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On the numerical investigation of a Luenberger type observer for infinite-dimensional vibrating systems

X.-D. LI * **C.-Z. XU** * **Y.-J. PENG** ** **M. TUCSNAK** ***

* *Laboratoire d'Automatique et de Génie des Procédés, CNRS UMR 5007 Université Claude Bernard Lyon 1, 43 Bd du 11 novembre, 69622 Villeurbanne, France (Tel: 33-0472431894; e-mail: lix@lagep.cpe.fr).*
** *Laboratoire de Mathématiques, CNRS UMR 6620 Université Blaise Pascal Clermont-Ferrand 2, 63177 Aubière, France.*
*** *Institut Elie Cartan & INRIA Lorraine, Université Henri Poincaré Nancy 1, BP 239, 54506 Vandoeuvre lès Nancy, France.*

Abstract: In this paper we present an infinite-dimensional Luenberger type observer for a class of vibrating systems. We undertake numerical investigations of the observer based on the Euler-Bernoulli model of elastic beam. The finite element method is adopted. The spatial interval is subdivided into a finite number N of smaller intervals. On each small interval we use hermitian shape functions of degree 3 to approximate the unknown function which is the solution of the system under study. Numerical simulations are carried out to illustrate the convergence of the designed observer.

Keywords: Observers, exponentially stable, linear systems, finite element method.

1. INTRODUCTION

An observer is an auxiliary dynamical system that reconstructs the state of the original system on the basis of its inputs and outputs. It produces an estimation of the current state of the given system on past observations. The observer construction is particularly important and useful in the context of infinite-dimensional dynamical systems because there is a challenge: the dimension of physical observations is always limited and finite in practice, while the state space is infinite-dimensional. For example, an attractive stabilizing feedback law is proposed in Coron and d'Andrea-Novel (1998) for a rotating body-beam system, but in order to be applied, it needs full information of the infinite-dimensional state. An attempt to implement such a feedback law consists to construct an observer to estimate the state and to feedback the estimated state using the separation principle (see Curtain and Pritchard (1978), Sontag (1990) and Gauthier and Kupka (1992)). An observer has also its proper interest in process supervision or monitoring.

An infinite-dimensional Luenberger type observer has been proposed in Celle et al. (1989) in the context of finite-dimensional nonlinear systems. Then the observer has been shown to be valid for infinite-dimensional dissipative bilinear systems with regularly persistent inputs, see Xu et al. (1995) and Gauthier et al. (1998). However, only weak stability has been guaranteed for the observer, which is the best to be expected. This is because by assumptions of the continuity and finite dimension for the observer operator, exponential stability, which is desirable for applications, is not achievable.

In the present paper, under the assumption of exact observability for the system, we propose a Luenberger type observer which is exponentially stable with some unbounded observation operators (typically these are boundary observations). In this sense our paper is an improvement of the results of Curtain and Pritchard (1978), Celle et al. (1989), Xu et al. (1995) and Gauthier et al. (1998) in the construction of observers. Our observer design is based on the results concerning the stabilization of systems with collocated control and observation. The exponential stability of such an observer is well-known in the finite dimension case, however a high gain of correction may cause exponential instability for an infinite-dimensional observer. For this reason we require a careful analysis.

Our paper is organized as follows: after some preliminary notions given in section 2, our main result will be presented in section 3. We prove that our proposed observer is exponentially stable if the gain of correction is small. Then to illustrate the potential application of our observer we work out a rotating body-beam system as an example in section 4. In this example we show how to get an arbitrary decay rate of the observer using a second step design. The observer has a simple collocated actuator/sensor structure so that it is applicable for many other vibrating systems, for more details, see Guo and Shao (2005), Guo and Zhang (2005), Curtain and Weiss (2006) and Weiss and Curtain (2006). In section 5 we present some simulation results.

2. PRELIMINARIES

Let U, X and Y be Hilbert spaces. We consider the following linear system on the state space X:

$$\begin{cases} \dot{w}(t) = Aw(t) + Bu(t), \\ w(0) = w_0, \\ y(t) = Cw(t), \end{cases} \tag{2.1}$$

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with A the generator of a C_0 unitary group e^{At} on X, w the state trajectory, u the input function on the input space U, y the output function on the output space Y, $B\in\mathcal{L}(U;X_{-1})$ and $C\in\mathcal{L}(X_1;Y)$, where X_{-1} denotes the completion of X with the norm $\|z\|_{-1}=\|(\beta I-A)^{-1}z\|$ and X_1 denotes $\mathcal{D}(A)$ with the norm $\|z\|_1=\|(\beta I-A)z\|$.

First we give some essential preliminary notions of regularity of a linear system.

Let A be the generator of a C_0 semigroup T on X.

Definition 1. A pair (A,B) is *admissible* if $\forall\,\tau>0$,

$$\int_0^{\tau} T_{\tau-t}Bu(t)dt\in X,\quad \forall\, u\in L^2_{loc}(\mathbb{R}^+;U).$$

Then B is called an *admissible control operator for T*.

Definition 2. A pair (A,C) is *admissible* if $\forall\tau>0$, $\exists\, K_\tau>0$ such that the following estimation holds:

$$\int_0^{\tau}\|\, CT_t w_0\|^2_Y dt\le K_\tau\,\|w_0\|^2_X,\quad \forall\, w_0\in\mathcal{D}(A).$$

Then C is called an *admissible observation operator for T*.

Definition 3. The linear system (2.1) or the triple (A,B,C) is *regular* if

i) (A,B) and (A,C) are admissible and

ii) for a $D\in\mathcal{L}(U;Y)$,

$$\begin{aligned}\lim_{s\to+\infty} G(s)u &\overset{def}{=} \lim_{s\to+\infty} Du + C_\Lambda(sI-A)^{-1}Bu\\ &= Du,\quad \forall\, u\in U,\end{aligned}$$

where the transfer function G of the system (2.1) belongs to $H^\infty(\mathbb{C}_\alpha,\mathcal{L}(U;Y))$, i.e. G is analytic and bounded in some complex right half-plan $\mathbb{C}_\alpha=\{s\in\mathbb{C}\mid\Re(s)>\alpha\}$, with $\alpha\in\mathbb{R}^+$ and C_Λ is the Λ-*extension* of C defined by

$$C_\Lambda x\overset{def}{=}\lim_{\lambda\to+\infty} C\lambda(\lambda I-A)^{-1}x,\ x\in\mathcal{D}(C_\Lambda),$$

where $\mathcal{D}(C_\Lambda)$ consists of all $x\in X$ for which the limit exists.

Definition 4. A pair (A,C) is *exactly observable* (*in time* τ) if it is admissible and there exist some positive constants τ and k_τ such that

$$k_\tau\|w_0\|^2_X\le\int_0^{\tau}\|CT_tw_0\|^2_Y dt,\quad \forall\, w_0\in\mathcal{D}(A).$$

These notions are quite well known since more than ten years, for details, see Weiss (1994), Staffans and Weiss (2000) and Curtain and Weiss (2006).

3. MAIN RESULT

Let X be an Hilbert space. With all the preliminary definitions in the last paragraph, we introduce here a main result concerning the stability of a Luenberger type observer for the dynamical system (2.1) from Deguenon et al. (2006).

The Luenberger type observer system they proposed is governed by

$$\begin{cases}\dot{\hat w}(t)=[A-\kappa\, C^*C\,]\hat w(t)+Bu(t)+\kappa C^*y(t),\ \kappa>0,\\ \hat w(0)=\hat w_0,\end{cases}\tag{3.1}$$

where C^* denotes the adjoint operator of C.

We need to define some stability concepts here.

Definition 5. The observer (3.1) is said *stable* if the error $\varepsilon(t)=\hat w(t)-w(t)$ between the trajectories of the estimated state and the real state converges to zero in the state space X as time t goes to infinity. We say that (3.1) is *exponentially stable* if there exist some positive constants $M>0$ and $\omega>0$ such that

$$\|\varepsilon(t)\|_X\le Me^{-\omega t}\|\varepsilon(0)\|_X,\ \forall\, t\ge 0.$$

The supremum of $\omega>0$ such that the above inequality is true is called the *decay rate* or *convergence rate* of the observer.

The main contribution of Deguenon et al. (2006) is the following result:

Theorem 1. Let A be the generator of a C_0 unitary group on X. Assume that (A,C^*,C) and (A,B,C) are regular and (A,C) is exactly observable. Then the observer system (3.1) on X is exponentially stable for $0<\kappa<1/K_{max}$, with

$$K_{max}=\sup_{\substack{\|v\|_Y=1\\ v\in Ran(C_\Lambda)}}\lim_{\beta\to+\infty}\beta\,\|(\beta I-A)^{-1}C^*v\|^2_X.$$

Moreover, if some number $\kappa>1/K_{min}$ is an admissible feedback for the triple (A,C^*,C), then the corresponding observer system (3.1) is exponentially unstable. Here

$$K_{min}=\inf_{\substack{\|\tilde v\|_Y=1\\ \tilde v\in Ran(C_\Lambda)}}\lim_{\beta\to+\infty}\beta\,\|(\beta I-A)^{-1}C^*\tilde v\|^2_X.$$

Remark 1. Theorem 1 is related to the collocated feedback exponential stabilization studied in Slemrod (1974) and the work Weiss and Curtain (1999), Curtain and Weiss (2006). The proof of the theorem is inspired by the paper Jurdjevic and Quinn (1978).

4. APPLICATION TO A ROTATING BODY-BEAM SYSTEM

The rotating body-beam model that we consider consists of a disk with a beam attached to its center and perpendicular to the disk plane. The disk can rotate freely around its axe which is fixed. The beam is supposed non extensible and is constrained to move in some fixed plane perpendicular to the disk plane. The model presents somewhat idealized situation in stabilizing the motion of the spacecrafts.

Many authors have elaborated stabilizing feedback law for similar models, see Bloch and Titi (1990), Xu and Baillieul (1993), Xu and Sallet (1992), Morgul (1994), Coron and d'Andrea-Novel (1998), Chentouf and Couchouron (1999), Laousy et al. (1996) and Conrad and Pierre (1990). In particular the feedback law in Coron and d'Andrea-Novel (1998) is non local, hence for applications one needs to measure all the state variables which are infinite-dimensional and so physically non measurable. However the moment force and the lateral force of the beam on the fixed end are physically measurable as well as the angular velocity of the disk.

Hence an observer design is desirable to estimate all the state variables from these measurements. Our future objective is, by applying the separation principle suggested in Gauthier and Kupka (1992), to cascade our observer and the feedback law of Coron and d'Andrea-Novel (1998) in order to achieve stabilization.

For this purpose we study a simplified model of rotating body-beam with constant angular velocity. Some physical coefficients as flexural rigidity, length and mass per unit length of the beam are set to be 1. The dynamic of the body-beam system is described by

$$\begin{cases} w_{tt}(x,t) + w_{xxxx}(x,t) = \omega_*^2 w(x,t), \quad t>0,\ x\in(0,1), \\ w(0,t) = w_x(0,t) = 0,\ w_{xx}(1,t) = w_{xxx}(1,t) = 0, \\ w(x,0) = w_0(x),\ w_t(x,0) = w_1(x), \\ y(t) = w_{xx}(0,t), \end{cases} \tag{4.1}$$

where the angular velocity ω_* is a real positive constant (possibly null). For observer construction we assume that $\omega_* < \sqrt{l_1}$, which l_1 indicates the smallest eigenvalue of the differential operator $D = \partial_{xxxx}$. Thus the system (4.1) is skew-adjoint and we can apply Theorem 1 with a Luenberger type observer as follows:

$$\begin{cases} \hat{w}_{1t}(x,t) = \hat{w}_2(x,t) - \kappa F(x)\{\hat{w}_{1xx}(0,t) - y(t)\}, \\ \hat{w}_{2t}(x,t) = -\hat{w}_{1xxxx}(x,t) + \omega_*^2\hat{w}_1(x,t), \\ \hat{w}_1(0,t) = \hat{w}_{1x}(0,t) = 0,\ \hat{w}_{1xx}(1,t) = \hat{w}_{1xxx}(1,t) = 0, \\ \hat{w}(x,0) = \hat{w}_0(x),\ \hat{w}_t(x,0) = \hat{w}_1(x), \end{cases} \tag{4.2}$$

where $\kappa > 0$ and $F(x)$ is the unique solution of the following differential equation:

$$\begin{cases} F^{''''}(x) - \omega_*^2 F(x) = 0, \\ F(0) = F^{''}(1) = F^{'''}(1) = 0, \\ F^{'}(0) = 1. \end{cases} \tag{4.3}$$

Set $H_L^2 = \{f \in H^2(0,1) \,|\, f(0) = f_x(0) = 0\}$. The state space for systems (4.1) and (4.2) is the Hilbert space $X = H_L^2 \times L^2(0,1)$ equipped with the inner product

$$<f,g>_X = \int_0^1 [\, f_{1xx}(x)\, g_{1xx}(x) + f_2(x)\, g_2(x) - \omega_*^2\, f_1(x)\, g_1(x)\,]\, dx.$$

The output space $Y = \mathbb{R}$ is equipped with the usual euclidian scalar product.

By Theorem 1 we have the following results:

Theorem 2. Assume that the constant $\omega_* < \sqrt{l_1}$. Then the system (4.1) and the observer (4.2) have a unique solution in $\mathcal{C}(\mathbb{R}^+; X^2)$ for all initial conditions $(w_0, w_1, \hat{w}_0, \hat{w}_1) \in X^2$. Moreover there exist some positive constants M and α such that

$$\left\| \begin{pmatrix} \hat{w}_1(\cdot,t) \\ \hat{w}_2(\cdot,t) \end{pmatrix} - \begin{pmatrix} w(\cdot,t) \\ w_t(\cdot,t) \end{pmatrix} \right\|_X \le M e^{-\alpha t} \left\| \begin{pmatrix} \hat{w}_0 \\ \hat{w}_1 \end{pmatrix} - \begin{pmatrix} w_0 \\ w_1 \end{pmatrix} \right\|_X.$$

We prove that the observer is exponentially convergent.

Theorem 3. The observer (4.2) is exponentially stable for every gain of correction $\kappa > 0$. Moreover its exponential decay rate is determined by the spectral bound of the generator $A^\kappa = A - \kappa C^* C$. It can be made as fast as we want, on replacing $\kappa\,[F(x)\ \ 0]^T$ by $\kappa\,[F(x)\ \ 0]^T + B(x)$ with some appropriate κ and $B(x)$.

5. SIMULATION RESULTS

In this section, we will apply the finite element method to simulate the observer system.

5.1 Hermitian Shape Functions

We propose a semi-discrete scheme in space. Assume that the interval $E = [0,1]$ is uniformly subdivided into N elements $E_i = [x_i, x_{i+1}]$, $i = 0, \dots, N-1$, we denote h the length of step.

Our original system has high order derivative items (spatial derivative of degree 4 in our beam example), furthermore the continuity of the first partial spatial derivative of the solution (slope of the elastic curve) between adjoining intervals is required as our solution should be in $H^2(0,1)$.

Thus we need to introduce the hermitian interpolation with the polynomial (so piecewise $\mathcal{C}^1$) functions defined on E_i (they can be extended to E with prolongation by zero on $E \setminus E_i$) as follows:

$$\begin{cases} H_1^i(x) = \frac{1}{4}(\xi_i^3(x) - 3\xi_i(x) + 2), \\ H_2^i(x) = \frac{h}{8}(\xi_i^3(x) - \xi_i^2(x) - \xi_i(x) + 1), \\ H_3^i(x) = \frac{1}{4}(-\xi_i^3(x) + 3\xi_i(x) + 2), \\ H_4^i(x) = \frac{h}{8}(\xi_i^3(x) + \xi_i^2(x) - \xi_i(x) - 1), \end{cases}$$

and

$$H_j^i|_{E\setminus E_i} = 0, \quad i = 0, \dots, N-1,\ j = 1,2,3,4,$$

where

$$\xi_i(x) = \frac{2x - x_i - x_{i+1}}{x_{i+1} - x_i},\ \forall\, x \in [x_i, x_{i+1}],\ i = 0, \dots, N-1,$$

are the *local coordinates* which allow to put all the operations into a standardized element $[-1,1]$. It is convenient to do these transformations since it simplifies the computation of global system matrices by concatenating the block matrices obtained on every standardized elements.

The hermitian shape functions satisfy the following conditions:

$$\begin{cases} H_1^i(x_i) = 1 \ \text{ and }\ H_{1x}^i(x_i) = H_1^i(x_{i+1}) = H_{1x}^i(x_{i+1}) = 0, \\ H_{2x}^i(x_i) = 1 \ \text{ and }\ H_2^i(x_i) = H_2^i(x_{i+1}) = H_{2x}^i(x_{i+1}) = 0, \\ H_3^i(x_{i+1}) = 1 \ \text{ and }\ H_3^i(x_i) = H_{3x}^i(x_i) = H_{3x}^i(x_{i+1}) = 0, \\ H_{4x}^i(x_{i+1}) = 1 \ \text{ and }\ H_4^i(x_i) = H_{4x}^i(x_i) = H_4^i(x_{i+1}) = 0. \end{cases}$$

Now we define on E the following functions:

$$\phi_1^0(x) = H_1^0(x),$$
$$\phi_2^0(x) = H_2^0(x);$$

where their supports are E_0;

$$\phi_1^i(x) = H_3^{i-1}(x) + H_1^i(x),$$
$$\phi_2^i(x) = H_4^{i-1}(x) + H_2^i(x);$$

where their supports are $E_{i-1} \cup E_i$, $i = 1, ..., N-1$ and

$$\phi_1^N(x) = H_3^{N-1}(x),$$
$$\phi_2^N(x) = H_4^{N-1}(x),$$

where their supports are E_N.

The set $\mathcal{B} = \{\phi_l^k,\ k = 0, ..., N,\ l = 1, 2\}$ forms a basis of its span $\tilde{V}_h$, which is a $(2N+2)$-dimensional discrete space (included in H_L^2).

5.2 Numerical Approximation

With the separation of variables, the approached solution $w_h \in V_h$ that we look for can be written as

$$w_h(t, x) = \sum_{k=0}^{N} w^k(t)\,\phi_1^k(x) + \tilde{w}^k(t)\,\phi_2^k(x).$$

The boundary conditions $w(0,t) = w_x(0,t) = 0$ of (4.1) imply

$$w^0\phi_1^0(0) + \tilde{w}^0\phi_2^0(0) + w^1\phi_1^1(0) + \tilde{w}^1\phi_2^1(0) = 0,$$
$$w^0\phi_{1x}^0(0) + \tilde{w}^0\phi_{2x}^0(0) + w^1\phi_{1x}^1(0) + \tilde{w}^1\phi_{2x}^1(0) = 0,$$

then we get $w^0 = \tilde{w}^0 = 0$. Thus we can choose the space $V_h = span\,(\phi_1^1, \phi_2^1, \dots, \phi_1^N, \phi_2^N)$ (which has dimension $2N$) as our discrete space.

Taking ϕ_l^k, $k = 1, ..., N$, $l = 1, 2$ as the shape functions, we apply the usual L^2-inner product to the discretized original system. By integration by parts this gives

$$\begin{aligned}\int_0^1 w_{1t}(x)\phi_l^k(x)dx &= \int_0^1 w_2(x)\phi_l^k(x)dx,\\ \int_0^1 w_{2t}(x)\phi_l^k(x)dx &= -\int_0^1 w_{1xx}(x)\phi_{lxx}^k(x)dx\\ &\quad + \omega_*^2\int_0^1 w_1(x)\phi_l^k(x),\end{aligned} \tag{5.1}$$

where $w_1 = w_h$ and $w_2 = w_{ht}$.

Thus our discretized problem consists to solve the following dynamical system:

$$\frac{d}{dt}\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{pmatrix} 0 & I \\ -J & 0 \end{pmatrix}\begin{pmatrix} W_1 \\ W_2 \end{pmatrix},$$

where $W_1(t) = [\,w^1(t), \tilde{w}^1(t), \dots, w^N(t), \tilde{w}^N(t)\,]^T$ and $W_2(t) = [\,w_t^1(t), \tilde{w}_t^1(t), \dots, w_t^N(t), \tilde{w}_t^N(t)]^T$ denote the unknowns, I is the $2N$-by-$2N$ identity matrix and $J = S^{-1}R - \omega_*^2 I$ with S and R, respectively, the mass matrix and the rigidity matrix given by

$$S_{p,q} = \int_0^1 \phi_l^k(x)\,\phi_{l'}^{k'}(x)dx,$$
$$R_{p,q} = \int_0^1 \phi_{lxx}^k(x)\,\phi_{l'xx}^{k'}(x)dx,$$

with $p = 2k+l-2$ and $q = 2k'+l'-2$, for $k, k' = 1, \dots, N$, $l, l' = 1, 2$.

It is clear that S and R are both $2N$-by-$2N$ symmetric pentadiagonal (sparse) matrices and R is invertible.

A similar computation gives the variational formulations for the observer system as follows:

$$\begin{aligned}\int_0^1 \hat{w}_{1t}(x)\phi_l^k(x)dx &= \int_0^1 \hat{w}_2(x)\phi_l^k(x)dx - \int_0^1 F(x)\phi_l^k(x)dx\\ &\quad \cdot\kappa(\hat{w}_{1xx}(0,t) - w_{1xx}(0,t)),\\ \int_0^1 \hat{w}_{2t}(x)\phi_l^k(x)dx &= -\int_0^1 \hat{w}_{1xx}(x)\phi_{lxx}^k(x)dx\\ &\quad + \omega_*^2\int_0^1 \hat{w}_1(x)\phi_l^k(x).\end{aligned} \tag{5.2}$$

Combing the variational formulations (5.1) and (5.2) corresponding to the original system and the observer, we can rewrite our discretized problem as follows:

Find $W_1, W_2, \hat{W}_1$ and $\hat{W}_2 \in V_h$ such that

$$\frac{d}{dt}\begin{pmatrix} W_1 \\ W_2 \\ \hat{W}_1 \\ \hat{W}_2 \end{pmatrix} = \begin{pmatrix} 0 & I & 0 & 0 \\ -J & 0 & 0 & 0 \\ K & 0 & -K & I \\ 0 & 0 & -J & 0 \end{pmatrix}\begin{pmatrix} W_1 \\ W_2 \\ \hat{W}_1 \\ \hat{W}_2 \end{pmatrix} \tag{5.3}$$

with $K = S^{-1}P$ where the elements of P are given by

$$P_{p,m} = \begin{cases} \kappa\,\phi_{mxx}^1(0)\displaystyle\int_0^1 F(x)\phi_l^k(x)dx, & m = 1, 2,\\ 0, & m = 3, \dots, N,\end{cases}$$

with $p = 2k + l - 2$, $k = 1, \dots, N$, $l = 1, 2$.

We notice that $P_{p,m} = 0$, $m = 3, \dots, N$, which follows from the facts that $w_{1xx}(0,t) = w^1(t)\,\phi_{1xx}^1(0) + \tilde{w}^1(t)\,\phi_{2xx}^1(0)$ and $\hat{w}_{1xx}(0,t) = \hat{w}^1(t)\,\phi_{1xx}^1(0) + \hat{\tilde{w}}^1(t)\,\phi_{2xx}^1(0)$.

For determinating $P_{p,m}$, $m = 1, 2$, we need to compute explicitly the solution $F(x)$ of the differential equation (4.3). If $\omega_* = 0$ it has a simple form $F(x) = x$. In the case where $\omega_* \neq 0$, we have

$$F(x) = \alpha(ch(\sqrt{\omega_*}x) - cos(\sqrt{\omega_*}x)) + \beta sh(\sqrt{\omega_*}x)$$
$$+(-\beta + \frac{1}{\sqrt{\omega_*}})sin(\sqrt{\omega_*}x),$$

where

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{1}{\sqrt{\omega_*}(2 + 2ch(\sqrt{\omega_*})cos(\sqrt{\omega_*}))}$$
$$\cdot\begin{pmatrix} ch(\sqrt{\omega_*})sin(\sqrt{\omega_*}) - sh(\sqrt{\omega_*})cos(\sqrt{\omega_*}) \\ (ch(\sqrt{\omega_*}) - sh(\sqrt{\omega_*}))sin(\sqrt{\omega_*}) + 1 \end{pmatrix}.$$

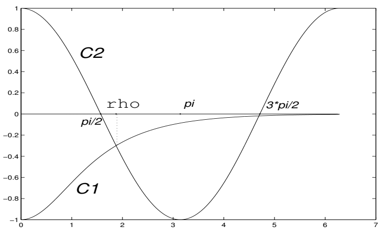

Fig. 1. Parameter ρ verifying equation $ch(\rho)\cdot cos(\rho)+1=0$; curves C_1: y=-1/ch(x); C_2: y=cos(x).

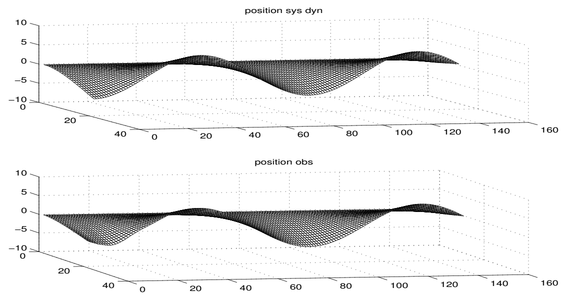

Fig. 2. Position vectors of the dynamical system (top) and of the observer (bottom), respectively, in the case where $\omega_* = 0$.

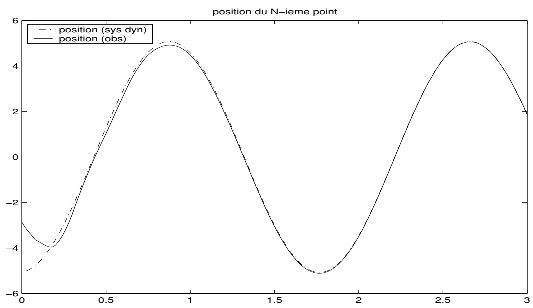

Fig. 3. Evolution of the positions at the N-th point for the dynamical system (dashed) and the observer (solid), respectively, in the case where $\omega_* = 0$.

5.3 Simulation Results

We take spatial grid size $N = 20$ and time step $dt = 0.02$. The parameters are set to $\kappa = 0.5$ and $\omega_* = 1.1$ in the case where $\omega_* \neq 0$. The initial conditions are given as follows:

$$\begin{cases} w(0,x) = -2\gamma[ch(\rho x) - cos(\rho x)] + 2(sh(\rho x) - sin(\rho x)), \\ w_t(0,x) = 0, \end{cases}$$

where $\gamma = -(sh(\rho) + sin(\rho))/(ch(\rho) + cos(\rho))$ with $\rho \simeq 1.8751$ (which satisfies $ch(\rho)cos(\rho) + 1 = 0$). (See Fig.1).

It is clear that with these initial conditions the oscillation of our original system is a simple harmonic motion.

For the observer system, we take $(\hat{w}_0\ \hat{w}_1)^T = 2(w_0\ w_1)^T$ as the initial conditions.

We use the subroutine *ode*15*s* in Matlab to solve the dynamical system (5.1) with the above initial conditions. The profiles $w(t,x)$ and $\hat{w}(t,x)$ represent, respectively, the positions of the original system and the constructed Luenberger type observer in the case where $\omega_* = 0$ (respectively

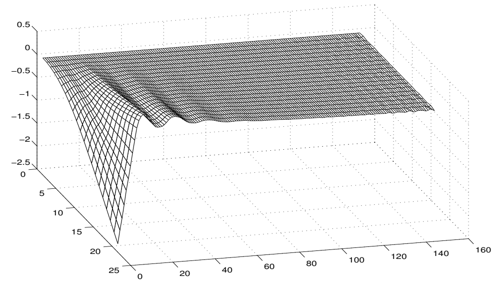

Fig. 4. Evolution of the observer error $w - \hat{w}$ in the case where $\omega_* = 0$.

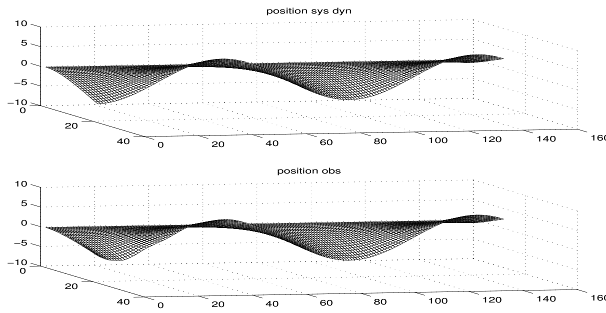

Fig. 5. Position vectors of the dynamical system (top) and of the observer (bottom), respectively, in the case where $\omega_* \neq 0$.

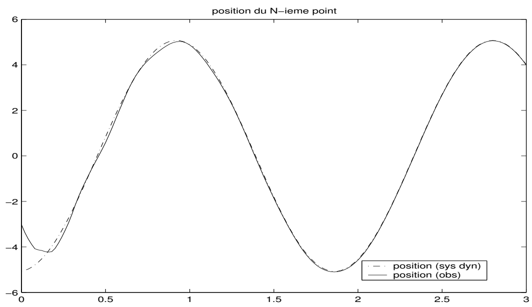

Fig. 6. Evolution of the positions at the N-th point for the dynamical system (dashed) and the observer (solid), respectively, in the case where $\omega_* \neq 0$.

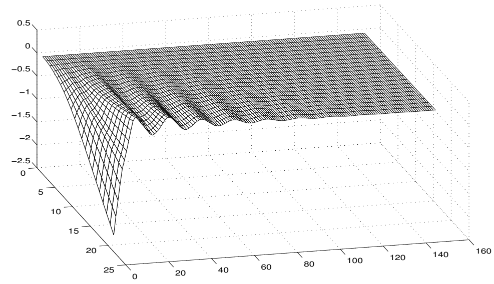

Fig. 7. Evolution of the observer error $w - \hat{w}$ in the case where $\omega_* \neq 0$.

$\omega_* \neq 0$). They are illustrated in Fig.2 (respectively Fig.4). In particular, Fig.3 and Fig.5 illustrate the error dynamics of the observer at the N-th point of the interval $[0,1]$, respectively, for the case where $\omega_* = 0$ and $\omega \neq 0$. For the error at each point of the interval $[0,1]$ two cases are also represented, respectively, in Fig.4 and Fig.7. We note that it takes more time for the observer to approach the real state in the ω_*-nonzero case than in the ω_*-zero case. We could improve the convergence rate by choosing better gain parameters, it is part of our future investigation.

6. CONCLUSIONS

We have presented the Luenberger type observer designed for a class of infinite-dimensional vibrating systems and the numerical simulation results on a concrete application. The future work is focused on extending the observer construction to other vibration systems studied in Guo and Shao (2005), Guo and Zhang (2005), Curtain and Weiss (2006) and Weiss and Curtain (2006). Theoretically we also wish to investigate the possibility of the construction in a more general context as in Lasiecka (1989). As we indicated in section 4, for the rotating body-beam system, our objective is to cascade our observer and the feedback law proposed in Coron and d'Andrea-Novel (1998) to achieve stabilization. The decay rate of our observer can be made as large as we want by a second step design using the Riesz basis (cf. Rao (1997)). Moreover we prove that the constructed observer is still valid for any time-varying angular velocity provided that its distance to some constant velocity is small in terms of L^∞ uniform norm. The observer is exponentially convergent for any angular velocity $\omega(t)$ located in a small L^∞ ball centered at a constant ω_*.

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