Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

A Global Solution to Economic Dispatch with Multiple Fuel Units Using a Function Merger

Kyung-Il Min*, Su-Won Lee and Young-Hyun Moon*****

*Electrical Engineering Department, Yonsei University,
Seoul, Korea, (e-mail: kimin@yonsei.ac.kr)
** Institute of TMS Information Technology, Yonsei University,
Seoul, Korea, (e-mail:swon@yonsei.ac.kr)
***Electrical Engineering Department, Yonsei University,
Seoul, Korea, (e-mail: moon@yonsei.ac.kr)*

Abstract: This paper presents a new systematic approach to find a global solution to economic dispatch (ED) with multiple fuel units using a function merger (FM). Currently, no systematic approach has been developed to find a global solution to economic dispatch with multiple fuel units. Various heuristic methods have been proposed, however it is almost impossible to guarantee a global solution by those methods yet. The proposed method uses the FM and λ-P functions. A FM merges several fuel cost functions into one that satisfies the optimal conditions of an ED. The FM procedures are described in detail with illustrative examples. The global optimality of the proposed method is checked with the ACM (All-Combination Method). The proposed method is tested with a 10-generator system. The results show that the global optimality is achievable by the proposed method.

1. INTRODUCTION

Economic dispatch (ED) is defined as finding an optimal distribution of system load to the generators, in order to minimize the total generation cost. In recent decades, a considerable number of studies have been conducted on ED with a non-smooth fuel cost function for the systems including multiple fuel units. Generally, ED problems are solved by the Lagrangean multiplier method (Wood and Wollenberg, 1996). However, this method cannot be applied to solve an ED problem that includes multiple fuel units due to its nonlinearity.

An ED containing multiple fuel units was introduced by C. E. Lin and G. L. Viviani in 1984, and further research has been published with the application of various approaches. Most of the researches are based on heuristic optimization techniques with distinct limitation in guaranteeing the global optimality (Park *et al*., 2005). A mixed integer programming (Tao Li and M. Shahidehpour, 2005) could be one of the ways to obtain globally optimal solution but it may cause problems of the “curse of dimension” if the number of generators increases considerably.

This paper proposes a new algorithm to find a global solution to ED with multiple fuel units using the function merger (FM) method. FM merges several fuel cost functions into one that satisfies the optimality. In order to merge the functions, the λ-P function method is applied, which inverts the P and λ axes of the incremental fuel cost function (Moon *et al*., 2000, Madrigal and Quintana, 2000 and Min *et al*., 2006).

This paper focuses on the only essential principle to attain optimality in the ED with multiple fuel units. A further direction of this study will be to apply the proposed algorithm to practical large systems including various constraints such as ramp rate, flow limits, etc.

2. FORMULATION OF ED PROBLEM WITH MULTIPLE FUEL UNITS

2.1 Formulation of the ED Problem

The ED can be formulated as an optimization as follows:

$$Min \sum_{i=1}^{n_g} F_i(P_i) \tag{1}$$

$$s.t. \sum_{i=1}^{n_g} P_i = P_D + P_{Loss} \tag{2}$$

$$P_i^{\min} \le P_i \le P_i^{\max} \qquad \text{for } i = 1, \ldots, n_g \tag{3}$$

where

F_i	fuel cost function of generator i
P_i	power output of generator i
P_D	total system demand
P_{Loss}	total system loss

$P_i^{\min}$ minimum output of generator i

$P_i^{\max}$ maximum output of generator i

n_g number of generators

For simplicity, P_{Loss} is often omitted with the assumption of P_D accounting for the system loss. The fuel cost function may have a high degree of nonlinearity, or may be impossible to express as a closed function. However, the cost function is usually approximated as a second order polynomial for practical field applications.

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \tag{4}$$

where a_i, b_i, and c_i are the cost coefficients of the generator i.

2.2 *ED with Multiple Fuel Units*

In the case of an ED with multiple fuel units, the ED problem can be formulated by using piecewise quadratic functions (Lin and Viviani, 1984). Piecewise quadratic and incremental cost functions are illustrated in Fig. 1 (Park *et al.*, 2005). In this case, the fuel cost has the following form.

$$F_i(P_i) = \begin{cases} a_{i1} + b_{i1}P_i + c_{i1}P_i^2 & \text{if } P_{i1}^{\min} \le P_i \le P_{i1}^{\max} \\ a_{i2} + b_{i2}P_i + c_{i2}P_i^2 & \text{if } P_{i2}^{\min} \le P_i \le P_{i2}^{\max} \\ \vdots & \vdots \\ a_{in} + b_{in}P_i + c_{in}P_i^2 & \text{if } P_{in}^{\min} \le P_i \le P_{in}^{\max} \end{cases} \tag{5}$$

where a_{ij}, b_{ij}, and c_{ij} are the cost coefficients of fuel j for unit i and $P_{ij}^{\min}$ is equal to $P_{i-1,j}^{\max}$.

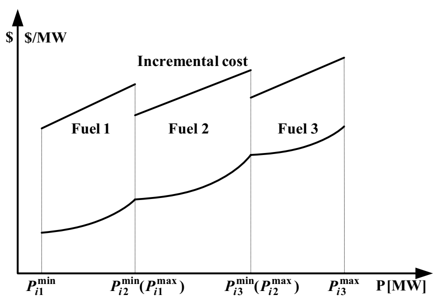


Fig. 1. Piecewise quadratic and incremental fuel cost functions

3. OVERVIEW OF ED ALGORITHM BY THE λ-P FUNCTION METHOD

The ED algorithm that uses the λ-P function method is found in some references (Moon *et al.*, 2000, Madrigal and Quintana, 2000 and Min *et al.*, 2006). The main feature of this method is to use the inverse of the incremental fuel cost functions based on the duality theory, as illustrated in Fig. 2. The inverse functions can be easily obtained because the incremental fuel cost functions are linear.

This method is developed on the basis that each output power of the generators can be determined by the incremental cost λ. Once the incremental cost λ is determined, then the total generating power, P_{Gttl}, can be directly calculated and can be denoted as a function of λ by

$$P_{Gttl}(\lambda) = \sum_{i=1}^{n_g} P_{Gi}(\lambda) \tag{6}$$

Here, it is noted that $P_{Gttl}(\lambda)$ is nondecreasing. Given the total demand of the system, the optimal incremental cost λ^* can be obtained by solving

$$P_{Gttl}(\lambda) = \sum_{i=1}^{n_g} P_{Gi}(\lambda) = P_D \tag{7}$$

where P_D is the total demand including the estimated system loss.

The nondecreasing property of P_{Gttl} allows utilization of the bisection or linear interpolation methods in order to obtain the optimal incremental cost λ^*. It should be noted that the Kuhn-Tucker conditions need not be checked, since $P_{Gi}(\lambda)$ provides all the information of the limitation of the generation outputs and the must-run conditions.

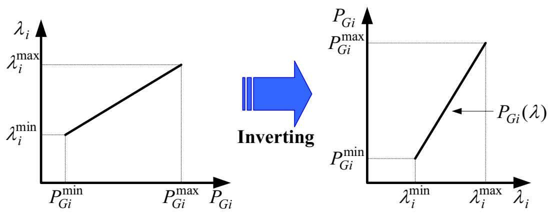


Fig. 2. Inverting process of the incremental cost function using duality theory

Fig. 3 shows an illustrative example with a 3-generator system. Gen. 1 and Gen. 3 are operated in the must-run condition where each generator must produce its minimum output, while Gen. 2 is stopped because its economical efficiency is below a certain marginal cost.

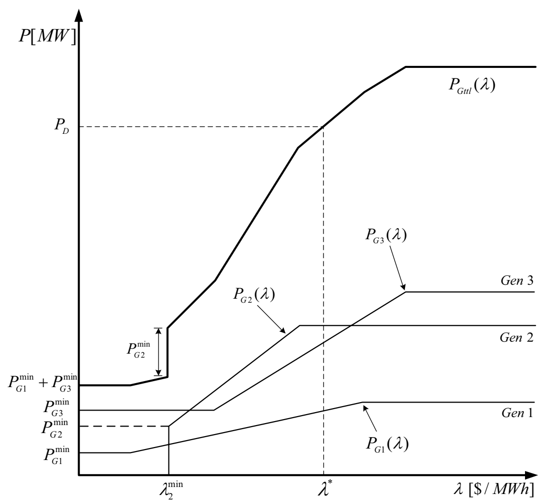


Fig. 3. The summation of three generators' output power

The λ-P function method is composed of the following 4 steps:

Step 1) Establish the λ-P functions by inverting the P-λ functions for all of the generators.

Step 2) Construct the total generation function $P_{Gttl}(\lambda)$ by summing up the λ-P functions for all the generators.

Step 3) Calculate the optimal λ^* by solving (7) and by using the bisection method and/or linear interpolation.

Step 4) Calculate the optimal dispatch for each generator with $P_{Gi}(\lambda^*)$.

4. GLOBAL SOLUTION TO ED PROBLEM WITH MULTIPLE FUEL UNITS

This study proposes a new systematic approach to finding a global solution to an ED with multiple fuel units using a function merger (FM) technique. A FM is defined as merging several fuel cost functions into one fuel cost function that satisfies all of the optimal conditions in an ED.

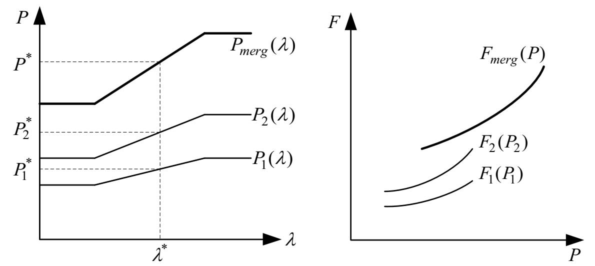
4.1 The FM with no consideration of generation limits

On the basis of the λ-P function method, the FM merges the inverse of the incremental cost functions for each generator into a fuel cost function. Consider two generators with the fuel cost functions as follows:

$$F_1(P_1) = \tfrac{1}{2}a_1P_1^2 + b_1P_1 + c_1 \tag{8}$$

$$F_2(P_2) = \tfrac{1}{2}a_2P_2^2 + b_2P_2 + c_2 \tag{9}$$

As indicated in Fig. 4(a), the merged λ-P function can be obtained by simply adding both functions. However, an explicit merged fuel cost function for the FM process also needs to be obtained in order to guarantee optimality of the solution.



(a) λ-P merging (b) Merged cost function generation

Fig. 4. Merged λ-P and cost function

Here, the goal is to find a merged fuel cost function for the two generators that satisfy optimality. The optimality condition requires

$$a_1P_1^* + b_1 = a_2P_2^* + b_2 = \lambda^* \tag{10}$$

$$P_1^* + P_2^* = P \tag{11}$$

where P is the required generation for both generators.

By solving (10) and (11), the following is obtained:

$$P_1^* = m_1P + n_1 \tag{12}$$

$$P_2^* = m_2P + n_2 \tag{13}$$

where $m_1 = \dfrac{a_2}{a_1 + a_2}$, $m_2 = \dfrac{a_1}{a_1 + a_2}$

$n_1 = \dfrac{b_2 - b_1}{a_1 + a_2}$, $n_2 = \dfrac{b_1 - b_2}{a_1 + a_2}$

By using these results, the following merged fuel cost can easily be obtained for the two generators.

$$F_{merg}(P) = F_1(P_1^*) + F_2(P_2^*) \tag{14}$$

Eq. (14) can be rewritten as

$$\begin{aligned} F_{merg}(P) &= F_1(P_1^*) + F_2(P_2^*) \\ &= \tfrac{1}{2}a_mP^2 + b_mP + c_m \end{aligned} \tag{15}$$

where $a_m = a_1m_1^2 + a_2m_2^2$

$b_m = a_1m_1n_1 + a_2m_2n_2 + b_1m_1 + b_2m_2$

$c_m = \tfrac{1}{2}a_1n_1^2 + \tfrac{1}{2}a_2n_2^2 + b_1n_1 + b_2n_2 + c_1 + c_2$

4.2 The FM process with multiple fuel cost functions

A FM with multiple fuel cost functions is similar to a FM with nonduplicated λ-P curves, which is explained in the previous section.

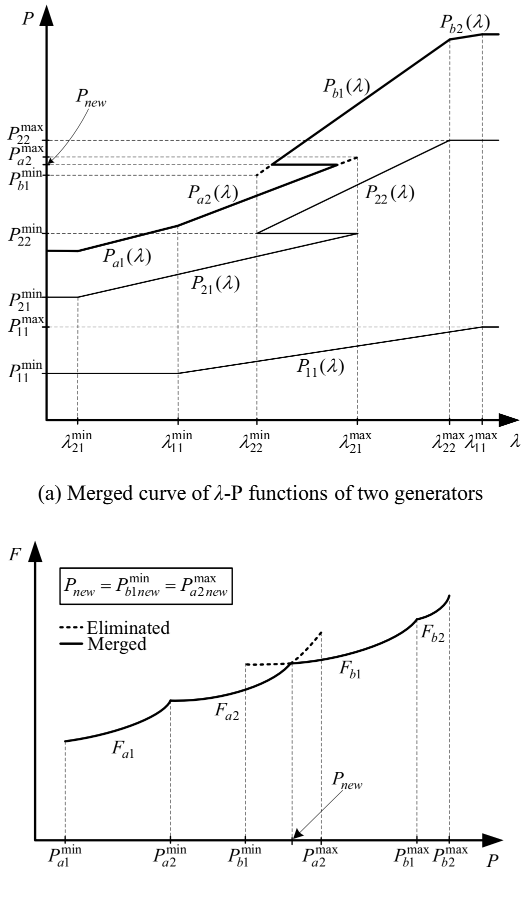
Consider two generators with these fuel cost functions:

$$F_1(P_1) = a_{11}P_1^2 + b_{11}P_1 + c_{11},\ P_{11}^{\min} \le P_1 \le P_{11}^{\max} \tag{16}$$

$$F_2(P_2) = \begin{cases} a_{21}P_2^2 + b_{21}P_2 + c_{21},\ P_{21}^{\min} \le P_2 \le P_{21}^{\max} \\ a_{22}P_2^2 + b_{22}P_2 + c_{22},\ P_{22}^{\min} \le P_2 \le P_{22}^{\max} \end{cases} \tag{17}$$
$$\text{with } P_{21}^{\max} = P_{22}^{\min}$$

Fig. 5(a) shows the λ-P functions of two generators and the merged cost function. The function merger can easily be performed with λ-P curves for the continuous parts. Here, it should be noted that the λ-P curves do have duplicate regions, with respect to λ, even though both fuel cost functions do not, with respect to P.

The duplicity problem of generation in the λ region, for example $[\lambda_{22}^{\min}, \lambda_{21}^{\max}]$ in Fig. 5(a), can be solved by selecting the smaller cost. A new breakpoint P_{new} is determined by the point that both fuel costs are same, where the merged cost functions F_{a2} and F_{b1} can be easily found by (15). Fig. 5(b) shows that the merged fuel cost curve is represented by solid lines and the eliminated parts by dotted lines. An intersection point of the fuel curves becomes a new minimum and/or a maximum P for the corresponding unit.



(a) Merged curve of λ-P functions of two generators

(b) merged fuel cost function

Fig. 5. Function merger of multiple fuel cost functions

There is no restriction in the number of functions for the merging procedure. However, only the two-functions merging technique with the one-by-one merging procedure is explained here and it will be applied to the entire proposed algorithm.

4.3 *Illustrative Examples*

Consider the example of a 3-generator system. The fuel cost functions of the generators are as follows:

$$F_1(P_1) = 13.73 - 0.2871P_1 + 0.002415P_1^2,\ 100 \le P_1 \le 230 \quad (18)$$

$$F_2(P_2) = \begin{cases} 39.24 - 0.4116P_2 + 0.001524P_2^2,\ 180 \le P_2 \le 306 \\ 132.71 - 0.8245P_2 + 0.001875P_2^2,\ 306 \le P_2 \le 390 \end{cases} \quad (19)$$

$$F_3(P_3) = \begin{cases} 22.96 - 0.07298P_3 + 0.000902P_3^2,\ 200 \le P_3 \le 364 \\ 220.36 - 0.9746P_3 + 0.00189P_3^2,\ 364 \le P_3 \le 450 \end{cases} \quad (20)$$

The λ-P functions are:

$$P_1(\lambda) = 207.0\lambda + 59.4, \qquad 0.196 \le \lambda \le 0.824 \quad (21)$$

$$P_2(\lambda) = \begin{cases} 328.1\lambda + 135.0, & 0.137 \le \lambda \le 0.521 \\ 266.7\lambda + 219.9, & 0.323 \le \lambda \le 0.638 \end{cases} \quad (22)$$

$$P_3(\lambda) = \begin{cases} 554.1\lambda + 40.4, & 0.288 \le \lambda \le 0.584 \\ 264.6\lambda + 257.8, & 0.401 \le \lambda \le 0.726 \end{cases} \quad (23)$$

Fig. 6 shows a graph of the λ-P functions in (21) to (23).

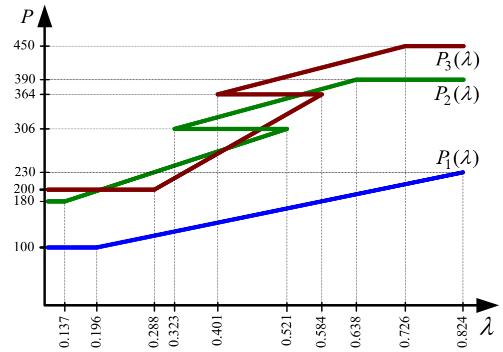


Fig. 6. The λ-P functions of the three generators

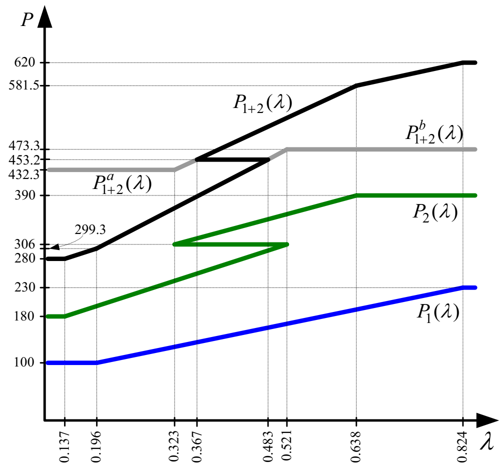


Fig. 7. The procedure of constructing the $P_{1+2}(\lambda)$

First, $P_1(\lambda)$ is merged with $P_{21}(\lambda)$ and $P_{22}(\lambda)$ and then, $P^a_{1+2}(\lambda)$ and $P^b_{1+2}(\lambda)$ are constructed, as illustrated in Fig. 7. The fuel cost functions of $P^a_{1+2}(\lambda)$ and $P^b_{1+2}(\lambda)$ are arranged in Table 1.

Second, $P_{1+2}(\lambda)$ is built by choosing the cheapest parts of the two curves for all P. The fuel cost functions of $P_{1+2}(\lambda)$ appear in Table 2. $P_3(\lambda)$ is also merged with $P_{1+2}(\lambda)$ in the same manner as described above. $P^a_{1+2+3}(\lambda)$, $P^b_{1+2+3}(\lambda)$, and $P_{1+2+3}(\lambda)$ are arranged in Table 1 and 2, respectively.

Finally, in order to calculate the dispatch of each generator, find the range involving P_D and calculate the optimal λ^* in the $P_{1+2+3}(\lambda)$. Using the fuel combination, if λ^* is set between $\lambda_{ij}^{\min}$ and $\lambda_{ij}^{\max}$ for a specific generator, its generation power is determined by the λ-P function of each generator for λ^*. If λ^* is smaller (larger) than $\lambda_{ij}^{\min}$ ($\lambda_{ij}^{\max}$) then the generator output becomes $P_{ij}^{\min}$ ($P_{ij}^{\max}$) of each generator.

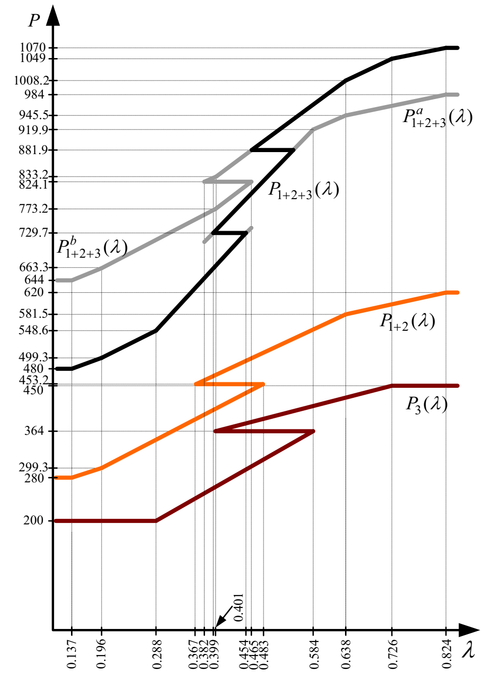


Fig. 8. Construction of $P_{1+2+3}(\lambda)$ in the example

Table 1. Fuel Cost Functions for Power Ranges

$P^a_{1+2}(\lambda)$, $P^b_{1+2}(\lambda)$, $P^a_{1+2+3}(\lambda)$, and $P^b_{1+2+3}(\lambda)$

	Rng	P_{min}	P_{max}	λ_{min}	λ_{max}	a	b	c	Fuel
$P^a_{1+2}(\lambda)$	1	280	299.31	0.137	0.196	104.81	-0.7164	0.001524	11
	2	299.31	473.33	0.196	0.521	51.99	-0.3634	0.000934	11
$P^b_{1+2}(\lambda)$	1	432.31	581.53	0.323	0.638	129.61	-0.5896	0.001056	12
	2	581.53	620	0.638	0.824	589.36	-2.1708	0.002415	12
$P^a_{1+2+3}(\lambda)$	1	480	499.31	0.137	0.196	353.51	-1.3260	0.001524	111
	2	499.31	548.59	0.196	0.288	206.51	-0.7372	0.000934	111
	3	548.59	729.71	0.288	0.454	63.46	-0.2157	0.000459	111
	4	729.71	919.94	0.399	0.584	118.49	-0.3111	0.000486	121
	5	919.94	945.53	0.584	0.638	600.04	-1.3580	0.001056	121
	6	945.53	984	0.638	0.824	1815.47	-3.9289	0.002415	121
$P^b_{1+2+3}(\lambda)$	1	644	663.31	0.137	0.196	683.53	-1.8259	0.001524	112
	2	663.31	773.24	0.196	0.401	424.10	-1.0437	0.000934	112
	3	773.24	824.08	0.401	0.465	239.28	-0.5656	0.000625	112
	4	824.08	833.42	0.382	0.401	600.11	-1.3580	0.001056	122
	5	833.22	1008.15	0.401	0.638	337.39	-0.7276	0.000677	122
	6	1008.15	1049.83	0.638	0.726	726.63	-1.4998	0.001060	122
	7	1049.83	1070	0.726	0.824	2219.78	-4.3443	0.002415	122

Table 2. Fuel Cost Functions for Power Ranges

$P_{1+2}(\lambda)$ and $P_{1+2+3}(\lambda)$

	Rng	P_{min}	P_{max}	λ_{min}	λ_{max}	a	b	c	Fuel
$P_{1+2}(\lambda)$	1	280	299.31	0.137	0.196	104.81	-0.7164	0.001524	11
	2	299.31	453.16	0.196	0.483	51.99	-0.3634	0.000934	11
	3	453.16	581.53	0.367	0.638	129.61	-0.5896	0.001056	12
	4	581.53	620	0.638	0.824	589.36	-2.1708	0.002415	12
$P_{1+2+3}(\lambda)$	1	480	499.31	0.137	0.196	353.51	-1.3260	0.001524	111
	2	499.31	548.59	0.196	0.288	206.51	-0.7372	0.000934	111
	3	548.59	729.71	0.288	0.454	63.46	-0.2157	0.000459	111
	4	729.71	881.87	0.399	0.547	118.49	-0.3111	0.000486	121
	5	881.87	1008.15	0.467	0.638	337.39	-0.7276	0.000677	122
	6	1008.15	1049.83	0.638	0.726	726.63	-1.4998	0.001060	122
	7	1049.83	1070	0.726	0.824	2219.78	-4.3443	0.002415	122

It should be noted that if the committed generators are not changed for another total demand, the procedures of constructing the merged function tables can be omitted. Once all of the FM procedures are executed, the dispatch of the generators can easily be obtained by the FM table previously constructed.

5. SIMULATION RESULTS

The proposed FM method is applied to the ED problems with a 10-generator system (Lin and Viviani, 1984). For simplicity, system losses are not considered. The fuel cost data of the generators is given in Table 5. During testing the total system demand is varied from 2400 MW to 2700 MW with 100 MW increments. The globally optimal solution to the ED with multiple fuel units is acquired by ACM and FM. Both ACM and FM were directly coded using real values and were implemented on a personal computer (Pentium D CPU 3.00 GHz) in Microsoft Visual C++ 6.0. The results of the proposed algorithms and the ACM are summarized in Table 3.

Table 3. Comparison of ACM and FM

S	U	FM		ACM		FM		ACM		FM		ACM		FM		ACM	
		F	GEN	F	GEN	F	GEN	F	GEN	F	GEN	F	GEN	F	GEN	F	GEN
1	1	1	189.74	1	189.74	2	206.52	2	206.52	2	216.54	1	216.54	2	218.25	2	218.25
	2	1	202.34	1	202.34	1	206.46	1	206.46	1	210.91	1	210.91	1	211.66	1	211.66
	3	1	253.90	1	253.90	1	265.74	1	265.74	1	278.54	3	278.54	1	280.72	1	280.72
	4	3	233.05	3	233.05	3	235.95	3	235.95	3	239.1	3	239.1	3	239.63	3	239.63
2	5	1	241.83	1	241.83	1	258.02	1	258.02	1	275.52	1	275.52	1	278.50	1	278.50
	6	3	233.05	3	233.05	3	235.95	3	235.95	3	239.1	3	239.1	3	239.63	3	239.63
	7	1	253.27	1	253.27	1	268.86	1	268.86	1	285.72	1	285.72	1	288.58	1	288.58
3	8	3	233.05	3	233.05	3	235.95	3	235.95	3	239.1	3	239.1	3	239.63	3	239.63
	9	1	320.38	1	320.38	1	331.49	1	331.49	1	343.49	1	343.49	3	428.52	3	428.52
	10	1	239.40	1	239.40	1	255.06	1	255.06	1	271.99	1	271.99	1	274.87	1	274.87
TP		2400		2400		2500		2500		2600		2600		2700		2700	
TC		481.723		481.723		526.239		526.239		574.381		574.381		623.809		623.809	

As shown in Tables 3, the FM provides the globally optimal solutions that are exactly equal to the results provided by ACM. Table 5 in Appendix shows the fuel cost functions and combinations of fuel type for a power range of 1353 MW to 3695 MW, which is the minimum to maximum feasible generating power in the system, respectively. The global solutions to the 10-generator system can be obtained by the fuel cost function involving the total demand in Table 5.

The simulation time is checked, in order to compare ACM with FM regarding the computational burden. The number of generators increases from 2 to 10. These results appear in Table 4.

Table 4. Simulation Time for the Number of Generators

Number of Gens.	ACM [sec]	FM [sec]
2	0.000722	0.001390
3	0.000819	0.001796
4	0.001019	0.002140
5	0.001671	0.002703
6	0.003859	0.003078
7	0.011421	0.003484
8	0.036710	0.003937
9	0.118282	0.004375
10	0.389060	0.004850

The middle value between the minimum and maximum feasible generating output is chosen as the total demand of the ACM for each case. The periods of constructing fuel cost functions for whole power ranges are measured as computation time in the FM. The results show that the computation time increases exponentially in the ACM, while linearly in FM, which verifies that the FM demonstrates predictable computational behaviour.

6. CONCLUSIONS

This paper presents an algorithm to find a global solution to ED with multiple fuel units. The proposed algorithm uses FM on the basis of the λ-P function method using duality theory. The global optimality is checked with ACM.

Conventional heuristic approaches cannot provide a global optimality to the ED problem with multiple fuel units. Moreover, these approaches have a crucial flaw, which is the "curse of dimension" for large systems.

The proposed FM method is applied to a sample case of a 10-generator system. The global solutions obtained by the FM are compared with the results of the ACM, which provide the global optimal solutions. By comparing simulation time, the FM is shown to overcome the problems of the curse of dimension.

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Appendix

Table 5. Fuel Cost Functions and Combination of Fuel Type for All Power Ranges in 10-generator System

	P_{min}	P_{max}	λ_{min}	λ_{max}	F_{min}	F_{max}	a	b	c	Fuel Combination
1	1353.00	1361.32	0.038	0.074	205.63	206.09	0.002176	-5.851	4138.0	1211121121
2	1361.32	1406.19	0.074	0.141	206.09	210.92	0.000747	-1.960	1490.2	1211121121
3	1406.19	1415.43	0.141	0.147	210.92	212.25	0.000336	-0.805	678.0	1211121121
4	1415.43	1473.07	0.147	0.177	212.25	221.58	0.000255	-0.574	514.3	1211121121
5	1473.07	1540.50	0.177	0.200	221.58	234.26	0.000171	-0.329	333.6	1211121121
6	1540.50	1553.41	0.168	0.177	234.26	236.48	0.000341	-0.882	784.4	1211121111
7	1553.41	1616.44	0.177	0.203	236.48	248.44	0.000207	-0.465	460.6	1211121111
8	1616.44	1672.09	0.195	0.219	248.44	259.97	0.000218	-0.511	503.5	1311121111
9	1672.09	1697.25	0.202	0.214	259.97	265.20	0.000251	-0.637	623.6	1312121111
10	1697.25	1715.87	0.208	0.217	265.20	269.16	0.000238	-0.600	597.4	1311121111
11	1715.87	1735.09	0.203	0.215	269.16	273.18	0.000294	-0.806	786.2	1312121211
12	1735.09	1754.01	0.208	0.218	273.18	277.21	0.000277	-0.753	746.5	1111121211
13	1754.01	1772.92	0.202	0.216	277.21	281.16	0.000356	-1.047	1018.4	1312111211
14	1772.92	1798.63	0.207	0.224	281.16	286.70	0.000331	-0.967	954.8	1111111211
15	1798.63	1889.87	0.196	0.271	286.70	308.02	0.000412	-1.285	1265.8	1112111211
16	1889.87	1950.78	0.271	0.310	308.02	325.73	0.000321	-0.942	941.9	1112111211
17	1950.78	1962.54	0.310	0.316	325.73	329.42	0.000249	-0.660	667.3	1112111211
18	1962.54	2002.55	0.283	0.310	329.42	341.28	0.000342	-1.061	1092.6	1113111211
19	2002.55	2015.36	0.310	0.317	341.28	345.30	0.000261	-0.737	768.4	1112131211
20	2015.36	2054.32	0.282	0.310	345.30	356.83	0.000367	-1.196	1266.7	1112131311
21	2054.32	2067.85	0.310	0.318	356.83	361.08	0.000275	-0.821	881.7	1112131311
22	2067.85	2068.21	0.318	0.318	361.08	361.19	0.000219	-0.587	639.9	1112131311
23	2068.21	2106.10	0.280	0.310	361.19	372.38	0.000395	-1.352	1469.8	1113131311
24	2106.10	2118.90	0.310	0.318	372.38	376.40	0.000291	-0.915	1009.5	1113131311
25	2118.90	2170.67	0.318	0.341	376.40	393.47	0.000229	-0.651	729.4	1113131311
26	2170.67	2436.35	0.341	0.442	393.47	497.54	0.000189	-0.480	544.4	1113131311
27	2436.35	2614.10	0.439	0.505	497.54	581.47	0.000187	-0.470	535.4	2113131311
28	2614.10	2744.62	0.479	0.520	581.47	646.72	0.000157	-0.344	404.5	2113131331
29	2744.62	2881.66	0.520	0.579	646.72	722.03	0.000212	-0.643	815.4	2113131331
30	2881.66	2918.90	0.568	0.584	722.03	743.48	0.000214	-0.666	862.5	2113132331
31	2918.90	2959.07	0.506	0.520	743.48	764.11	0.000177	-0.524	769.9	2123131331
32	2959.07	3077.97	0.520	0.579	764.11	829.50	0.000248	-0.948	1396.6	2123131331
33	3077.97	3161.48	0.567	0.609	829.50	878.62	0.000251	-0.977	1460.9	2123132331
34	3161.48	3189.09	0.594	0.609	878.62	895.24	0.000272	-1.127	1720.4	2123232331
35	3189.09	3290.50	0.573	0.601	895.24	954.78	0.000139	-0.313	479.6	2123133331
36	3290.50	3303.30	0.601	0.609	954.78	962.53	0.000320	-1.503	2439.0	2123133331
37	3303.30	3318.75	0.597	0.601	962.53	971.79	0.000145	-0.362	574.8	2123233331
38	3318.75	3351.54	0.601	0.625	971.79	991.89	0.000355	-1.757	2889.1	2123233331
39	3351.54	3385.51	0.625	0.654	991.89	1013.62	0.000439	-2.319	3830.6	2123233331
40	3385.51	3462.58	0.577	0.601	1013.62	1059.01	0.000159	-0.499	881.8	2123133332
41	3462.58	3474.40	0.601	0.612	1059.01	1066.18	0.000451	-2.519	4379.2	2123133332
42	3474.40	3490.84	0.596	0.601	1066.18	1076.02	0.000167	-0.566	1013.9	2123233332
43	3490.84	3513.06	0.601	0.625	1076.02	1089.64	0.000524	-3.059	5365.7	2123233332
44	3513.06	3537.47	0.625	0.660	1089.64	1105.33	0.000730	-4.504	7903.3	2123233332
45	3537.47	3567.33	0.660	0.713	1105.33	1125.83	0.000884	-5.592	9827.7	2123233332
46	3567.33	3607.37	0.586	0.593	1125.83	1149.44	0.000083	-0.007	91.8	2123333332
47	3607.37	3629.59	0.593	0.601	1149.44	1162.70	0.000187	-0.754	1439.8	2123333332
48	3629.59	3644.51	0.601	0.625	1162.70	1171.85	0.000780	-5.064	9262.2	2123333332
49	3644.51	3657.77	0.625	0.660	1171.85	1180.37	0.001344	-9.174	16750.4	2123333332
50	3657.77	3695.00	0.660	0.808	1180.37	1207.69	0.001978	-13.812	25234.0	2123333332