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Dissipative Control for Singularly Perturbed Fuzzy Systems

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Abstract: Dissipative stabilizing state feedback controllers are investigated for singularly perturbed fuzzy systems. We derive sufficient conditions for the existence of (Q, S, R)-dissipative controllers for a class of nonlinear systems represented by T-S fuzzy model. Based on Lyapunov theory, the main results are cast in LMI formulation solvable by existing LMI solvers. We demonstrate that many existing technical papers are a specialized case of the general quadratic dissipative control problem involving singularly perturbed fuzzy systems. Finally the utility of the proposed method is illuminated by an example.

Keywords: Dissipative control, Singularly perturbed systems, TS fuzzy models, Linear matrix inequality (LMI)

1. INTRODUCTION

Over the decades, a large number of important results on stabilizations have been derived for TS fuzzy models, which rely on the existence of a common P to a quadratic Lyapunov function. Following the progress, the robust fuzzy control problems for uncertain fuzzy systems characterized by 2-norm structure have been the main topic of research [Tanaka and Wang [2001]]. In addition to stability and robust issues, performance of a control system also attracts attentions. Generally speaking, robust stability can be categorized into two areas: Stability robustness concerns stability problems of a control system subject to parameter uncertainties [Kang et al. [1998]-Kiriakidis [2001]]. Performance robustness focuses on performance problems such as H_∞ and/or H_2 while the system is under plant parametric variations [Tanaka et al. [1996]-Lo and Lin [2004]].

Many physical systems contains complex multiple time-scales dynamics due to presence of small parameters [Fridman [2001], Pan and Basar [1996]]. For a two-time-scale systems, both slow and fast dynamic behaviors arise and may lead to controller being ill-conditioning. To tackle such problems, multiple-time-scale processes are addressed in the framework of singularly perturbed systems which alleviates the ill-conditioning resulted from interaction of slow and fast dynamic modes.

The theory of dissipative systems has played an important role in systems and control problems, incorporating basic tools – bounded real lemma, passitivity lemma and circle criterion – into one general quadratic form known as supply rate. A seminal paper for dissipative control is introduced by Willems [Willems [1972a,b]] and subsequently generalized by Hill and Moylan [Hill and Moylan [1980]]. Literature on dissipative systems theory can be found on [Yuliar and James [1996]-Xie et al. [1998]] and references therein. Although there has been a considerable research results on dissipative nonlinear systems, the results all require solving a nonlinear partial differential equation, imposing difficulties in controller synthesis.

Although [Liu et al. [2005a]] studied an H_∞ control problem for nonlinear singularly perturbed systems where state feedback and static output feedback are investigated, an iterative LMI approach was utilized to search for the static output feedback gain.

Motivated by the work [Assawinchaichote and Nguang [2004a,b], Liu et al. [2005b,a]], we generalize the notion to dissipative control theory, investigating a unified approach to synthesize stabilizing controllers for nonlinear systems represented by TS fuzzy models. The proposed stabilizing controllers guarantee that the closed-loop systems are asymptotically stable with respect to a specified supply rate.

The organization of this paper is as follows. Section II rederives fuzzy version of dissipative theorem expressed in an LMI formulation. In section III, stabilization problems via state feedback control is addressed. One illustrative example is considered in Section IV and concluding remarks are made in Section V.

Notations: The symbol $\star$ is used for terms that are induced by symmetry. The notations Y_μ and $Y_{\mu\mu}$ stand

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for $\sum_{i=1}^{r}\mu_i Y_i$ and $\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j Y_{ij}$, where $\mu_i \geq 0$ and $\sum_{i=1}^{r}\mu_i = 1$. Also $\sigma x = \dot{x}(t)$ is for continuous-time systems. The indexes $i, j = 1, 2, \cdots, r$(number of fuzzy rules) are used throughout this paper.

2. PRELIMINARIES

2.1 General nonlinear systems

Consider the following nonlinear systems described by

$$\sigma x(\cdot) = A(x) + B(x)w(\cdot), \quad x(0) = x_0 \tag{1}$$

$$z(\cdot) = C(x) + D(x)w(\cdot) \tag{2}$$

where $\sigma x(\cdot) = \dot{x}(t)$ denotes a continuous-time system and $x(k)$ represents a discrete-time system. The vector $x \in R^n$ denotes the state vector, available for control purpose. The vector $z \in R^q$ stands for the controlled variables. The disturbance is signal $w \in R^d$ of $L_2[0,\infty)$ and the vector $u \in R^m$ represents the control input. We begin our analysis by making the following definitions [Xie et al. [1998]].

Definition:

A continuous system is called dissipative if

$$\int_0^T r(z(t), w(t))dt + \beta(x_0) \geq 0, \quad \forall w, \forall T \geq 0$$

for some finite function β. Furthermore, a continuous system is called strictly dissipative if for some sufficiently small scalar $\alpha > 0$

$$\int_0^T r(z(t), w(t))dt + \beta(x_0) \geq \alpha \int_0^T w(t)w(t)dt, \quad \forall w, \forall T \geq 0$$

The $r(z, w)$ function is known as the supply rate (or power function) where z and w are the system output and input respectively.

We consider a general supply rate of the following form (a.k.a. $(Q, S, R) - dissipativity$)

$$r_q(z, w) = \frac{1}{2}(w'Qw + 2w'Sz + z'Rz) \tag{3}$$

where Q, $R \leq 0$ are symmetric matrices. The theory of dissipative systems generalize the system theory, including the bounded real (small gain) theorem, passivity theorem, circle criterion, and sector bounded nonlinearity. To see this, a few special cases fall out immediately by setting the Q, S, R parameters. For example,

(1) H_∞ performance: $Q = \gamma^2 I, \gamma > 0$, $S = 0$, and $R = -I$.
(2) Positive real performance: $Q = 0$, $S = I$, and $R = 0$.
(3) Mixed performance: $Q = \theta\gamma^2 I$, $S = (1-\theta)I, \theta \in [0,1]$, and $R = -\theta I$.
(4) Sector bounded performance: $Q = -\frac{1}{2}(K_1'K_2 + K_2'K_1)$, $S = \frac{1}{2}(K_1 + K_2)'$, and $R = -I$, for some constant matrices K_1, K_2.

2.2 Nonlinear singularly perturbed fuzzy (control) systems

To facilitate the presentation, we assume that the readers are familiar with the basic fuzzy setup consisting of r fuzzy rules whose consequent parts are characterized by local Takagi-Sugeno type linear models. For example, given the following fuzzy plant rules $i = 1, 2, \cdots, r$. [Tanaka and Wang [2001]]

$$If \quad z_1 \ is \ M_{i1} \ and \ z_2 \ is \ M_{i2} \ and \ \cdots \ and \ z_p \ is \ M_{ip}$$
$$then$$
$$\begin{aligned}\sigma\ x_1 &= A_{11i}x_1 + A_{12i}x_2 + B_{11i}w + B_{21i}u \\ \epsilon\sigma\ x_2 &= A_{21i}x_1 + A_{22i}x_2 + B_{12i}w + B_{22i}u \\ z &= C_{11i}x_1 + C_{12i}x_2 + D_{11i}w + D_{12i}u\end{aligned}$$

by using a standard fuzzy inference method – singleton fuzzifier, product fuzzy inference and weighted average defuzzifier, the inferred fuzzy model of interest, after normalization, is described by the following open loop, singularly perturbed fuzzy system

$$\begin{bmatrix} E_\epsilon \sigma x \\ z \end{bmatrix} = \left[\begin{array}{cc|c} \mathcal{A}_\mu & \mathcal{B}_{1\mu} & \mathcal{B}_{2\mu} \\ \mathcal{C}_{1\mu} & \mathcal{D}_{11\mu} & \mathcal{D}_{12\mu} \end{array}\right] \left[\begin{array}{c} x \\ w \\ \hline u \end{array}\right] \tag{4}$$

where

$$E_\epsilon = \begin{bmatrix} I_{n\times n} & 0 \\ 0 & \epsilon I_{m\times m} \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\mathcal{A}_\mu = \begin{bmatrix} A_{11\mu} & A_{12\mu} \\ A_{21\mu} & A_{22\mu} \end{bmatrix}, \quad \mathcal{B}_{1\mu} = \begin{bmatrix} B_{11\mu} \\ B_{12\mu} \end{bmatrix}, \quad \mathcal{B}_{2\mu} = \begin{bmatrix} B_{21\mu} \\ B_{22\mu} \end{bmatrix}$$

$$\mathcal{C}_{1\mu} = [\, C_{11\mu} \ C_{12\mu} \,], \quad \mathcal{D}_{11\mu} = D_{11\mu}, \quad \mathcal{D}_{12\mu} = D_{12\mu}$$

and $x \in R^n$, $z \in R^m$, $y \in R^s$ and $\mu_i \geq 0$ are the state variable, controlled output, measured output and grade of membership, respectively. $\mathcal{A}_i$, $\mathcal{B}_{1i}$, $\mathcal{B}_{2i}$, $\mathcal{C}_{1i}$, $\mathcal{C}_{2i}$ $\mathcal{D}_{11i}$, and $\mathcal{D}_{12i}$ are real constant matrices of appropriate dimensions. We assume $(\mathcal{A}_i, \mathcal{B}_{1i})$ is completely controllable and $(\mathcal{A}_i, \mathcal{C}_{2i})$ is completely observable.

Remarks:

(1) Although the singularly perturbed fuzzy system (4) is displayed in a general form, we consider a simple case by removing the column and row associated with u for now (i.e, $u = 0$), the underlying fuzzy system (4) can be viewed as the general nonlinear systems characterized by (1)-(2). To see this, let

$$A(x) = \mathcal{A}_\mu x = \sum_i^r \mu_i A_i x, \quad B(x) = \mathcal{B}_\mu = \sum_i^r \mu_i B_i$$

and

$$C(x) = \mathcal{C}_\mu x = \sum_i^r \mu_i C_i x, \quad D(x) = \mathcal{D}_\mu = \sum_i^r \mu_i D_i.$$

Theorem 1. (Systems). There exists an $\epsilon^* > 0$ such that the fuzzy system is dissipative and asymptotically stable with respect to a specified $r_q(w, z)$ for $\epsilon \in (0, \epsilon^*]$, if, given the supply rate (3), there exists a common matrix $\mathcal{P}$ displayed below

$$\mathcal{P} = \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix} > 0$$

such that

$$\begin{bmatrix} \mathcal{A}_i'\mathcal{P} + \mathcal{P}'\mathcal{A}_i & * & * \\ \mathcal{B}_{1i}\mathcal{P} - S\mathcal{C}_{1i} & -Q - (S\mathcal{D}_{11i} + *) & * \\ \mathcal{C}_{1i} & \mathcal{D}_{11i} & U \end{bmatrix} < 0 \tag{5}$$

where $0 < P_{11} \in R^{n\times n}$ and $0 < P_{22} \in R^{m\times m}$.

Proof: Since $P_{11} > 0$ and $P_{22} > 0$, it is obvious that there exists a scalar $\epsilon_1 > 0$ such that $P_{11} - \epsilon \cdot P_{21}^{'} P_{22}^{-1} P_{21} > 0$ for $\epsilon \in (0, \epsilon_1]$. Let

$$\mathcal{P}_\epsilon = \begin{bmatrix} P_{11} & \epsilon P_{21}^{'} \\ P_{21} & P_{22} \end{bmatrix}$$

and then

$$E_\epsilon \mathcal{P}_\epsilon = \begin{bmatrix} P_{11} & \epsilon P_{21}^{'} \\ \epsilon P_{21} & \epsilon P_{22} \end{bmatrix} > 0$$

for $\epsilon \in (0, \epsilon_1]$. Choose a quadratic Lyapunov function as $V(x(t)) = \frac{1}{2} x^{'}(t) E_\epsilon \mathcal{P}_\epsilon x(t)$ with quadratic supply rate $r_q(z, w) = \frac{1}{2}(w'Qw + 2w'Sz + z'Rz)$ to show that (4) is asymptotically stable. To this end, derive a stability condition for following inequality

$$\dot{V}(t) - r_q(z, w) < 0$$

where $\dot{V}(t)$ is the time derivative of V function along the state trajectory. To continue the stability proof, it is straightforward to verify the following identity

$$\begin{aligned}
&\dot{V}(t) - \frac{1}{2}(w(t)'Qw(t) + 2w(t)'Sz(t) + z(t)'Rz(t)) \\
&= x^{'}(\mathcal{A}_\mu^{'} \mathcal{P}_\epsilon + \mathcal{P}_\epsilon^{'} \mathcal{A}_\mu)x + w^{'} \mathcal{B}_{1\mu}^{'} \mathcal{P}_\epsilon x + x^{'} \mathcal{P}_\epsilon^{'} \mathcal{B}_{1\mu} w \\
&\quad - x^{'} \mathcal{C}_{1\mu}^{'} R \mathcal{C}_{1\mu} x - w^{'}(S + \mathcal{D}_{11\mu}^{'} R)\mathcal{C}_{1\mu} x \\
&\quad - x^{'} \mathcal{C}_{1\mu}^{'} (S + \mathcal{D}_{11\mu}^{'} R)^{'} w - w^{'}(Q + S\mathcal{D}_{11\mu} \\
&\quad + \mathcal{D}_{11\mu}^{'} S^{'} + \mathcal{D}_{11\mu}^{'} R \mathcal{D}_{11\mu})w \\
&= x^{'}(\mathcal{A}_\mu^{'} \mathcal{P} + \mathcal{P}^{'} \mathcal{A}_\mu + O(\epsilon))x \\
&\quad + w^{'}(\mathcal{B}_{1\mu}^{'} \mathcal{P} + T(\epsilon))x + x^{'}(\mathcal{B}_{1\mu}^{'} \mathcal{P} + T(\epsilon))^{'} w \\
&\quad - x^{'} \mathcal{C}_{1\mu}^{'} R \mathcal{C}_{1\mu} x - w^{'}(S + \mathcal{D}_{11\mu}^{'} R)\mathcal{C}_{1\mu} x \\
&\quad - x^{'} \mathcal{C}_{1\mu}^{'} (S + \mathcal{D}_{11\mu}^{'} R)^{'} w - w^{'}(Q + S\mathcal{D}_{11\mu} \\
&\quad + \mathcal{D}_{11\mu}^{'} S^{'} + \mathcal{D}_{11\mu}^{'} R \mathcal{D}_{11\mu})w \\
&= \begin{bmatrix} x \\ w \end{bmatrix}^{'} \left(M + \bar{O}(\epsilon)\right) \begin{bmatrix} x \\ w \end{bmatrix} \qquad (6)
\end{aligned}$$

where

$$M = \begin{bmatrix} \mathcal{A}_\mu^{'} \mathcal{P} + \mathcal{P}^{'} \mathcal{A}_\mu - \mathcal{C}_{1\mu}^{'} R \mathcal{C}_{1\mu} & * \\ \mathcal{B}_{1\mu}^{'} \mathcal{P} - (S + \mathcal{D}_{11\mu}^{'} R)\mathcal{C}_{1\mu} & \begin{pmatrix} -Q - (S\mathcal{D}_{11\mu} + *) \\ -\mathcal{D}_{11\mu}^{'} R \mathcal{D}_{11\mu} \end{pmatrix} \end{bmatrix}$$

and $\bar{O}(\epsilon)$ is defined below

$$\begin{bmatrix} O(\varepsilon) & * \\ T(\varepsilon) & 0 \end{bmatrix}$$

Obviously, the inequality (5) implies there exists a scalar $\epsilon_2 > 0$ such that $(M + \bar{O}(\epsilon)) < 0$ for $\epsilon \in (0, \epsilon_2]$. Let $\epsilon^* = \min(\epsilon_1, \epsilon_2)$, and then we have both $E_\epsilon \mathcal{P}_\epsilon > 0$ and inequality (6) < 0 hold true. Now we get

$$M = \begin{bmatrix} \mathcal{A}_\mu^{'} \mathcal{P} + \mathcal{P}^{'} \mathcal{A}_\mu & * \\ \mathcal{B}_{1\mu}^{'} \mathcal{P} - S\mathcal{C}_{1\mu} & -Q - (S\mathcal{D}_{11\mu} + *) \end{bmatrix} - \begin{bmatrix} \mathcal{C}_{1\mu}^{'} \\ \mathcal{D}_{1\mu}^{'} \end{bmatrix} R \left[\mathcal{C}_{1\mu} \ \mathcal{D}_{1\mu} \right] < 0 \qquad (7)$$

where $R^{-1} = U$. To cast (7) into a feasibility problem solvable by convex algorithms, performing Schur complement on (7) and factoring out $\sum_{i=1}^{r} \mu_i$ yields the sufficient condition LMIs (5). To prove the (Q, S, R)-dissipativity, integrating $\dot{V} - r_q(z, w) < 0$ from 0 to T, we have

$$V(T) - V(0) - \int_0^T r_q(z(t), w(t))dt < 0$$

leading to

$$\int_0^T r_q(z(t), w(t))dt + V(0) > V(T)$$

where $\beta(x_0) = V(0)$ is a finite value, and $V(T) \geq 0$. Thus the system is dissipative and the proof is completed. □

Continuing along the line of analysis, consider a state feedback controller

$$u = \sum_{i=1}^{r} \mu_i \mathcal{K}_i x$$

and the control system (4). The closed-loop system

$$\begin{bmatrix} E_\epsilon \sigma x \\ z \end{bmatrix} = \begin{bmatrix} \mathcal{G}_\mu & \mathcal{B}_{1\mu} \\ \mathcal{H}_{1\mu} & \mathcal{D}_{11\mu} \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} \qquad (8)$$

where $\mathcal{G}_{\mu\mu} = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j (\mathcal{A}_i + \mathcal{B}_{2i} \mathcal{K}_j)$ and $\mathcal{H}_{\mu\mu} = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j (\mathcal{C}_{1i} + \mathcal{D}_{12i} \mathcal{K}_j)$.

Theorem 2. (Stabilization). There exists an $\epsilon^* > 0$ such that fuzzy system is dissipative and asymptotically stable with respect to a specified $r_q(w, z)$ for $\epsilon \in (0, \epsilon^*]$, if, given the supply rate (3), there exists a common matrix $\mathcal{P}$

$$\mathcal{P} = \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix} > 0 \qquad (9)$$

such that

$$\mathcal{M}_{ij} < 0 \qquad (10)$$

where

$$\mathcal{M}_{ij} = \begin{bmatrix} (\mathcal{X}^{'} \mathcal{A}_i^{'} + \mathcal{N}_j^{'} \mathcal{B}_{2i}^{'}) + * & * & * \\ \mathcal{B}_{1i} - S\mathcal{C}_{1i}\mathcal{X} - S\mathcal{D}_{12i}\mathcal{N}_j & -Q - (S\mathcal{D}_{11i} + \star) & * \\ \mathcal{C}_{1i}\mathcal{X} + \mathcal{D}_{12i}\mathcal{N}_j & \mathcal{D}_{11i} & U \end{bmatrix}$$

and $\mathcal{X} = \mathcal{P}^{-1}, \mathcal{N}_j = \mathcal{K}_j \mathcal{X}$, and $0 < P_{11} \in R^{n\times n}$, $0 < P_{22} \in R^{m\times m}$.

Proof: Given (8), Theorem 1 is utilized to prove the results, thus the proof being omitted to save space. □

It is noted that when solving the stabilization problem, we use the relaxed method

$$M_{ij} + M_{ji} < 0, 1 \leq i \leq j \leq r.$$

3. AN ILLUSTRATIVE EXAMPLE

Consider a tunnel diode circuit borrowed from [Assawinchaichote et al. [2004]] where the tunnel diode is characterized by $i_D = -0.2v_D - 0.05v_D^3$. Assume that ϵ is a parasitic inductance. Let $x_1 = v_C$ be the capacitor voltage and $x_2 = i_L$ be the inductor current. Then, the circuit can be modelled by the following state equations:

$$\begin{aligned}\dot{x}_1 &= 2x_1 + 0.5x_1^3 + 10x_2,\\ \epsilon\dot{x}_2 &= -0.1x_1 - 10x_2 + 0.1w + 0.1u,\\ z &= \left[\, x_1^{'}\ x_2^{'} \,\right]'\end{aligned} \qquad (11)$$

where u is the input, w is the process noise which may represent un-modelled dynamics, z is the controlled output, $x = [x_1^{'}\ x_2^{'}]'$. Note that the variables x_1 and x_2 are the deviation variables. The nonlinear network system (11) can be exactly represented by a two-rule T-S fuzzy model:

$$\begin{aligned}E_\epsilon \dot{x} &= \sum_{i=1}^{2} \mu_i [A_i x + B_1 w + B_{2i} u]\\ z &= C_1 x\end{aligned}$$

where

$$A_1 = \begin{bmatrix} 2 & 10 \\ -0.1 & -10 \end{bmatrix}, B_{21} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 6.9 & 10 \\ -0.1 & -10 \end{bmatrix}, B_{22} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$$

and $\mu_1 = 1 - \frac{x_1^2}{9}, \mu_2 = 1 - \mu_1$. The state feedback fuzzy controller is designed as

$$u = \sum_{i=1}^{2} \mu_i K_i x.$$

The notion of supply rate in this paper is viewed as a performance index and the state feedback controller is to stabilize the underlying system in an H_∞ sense. As such, we choose $\epsilon = 0.05H$, and

$$Q = diag[0.64\ 0.64], S = 0, R = -I.$$

to achieve a designated H_∞ performance where γ=0.8. Solving the LMIs (9) and (10) via the LMI solver in the MATLAB, we find

$$X = \begin{bmatrix} 0.4885 & 0 \\ -0.4639 & 0.8572 \end{bmatrix}, \quad P = \begin{bmatrix} 2.0471 & 0 \\ 1.1078 & 1.1666 \end{bmatrix}$$

$$N_1 = [-126.6416\ 60.4045], \quad K_1 = [-89.886\ 51.78]$$

$$N_2 = [-126.8804\ 60.3660], \quad K_2 = [-89.985\ 51.75]$$

The disturbance input signals w is a rectangular signal as shown in Figure 1. The state trajectories with initial condition $x(0) = [1.5 -1]'$ is depicted in Figure 2 where the rectangular disturbance is activated while Figure 3 shows the trajectories where the disturbance is deactivated.

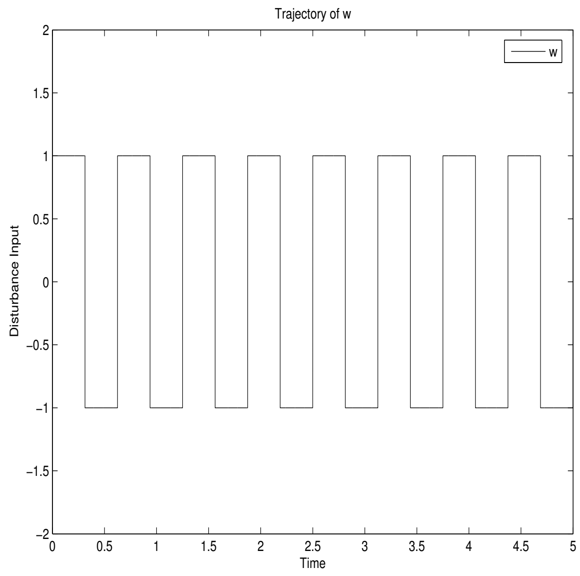

Fig. 1. Disturbances w

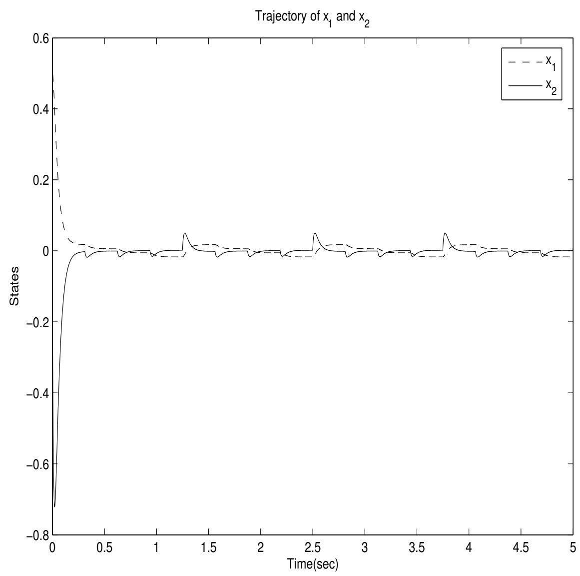

Fig. 2. Trajectories of states x_1 and x_2 with disturbances

4. CONCLUSION

A complete solution to a state feedback stabilization problem involving singularly perturbed systems is presented. The focus of this paper is to incorporate the (Q, S, R) parameter into the system, known as supply rate, which can be various existing performance indexes when specialized. The dissipative control scheme is applied to derive an LMI test condition. Although only continuous-time systems are investigated in detail, a unified treatment applicable to discrete-time counterparts is readily obtainable. An example is demonstrated to validate the theorem derived.

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Fig. 3. Trajectories of states x_1 and x_2 without disturbances

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