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# Nonlinear Modeling of Double-Loop Hysteretic Systems ⋆

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**Abstract:** This paper presents two new dynamic hysteresis models obtained from the Bouc-Wen model by incorporating position and acceleration information. On one hand, the model employing position information is rate-independent and it is able to reproduce some kind of double hysteretic loops unable to be reproduced with the original Bouc-Wen model. On the other hand, the model employing acceleration information is insensitive to linear time-scale variations. Double hysteretic loops have been experimentally reported and seen in shape memory alloys, reinforced concrete structures, wood structures and lightweight steel shear wall structures. The proposed hysteretic models represent a prominent use in the field of structural dynamics and earthquake engineering because they can capture the non-linear dynamics of the materials and structures presented earlier when they are subjected to dynamic loads as an earthquake excitation, using the position and acceleration information, being the last one an available source in the field with the use of accelerometers. 

## 1. INTRODUCTION

Hysteresis is a property of systems (usually physical systems) that do not instantly follow the forces applied to them, but react slowly, or do not return completely to their original state, that is, systems whose states depend on their immediate history. In other words, hysteretic systems have been recognized as systems with memory, and they generate loops (Visintin, 1991). Also, and according with Oh and Bernstein (2005), there is no precise definition of hysteresis, but it is adopted that hysteresis is effectively a nontrivial quasi-dc input-output closed curve in the input-output map that persists when a periodic input is applied and its frequency content approaches to dc value. Hysteresis phenomena occurs in biology, optics, electronics, ferroelectricity, magnetism, structural mechanics, among other areas (Oh and Bernstein, 2005; Krasnosel'skii and Pokrovskii, 1989). Modeling of hysteresis phenomenon has been reported, for instance, in Oh and Bernstein (2005); Chua and Steven (1972); Clarke (2005); Song and Kiureghian (2006), and reference there in. This paper is primarily concerned with modeling of a hysteretic case where double hysteretic loops have been experimentally observed, as seen in smart materials, such as shape memory alloys (SMA) (Aiken et al., 1992; Bruno and Valente, 2002; Dolce et al., 2000, 2005; Saadat et al., 2002); and mechanical and structural systems, such as reinforced concrete structures (Kunnath et al., 1997), wood structures (Foliente, 1995), and lightweight steel shear wall structures (Pastor and Rodríguez-Ferran, 2005). Modeling of asymmetric hysteretic loop resembling double-loop behavior has been reported in Dobson et al. (1997), where hysteresis modeling is based on a modification to a previously reported model which uses hysteretic cycle decoupling by making the shape parameters system dynamic dependent, but no position or acceleration information are considered. Moreover, in Li et al. (2004), a model of a kind of double-loop hysteretic behavior is also shown. Again, no position or acceleration is invoked. It is convenient to note that in both models Dobson et al. (1997) and Li et al. (2004), the internal dynamic model of hysteresis is converted from first to second order. Traditionally, the internal dynamic of hysteresis model is a first order differential equation with velocity information supplied as its input, which is the case of the Bouc-Wen model (Wen, 1976). Nevertheless, hysteresis model can depend on position and velocity, as is shown in Oh and Bernstein (2005), Chua and Steven (1972), Song and Kiureghian (2006), and Chatterjeea and Basub (2006). The main objective of the present work is to bear witness that double-loop hysteretic behavior can be obtained from the Bouc-Wen when just the signum of the position or the signum of the acceleration is incorporated. So, this paper presents two new dynamic hysteresis models obtained from the Bouc-Wen model (Wen, 1976) by, in the first case, incorporating acceleration information, and in the second one, position information. It is shown by numerical simulations that double hysteretic loops can be captured by using the proposed models. It is also shown

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that the proposed model using position information is rate-independent meanwhile the second one is insensitive to linear time-scale variations. Even so, asymmetric double-loop behavior can be obtained from the position case when dc value is supplied to the position input.

The paper is organized as follows. In Section 2, the new dynamic hysteresis model obtained from the Bouc-Wen model by incorporating acceleration information is presented. Section 2 also presents numerical simulations of an isolated structure using passive devices like SMA cables as seen in Wilde et al. (2000) and Yamashita (2004), to give us an overlook of the simulated response of the new hysteretic model. In Section 3, the position case is then shown with the same numerical simulation platform employed in Section 2. Finally, in Section 4 the conclusions are drawn.

## 2. HYSTERETIC MODEL USING ACCELERATION INFORMATION

Consider the Bouc-Wen model Smith et al. (2002):

$$\dot{z} = D^{-1}(A\dot{x} - \beta|\dot{x}||z|^{n-1}z - \lambda\dot{x}|z|^n), \tag{1}$$

where $A$, $\beta$, $\lambda$ are dimensionless parameters which control the shape and the size of the hysteresis loop, $n \geq 1$ is a scalar that governs the smoothness of the transition from elastic to plastic response. Conceive now the following model:

$$\begin{aligned}\dot{z} =& f(z, \dot{x}, \ddot{x}) \\ =& D^{-1}(A\dot{x} - \beta|\dot{x}||z|^{n-1}z - \lambda\dot{x}|z|^n + \gamma\text{sgn}(\ddot{x})|\dot{x}|), \\ & z(0) = z_0, \quad t \geq 0,\end{aligned} \tag{2}$$

where the acceleration information has been introduced by means of the signum term, where $\text{sgn}(0) = 0$, $\text{sgn}(x) = 1$ if $x > 0$, and $\text{sgn}(x) = -1$ if $x < 0$. Acceleration dependence for a kind of hysteretic dynamic system has already been reported in Benítez et al. (2006) by employing the LuGre friction model. Following this strategy, we have incorporated the acceleration information in the same way as in Benítez et al. (2006).

*Proposition 1.* The dynamic system (2) is insensitive to linear time-scale variations.

**Proof.** For simplicity, let $D = 1$. Let $\tau(t) = \alpha t$, $\alpha > 0$ be a positive linear time-scale (Oh and Bernstein, 2005). Then $\tau(0) = 0$, $\dot{\tau} = \alpha$, $\ddot{\tau} = 0$ and, thus, $z_\tau(0) = z(\tau(t)) = z(0) = z_0$. Now, for all $t > 0$, consider

$$\begin{aligned}\frac{dz_\tau(t)}{dt} &= A\frac{dx_\tau(t)}{dt} - \beta\left|\frac{dx_\tau(t)}{dt}\right||z_\tau(t)|^{n-1}z_\tau(t) \\ &\quad - \lambda\frac{dx_\tau(t)}{dt}|z_\tau(t)|^n + \gamma\text{sgn}\left(\frac{d}{dt}\left(\frac{dx_\tau(t)}{dt}\right)\right)\left|\frac{dx_\tau(t)}{dt}\right|\end{aligned}$$

$$\begin{aligned}\dot{\tau}\frac{dz(\tau)}{d\tau} &= A\dot{\tau}\frac{dx(\tau)}{d\tau} - \beta\left|\dot{\tau}\frac{dx(\tau)}{d\tau}\right||z(\tau)|^{n-1}z(\tau) \\ &\quad - \lambda\dot{\tau}\frac{dx(\tau)}{d\tau}|z(\tau)|^n \\ &\quad + \gamma\text{sgn}\left(\ddot{\tau}\frac{dx(\tau)}{d\tau} + \dot{\tau}^2\frac{d}{d\tau}\left(\frac{dx(\tau)}{d\tau}\right)\right)\left|\dot{\tau}\frac{dx(\tau)}{d\tau}\right|\end{aligned}$$

Since $\tau(t)$ is a linear positive time-scale, $\dot{\tau} = \alpha > 0$ and $\ddot{\tau} = 0$. Hence, it follows that:

$$\begin{aligned}\dot{\tau}\frac{dz(\tau)}{d\tau} &= A\dot{\tau}\frac{dx(\tau)}{d\tau} - \beta\dot{\tau}\left|\frac{dx(\tau)}{d\tau}\right||z(\tau)|^{n-1}z(\tau) \\ &\quad - \lambda\dot{\tau}\frac{dx(\tau)}{d\tau}|z(\tau)|^n \\ &\quad + \gamma\text{sgn}\left(\frac{d}{d\tau}\left(\frac{dx(\tau)}{d\tau}\right)\right)\dot{\tau}\left|\frac{dx(\tau)}{d\tau}\right| \\ \frac{dz(\tau)}{d\tau} &= A\frac{dx(\tau)}{d\tau} - \beta\left|\frac{dx(\tau)}{d\tau}\right||z(\tau)|^{n-1}z(\tau) \\ &\quad - \lambda\frac{dx(\tau)}{d\tau}|z(\tau)|^n + \gamma\text{sgn}\left(\frac{d}{d\tau}\left(\frac{dx(\tau)}{d\tau}\right)\right)\left|\frac{dx(\tau)}{d\tau}\right|,\end{aligned}$$

as required. ■

Moreover, we have the next property.

*Proposition 2.* Consider system (2) with input $\dot{x}$ and output $z$. If $\beta > 0$, then this system is bounded-input bounded-output (BIBO).

**Proof.** Taking the Lyapunov function $V(z) = \frac{1}{2}z^2$, its time derivative along the trajectories of system (2) yields:

$$\begin{aligned}\dot{V}(z) &= -|\dot{x}||z|[|z|\beta - \text{sgn}(\dot{x})\text{sgn}(z) - \gamma\text{sgn}(\ddot{x})\text{sgn}(z)] \\ &\leq -|\dot{x}||z|[|z|\beta - 1 - |\gamma|]\end{aligned}$$

The above is non-positive if

$$|z| \geq \frac{1 + |\gamma|}{\beta},$$

with $|\dot{x}|$ bounded, which implies that $z(t)$ is bounded. ■

### *2.1 Numerical simulations*

Consider the following system

$$m\ddot{x} + c\dot{x} + \Phi(x, t) = u(t),$$

where $\Phi(x, t)$ is the restoring force with hysteretic behavior, $x(t)$ the position, $u(t)$ the control input, and $m$ and $c$ the mass and the damping coefficients, respectively.

This system is composed of a base-isolated structure with passive and active devices like SMA cables, which can act as a passive device (previously treated - superelasticity or shape-memory effect (Bruno and Valente, 2002)) or as an active device (temperature-induced transformation - shape-memory effect (Saadat et al., 2002; Elahinia and Ashrafiuon, 2002)). This feature makes the structure very versatile, due to the fact that for near-fault earthquakes, the maximum displacements induced by this impulse-like seismic excitations, can be controlled by this SMA cables, to keep the structure in the elastic range. The base-isolated system will stay out of its ultimate failure limit due to the superelastic and recentering properties of the SMA cables.

In order to study the behavior of our hysteretic model, the restoring force is implemented as follows:

$$\begin{aligned}\Phi(x, t) &= x(t) + z(t), \\ \dot{z} &= \dot{x} - \beta|\dot{x}|z - \text{sgn}(\ddot{x})|\dot{x}|.\end{aligned} \tag{3}$$

In numerical simulations, we set $m = c = 1$. We take $u(t) = \sin((0.03t + 0.2)t)$ as in Smith et al. (2002). Plots of $z(t)$ versus $x(t)$ are depicted in Figure 1 for two different values of $\beta$. As we can see, the double-hysteretic loops are seen in Figure 1 with an asymmetric behavior with large residual strains for $\beta = 4$ than for $\beta = 8$ for the first load cycles. As the load cycles increase, there exist a residual strain prior to the loading curve or an offset with respect to the initial cycles. Furthermore, this behavior may represent degrading systems and pinching as seen in reinforced concrete structures, which encounter large residual strains at the end of their loading cycles and the energy dissipated diminishes its larger initial value with the incremental loading cycles.

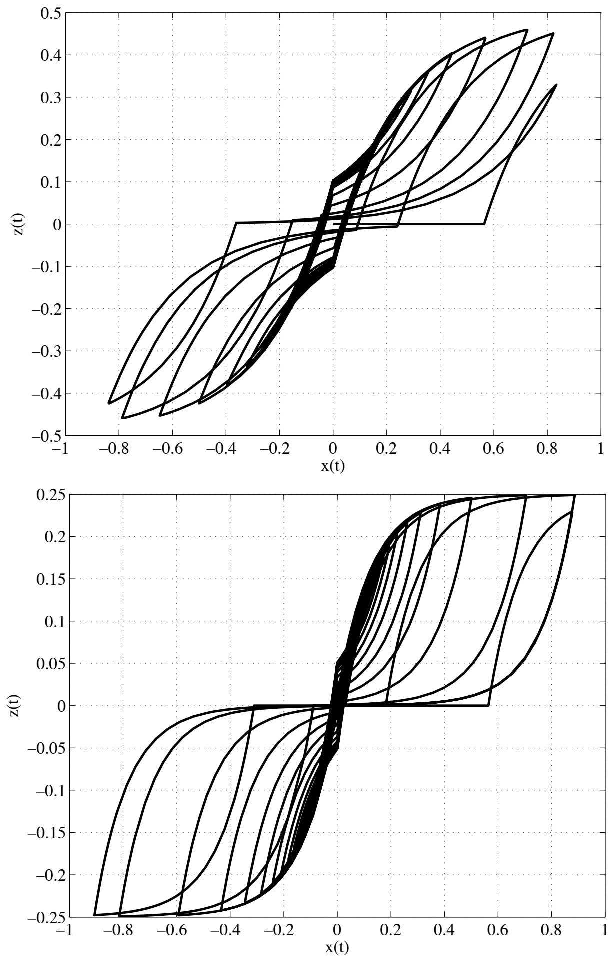


Fig. 1. Simulation results. Plot of $z(t)$ versus $x(t)$ with $\beta = 4$ (up) and $\beta = 8$ (down).

## 3. HYSTERETIC MODEL USING POSITION INFORMATION

Consider the modified Bouc-Wen model (for simplicity, we set $D^{-1} = 1$).

$$\begin{aligned}\dot{z} &= f(z, x, \dot{x}) \\ &= A\dot{x} - \beta|\dot{x}||z|^{n-1}z - \lambda\dot{x}|z|^n + \gamma\mathrm{sgn}(x)|\dot{x}|, \qquad (4)\end{aligned}$$

where the position information has been introduced by means of the signum term.

*Proposition 3.* The dynamic system (4) is *rate-independent.*

**Proof.** Let $\tau(t)$ be a positive time-scale (Oh and Bernstein, 2005). Then $\tau(0) = 0$ and, thus, $z_\tau(0) = z(\tau(0)) = z(0) = z_0$. Now, for all $t > 0$, consider

$$\begin{aligned}\frac{dz_\tau(t)}{dt} &= A\frac{dx_\tau(t)}{dt} - \beta\left|\frac{dx_\tau(t)}{dt}\right||z_\tau(t)|^{n-1}z_\tau(t) \\ &\quad - \lambda\frac{dx_\tau(t)}{dt}|z_\tau(t)|^n + \gamma\mathrm{sgn}(x_\tau(t))\left|\frac{dx_\tau(t)}{dt}\right| \\ \dot{\tau}\frac{dz(\tau)}{d\tau} &= A\dot{\tau}\frac{dx(\tau)}{d\tau} - \beta\left|\dot{\tau}\frac{dx(\tau)}{d\tau}\right||z(\tau)|^{n-1}z(\tau) \\ &\quad - \lambda\dot{\tau}\frac{dx(\tau)}{d\tau}|z(\tau)|^n + \gamma\mathrm{sgn}(x(\tau))\left|\dot{\tau}\frac{dx(\tau)}{d\tau}\right|.\end{aligned}$$

Since $\tau$ is a positive time scale, $\dot{\tau}(t) \geq 0$. Hence, it follows that:

$$\begin{aligned}\frac{dz(\tau)}{d\tau} &= A\frac{dx(\tau)}{d\tau} - \beta\left|\frac{dx(\tau)}{d\tau}\right||z(\tau)|^{n-1}z(\tau) \\ &\quad - \lambda\frac{dx(\tau)}{d\tau}|z(\tau)|^n + \gamma\mathrm{sgn}(x(\tau))\left|\frac{dx(\tau)}{d\tau}\right|,\end{aligned}$$

as required. ■

Moreover, we have the next property.

*Proposition 4.* Consider system (4) with input $\dot{x}$ and output $z$. If $\beta > 0$, then this system is bounded-input bounded-output (BIBO).

**Proof.** Taking the Lyapunov function $V(z) = \frac{1}{2}z^2$, its time derivative along the trajectories of system (4) yields:

$$\begin{aligned}\dot{V}(z) =& -|\dot{x}||z|\left[-A\mathrm{sgn}(\dot{x})\mathrm{sgn}(z) + \beta|z|^n\right. \\ &\left.+\lambda|z|^n\mathrm{sgn}(\dot{x})\mathrm{sgn}(z) - \gamma\mathrm{sgn}(x)\mathrm{sgn}(z)\right] \\ \leq& -|\dot{x}||z|\left[-A - |\gamma| + (\beta+\lambda)|z|^n\right]\end{aligned}$$

The above is non-positive if $|z| \geq \sqrt[n]{\frac{A+|\gamma|}{\beta+\lambda}}$ with $|\dot{x}|$ bounded, which implies that $z(t)$ is bounded. ■

### 3.1 Numerical simulations

Consider again the system

$$m\ddot{x} + c\dot{x} + \Phi(x, t) = u(t). \qquad (5)$$

In order to study the behavior of our hysteretic model, now the restoring force is implemented as follows:

$$\begin{aligned}\Phi(x, t) &= x(t) + z(t), \\ \dot{z} &= \dot{x} - \beta|\dot{x}|z + \mathrm{sign}(x)|\dot{x}|. \qquad (6)\end{aligned}$$

In numerical simulations, we set $m = c = 1$. We take $u(t) = \sin((0.03t + 0.2)t)$. Simulation results of $z(t)$ versus $x(t)$ are pictured in Figure 2.

Moreover, asymmetric double-loop behavior can be reproduced by introducing a DC component to the position information, as follows:

$$\begin{aligned}\Phi(x, t) &= x(t) + z(t), \\ \dot{z} &= \dot{x} - \beta|\dot{x}|z + \mathrm{sign}(x + DC_{value})|\dot{x}| \qquad (7)\end{aligned}$$

where $DC_{value}$ is the position offset component. Repeating simulation experiment but invoking (7) with $\beta = 10$ and $DC_{value} = 0.2$, the output is described in Figure 3.

*Remark 1.* The system (4) is dependent of DC value in position. This dependence can be eliminated by doing the following modification:

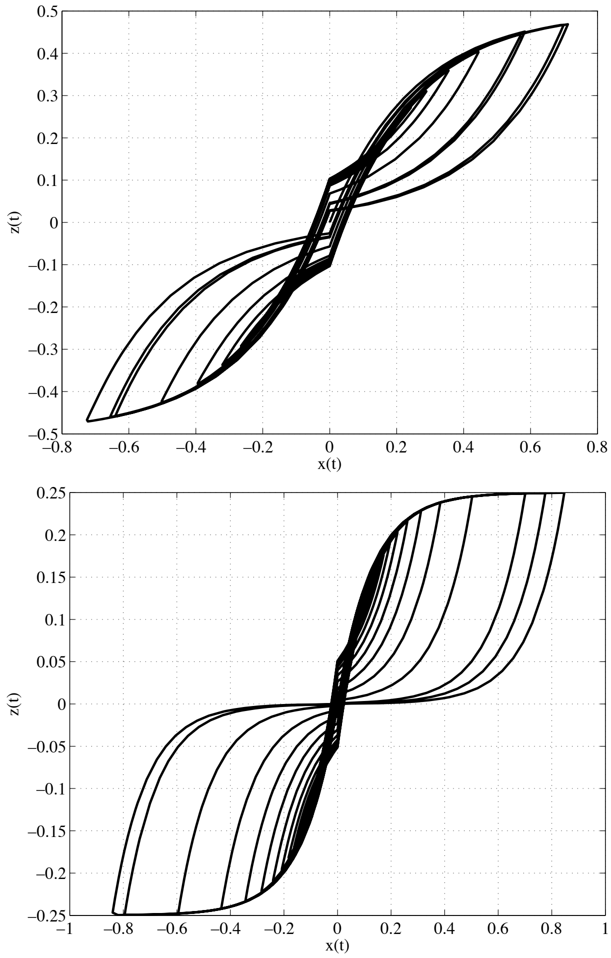


Fig. 2. Simulation results. Plot of $z(t)$ versus $x(t)$ with $\beta = 4$ (up) and $\beta = 8$ (down).

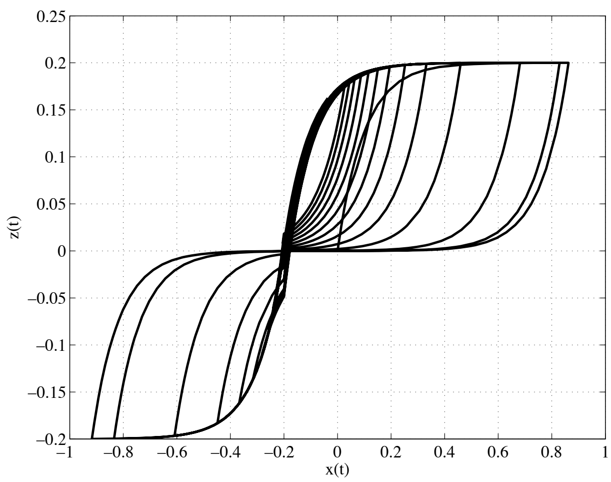


Fig. 3. Simulation results. Plot of $z(t)$ versus $x(t)$.

$$
\begin{aligned}
\dot{z} =& f(z, x, \dot{x}) \\
=& A\dot{x} - \beta|\dot{x}||z|^{n-1}z - \lambda\dot{x}|z|^n + \gamma \mathrm{sgn}(w)|\dot{x}|, \\
& \dot{w} = \dot{x}, \quad w(0) = 0.
\end{aligned} \tag{8}
$$

As we can see in Figure 2 the double hysteretic loops present smaller values of residual strains per cycle than as seen in Figure 1. Therefore, it represents in a more feasible manner the behavior of recentering materials like SMA, which have very small residual strains under many load cycles. We can conclude about this that for $\beta \to \infty$ the residual strain in the double hysteretic loops will be close to 0. At the same time, for large values of $\beta$, the energy dissipated per cycle is larger due to the large area of the loop. Moreover, a large pre-yield stiffness is seen in the initial loading slope for large values of $\beta$ and consequently smaller values of post-yield stiffness is encountered. This may be beneficial when modeling materials and systems that have recentering properties, small residual strains, superelastic properties, shape memory effects, and need to dissipate large energy per cycle under a dynamic load. At the same time, as we can see in Figure 3, an initial offset is perceived and similar recentering behavior shown in Figure 1 and Figure 2 is captured. This type of responses may be useful to represent systems with a mass distribution offset under dynamic loads.

## 4. CONCLUSIONS

This paper has dealt with the modeling problem of double-loop hysteretic systems by incorporating the signum information of acceleration and position. Two new models have been proposed. Numerical simulations were performed in a system incorporating a base-isolated with a passive or active device like SMA cables. The numerical simulations represent in a feasible way the double-loop hysteresis. Future work using these two models can be used to pinching and degrading hysteretic systems (Baber and Nouri, 1986; Baber and Wen, 1981; Mostaghel, 1999), or more complex structural systems including reinforced concrete structures, wood, and lightweight steel shear wall structures under earthquake excitations.

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