Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Real-time 2DoF Control of a Quadruple Tank System with Integral Action

M. Herceg [*], **J. Mikleš** [*], **M. Fikar** [*], **M. Kvasnica** [*], **Ľ. Čirka** [*]

[*] *Institute of Information Engineering, Automation and Mathematics, Slovak University of Technology, 81237 Bratislava, Slovakia, (fax: +421-2-52496469; email: martin.herceg@stuba.sk)*

Abstract: In this paper we aim at polynomial synthesis of a two-degree-of-freedom (2DoF) controller with integral action for a coupled tank system using the Polynomial Toolbox. The main advantage of the proposed 2DoF controller is in computational savings compared to classical feedback design. It will be shown that the procedure leads to solving of two spectral factorizations where the closed loop poles are assigned with respect to given optimization weights. This makes the controller easy to tune. Moreover, experimental results show that desired tracking performance is fulfilled.

Keywords: two-degree-of-freedom controller, polynomial methods, optimal control, integral action

1. INTRODUCTION

Linear techniques are undoubtedly well-established control approaches with strong theoretical background. Their application is not only limited to linear systems but they can be extended to nonlinear systems as well. The most natural way of such extension is the linearization of the nonlinear system around the operating point. If the model is obtained in this way, then the validity of the linear model is limited to some neighborhood of the operating point and the control policy has to take this into account. There are also another important aspects to be considered. Possible external disturbances as well as measurement noise might cause unwanted performance loss. To be able to cope with these issues, traditionally, the controller must provide integral action.

One method of including integral action is to incorporate derivatives of control variables into the objective functional of state optimal control problem instead of standard control variables. This approach has been investigated by Dostál et al. [1994] and it leads to an optimal control synthesis where the augmented state vector contains $2n$ elements. Another option of embedding the integration property to control is to include integrators only on the states of interest. By this way the dimension of the augmented state becomes $n + n_d$ where n_d is the number of desired integrators. Since the dimension of the augmented state is reduced, this approach offers potential computational savings. Moreover, as pointed by Kučera [1981], if the polynomial algebra is applied, the approximate total amount of operations is $30(n + n_d)^3$ which contrasts to $75(n + n_d)^3$ operations estimated for solving Riccati equations. Therefore, the main result of this paper is a polynomial design of computationally effective state optimal controller with observer which includes integral action.

In this paper we aim at 2DoF controller design and its implementation to a coupled tank system, which exhibits nonlinear behavior. Several publications are devoted to this narrow area, more specifically contributed by Grimble [2002, 1988], where theoretical background is explained in more details and close relations with $\mathcal{H}_2$ optimization are given.

Whilst the theory of polynomial 2DoF control is sufficiently covered in the literature, there is still lack of practical implementations. Some examples include adaptive polynomial control [Bobál et al., 2004, Kubalčík and Bobál, 2002, Kubalčík et al., 2005], polynomial control of time-delay systems [Dostál et al., 2002] or $\mathcal{H}_2$ control [Mikleš et al., 2005]. In this contribution we implement the 2DoF controller with integral action in Matlab's Real Time Workshop (RTW) environment and furthermore, the overall procedure will be tested on a laboratory equipment consisting of a quadruple coupled hydraulic tank system.

The paper has the following structure. In the second section a description of the physical setup for the pneumatic-hydraulic system is given and moreover, modeling issues are discussed. The section three focuses on polynomial synthesis of the 2DoF controller with integral action. Finally, the controller synthesis using Polynomial Toolbox and its on-line implementation using the Real Time Workshop is given in section four and experimental results are presented in the section five.

2. DESCRIPTION OF THE PLANT

2.1 Physical setup

The plant consist of interconnected four water tanks, one storage reservoir and two pumps. A front view of the plant is depicted in Fig.1 whereas the sketch of the system is illustrated in Fig. 2. In between the water tanks there is an air tank with orifice at the bottom. It represents the

Figure 1. A front view of the quadruple tank system.

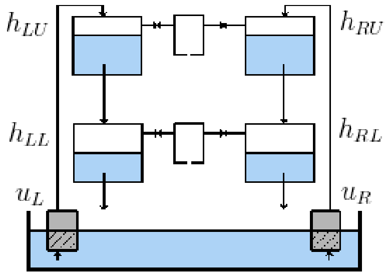

Figure 2. Scheme of the coupled tank system.

air-path between the tanks because the air is pushed out of this tank if the levels increase. Manipulating signal is a voltage supplied to the left and right pump located at the bottom tank which serves as a storage reservoir. Measured signals are the levels of the water in left lower and right lower tanks, provided by a differential-pressure indicator. Upper tanks do not have a measurement sensor. Water can be pumped only into a tank located just above that pump since there is no water connection between the left and right side. Because air spaces above the water levels are connected, this influences the dynamical behavior of the system. If required, this influence can be totally turned off by closing a valve in the air-path.

More detailed construction issues, as well as insights of the signal properties are addressed in Macháček et al. [2004]. Practically the user communicates with the plant via a personal computer with installed Matlab's RTW and an input/output (I/O) card. Signals from the I/O card are transferred to a control unit of the plant which has two inputs and two outputs.

2.2 Nonlinear model of the plant

General model of the quadruple tank system as depicted in Fig. 2 can be derived from the mass and energy conservation laws. In the equations we will refer to the locations of the respective tanks using the following subscript notation:

LU = left upper   RU = right upper
LL = left lower   RL = right lower.

The mathematical model was inferred by Macháček et al. [2005] and we will use only the final version. We denote by h the level of the liquid in the tank, h_{max} is the maximum allowed tank level, p corresponds to the pressure above the level of the liquid, p_A is the ambient pressure, ρ the mass density of the liquid, T_A the ambient temperature, S the cross-section area of the tank, S_0 the cross-section area of the air orifice, V_0 the volume of the air tank, g the acceleration of gravity, k the discharge coefficient, k_0 air discharge coefficient and R is the gas constant.

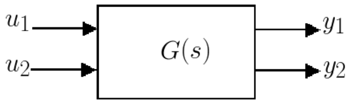

Figure 3. The plant is identified in a continuous time domain as a linear system with two inputs two outputs.

The time derivatives are given as

$$
\begin{aligned}
\dot{h}_{LU} &= -\frac{k_L}{\rho S}\sqrt{h_{LU}\rho g + p_H - p_L} + \frac{1}{\rho S}Q_L \\
\dot{h}_{LL} &= \frac{k_L}{\rho S}\sqrt{h_{LU}\rho g + p_H - p_L} - \frac{k_L}{\rho S}\sqrt{h_{LL}\rho g + p_L - p_A} \\
\dot{h}_{RU} &= -\frac{k_R}{\rho S}\sqrt{h_{RU}\rho g + p_H - p_L} + \frac{1}{\rho S}Q_R \\
\dot{h}_{LL} &= \frac{k_R}{\rho S}\sqrt{h_{RU}\rho g + p_H - p_L} - \frac{k_R}{\rho S}\sqrt{h_{RL}\rho g + p_L - p_A} \\
\dot{p}_H &= \frac{p_H S}{\psi_H}\left(\dot{h}_{LU} + \dot{h}_{RU}\right) - \frac{k_0 S_0 R T_A (p_H - p_A)}{\psi_H} \\
\dot{p}_L &= \frac{p_L S}{\psi_L}\left(\dot{h}_{LL} + \dot{h}_{RL}\right) - \frac{k_0 S_0 R T_A (p_L - p_A)}{\psi_L}
\end{aligned}
$$

where

$$
\begin{aligned}
\psi_H &= 2Sh_{max} - Sh_{LU} - Sh_{RU} + V_0 \\
\psi_L &= 2Sh_{max} - Sh_{LL} - Sh_{RL} + V_0
\end{aligned}
$$

The manipulating variables Q_R and Q_L are the liquid flows generated by the pumps and they depend polynomially on the input voltage signals u_R and u_L. Measured signals are the levels of the liquid in the bottom tanks and their conversion to voltage outputs y_L and y_R is an affine function. To avoid possible numerical problems, input variables u_L, u_R and output variables y_L, y_R are normalized in the scale [0, 10] V.

2.3 Linearized model of the plant

Since the full model of the plant exhibits several nonlinearities and some parameters of the plant are tedious to obtain, we use an approximate linearized model by employing continuous identification. Here the structure and dimensions of nonlinear model will be exploited as a hot start for recursive least square method Kulhavý and Kárný [1984]. An effective tool for performing continuous identification is IDTOOL Toolbox [Čirka et al., 2006] for Matlab which implements an improved version of the LDDIF algorithm, which was originally proposed by Kulhavý and Kárný [1984].

The plant is considered as a two input two output (TITO) system, as illustrated in Fig. 3, which can be modeled by the following transfer function

$$
G(s) = \begin{bmatrix} G_1(s) & G_2(s) \\ G_3(s) & G_4(s) \end{bmatrix}. \quad (1)
$$

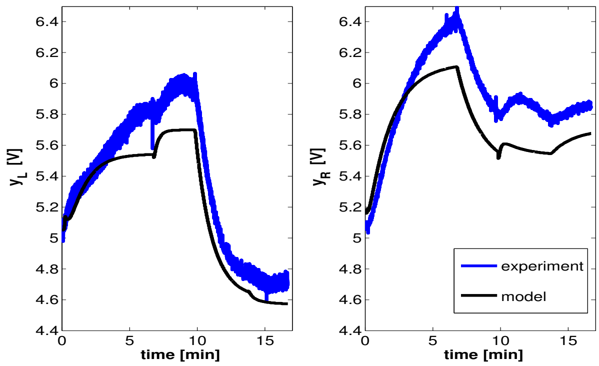

Figure 4. Comparison between the linearized model and the set of measurement data.

The identification is performed around the operating point

$$u_L^s = 4\,V,\ u_R^s = 4\,V,\ y_L^s = 5.05\,V,\ y_R^s = 5.16\,V \qquad (2)$$

which corresponds to the water levels approximately in the middle. Consequently, by defining deviation variables as

$$u = \begin{pmatrix} u_L - u_L^s \\ u_R - u_R^s \end{pmatrix},\ y = \begin{pmatrix} y_L - y_L^s \\ y_R - y_R^s \end{pmatrix} \qquad (3)$$

the linearized model obtained by IDTOOL can be expressed using the partial transfer functions as

$$G_1(s) = \frac{1.9}{2.7s^3 + 62.8s^2 + 73.9s + 1} \qquad (4a)$$

$$G_2(s) = \frac{1.2s - 0.31}{2.1s^3 + 16.3s^2 + 19.8s + 1} \qquad (4b)$$

$$G_3(s) = \frac{0.66s - 0.22}{1.6s^3 + 12.3s^2 + 11.7s + 1} \qquad (4c)$$

$$G_4(s) = \frac{1.77}{2.2s^3 + 33.0s^2 + 104.0s + 1} \qquad (4d)$$

2.4 Model validation

In order to validate the effect of linearization we compared the differences between the linearized model and the collection of measurements data. As it can be seen from Fig. 4, the approximated linear model posses similar dynamics as the original nonlinear plant but the steady state gain is not exactly captured. This is the main drawback of the linearization approach because the approximated model is valid only near the selected operating point. However, the controller must provide satisfactory tracking properties despite these inaccuracies. To attain this goal we embed an integrator in the closed loop and in the next section the 2DoF controller will be derived.

3. 2DOF CONTROL WITH INTEGRAL ACTION

3.1 Derivation of the 2DoF controller

In this section, the derivation of the 2DoF controller is based on the idea of the state-space solutions to the $\mathcal{H}_2$ problem, originally published by Doyle et al. [1989] and the polynomial matrix solutions, given by Hunt et al. [1991]. It was shown by Kwakernaak [2000] that under mild assumptions the $\mathcal{H}_2$ problem can be reduced to an LQG setup. Here we formulate similar assumptions under which the problem can be solved by two spectral factorizations.

We assume the input-state-output model of the plant, which corresponds to the real plant depicted in Fig.2, of the form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 v(t) + Bu(t) \\ z(t) &= C_1 x(t) + D_{11} v(t) + D_{12} u(t) \\ y(t) &= Cx(t) + D_{21} v(t) + D_{22} u(t) \end{aligned} \qquad (5)$$

where A, B, B_1, C, C_1, D_{12}, D_{21} are matrices of appropriate dimensions and $x(t)$ is the state vector, $u(t)$ denotes a vector of manipulated inputs, $v(t)$ is an exogenous input signal, not manipulated by the controller, $y(t)$ denotes the measurable outputs, and $z(t)$ corresponds to vector of performance variables. Since the controller must provide integral action, we embed this property by augmenting the state space model by adding new states

$$\dot{\bar{x}} = w - y \qquad (6)$$

which add an integrator on the output errors. Here, w denotes the reference (setpoint) signal. By doing this, the augmented state becomes $\tilde{x} = [x,\ \bar{x}]^T$ and the general input-state-output model of the plant is given by

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{A}\tilde{x}(t) + \tilde{B}_1 v(t) + \tilde{B}u(t), \\ \tilde{z}(t) &= \tilde{C}_1 x(t) + \tilde{D}_{11} v(t) + \tilde{D}_{12} u(t), \\ y(t) &= \tilde{C}\tilde{x}(t) + \tilde{D}_{21} v(t) + \tilde{D}_{22} u(t). \end{aligned} \qquad (7)$$

where the matrices are given in a compact form

$$\left[\begin{array}{c|c|c} \tilde{A} & \tilde{B}_1 & \tilde{B} \\ \hline \tilde{C}_1 & \tilde{D}_{11} & \tilde{D}_{12} \\ \hline \tilde{C} & \tilde{D}_{21} & \tilde{D}_{22} \end{array}\right] = \left[\begin{array}{cc|ccc|c} A & 0 & B_1 & 0 & 0 & B \\ -C & 0 & 0 & 0 & 0 & 0 \\ \hline Q^{0.5} & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{Q}^{0.5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & R^{0.5} \\ \hline C & 0 & 0 & 0 & D_{21} & 0 \end{array}\right] \qquad (8)$$

Here, Q, $\bar{Q}$, and R are performance tuning matrices which correspond to the vector of performance variables $\tilde{z}(t)$ in (7). The 2DoF design procedure then requires following assumptions to hold:

(A1) The system $\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}\tilde{u}(t)$ is stabilizable.
(A2) The system $\dot{x}(t) = Ax(t) + Bu(t)$, $z(t) = C_1 x(t)$ is detectable.
(A3) The system $\dot{x}(t) = Ax(t) + B_1 v(t)$, $y(t) = Cx(t)$ is stabilizable and detectable.
(A4) The matrix $\tilde{D}_{12}^T \tilde{D}_{12}$ is nonsingular.
(A5) The matrix $D_{21} D_{21}^T$ is nonsingular.
(A6) The equalities $\tilde{D}_{12}^T C_1 = 0$ and $B_1 D_{21}^T = 0$ are satisfied.

Define matrix transfer functions as

$$(sI - \tilde{A})^{-1}\tilde{B} = \tilde{B}_{Rs}(s)\tilde{A}_R^{-1}(s) \qquad (9)$$

where $\tilde{A}_R$, $\tilde{B}_{Rs}$ are right coprime polynomial matrices of $\tilde{A}$, $\tilde{B}$, with $\tilde{A}_R$ being column reduced, and

$$C(sI - A)^{-1} = A_L^{-1}(s) B_{Ls}(s) \qquad (10)$$

with A_L, B_{Ls} being left coprime polynomial matrices of C, A where A_L is row reduced. Then the following theorem gives expressions for the 2DoF controller.

Theorem 1. If assumptions A1–A6 are met, the 2DoF control law as depicted in Fig. 5 exists and is given by

$$u = R_b(s)y + R_f(s)w \qquad (11)$$

where the feedback and feedforward transfer functions can be expressed, respectively, as

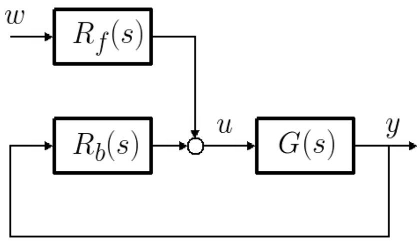

Figure 5. 2DoF control configuration.

$$R_b(s) = -\tilde{K}(sI - \tilde{A}_k)^{-1}B_b \quad (12a)$$
$$R_f(s) = -\tilde{K}(sI - \tilde{A}_k)^{-1}B_f. \quad (12b)$$

where the matrices in equations (12) are constructed as

$$\tilde{K} = \begin{bmatrix} K & \bar{K} \end{bmatrix}, \qquad \tilde{A}_k = \begin{bmatrix} A - LC - BK & -B\bar{K} \\ 0 & 0 \end{bmatrix},$$
$$B_b = \begin{bmatrix} L \\ -I \end{bmatrix}, \qquad B_f = \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

The matrix $\tilde{K}$ is determined via

$$\tilde{K} = \tilde{X}_L^{-1}\tilde{Y}_L, \quad (13)$$

and $\tilde{X}_L$, $\tilde{Y}_L$ are a solution of the Diophantine equation

$$\tilde{X}_L\tilde{A}_R(s) + \tilde{Y}_L\tilde{B}_{Rs}(s) = \tilde{F}_R(s) \quad (14)$$

where the term $\tilde{F}_R(s)$ is obtained by solving the following spectral factorization

$$\tilde{F}_R^T(-s)\tilde{F}_R(s) = \tilde{A}_R^T(-s)(\tilde{D}_{12}\tilde{D}_{12}^T)\tilde{A}_R(s) + \tilde{B}_{Rs}^T(-s)(\tilde{C}_1^T\tilde{C}_1)\tilde{B}_{Rs}(s). \quad (15)$$

Subsequently, $\bar{K}$ is determined by decomposing the matrix $\tilde{K}$ as

$$\tilde{K} = [K\ \bar{K}] \quad (16)$$

such that the parts K, $\bar{K}$ correspond to x and $\bar{x}$ respectively. The observer gain L is obtained from

$$L = Y_R X_R^{-1} \quad (17)$$

where X_R, Y_R are a solution of the Diophantine equation

$$A_L(s)X_R + B_{Ls}Y_R = O_L(s). \quad (18)$$

and the spectral factor $O_L(s)$ is given as a solution of

$$O_L(s)O_L^T(-s) = A_L(s)(D_{21}D_{21}^T)A_L^T(-s) + B_{Ls}(s)(B_1B_1^T)B_{Ls}^T(-s). \quad (19)$$

Proof: Since assumption A1 is satisfied and by now $w = 0$ then there exist a control law

$$u = -\tilde{K}\tilde{x} \quad (20)$$

which stabilizes the closed loop system

$$\dot{\tilde{x}} = (\tilde{A} - \tilde{B}\tilde{K})\tilde{x}$$
$$\begin{pmatrix} \dot{x} \\ \dot{\bar{x}} \end{pmatrix} = \begin{pmatrix} A - BK & -B\bar{K} \\ -C & 0 \end{pmatrix}\begin{pmatrix} x \\ \bar{x} \end{pmatrix} \quad (21)$$

and minimizes the LQ criterium

$$I = \int_0^\infty (\tilde{x}^T\tilde{C}_1^T\tilde{C}_1\tilde{x} + u^T\tilde{D}_{12}\tilde{D}_{12}^T u)\,\mathrm{d}t \quad (22)$$

where A6 eliminates the cross product and A4 underlines feasibility. Following Kučera [1981] and Mikleš and Fikar [2007] the matrix $\tilde{K}$ can be obtained from (13), (14) and leads to spectral factorization (15) which places the closed loop poles optimally in the left half plane of the imaginary axis. Decomposing the matrix $\tilde{K}$ as in (16) and considering $w \neq 0$ from the closed loop system (21) we obtain the relation between plant outputs and reference signal w given as

$$\begin{pmatrix} \dot{x} \\ \dot{\bar{x}} \end{pmatrix} = \begin{pmatrix} A - BK & -B\bar{K} \\ -C & 0 \end{pmatrix}\begin{pmatrix} x \\ \bar{x} \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} w \quad (23a)$$
$$y = (C\ \ 0)\begin{pmatrix} x \\ \bar{x} \end{pmatrix} \quad (23b)$$

with gain equal the identity matrix. Hence, the plant outputs will always reach their respective setpoints such that the transition energy given by (22) is minimized. Because a perfect state-feedback (20) is not possible, a Kalman filter is employed

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}). \quad (24)$$

and assumptions A2, A3, and A5 presume that such matrix L exists. Consequently, as described in Kučera [1981] and Mikleš and Fikar [2007] the observer gain L can be determined from (17), (18) which leads to spectral factorization (19). The output feedback law can be now expressed as

$$u = -K\hat{x} - \bar{K}\bar{x} \quad (25)$$

and by plugging (25) into (24) we obtain

$$\dot{\hat{x}} = (A - LC - BK)\hat{x} - B\bar{K}\bar{x} + Ly. \quad (26)$$

Due to the definition of $\bar{x}$ in (6), the closed loop system (26) can be written in the form

$$\begin{pmatrix} \dot{\hat{x}} \\ \dot{\bar{x}} \end{pmatrix} = \begin{pmatrix} A - LC - BK & -B\bar{K} \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \hat{x} \\ \bar{x} \end{pmatrix} + \begin{pmatrix} L \\ -I \end{pmatrix} y + \begin{pmatrix} 0 \\ I \end{pmatrix} w \quad (27)$$

which can be further simplified to

$$\dot{\xi} = A_k\xi + B_b y + B_f w \quad (28a)$$
$$u = -\tilde{K}\xi \quad (28b)$$

where $\xi = [\hat{x},\ \bar{x}]^T$ and (12) are the transfer functions of the system (28). □

Remark 2. The augmentation form (7) determines the number of integration terms to be included in the 2DoF controller. By this way the dimension of the augmented state becomes $n + n_y$, where n is the dimension of the state x and n_y usually stands for the number of measurable outputs, but it can be modified to only include outputs of interest. Therefore, the proposed procedure is computationally more efficient.

In summary, the 2DoF control problem can be recast as a pole placement problem in which the closed loop dynamics is determined by two spectral factors $\tilde{F}_R(s)$ and $O_L(s)$. Matrices $\tilde{D}_{12}$, $\tilde{C}_1$, D_{21}, and B_1 can be considered as weighing parameters and their relation to plant outputs will be explained in the next section.

3.2 Tuning the 2DoF controller

Equation (15), when represented in a state space form, is associated with a solution to the algebraic Riccati equation which stems from minimizing the infinite horizon control problem (22) and is equivalent to

$$I = \int_0^\infty (x^TQx + \bar{x}^T\bar{Q}\bar{x} + u^TRu)\,\mathrm{d}t \quad (29)$$

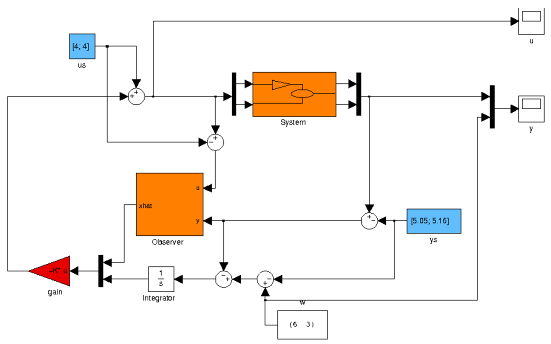

Figure 6. Control loop in Simulink.

where, besides the states x and inputs u, also the integral of output errors $\bar{x}$ is being penalized. By an appropriate choice of Q, $\bar{Q}$ and R one can influence the behavior of the 2DoF controller with straightforward consequences on the outputs. For instance if one requires fast tracking, then the $\bar{Q}$ weight for output errors shall be increased and R lowered.

Selection of the matrices D_{21} and B_1 can be similarly related to the theory of Kalman filtering whereas in this case these matrices account for the spectral intensities of the white noise signal v.

4. CONTROLLER SETUP

All polynomial calculations related to this work have been performed by the Polynomial Toolbox for MATLAB, available at `www.polyx.com`. An excellent demonstration of the recent version is given for instance given by Hromčík and Kučera [2007]. The state-feedback gain as well $\tilde{K}$ as the observer gain L, which are both parts of the controller (11), can be found e.g. by following the Matlab scripts introduced in [Mikleš and Fikar, 2007, Mikleš et al., 2005, 2006]. In the second step, the control loop has to be drawn in Simulink, for instance as illustrated in Fig. 6. The subroutines under the block "observer" are coded as given in (24) and the RTW blocks are hidden inside the block named "system". Subsequently, in order to control the plant in real-time, the I/O card has to be properly configured and connected to the device. An excellent tutorial for this experimental setup is given by Dušek et al. [2006].

Before proceeding to the last section with experimental results, we shall first test the 2DoF controller in a simulation scenario. Since the controller is based on a linearized model (4), the scenario will start from the steady state point (2). Reference signal w is kept piecewise constant and it corresponds to dashed lines in Fig. 7. Simulation results indicate that the desired tracking of the varying reference is achieved and controller is suitable for real-time application.

5. EXPERIMENTAL RESULTS

We have applied the 2DoF controller with integral action to a real coupled tank system using the RTW environment. The sampling time was selected as one second and initial conditions for the observer were generated randomly. The acquired experimental data are depicted in Fig. 8.

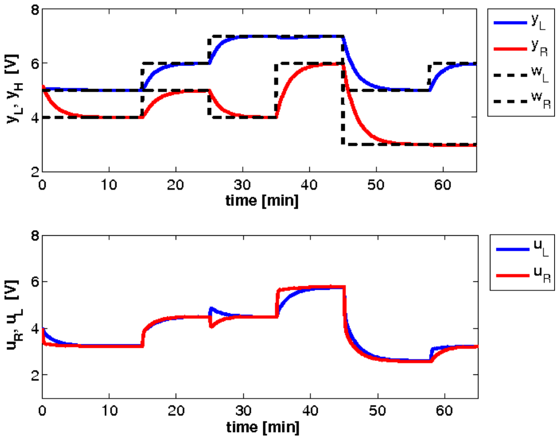

Figure 7. Simulation results for the 2DoF controller.

Although the experiment starts from a steady state point, one can clearly notice that in the first 10 minutes the trajectories differ significantly from the setpoints. This is due to the initialization phase of the observer where the estimated states recover from the startup error. After the initial period of ten minutes, the observer provides satisfactory estimates of the states and desired tracking is achieved. In the period between 40 and 50 minutes after the start the plant is manually driven far from the linearized point, which results in noticeable overshoots in the control profile. The integral action of the controller compensates these effects and the setpoints are reached after 5 minutes. Comparing to simulation results in Fig. 7 one can conclude that although integration property is present, the expected control performance is valid only for some neighborhood of the operating point. If the setpoints are driven away, the controller spends more effort to compensate the model/plant mismatch as to tracking errors and the overall performance degrades. Thus, a possible approach how to achieve better control performance is to instantaneously supply the information about the process model using the IDTOOL Toolbox [Čirka et al., 2006] and recalculating the gains $\tilde{K}$, L on-line. Since the polynomial approach is computationally effective, the on-line computational demand would not be problematic.

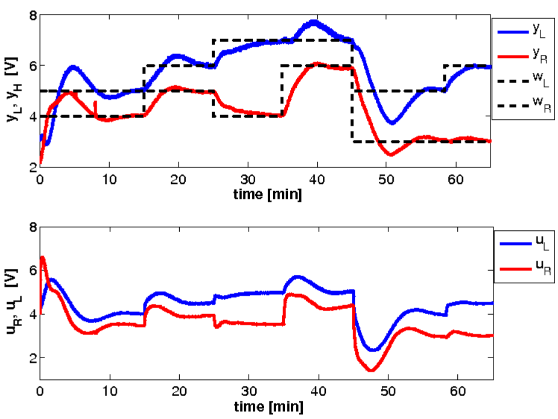

Figure 8. Experimental results for the 2DoF controller.

6. CONCLUSIONS

In this paper we have shown 2DoF control design with integral action applied to the quadruple tank system within the RTW environment. The synthesis is performed via the polynomial approach where the problem of adding integrator poles is efficiently treated using the Polynomial Toolbox. We have applied the identification toolbox IDTOOL to obtain a linearized model of the quadruple tank process and we have compared this model with the measured data. Although the gains in the model are not exact, the selected 2DoF structure is able to cope with this mismatch. Experimental results obtained with 2DoF controller show that adding the integration property provides satisfactory treatment of the model uncertainty while suppressing the measurement noise and moreover, satisfactory reference tracking is achieved. Despite this fact, further improvements of the proposed 2DoF control structure are possible which can help increase the performance for different operating regions.

ACKNOWLEDGMENT

The authors are pleased to acknowledge the financial support of the Scientific Grant Agency of the Slovak Republic under grants No. 1/3081/06 and 1/4055/07, and within the framework of the European Social Fund (PhD Students for Modern Industrial Automation in SR, JPD 3 2005/NP1-047, No 13120200115).

REFERENCES

V. Bobál, P. Chalupa, and P. Dostál. Application of polynomial methods for design of adaptive decentralized control. In *Proc. of the IEEE Int. Symposium on Comp. Aided Control Systems Design*, pages 115–120, Taipei, Taiwan, September 2004.

P. Dostál, A. Mészáros, and J. Mikleš. A modified LQ tracking problem. *Journal of Electrical Engineering*, 45 (4):129–133, 1994.

P. Dostál, V. Bobál, and M. Sysel. Design of controllers for integrating and unstable time delay systems using polynomial method. In *Proc. of the American Control Conference*, volume 4, pages 2773–2778, Anchorage, USA, May 2002.

J.C. Doyle, K. Glover, P.P Khargonekar, and B.A. Francis. State-space solutions to standard $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems. *IEEE Trans. on Automatic Control*, 34:831–847, August 1989.

F. Dušek, D. Honc, and L. Havlíček. Jednoduché řízení TITO soustavy - praktické aspekty. In *Proc. of 7th Int. Scientific – Technical Conference Process Control*, Kouty nad Desnou, Czech Republic, June 2006. R061.pdf (in czech).

M.J. Grimble. Polynomial matrix solution of $\mathcal{H}_2$ optimal control problem for state-space systems. *Optimal Control Applications and Methods*, 23:59–89, 2002.

M.J. Grimble. Two-degrees of Freedom Feedback and Feedforward Optimal Control of Multivariable Stochastic Systems. *Automatica*, 24(6):809–817, 1988.

M. Hromčík and V. Kučera. Polynomial Toolbox 3.0 and an $\mathcal{H}_2$ optimal control case study. In *Proc. of the 16th Int. Conference on Process Control*, Štrbské Pleso, Slovakia, June 11-14 2007. paper 123f.pdf.

K.J. Hunt, M. Šebek, and V. Kučera. Polynomial approach to $\mathcal{H}_2$ -optimal control: themultivariable standard problem. In *Proc. of the 30th IEEE Conf. on Decision and Control*, volume 2, pages 1261–1266, Brighton, England, December 1991.

M. Kubalčík and V. Bobál. Adaptive control of coupled drives apparatus based on polynomial theory. In *Proc. of the Int. Conference on Control Application*, volume 1, pages 594–599, Glasgow, Scottland, September 2002.

M. Kubalčík, V. Bobál, and P. Chalupa. Adaptive control of coupled drives apparatus based on polynomial theory. In *Proc. of the IFAC 16th World Congress*, Prague, Czech Republic, 2005.

R. Kulhavý and M. Kárný. Tracking of slowly varying parameters by directional forgetting. In *Proceedings of the 9th IFAC World Congres*, Budapest, Hungary, 1984.

V. Kučera. New results in state estimation and regulation. *Automatica*, 17(5):745–748, 1981.

H. Kwakernaak. $\mathcal{H}_2$-optimization: Theory and application to robust design control. In *Proceedings of the 2nd EUROPOLY Workshop, IFAC Symposium on Robust Control Design*, Prague, Czech Republic, 21–23 June 2000.

J. Macháček, F. Dušek, and D. Honc. Laboratory multivariable hydraulic-pneumatic process. In *International Carpathian Control Conference*, volume I, pages 133–138, Zakopane, Poland, May 2004.

J. Macháček, D. Honc, and F. Dušek. Nonlinear and linearized mathematical-physical model of hydraulic-pneumatic process. In *Proc. of the 15th Int. Conference on Process Control*, page 059f.pdf, Štrbské pleso, Slovakia, June 7–10 2005.

J. Mikleš and M. Fikar. *Process Modelling, Identification, and Control*. Springer Verlag, Berlin Heidelberg, 2007.

J. Mikleš, M. Fikar, Ľ. Čirka, and L. Dermíšek. Implementation of $\mathcal{H}_2$ Optimal Controller with Integral Action. In M. H. Hamza, editor, *Proceedings of the 24th IASTED International Conference on Modelling, Identification, and Control*, pages 457–175.pdf, Innsbruck, Austria, 2005. ACTA Press.

J. Mikleš, Ľ. Čirka, and M. Fikar. $\mathcal{H}_2$ Optimal Controller with Integral Action for a Chemical Reactor. In *Proceedings of the 2006 IEEE International Conference on Control Applications*, pages 2127–2131, Munich, Germany, October 4-6 2006.

Ľ. Čirka, M. Fikar, and P. Petruš. IDTOOL 4.0 - A Dynamical System Identification Toolbox for MATLAB/Simulink. In *14th Annual Conference Proceedings: Technical Computing Prague 2006*, pages 29–29. The MathWorks, Inc. & HUMUSOFT s.r.o. & Institute of Chemical Technology in Prague, October 2006. Available at `http://www.kirp.chtf.stuba.sk/~cirka/idtool/?IDTOOL`.