

## A RECEDING-HORIZON FORMATION TRACKING CONTROLLER WITH LEADER-FOLLOWER STRATEGIES

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**Abstract:** This paper presents a receding-horizon leader-follower (RH-LF) controller for addressing the formation control problem of multiple nonholonomic mobile robots. The issues to be investigated include separation, bearing, and orientation deviation control between the leader and the followers, where the orientation deviation control is especially important to control precision. After the leader-follower formation problem is posed to a formation tracking problem, a receding-horizon formation tracking controller is proposed, which guarantees asymptotic convergence of the formation tracking error to zero. Simulations are performed to verify the effectiveness of the proposed control strategy. *Copyright © 2002 IFAC*

**Keywords:** Mobile robots; Formation control; Receding-horizon control; Leader-follower Strategy.

### 1. INTRODUCTION

Control and coordination of multiple mobile robots have been considerably done during the past ten years. It is generally accepted that multiple mobile robots, if cooperatively work under high efficient organization and principles, can behave as a whole, and also guarantee the robot team with fault tolerance and robust properties (Liu and Wu, 2001). Mobile robot team can perform tasks that are difficult for one single robot, for example, group hunting (Cao *et al.*, 2006), large area exploration (Burgard *et al.*, 2005), surveillance (Tang and Ozguner, 2005), object transportation (Yamashita

*et al.*, 2003; Berman *et al.*, 2004), and spacecraft interferometry tasks (Beard *et al.*, 2001).

Robot formation control has received much attention in multi-robot coordinations, since robots moving in formation can “reduce the system cost, increase the robustness and efficiency of the system” (Chen and Wang). The formation control approaches can be roughly categorized into three (Chen and Wang; Lawton *et al.*, 2003; Beard *et al.*, 2001): behaviour-based, leader-follower, and virtual structure methods. In behaviour-based formation, a group behaviour (or mission) comprised of some low-level actions (or sub-tasks), is constructed to achieve the global objective, where the individual robot needs to perform low-level actions to make the group behaviour accomplished (Balch and Arkin, 1998; Berman *et al.*, 2004; Long *et al.*, 2005). In leader-follower formation,

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one robot is designated as the leader, whose motion defines the group bulk motion, and the other robots are controlled to follow their respective leader with given separations and bearings (Desai *et al.*, 2001; Das *et al.*, 2002; Tanner *et al.*, 2004; Huang *et al.*, 2006). In virtual structure formation, robots behave like particles embedded in a rigid virtual structure, and this method is usually applied in high precision formation tasks (Lewis and Tan, 1997; Ren and Beard, 2004). In leader-follower formation strategy, one leader “leads” the group and the followers “follow” their respective leaders to maintain desired geometric relationship. Traditionally, leader-follower controller concentrates on the explicit control of the separation and bearing between robots, and orientations of the followers are treated as internal dynamics, which makes the robot group move on the same direction. However, formation tasks such as switch between formations and curved path tracking usually require the follower to hold an orientation deviation with respect to the leader.

In this paper, a framework to deal with nonholonomic mobile robot formation problems is investigated, which investigates the issues of separation, bearing and orientation deviation control between the leader and follower. After transforming the leader-follower formation problem to a formation tracking problem, receding-horizon (RH) controllers are applied, which guarantees the formation tracking error asymptotically converged under a suboptimal strategy.

Receding-horizon (RH) method is also known as model-predictive-control (MPC), and has been frequently applied in industry. RH controller mainly concentrates on solving optimization problems of a predictive control horizon with input and state constraints (Keerthi and Gilbert, 1988). Due to the use of predictive horizon, the stability of the open loop system becomes the main problem. It was shown that when applying an infinitive control predictive horizon, the stability can be guaranteed for even nonlinear systems (Keerthi and Gilbert, 1988), but infinitive predictive horizon is not applicable in practice. The stability can also be guaranteed by adding a terminal state constraint to the system, e.g. force the terminal state equal to zero (Rawlings and Muske, 1993), but this approach is time consuming (Gu and Hu, 2005). Further researches shown that the system stability can be guaranteed by converting the terminal state equality into an inequality, e.g. a terminal state penalty function (cost function), which should be larger than the cost function of the linear feedback controller applied to the system when entering the terminal state region (Chen and Allgower, 1998; De Nicolao *et al.*, 1998). The recent works suggest that the terminal state linear feedback controller was not necessary, and the system stability could be guaranteed as long as the terminal state inequality constraint was met. Thus receding horizon control was applied to solve the nonholonomic point-wise stabilization problem (Fontes, 2001; Gu and Hu, 2005).

The paper is organized as follows. Section 2 will propose the new leader-follower formation strategies: separation-bearing-orientation control (SBOC) and separation-separation-orientation control (SSOC), also in this section we transform the formation problem to a formation tracking problem, which will be solved in section 3 using receding-horizon controllers. Simulations will be performed in section 4 to verify the effectiveness of the proposed control strategy. Conclusions of this work will be given in section 5.

## 2. LEADER-FOLLOWER FORMATION

### 2.1 Formation Problem Definition

We first give some necessary assumptions regarding to the robot group as follows:

*Assumption A1:* The robots are assumed labelled at the very beginning, and the robot whose motion defines the group bulk motion is designated as the group leader  $R_1$ .

*Assumption A2:* Robot’s configuration and velocity are measurable.

*Assumption A3:* Every robot owns an embedded obstacle avoiding program, which should be triggered by sonar sensors when the robot is too close to an obstacle or another robot.

Define  $F(t)$  as the reference formation for the robot group at time  $t$ , which should be determined by the following formation parameters:

1. The configuration of the group leader  $p_{GL}$ .
2. An interaction graph  $G_{LF}(G_i), (i = 2, 3, \dots, n)$ , which defines the desired relationship between every leader-follower pair.  $n$  is the number of robots, and  $G_i = LF(R_j \rightarrow R_i(R_{i+1}))$ , where  $R_j$  denotes the follower and  $R_i(R_{i+1})$  the leader.
3. A kinematic graph  $v_{LF}(v_i)$ , which defines the desired velocity for every formation robot, including both the linear and angular velocities.

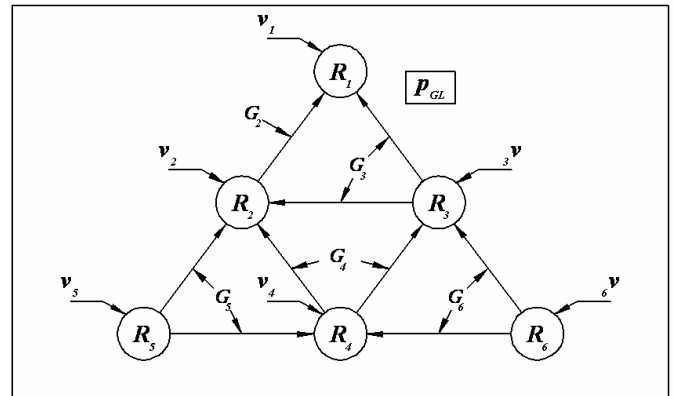


Fig. 1. An example of  $F(t)$ .

Fig. 1 illustrates the reference formation of a group of six robots. The formation problem can be formulated as follows.

*Leader-Follower Formation Problem:* Suppose a group of  $n$  robots, randomly placed at the beginning, are driven to track desired formation trajectory  $FT(t)$ .  $FT(t)$  is comprised of  $k$  curved paths  $l_i(t_0, t_f) = (x_i(t), y_i(t))$ ,  $(i = 1, 2, \dots, k)$ , where  $(t_0, t_f)$  denotes the time interval that the group are desired to move on  $l_i$ , and  $l_i$  defines the group bulk motion, i.e. the path travelled by the leader. During the tracking, the robots are required to form and maintain the desired time-varying formation shape denoted by  $F(t)$ .

The robots here are typical nonholonomic mobile robots, with the following kinematics described by a unicycle model:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta \\ \sin \theta \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega = g_1 v + g_2 \omega \quad (1)$$

where  $(x, y)$  is the position of the centre point of the robot wheels,  $\theta$  is the robot orientation,  $v$  and  $\omega$  are the robot linear and angular velocity inputs. Denote  $p = (x, y, \theta)^T$  as the robot configuration. The nonlinear system is subject to a constraint that the robot wheels must roll without slipping on the lateral direction

$$\dot{x} \sin \theta - \dot{y} \cos \theta = 0 \quad (2)$$

Since (2) is non-integratable, the constraint cannot be converted into an explicit geometric form, which makes (1) has more state variables than control inputs. The nonholonomic system cannot be stabilized to the origin by continuous time-invariant feedback laws (Brockett, 1983). However, since the accessibility rank condition is globally satisfied

$$\text{rank}([g_1 \ g_2 \ [g_1, g_2]]) = 3 \quad (3)$$

where  $[g_1, g_2]$  is Lie bracket of  $g_1$  and  $g_2$ , the non-holonomic system can be stabilized using nonlinear or time-variant controllers (Bloch *et al.*, 1992).

## 2.2 Leader-Follower Formation Strategies: SBOC & SSOC

The follower robots are controlled with given separation, bearing, and relative orientation deviation to the leader. Two formation control strategies, *Separation-Bearing-Orientation Controller (SBOC)* and *Separation-Separation-Orientation Controller (SSOC)*, will be proposed in the following. Note that the robot which is the closest to the leader, denoted by  $R_2$ , can only be controlled using SBOC.

Consider a two robot formation problem as shown in Fig. 2. Under the leader-follower control strategy SBOC,  $R_j$  follows  $R_i$  with a desired separation  $l_{i,j}$  and bearing  $\psi_{i,j}$ , and  $R_j$  is required to maintain a desired relative orientation  $\beta_{i,j} = \theta_i - \theta_j$  with respect to  $R_i$ , where  $\theta_i$  and  $\theta_j$  are the orientation of  $R_i$  and  $R_j$ , respectively. Note that the reference configuration of the follower robot is precisely defined at every time instant.

SSOC concerns the three robot formation problems as shown in Fig. 3. Under SSOC, the third robot  $R_j$  is required to maintain a desired separation pair  $l_{i,j}$  and  $l_{i+1,j}$  to  $R_i$  and  $R_{i+1}$  respectively, and meanwhile  $R_j$  has to maintain a desired relative orientation  $\beta_{i,j} = \theta_i - \theta_j$  with respect to  $R_i$ . Both SBOC and SSOC can be used to solve the formation tracking problem for each robot. As in Fig. 2, the desired position of the follower  $R_j$  is given by

$$p_j^d = p_i + [l_{i,j}^d \cos \psi_{i,j}^d \quad l_{i,j}^d \sin \psi_{i,j}^d \quad \beta_{i,j}^d]^T \quad (4)$$

where  $l_{i,j}^d$ ,  $\psi_{i,j}^d$  and  $\beta_{i,j}^d$  are the desired separation, bearing and relative orientation between  $R_i$  and  $R_j$ ,  $p_i$  is the posture of the leader. The formation tracking error of the robot  $R_j$  can be defined as (Kanayama, 1990)

$$p_j^e = \begin{bmatrix} x_j^e \\ y_j^e \\ \theta_j^e \end{bmatrix} = \begin{bmatrix} \cos \theta_j & \sin \theta_j & 0 \\ -\sin \theta_j & \cos \theta_j & 0 \\ 0 & 0 & 1 \end{bmatrix} (p_j^d - p_j) \quad (5)$$

where  $p_j$  is the posture of the follower. The control objective of SBOC for  $R_j$  is to stabilize the tracking error  $p_j^e$  towards zero.

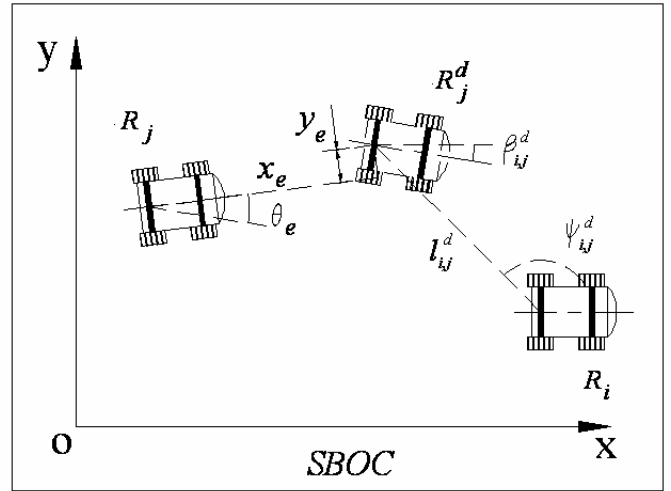


Fig. 2. Formation tracking of SBOC

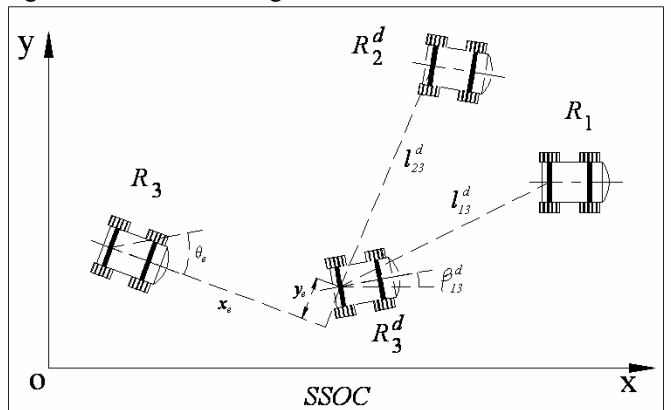


Fig. 3. Formation tracking of SSOC

## 3. RECEDING-HORIZON LEADER-FOLLOWER FORMATION CONTROLLER

### 3.1 Robot path tracking and control framework

From (1) and (5), the robot tracking error can be expressed as

$$\begin{bmatrix} \dot{x}^e \\ \dot{y}^e \\ \dot{\theta}^e \end{bmatrix} = \begin{bmatrix} v^r \cos \theta^e \\ v^r \sin \theta^e \\ \omega^r \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} v + \begin{bmatrix} y^e \\ -x^e \\ -1 \end{bmatrix} \omega \quad (6)$$

where  $v^r$  and  $\omega^r$  are the predefined reference linear and angular velocities. Rewrite (6) as a nominal nonlinear plant

$$\dot{q}_e(t) = f(t, q_e(t), u(t)) \quad (7)$$

where  $q_e(t) = p^e(t)$  and  $u(t)$  are the  $n$  dimensional tracking error state vector and  $m$  dimensional input vector, respectively.  $f$  is assumed to be a continuous function. The input vector is constrained in a compact and convex set  $0 \in U \in R^m$ .

The RH strategy is performed with the steps as shown in Fig. 4. Suppose that the RH controller uses a predictive horizon length  $T$ . The following procedures will be used:

- (1) Measure the plant state at time instant  $t_i$ .
- (2) Compute the input  $\bar{u}(t) : (t_i \leq t \leq t_i + T)$  under an optimal strategy.
- (3) Apply  $u(t) = \bar{u}(t) : (t_i \leq t \leq t_i + \delta)$  to the plant. (Note that  $\delta$  is much smaller than  $T$ , and the remaining  $\bar{u}(t) : (t_i + \delta \leq t \leq t_i + T)$  is discarded.)
- (4) Repeat (1) to (3) at the next time instant  $t_{i+1} = t_i + \delta$ .

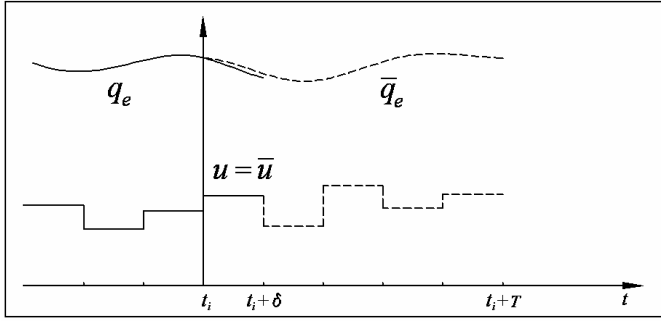


Fig. 4. General receding-horizon framework

Applying an input vector to the plant over a period  $(t_i, t_i + \delta)$ , the state vector at  $t_i + \delta$  can be predicted as

$$\dot{q}_e = f(\tau, q_e(\tau), u(\tau)), (\tau \in (t_i, t_i + \delta)) \quad (8)$$

where  $q_e(0) = q_e(t_i)$ , and  $u(\tau) \in U$ .

During the predictive horizon, the tracking error state should be optimized. Define the terminal state cost function measured during a period of  $T$  as

$$J(t, q_e(t), u(t)) = g(q_e(t+T)) + \int_t^{t+T} L(\tau, q_e(\tau), u(\tau)) d\tau \quad (9)$$

where  $L(t, q_e(t), u(t)) = q_e(t)^T Q q_e(t) + u(t)^T R u(t)$  is the state and input cost function measured during the predict horizon,  $Q$  is a positive symmetric weight matrix, and  $R = 0$  since the input energy cannot be minimized as long as the reference velocity does not tend to zero, which usually occurs in the formation task.  $g(q_e(t+T))$  is the terminal state penalty function which is continuous and differentiable, and  $g(0) = 0$ ,  $g(q_e(t)) > 0$  if  $q_e \neq 0$ . Define the open loop optimal problem (OP) at time  $t$ , which concentrates on the terminal state cost function, as

$$\min J(t, q_e(t), u(t)) \quad (10)$$

Note that (10) is subject to (8) and the input constraints.

### 3.2 RH convergent properties and control parameters design

It is also a requirement that the input sequence of the RH controllers drive  $q_e$  towards zero. It was proved that the system stability can be guaranteed if a terminal state penalty term is added to the cost function and a terminal state inequality constraint can be satisfied (Fontes, 2001; Gu and Hu, 2005). As stated above,  $g(q_e(t+T))$  is the terminal state penalty term and the terminal state inequality constraint is given by

$$\dot{g}(q_e(t)) + L(t, q_e(t), u(t)) \leq 0 \quad (11)$$

*Theorem 1:* Consider the optimal control problem presented in (10). If the input  $u(t)$  satisfies (11), the RH controller can asymptotically stabilize the system, i.e.  $q_e(t) \rightarrow 0$  as time  $t \rightarrow \infty$  (Fontes, 2001).

The proof of *Theorem 1* can be found in Fontes (2001). We now design the feedback input parameters.

Define  $g(q_e(t))$  as a Lyapunov-like function of the predicted state

$$g(q_e(t+T)) = \frac{1}{2} x_e^2(t+T) + \frac{1}{2} y_e^2(t+T) + \frac{1}{2} \theta_e^2(t+T) \quad (12)$$

Select  $L(t, q_e(t), u(t))$  in (9) as

$$L(t, q_e(t), u(t)) = q_1 x_e^2(t) + q_2 y_e^2(t) + q_3 \theta_e^2(t) \quad (13)$$

Note that  $(q_1 > 0, q_2 > 0, q_3 > 0)$  are the weight factors of the state errors. We design the applied feedback velocity input as

$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} v_r(t) \cos \theta_e + k_1 x_e(t) \\ \omega_r(t) + k_2 y_e(t) + k_3 \theta_e(t) \end{bmatrix} \quad (14)$$

Substituting (6) and (12~14) into (11), the stability condition becomes

$$\begin{aligned} & \dot{g}(q_e(t)) + L(t, q_e(t), u(t)) \\ &= x_e \dot{x}_e + y_e \dot{y}_e + \theta_e \dot{\theta}_e + q_1 x_e^2 + q_2 y_e^2 + q_3 \theta_e^2 \\ &= x_e (v_r \cos \theta_e - v + y_e \omega) + y_e (v_r \sin \theta_e - x_e \omega) + \\ & \theta_e (\omega_r - \omega) + q_1 x_e^2 + q_2 y_e^2 + q_3 \theta_e^2 \\ &= -(k_1 - q_1) x_e^2 - (k_3 - q_3) x_e^2 + q_2 y_e^2 + v_r y_e \sin \theta_e \\ & \quad - k_2 y_e \theta_e \end{aligned} \quad (15)$$

To make (15) satisfy the inequality condition (11), we choose the following inequality constraints for the feedback parameters

$$\begin{aligned} k_1 &> q_1 + q_2 y_e^2 / x_e^2 \\ k_2 &> v_r \sin \theta_e / \theta_e \\ k_3 &> q_3 \end{aligned} \quad (16)$$

In the actual implementation of the formation task,  $y_e$  is usually much smaller than  $x_e$ . However, critical conditions may occur when  $x_e$  converges much faster than  $y_e$ , and thus makes  $k_1$  too big, but under this situation, we can switch to a linear feedback controller. Suppose  $|x_e| \leq x_\epsilon$  and  $|y_e| \geq y_\epsilon$ , where  $y_\epsilon \gg x_\epsilon$  and both of them are predefined boundary parameters, we then apply following feedback parameters  $k_1 = -y_e \omega / x_e$ ,  $k_2 v_r > 0$  and  $k_3 > 0$ , which generate  $\dot{x}_e = 0$  and

$$\begin{bmatrix} \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} 0 & v_r \\ -k_2 & -k_3 \end{bmatrix} \begin{bmatrix} y_e \\ \theta_e \end{bmatrix}$$

It is easy to verify that as soon as  $v_r \neq 0$ ,  $y_e$  and  $\theta_e$  are exponential convergent. Thus we can hold  $x_e$ , converge  $y_e$  and  $\theta_e$  first and switch back to RH-LF then.

We now consider the terminal state region of the RH controller. Note that the input constraints must be satisfied when performing the RH-LF, i.e.  $u \in U$ ,

$$\begin{cases} -v_{\min} \leq v \leq v_{\max} \\ -\omega_{\min} \leq \omega \leq \omega_{\max} \end{cases} \quad (17)$$

Substituting (14) into (17) yields the terminal stated region

$$\begin{cases} -v_{\min} \leq v_r(t) \cos \theta_e + k_1 x_e \leq v_{\max} \\ -\omega_{\min} \leq \omega_r + k_2 y_e + k_3 \theta_e \leq \omega_{\max} \end{cases} \quad (18)$$

Since  $-\pi < \theta \leq \pi$ , the terminal state region can be defined as

$$\begin{cases} \frac{-v_{\min} - v_r \cos \theta_e}{k_1} \leq x_e \leq \frac{v_{\max} - v_r \cos \theta_e}{k_1} \\ \frac{-\omega_{\min} - \omega_r + k_3 \pi}{k_2} \leq y_e \leq \frac{\omega_{\max} - \omega_r - k_3 \pi}{k_2} \\ -\pi < \theta \leq \pi \end{cases} \quad (19)$$

*Remark 1:* From (14), it is easy to verify if the RH-LF controller guarantees asymptotical stability, i.e.  $q_e(t)$  go to zero as time  $t$  goes to infinity, the robot velocity will finally converge to the reference velocity to maintain the desired geometric relationship with the leaders.

In the terminal state region constraints (19), the condition of the orientation error is easy to satisfy. The robot reference velocity should be properly designed to meet the robot kinematic limitations. Otherwise the terminal state region does not exist, and the RH-LF controller cannot be applied.

### 3.3 Feasible RH solutions and suboptimality

In the above discussions, only stability properties of the proposed control approach are concerned, while the optimal control is not considered. It is stated in Scokaert (1999) that the feasible RH controllers can also guarantee the system suboptimal properties as well. We now discuss the suboptimal conditions of the proposed RH-LF controller.

Define  $V(t, q_e(t)) = g(q_e(t+T)) + \int_t^{t+T} L(\tau, q_e(\tau), u(\tau)) d\tau$  as the RH value function. From (12) and (13), it is easy to verify

$$V(t, q_e(t)) \geq \alpha(\|q_e(t)\|) \quad (20)$$

where  $\alpha$  is a  $K$ -function. From (20), Condition 1 of *Theorem 1* in Scokaert (1999) holds. The proof of Condition 2 of *Theorem 1* in Scokaert (1999) is too complex and the interested readers should refer to Scokaert (1999). As a result, the robot velocities converge to their reference values finally, and hence,

$$\lim_{t \rightarrow \infty} (\|u(t) - u_r(t)\|) = k_1 \|x_e(t)\| + k_2 \|y_e(t)\| + k_3 \|\theta_e(t)\| \leq \sigma(\|q_e(t)\|) \quad (21)$$

where  $\sigma$  is a  $K$ -function. Eq. (21) demonstrates the validity of the Condition 3 of *Theorem 1* in Scokaert (1999). Therefore, the proposed RH-LF controller guarantees the asymptotic convergence of the formation tracking system as well as the suboptimal solutions.

## 4. SIMULATIONS

In this section, simulations of using the RH-LF controller are presented. The simulation was performed on the formation task as shown in Fig. 5, where the three robots were required to switch from a collateral formation to a sequential formation, bypass the garden corner and then reform back to a collateral formation. Note that when applying leader-follower strategy here, the control of the relative orientation deviation between every follower and their respective leaders are especially important. A precise formation geometric shape must be maintained when the robot group switches from one formation to the other.

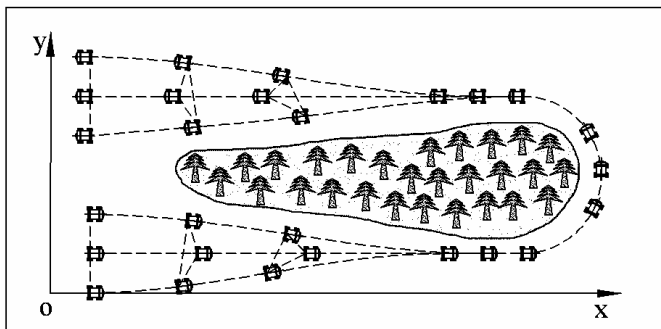


Fig. 5. The target formation tasks

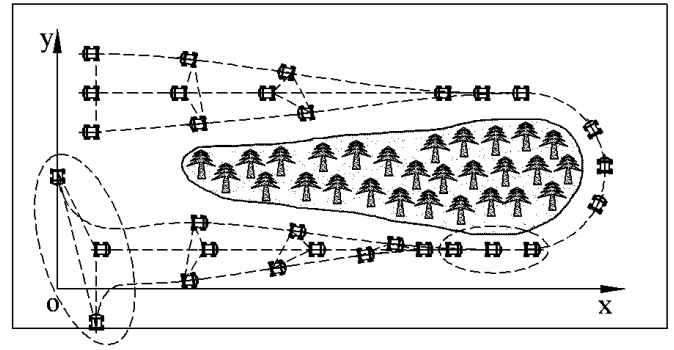


Fig. 6. Simulation results applying RH-LF controller. The straight path has a length of 9m, and the circular path has a radius of 2m. Every robot is subject to following velocity constraints

$$\begin{cases} -0.5 \leq v \leq 0.6(m/s) \\ -0.5 \leq \omega \leq 0.5(rad/s) \end{cases} \quad (22)$$

The constraints must be satisfied when designing the reference formation motions.

In the simulation, it was assumed that robots were randomly placed at the beginning with the initial postures of  $(1m, 1m, 0)$ ,  $(0m, 3m, 1.5\pi)$ ,  $(1m, 0m, 0.5\pi)$ .

The objective is to control the group to track the reference formation shapes. Weight factors  $q_1, q_2$  and  $q_3$  were chosen as  $q_1 = q_2 = 0.1$ ,  $q_3 = 0.05$ . To satisfy (16), the feedback parameters were chosen as

$$\begin{aligned} k_1 &= 0.1 + q_1 + q_2 y_e^2 / x_e^2 \\ k_2 &= 0.1 + v_r \sin \theta_e / \theta_e \\ k_3 &= 0.1 + q_3 \end{aligned} \quad (23)$$

Simulation results applying the proposed RH-LF controller are shown in Fig. 6. The initial postures of robots are depicted by the ellipse on the left, while the ellipse on the right depicts the robots postures when the group finished tracking the first path.

Fig. 7 shows the formation geometric condition for the leader-follower pairs. The upper figure illustrates the *SBOC* condition between  $R_1$  and  $R_2$ , where separation-12, bearing-12 and orientation deviation-12 denote the separation, bearing and orientation deviation conditions between  $R_1$  and  $R_2$ . The below figure illustrates the *SSOC* condition for  $R_1, R_2$  and  $R_3$ , which shows the separation between robot pair  $R_1$  and  $R_3, R_2$  and  $R_3$ , and the orientation deviation between  $R_1$  and  $R_3$ . The solid lines of Fig. 7 show the leader-follower formation geometric conditions when applying the proposed RH-LF controller, and the dashed lines represent the desired formation geometric relationships. Fig. 8 shows the errors between the desired and RH-LF applied formation parameters. From these figures we conclude that the desired leader-follower formation objective can be achieved.

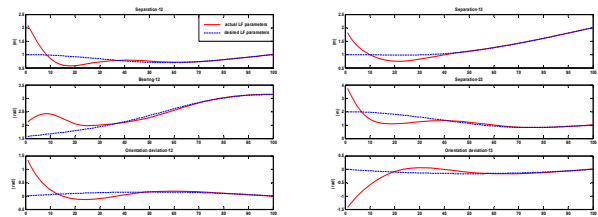


Fig. 7 *SBOC* & *SSOC* formation conditions

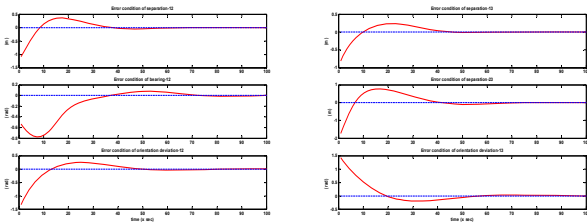


Fig. 8. RH-LF formation tracking errors.

## 5. CONCLUSIONS

This paper presents a RH-LF controller for multiple non-holonomic mobile robots in formations. A new leader-follower strategy concerning separation, bearing and orientation deviation between the leader and follower, is discussed first. Then both controllers of *SBOC* and *SSOC* are proposed with investigations of formation error tracking conditions for each robot in the team. It is shown that the proposed RH-LF controller guarantees asymptotic stability of the robot system. The proposed control strategy also ensures that the velocity input generates suboptimal solutions to the formation tracking system. Finally, simulations are performed to demonstrate the effectiveness of the proposed RH-LF formation controller.

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