e}_\lambda^\star$, we obtain the following sequence:

$$\begin{aligned} \Delta\mathbf{e}_0 &= -\mathbf{e}_\lambda^\star \\ \Delta\mathbf{e}_k &= \left(I - \alpha G_\lambda^t G_\lambda\right)^k \Delta\mathbf{e}_0 \end{aligned}$$

with $\alpha = \widetilde{\alpha}/2$.

We are looking for the minimum number of iterations k_{opt} so that the error in every singular direction of G_λ is lower than μ times the initial error:

$$\forall i = 1 \ldots n,\ \left(1 - \alpha \sigma_i^2(G_\lambda)\right)^{k_{opt}} < \mu \tag{16}$$

which gives, according to (12):

$$\begin{aligned} \left(1 - \alpha \lambda^2 \underline{\sigma}^2(D)\right)^{k_{opt}} &\leq \mu \\ k_{opt} &\geq \frac{ln(\mu)}{ln\left(1 - \alpha \lambda^2 \underline{\sigma}^2(D)\right)} \end{aligned} \tag{17}$$

Unless λ is large enough, this condition would lead to an excessive number of iterations and it would be necessary to define a maximum number of iterations k_{max} to compare with k_{opt}.

Fast matrix-vector product As the size of the exploited data may be large, matrix-vector products may take a larger amount of computational resources than necessary. One more time, the Tœplitz structure of the matrices G_i will help us to accelerate drastically the computation speed.

A method given in Björck (1996), and that requires only two fast Fourier transforms and one multiplication with a diagonal matrix can be performed to make the cost of the operation be reduced from $O(n^2)$ to $O(n \log_2 n)$, which is quite significant when n is large. Since the transpose of a Tœplitz matrix is also Tœplitz, a similar scheme can be used. For instance, this method allows to go 15 to 20 times faster (depending on the computer) than the regular product for $n \approx 2000$.

3. APPLICATION TO THE AIRBRAKES COMPENSATION OF AN AIRCRAFT

We applied this methodology to the airbrakes compensation of an aircraft, because it has shown to be very tedious to tune throughout many aircraft development programmes. As a matter of fact, the aerodynamic effect involved is not modelled precisely enough on pre-flight test

[1] This is the case for the identity operator, for the first and second discrete derivative operator, which are typical regularization operators.

CFD[2] models, the law design with these models is not satisfactory. With this methodology, we propose to tune the law using real aircraft responses.

3.1 Airbraking function and compensation

Usually on a large-scale civilian aircrafts, the airbraking function is performed by a symmetrical deflection of the spoilers, which are the control surfaces located on the upper wing surface as shown in Fig. 3.

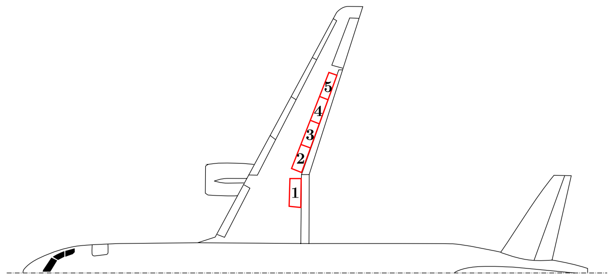

Fig. 3. Spoilers location

The airbraking function is meant to make the aircraft decelerate on its initial trajectory, by increasing the drag. However, the spoilers extension yields an additional loss of lift ΔL located behind the center of gravity, yielding a pitch-up, i.e. nose-up, moment ΔCm as shown in Fig. 4. This effect is a function of the aircraft airspeed V_c, altitude z_p, and center of gravity position x_{cg}.

In order to soften this pitch-up effect, there exists an open-loop control law that uses the orders δp_{sp} sent to the spoilers to compute an adequate deflection δq_{comp} of the elevator, creating the force ΔL_{elev} that will yield the compensating moment ΔCm_{elev}.

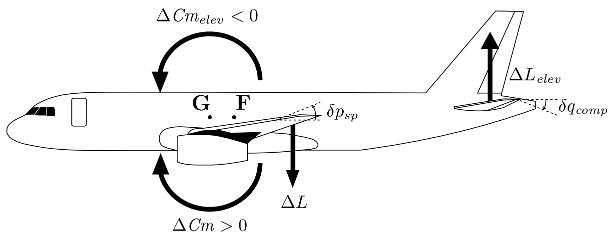

Fig. 4. Pitching moment due to airbrakes deflection

3.2 Specifications and constraints

A template for the desired behaviour $z^{\star}(t)$ to a rate-limited step order $w(t)$, is given by industrial specifications:

- aircraft vertical speed V_z variations must not exceed 200 ft/mn in absolute value
- aircraft flight path angle[3] γ variations must not exceed 0.2° in absolute value
- both criteria must be satisfied during at least 15 s, after the airbrakes extension

We must also make sure to limit the variation of the vertical load factor Nz at the center of gravity for comfort aspects. These specifications give us the constitution of the vector of outputs $z(t)$:

$$z(t) = (\gamma(t)\ V_z(t)\ Nz(t))^t$$

Regarding the control law synthesis, the main constraint is to design a simple open-loop structure (static or low-order dynamic system) having the δp_{sp_i} as inputs and its computed order must be added to the one generated by the main flight control law. The open-loop structure is a choice made by the industrial for this specific application.

3.3 Implementation

The method was tested using a Simulink model of a closed-loop aircraft with nonlinear flight dynamics, actuator and sensor modelling, and extensive flight control laws.

The method relies on three fundamental steps:

- the identification of the impulse responses g_i of the closed-loop aircraft
- the inversion with the auto-tuning of the regularization parameter
- the control law synthesis

Let us note that for each flight case where the method is applied, we need the behaviour of the aircraft disturbed by the airbrakes extension and its response when both disturbed and excited by the identification signal.

4. RESULTS

The results are presented for each part of the process which are the inversion and the synthesis for a couple of distinct flight cases: one at $V_c = 250$ kts and $z_p = 10000$ ft, and the other at $V_c = 330$ kts and $z_p = 30000$ ft. Both points are at a "neutral" position of the center of gravity. The method is performed for a full extension of the airbrakes.

4.1 Inversion

In Fig. 5 and Fig. 6, the dash-dotted line represents the behaviour aircraft without compensation, the solid line its behaviour with the computed compensation. The dashed lines mark the constraints that must not be violated in γ and V_z.

We can see that the results are very satisfactory except for the last 5 seconds of the simulation where $\delta q^{\star}_{comp}$ suddenly changes its behaviour. This is due to the regularization process that seems to find it more effective with respect to the balance between the norm of the error and the norm of the solution.

4.2 Synthesis

Once all compensation commands have been computed on the whole flight domain, we must find a structure for the controller so that it can reproduce these signals for any flight case. By watching carefully these signals, we found some similarities between them:

- non-minimum phase effect appearing as a function of z_p (see Fig. 5); we may then have an unstable zero
- first order dynamic during transient

[2] Computational Fluid Dynamics
[3] Longitudinal angle between the airspeed vector and the ground

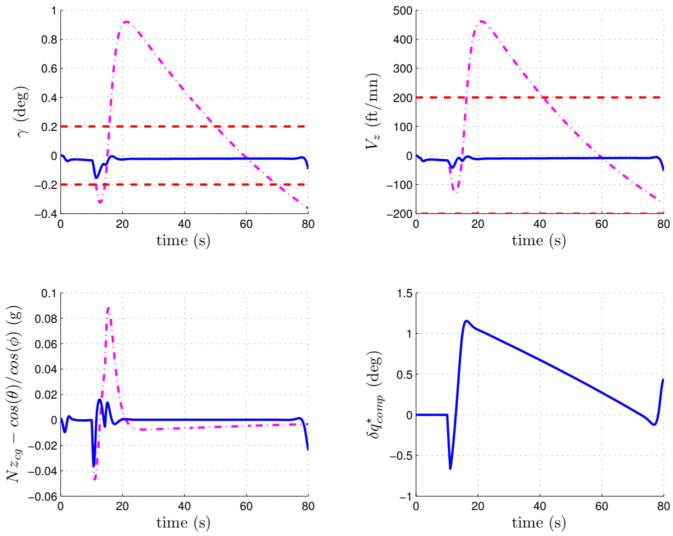

Fig. 5. Inversion results - V_c = 250 kts, z_p = 10000 ft

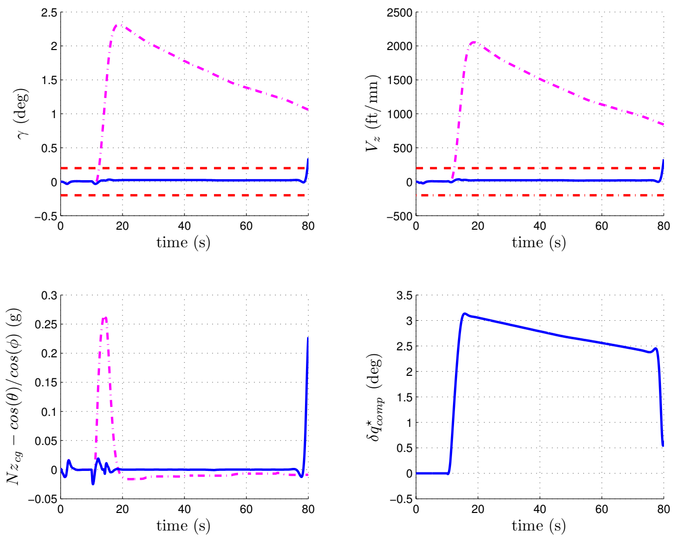

Fig. 6. Inversion results - V_c = 330 kts, z_p = 30000 ft

- decrease in the order function of V_c; we may then add an integrator
- various amplitudes function of V_c and z_p; we may then schedule the input gains

We thus obtain the following structure:

$$\delta q_{comp} = \left(a_1 \cdot \frac{1 - \tau_1 s}{1 + \tau_2 s} + a_2 \cdot \frac{1}{s} \right) \cdot \sum_{i=1}^{n_w} K_i \delta p_{sp_i} \quad (18)$$

$$= K(s, \Theta) \cdot w \quad (19)$$

where s is the Laplace variable and:

$$\Theta = (\tau_1, \tau_2, a_1, a_2, K_1, \ldots, K_{n_w})$$
$$w = \left(\delta p_{sp_1}, \ldots, \delta p_{sp_{n_w}}\right)$$

where n_w is the size of the order sent to the airbrakes.

We must now solve the following nonlinear optimization problem:

$$\min_{\Theta} \|K(t, \Theta) * w(t) - \Delta u^{\star}(t)\|_2 \quad (20)$$

Fig. 7 and Fig. 8 show the results for the flight cases introduced in the previous section. The solid line still represents the aircraft behaviour with the compensation computed by inversion, and the solid line with circle markers the aircraft behaviour with the control law.

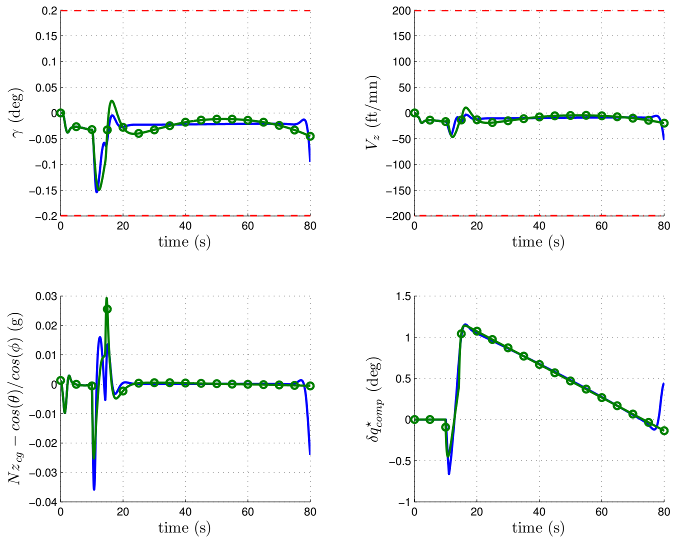

Fig. 7. Synthesis results - V_c = 250 kts, z_p = 10000 ft

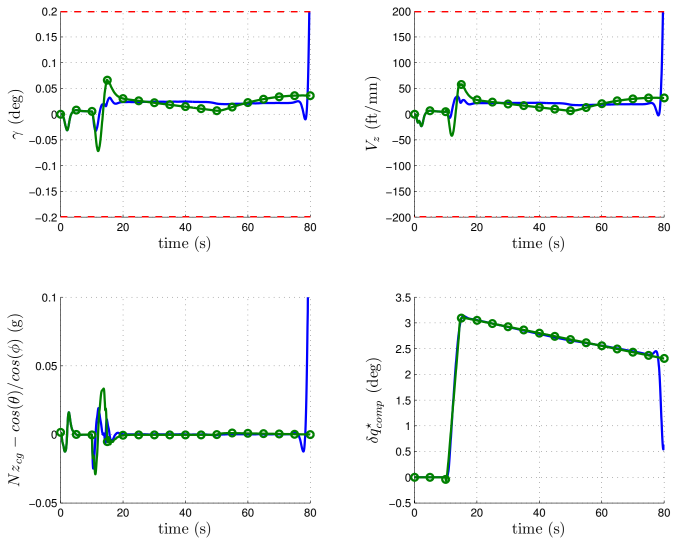

Fig. 8. Synthesis results - V_c = 330 kts, z_p = 30000 ft

The results we obtained are still satisfactory though a bit degraded. The issue that is raised is that the values of the parameter vector Θ at the various flight cases are not very consistent with each other, making their interpolation with respect to scheduling parameters V_c and z_p, not really relevant. In this case we'd rather interpolate the output of the different control laws.

4.3 Validation

The two previous flight cases not being really representative of the aircraft behaviour, the method was tested on whole the flight domain (from V_c=220 to 330 kts and z_p=10000 to 40000 ft). Fig. 9 shows the maximum error on $\gamma(t)$ compared to $\gamma^{\star}(t)$ during the 15 seconds following the airbrakes extension. The red circles shows the maximum allowed error (i.e. 0.2°) and the radiuses of the blue (inversion) and green (designed law) circles are proportionnal to the ratio between the maximum simulation error and the maximum allowed error.

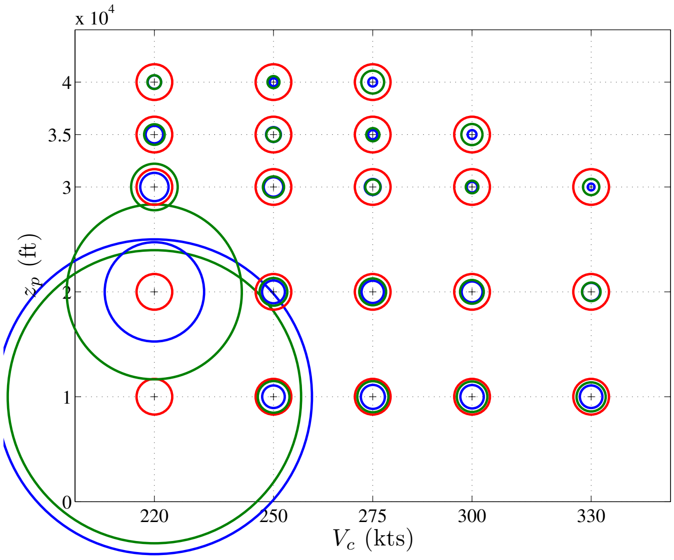

Fig. 9. Maximum errors on the flight domain

These results confirm the good performance of the method, except for cases where the aircraft is too slow to fly steadily with the airbrakes extended.

Finally, in order to test the robustness of the inversion, we checked its sensitivity to sensor noise by adding a gaussian white noise $\nu(t)$ of mean zero and standard deviation of 50% of the standard deviation of $z(t)$. The results are shown in Fig. 10.

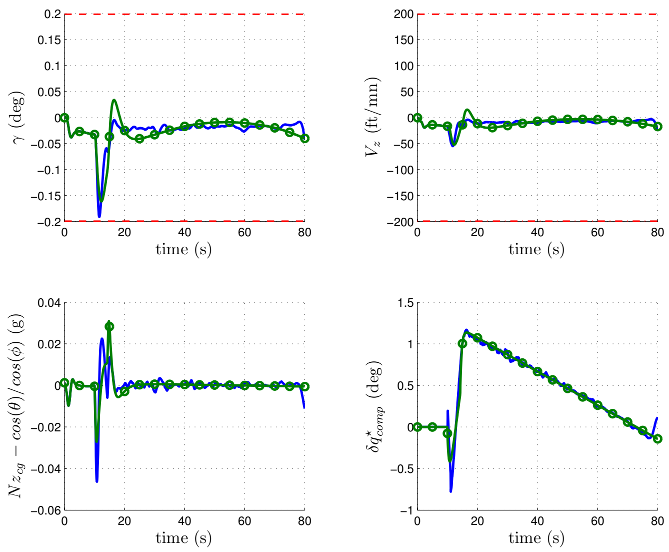

Fig. 10. Noise effect - $V_c = 250$ kts, $z_p = 10000$ ft

We can see that the results are still good and this is because of the regularization that plays a filtering role by minimizing the norm of the solution. As a matter of fact, in many least squares applications, regularization is often used to bypass the effects of noisy data.

5. CONCLUSION AND PERSPECTIVES

In this paper, we studied the application of an inversion-based synthesis of an open-loop controller for a civilian aircraft, by using a three step method. First step is the identification of the transfer between the elevator and the various outputs considered, by estimation of their impulse responses. In the second step, we showed that using specific aircraft responses and an iterative inversion algorithm, we can perform a fast computation of the optimal command signal to obtain a specified behaviour of the aircraft. Finally, with a qualitative reasoning based on the results of the previous step, we designed an open-loop control law so as to reproduce the optimal command signals. The synthesis itself may be user- and application-dependent but given the quickness of the method, several architectures may then be tested. It should be noted that this method is completely off-line: we must wait for specific flight test data to be available before applying the method. To gain even more time, we are currently working on its modification so that it can be included in an adaptive control scheme.

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