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Network Inversion Based Controller Design for Discrete T-S Fuzzy Model*

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Abstract:
This paper presents an elegant method for controlling nonlinear systems by modeling them in terms of a Takagi-Sugeno(T-S) fuzzy model. The concept of network inversion is used to design the controller for such a system. The proposed controller is shown to make the closed loop system stable in the sense of Lyapunov. The existing controller design techniques for T-S fuzzy model, like LMI techniques, robust control techniques are based on a sufficient or prerequisite condition for closed loop stability whereas in the present scheme no such sufficient condition is necessary. Moreover the present approach greatly simplifies the process of controller design compared to the earlier techniques. Simulation results on three nonlinear systems show the efficacy of the proposed control scheme. The proposed controller has also been implemented on the cart pole system in real time and the results are provided with a qualitative comparison with the well established LQR control.

Keywords: T-S Fuzzy model, Fuzzy Neural Network, Stability analysis, Network Inversion

1. INTRODUCTION

Takagi and Sugeno [Takagi and Sugeno, 1985] proposed a fuzzy modeling approach wherein the input space of the nonlinear system to be modeled is divided into different fuzzy regions with a local linear model being used in each region. The overall model output is obtained by defuzzification using the center of gravity (COG) method. Using the same concept, Johansen and Foss [Johansen and Foss, 1993] designed an ARMAX model in each of the operating regions. The global model was then obtained by interpolation. Since there are numerous established methods to design controllers for linear systems, one would definitely be attracted to simplify the nonlinear control problem by finding a global control law from the local linear controllers that will stabilize the entire fuzzy model. Thus the stability analysis and controller design for T-S fuzzy systems have attracted many researchers [Tanaka, 1995, Cao et al., 1998] in recent years.

In general, asymptotic stability of fuzzy system is not guaranteed even if individual subsystems are stable. Tanaka [Tanaka and Sugeno, 1992, Tanaka, 1995] analyzed the stability of this kind of fuzzy systems where stability can be ensured by finding a common Lyapunov function for all the ARMA models. The conditions for existence of common Lyapunov function are derived and the common Lyapunov function is obtained by solving a Linear Matrix Inequality (LMI). But it is generally difficult to find a common Lyapunov function, moreover it does not exist always. In 1998, these stability conditions have further been relaxed by Tanaka et al. [1998]. Feng et al. [1997] also discussed stability of fuzzy systems in continuous time case based on the same Lyapunov function approach. There, they have ensured global stability by introducing a compensating term by variable structure control along with the local controllers. For continuous time T-S fuzzy system, Zak [Zak, 1999] has proposed a fixed gain state feedback controller for the global system by expressing the T-S fuzzy model as a linear plant with a nonlinear disturbance term. In a recent work [Premkumar et al., 2006], we have proposed variable gain controllers using a similar approach. However these controllers need a tedious design procedure before applying.

In this paper, we propose a controller design scheme based on the inversion of the T-S fuzzy model of a nonlinear system. The local linear models of the T-S fuzzy system can either be identified from the input output data set using a fuzzy neural network or be computed from the nonlinear dynamics using a linearization technique. After identifying the system as a fuzzy cluster of local linear models, the required input to achieve a desired output, is predicted from the learned model by network inversion. Linden and Kindermann [Linden and Kindermann, 1989] first proposed a method of inversion for arbitrary continuous multilayer nets (MLN) based on iterative gradient search in the input space. This technique has been extended by Hoskins et al. [1992] where the forward dynamic model of the plant is learned using MLN, and iterative constrained inversion is performed online to generate control commands. Behera *et al.* have proposed an extended Kalman filter (EKF) based inversion algorithm for radial basis function (RBF) networks [Behera et al., 1995, 1996] to predict the control input. Above mentioned works motivated us to use this network inversion technique for T-S fuzzy model where given a desired output pattern, the control input is predicted through direct inversion of the fuzzy model using Lyapunov function approach.

Further the network inversion based control design scheme is extended to derive the feedback gains of a linearly parametrized control input for the T-S fuzzy model. The control law $u(k)$ in this case is assumed to have a simple representation $u(k) = -\sum_j \sigma_j F_j x(k)$ where F_j's are designed online through net-

⋆ This work was supported by Department of Science and Technology (DST), Govt. of India under the project titled "Intelligent Control Schemes and applications to dynamics and visual control of redundant manipulator systems". The project number is DST/EE/2005033. Laxmidhar Behera is the corresponding author of this manuscript. Email-id: lbehera@iitk.ac.in, l.behera@ulster.ac.uk work inversion. The advantage of such a scheme is that it avoids the need of any sufficient condition as in LMI techniques [Tanaka and Sugeno, 1992, Tanaka, 1995, Tanaka et al., 1998] or any prerequisite constraint as in robust control techniques [Zak, 1999, Premkumar et al., 2006]. It also simplifies the process involved in controller design. Furthermore, this design procedure can be used both for regulation as well as tracking problems. The applicability of the proposed scheme is demonstrated through simulation as well as experimental results.

2. MODELING OF NONLINEAR SYSTEMS USING T-S FUZZY SYSTEM

2.1 T-S fuzzy model

Let us consider a class of discrete time nonlinear system as,

$$x(k+1) = f(x(k), u(k)) \tag{1}$$

where $x(k)$ is the n-dimensional state vector, $u(k)$ the p-dimensional input vector.

The above system can be effectively modeled as a T-S fuzzy system where the nonlinear system is approximated by a fuzzy cluster of M local linear models where j^{th} fuzzy rule has the following form:

IF $x_1(k)$ is F_1^j AND $\cdots$ AND $x_n(k)$ is F_n^j THEN

$$x(k+1) = A_j x(k) + B_j u(k) \tag{2}$$

where $x = [x_1, x_2 \cdots, x_n]^T$, $j = 1, \cdots M$. Now onwards, we will term a local linear model as *subsystem*.

Given an input-output pair $(x(k), u(k))$, the fuzzy model around this operating point is constructed as the weighted average of the linear subsystems and has the form

$$x(k+1) = \frac{\sum_{j=1}^{M} \mu_j (A_j x(k) + B_j u(k))}{\sum_{j=1}^{M} \mu_j} \tag{3}$$

$$\text{where} \quad \mu_j = \prod_{i=1}^{n} \mu_j^i(x_i)$$

$\mu_j^i(x_i)$ is the membership function of the fuzzy term F_i^j, $j = 1, 2, \cdots, M$. The fuzzy system (3) can be rewritten as,

$$x(k+1) = \sum_{j=1}^{M} \sigma_j (A_j x(k) + B_j u(k)) \tag{4}$$

$$\text{where,} \quad \sigma_j = \frac{\mu_j}{\sum_{j=1}^{M} \mu_j}, \quad \sum_{j=1}^{M} \sigma_j = 1$$

2.2 Identification of local linear models

The linear model parameters A_j's and B_j's can be found either by linearizing the nonlinear system dynamics or from the input-output data set using a fuzzy neural network. The linearization at origin can be done using the standard Taylor series expansion and at other operating points the procedure described in [Zak, 2003] can be adopted. A typical fuzzy neural network is shown in Figure 1 where $x_i(k+1)$ is the individual state, $i = 1, \cdots n$. The four layers are fuzzification layer, rule layer, linear model layer and defuzzification layer. The specific function of each layer is given in [Zhang and Morris, 1995]. Detailed description of the identification method including the weight update law can be found in [Behera and Anand, 1999].

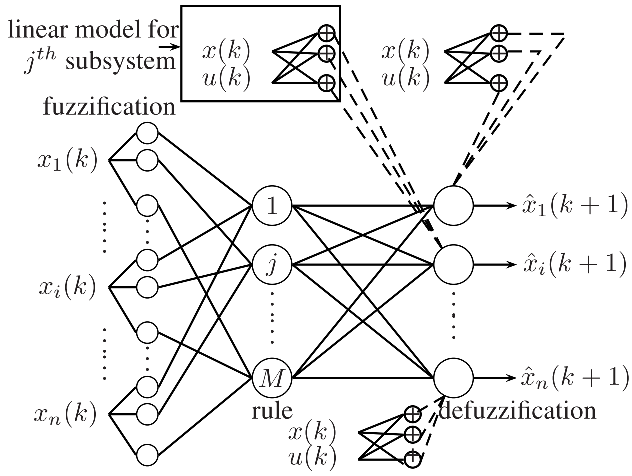

Fig. 1. Fuzzy neural network model of a nonlinear system with n states

3. STABILITY ANALYSIS AND CONTROLLER DESIGN FOR A T-S FUZZY MODEL

3.1 Parallel Distributed Fuzzy Compensator

In parallel distributed compensation (PDC) [Tanaka et al., 1998], each control rule is designed based on the corresponding rule of a T-S fuzzy model. The designed fuzzy controller uses the same fuzzy sets of the model. For the fuzzy model (4), a fuzzy regulator can be designed as follows:

Regulator Rule j:

IF $x_1(k)$ is F_1^j AND $\cdots$ AND $x_n(k)$ is F_n^j THEN

$$u(k) = -F_j x(k), \quad j = 1, 2 \cdots M \tag{5}$$

The overall fuzzy regulator is represented by

$$u(k) = -\sum_{j=1}^{M} \sigma_j F_j x(k) \tag{6}$$

Thus the design problem simplifies to designing the local feedback gains F_j.

If we put the control law (6) in equation (4), the closed loop system becomes

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{M} \sigma_i (A_i - B_i \sum_{j=1}^{M} \sigma_j F_j x(k)) \\ &= \sum_{i=1}^{M} \sum_{j=1}^{M} \sigma_i \sigma_j (A_i - B_i F_j) x(k) \end{aligned} \tag{7}$$

The above system will be asymptotically stable if there exists a common P for all the subsystems such that

$$H_{ij}^T P H_{ij} - P < 0$$

where $H_{ij} = A_i - B_i F_j$. Tanaka *et. al.* presented relaxed stability conditions [Tanaka et al., 1998] for the closed loop T-

S fuzzy system which provide a mean to design the controller gains by use of LMI techniques.

3.2 Inversion Based Controller Design Scheme

In this section we provide a new scheme for designing the controller gains which uses the concept of network inversion [Linden and Kindermann, 1989, Behera et al., 1995, Behera, 2003]. The inverse mapping of any network generates an input pattern for a desired output pattern. It is possible to obtain the required control input for the desired output via inversion process.

Control Scheme 1: In this control design scheme, given a desired state vector and the present state vector of the system model, a control input is generated such that the closed loop system is Lyapunov stable. The Lyapunov function for the system is taken as

$$V = \frac{1}{2}\tilde{x}^T\tilde{x}$$

where $\tilde{x} = x_d - \hat{x}$. Here, x_d is the desired state vector, $\hat{x}$ is the actual state vector of the FNN model. u is the predicted input using the inversion of the model which is fed to the actual system. Since during the inversion process, the present states are kept constant, $\hat{x}(k+1)$ can be considered as a function of only u. Thus the time derivative of the Lyapunov function can be written as

$$\begin{aligned}\dot{V} &= -\tilde{x}^T\frac{\partial\hat{x}}{\partial u}\dot{u}\\ &= -\tilde{x}^T J\dot{u}\end{aligned} \tag{8}$$

where $J = \frac{\partial\hat{x}}{\partial u} \in R^n$.

Theorem 1. If arbitrary initial input $u(0)$ is updated by

$$u(T) = u(0) + \int_0^T \dot{u}dt \tag{9}$$

where $\dot{u}$ is given by

$$\dot{u} = -\frac{\|\tilde{x}\|^2}{\|J^T\tilde{x}\|^2}J^T\tilde{x} \tag{10}$$

then $\tilde{x}$ converges to zero provided $\dot{u}$ exists along the convergence trajectory.

Proof: Substituting $\dot{u}$ from equation (9) into (8) we get

$$\dot{V} = -\|\tilde{x}\|^2$$

The iterative inversion based input update law is given by the following equation.

$$u(t+1) = u(t) + \delta\dot{u} \tag{11}$$

where t is the iterative index and δ is a small positive constant which represents the update rate. The possible numerical instability associated with the weight update law can be avoided by adding a small positive constant in the denominator. In this $\dot{V}$ will be negative semi definite and the error will not converge to zero asymptotically. Instead it will bounded by ball of small radius. The schematic diagram of the control scheme is shown in figure 2.

The inversion process can also be carried out using gradient search. In this case the iterative input update law is given as:

$$u(t+1) = u(t) + \delta\frac{\partial\tilde{x}}{\partial u} \tag{12}$$

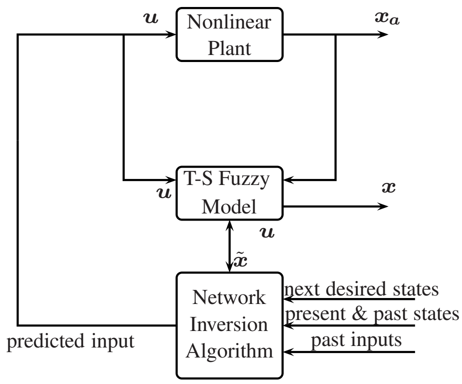

Fig. 2. Network Inversion Based Control Scheme

Control Scheme 2: The similar idea can be used to get the feedback gains F_j when the control input $u(k)$ is represented by the following parametrized form.

$$u(k) = -\sum_j \sigma_j F_j x(k) \tag{13}$$

Since the above form of control law is used for regulation purpose, we will first derive the feedback gains F_j for regulation only. For a regulation problem, the Lyapunov function candidate is chosen as:

$$V = \frac{1}{2}x^T x$$

We can get the required control input $u(k)$ by recursively updating F_j while keeping $x(k)$ fixed. Thus the control action can be viewed as a function of feedback gains only. Thus the time derivative of the Lyapunov function candidate can be derived as:

$$\dot{V} = x^T\dot{x} = x^T\frac{\partial x}{\partial u}\left(\sum_{j=1}^{M}\sigma_j\frac{\partial u}{\partial F_j}\dot{F}_j\right) = \sum_{j=1}^{M}\sigma_j(x^T D_j\dot{F}_j)$$

where $D_j = \frac{\partial x}{\partial u}\frac{\partial u}{\partial F_j}$. It is assumed here that the state changes due to the change in input only.

Let us select the incremental feedback gain to be

$$\dot{F}_j = -\frac{\|x\|^2}{\|D_j^T x\|^2}D_j^T x \tag{14}$$

Thus $\dot{V}$ becomes

$$\dot{V} = -\sum_{j=1}^{M}\sigma_j\|x\|^2 = -\|x\|^2$$

which is negative definite. Thus the following theorem ensures the asymptotic stability of the overall system.

Theorem 2. If an arbitrary initial feedback gain $F_j(0)$ is updated by

$$F_j(t) = F_j(0) + \int_0^t \dot{F}_j dt \tag{15}$$

where $\dot{F}_j$ is given in (14), then x converges to zero provided $\dot{F}_j$ exists along the convergence trajectory.

The possible numerical instability associated with the weight update law can be avoided by adding a small positive constant in the denominator.

Since the LMI techniques try to design the gains by finding out a common P and the existence of the common P is not guaranteed always, there may be cases where it fails to find out any feasible set of gains. But in the presented design technique no such sufficient condition is needed to be satisfied. The gains can be found out online while maintaining the closed loop stability through Lyapunov approach. We will show in one of the simulation results that the feedback gains designed using the proposed scheme actually satisfy the relaxed stability condition as given in [Tanaka et al., 1998].

The inversion process requires the training data set to be dimensionally sufficient to obtain an accurate T-S fuzzy model of the system. For open loop unstable nonlinear systems, the training data sets are obtained using a PD controller. Thus the input data depends on the states of the system which makes the data dimensionally insufficient. Random sinusoid trajectories are taken as the desired trajectories for the PD controller. Various dither signals like white noise, impulses, step functions are added to the output of the PD controller to make input data independent of the states of the system.

4. SIMULATION AND EXPERIMENTAL RESULTS

In this section we provide the simulation results for three nonlinear systems namely single link manipulator, cart pole system and two link manipulator to establish the validity of the proposed control schemes. One of the proposed controllers has also been implemented in real time for the cart pole system.

Single Link Manipulator The dynamic model of a single link manipulator is given as:

$$ml^2\ddot{\theta} + mgl\cos\theta = \tau$$

where $m = 11.36$ kg, $l = 0.432$ meter, $g = 9.81$ meter/sec^2. Considering the systems states as $z_1 = \frac{\pi}{2} - \theta, \quad z_2 = \dot{\theta}$, the state space model becomes

$$\begin{aligned} \dot{z}_1 &= -z_2 \\ \dot{z}_2 &= -22.7 \ \sin \ z_1 + 0.47 \ \tau \end{aligned} \tag{16}$$

Input (τ) and output (θ, $\dot{\theta}$) data are generated within the workspace $0 < z_1 < 2$ and $-2 < z_2 < 2$. Since the single link manipulator is an open loop unstable system, a PD controller is used for data generation. Random sinusoidal trajectories are taken as the desired trajectories. Various dither signals like noise, impulse, step are added to the PD controller output to make the identification proper. Joint angle z_1 and joint velocity z_2 are fuzzified in three equally spaced regions in the specified workspace. Gaussian function is chosen as fuzzy membership function. Thus the T-S fuzzy model is having 9 fuzzy rules. Each rule has been identified from the input-output data using the fuzzy neural network described in chapter 2. Out of 9 fuzzy rules, one fuzzy rule is given below:

Rule 1: If $x(k)$ is around $[0\ 0]^T$

Then $x(k+1) = \begin{bmatrix} 1.0000 & -0.0050 \\ 0.0913 & 0.9995 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0.0330 \end{bmatrix} u(k)$

The identification result is shown in Figure 3. For model validation, we have carried out network inversion using test data pairs. The result is plotted in figure 4.

After identifying the system, the control law is designed for regulation purpose using both control schemes described in section 3.2. In the first control scheme, u is directly updated using Lyapunov function based iterative inversion technique

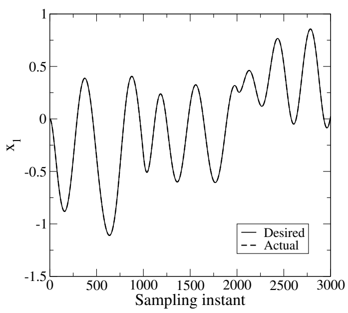

Fig. 3. Model prediction for Single Link Manipulator

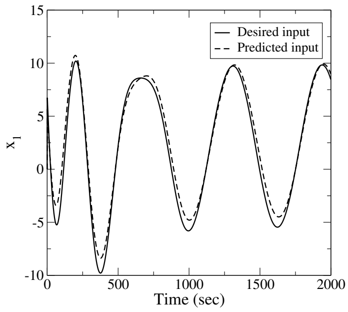

Fig. 4. Model Validation for Single Link Manipulator

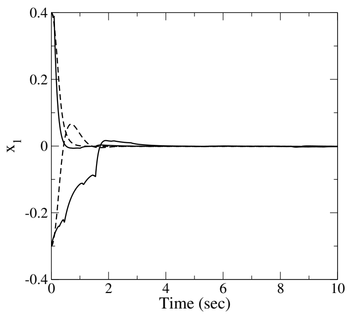

Fig. 5. Regulation result for Single Link Manipulator: Solid line represents control scheme 1 and dashed line represents control scheme 2

(equation (9)). In the second control scheme, u is a function of different feedback gains (equation (13)). The initial feedback gains F_j in (13) are taken as small random numbers and F_j's are updated online using the update law (14). The feedback gains F_j of only those subsystems are actually updated which are fired along the convergence trajectory. Gains of rest of the systems remain almost unchanged. The convergence results for two different state trajectories, starting from the initial conditions 0.4 and -0.3 respectively, are shown in figure 5. The solid line represents the control scheme one where the dashed line represents control scheme 2.

Though the feedback gains are designed using the proposed control scheme, they also satisfy the relaxed stability condition as described in [Tanaka et al., 1998]. Using the LMI technique, the common Lyapunov matrix P for the initial condition 0.4 is found to be $P = \begin{bmatrix} 0.15 & -0.01 \\ -0.01 & 0.02 \end{bmatrix}$ along the state trajectory starting from the initial conditions 0.4.

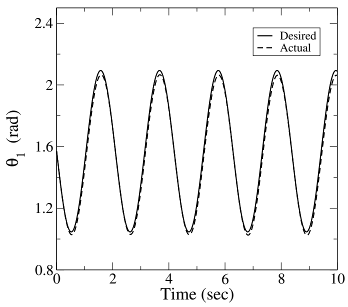

Fig. 6. Tracking result for two link manipulator based on Lyapunov based inversion technique

4.1 Two Link Manipulator

For a two-link robotic manipulator, the dynamical equations which relate the joint torques [τ_1, τ_2] to the joint angles [θ_1, θ_2] of the links are given as

$$\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} = \frac{1}{D}\begin{bmatrix} m_{22} & -m_{12}\\ -m_{21} & m_{11}\end{bmatrix}\begin{bmatrix}\tau_1 - v_1\\ \tau_2 - v_2\end{bmatrix} \quad (17)$$

$a_1 = 3.82, a_2 = 2.12, a_3 = 0.71, a_4 = 81.82, a_5 = 24.06.$

$$m_{11} = a_1 + a_2\cos\theta_2,\ m_{12} = m_{21} = a_3 + \frac{a_2}{2}\cos\theta_2,\ m_{22} = a_3$$

$$v_1 = a_4\cos\theta_1 - (a_2\sin\theta_2)(\dot{\theta}_1\dot{\theta}_2 + \frac{\dot{\theta}_2{}^2}{2}) + a_5\cos(\theta_1+\theta_2)$$

$$v_2 = (a_2\sin\theta_2)\frac{\dot{\theta}_1{}^2}{2} + a_5\cos(\theta_1+\theta_2)$$

$$D = m_{11}\, m_{22} - m_{12}\, m_{21}$$

Since the system dynamics exhibits a nonlinear behavior mainly because of the joint angles θ_1 and θ_2, we have fuzzified these two states only. Each state is fuzzified in seven equally spaced regions in the range $[\frac{\pi}{3}, \frac{2\pi}{3}]$ and $[\frac{-\pi}{6}, \frac{\pi}{6}]$ respectively. Gaussian function is chosen as fuzzy membership function. One of the total 49 fuzzy rules is given as follows:

Rule 1: If x is around $[\frac{\pi}{2}, 0, 0, 0]^T$, then

$$x(k+1) = \begin{bmatrix} 1.0 & .005 & 0 & 0\\ 0.15 & 1.0 & .118 & 0\\ .001 & 0 & 1.00 & .005\\ .205 & 0.0 & .463 & 1.00\end{bmatrix} x(k) + \begin{bmatrix} 0 & 0\\ -0.003 & .008\\ 0 & 0\\ -0.008 & .027\end{bmatrix} u(k)$$

$x(k) = [x_1\ x_2\ x_3\ x_4]^T = [\theta_1\ \theta_2\ \theta_3\ \theta_4]^T$ and $u(k) = [\tau_1\ \tau_2]^T$.

The above linear model is computed using Taylor series expansion for the discrete time approximation of the model and the linear models at other operating points are obtained by the linearization technique described in chapter eight of [Zak, 2003]. The controller is designed using Lyapunov approach based direct inversion algorithm for the desired trajectories,

$$\theta_{1d} = \frac{\pi}{2} - \frac{\pi}{6}\ \sin(3t), \quad \theta_{2d} = \frac{\pi}{6}\ \cos(3t)$$

The tracking results are shown in figure 6. Figure 7 shows the required input torques for the system.

4.2 Cart-Pole System

The proposed control algorithms are applied to stabilize an inverted pendulum mounted on a cart. The dynamics of the system can be written as

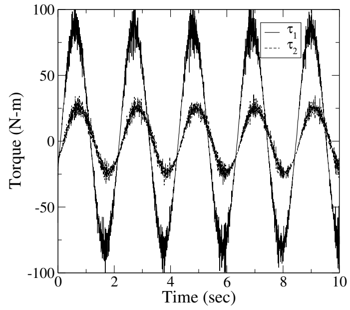

Fig. 7. Control torques for two link manipulator

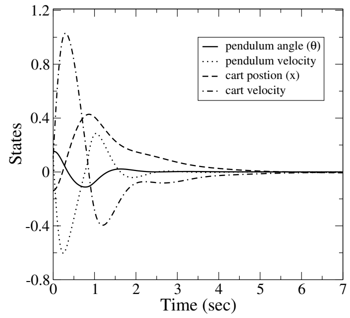

Fig. 8. Simulation results for cart pole system: Lyapunov based iterative inversion algorithm

$$\begin{aligned}
\dot{x}_1 &= x_2\\
\dot{x}_2 &= \frac{g\ sin(x_1) - amlx_2^2\ sin(2x_1)/2 - c_1\ a\ x_4\ cos(x_1)}{4l/3 - aml\ cos^2(x_1)}\\
&\quad - \frac{c_2\ a\ cos(x_1)}{4l/3 - aml\ cos^2(x_1)}u\\
\dot{x}_3 &= x_4\\
\dot{x}_4 &= \frac{-\frac{mag}{2}sin(2x_1) + \frac{4}{3}mlax_2^2 sin(x_1) + \frac{4}{3}ac_1x_4 + \frac{4}{3}ac_2u}{4/3 - am\ cos^2(x_1)}
\end{aligned} \quad (18)$$

where x_1 is the angle of the pendulum from vertical, x_2 is the angular velocity of the pendulum, x_3 is cart position, x_4 is the cart velocity and u is the input voltage. The system parameters are m, mass of the pendulum, M, mass of the cart, $2l$, length of the pendulum. g is acceleration due to gravity. a, c_1 and c_2 are three constants. The controller has been implemented both in simulation and real time. The real time experimental set up is a Quanser product for which the system parameters are given as $m = 0.23\ kg$, $M = 0.5\ kg$, $l = 0.321\ m$.

Pendulum angle x_1 is fuzzified in 7 equally spaced regions within the operating region $[-\pi/6,\ \pi/6]$. Gaussian function is chosen as fuzzy membership function. Around the equilibrium point (0,0), we have linearized the discrete time approximation of the system model using standard Taylor series expansion. At other operating points, linear models are obtained by the technique, described in chapter eight of [Zak, 2003].

The control objective here is to stabilize the system states at the equilibrium point $x = [0\ 0\ 0\ 0]^T$. Once the system is expressed as a fuzzy cluster of 7 linear models, the control input is computed using direct iterative inversion as given in (10) - (11). The desired output is taken as the output of a

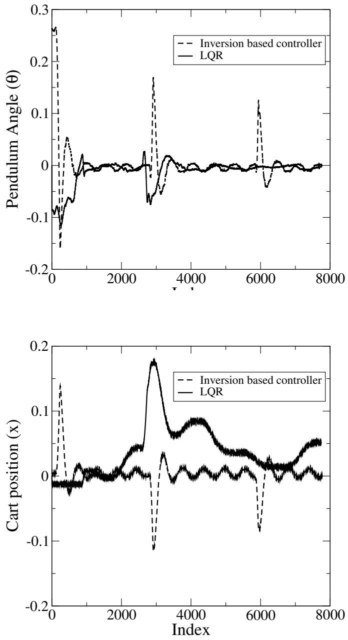

Fig. 9. Experimental results for cart pole system: Solid line represents the LQR control scheme and dashed line represents the proposed inversion based control scheme

reference linear model. The reference model is regulated using LQR control strategy. Simulation result is shown in figure 8.

Next we have applied the proposed controller to the real time set up of the cart pole system. The control input is obtained from the fuzzy model of the system using iterative gradient search. The results are compared with the well established LQR control where the control input is obtained from the linear model at the equilibrium point. Since the T-S fuzzy model represents the system for a wide operating range, it is expected that the proposed controller will work for a wide initial angle. This fact was indeed observed in the real time experiment. It was also observed that the proposed controller is more robust in the sense that it can tolerate disturbance of higher magnitude compared to LQR. The same observation can be made from the experimental results, shown in figure 9.

5. CONCLUSION

This work discusses the methods to design controllers for nonlinear systems when they are expressed as a discrete time T-S fuzzy model. The linear models of the T-S fuzzy model are obtained either from the input output data set of the nonlinear system using a fuzzy neural network or using a linearization technique from the nonlinear system model. In this work we use the concept of network inversion for the designing the controller. In one of the control schemes we have used the T-S fuzzy model to predict the control input directly for a desired output using inversion process. In the other scheme, a linearly parametrized form, as in the earlier design techniques, of the control input is used where the parameters are updated online using the inversion of the system model. Since the inversion process uses the Lyapunov function approach, it also guarantee the stability of the closed loop system. The advantage of such a model is that it does not require any prerequisite condition to be satisfied while designing the controller. The feedback gains are calculated online and the design process is much simplified in comparison with the earlier techniques.

Three simulation examples have been demonstrated which show the applicability of the proposed controllers. In first example, the T-S fuzzy model of a single link manipulator is identified from the input-output data and a stabilizing controller is designed using the proposed schemes. It has been found that the feedback gains for the second control scheme also satisfy the LMI condition. In the second example, two link manipulator system is considered for which a tracking controller is designed using the direct inversion technique. In the third example the nonlinear dynamics of a cart pole system is considered. The proposed direct inversion based controller is implemented both in simulation and real time. The experimental results are compared with the established linear control law (LQR) and it is seen that the proposed controller can work for a wide range of initial pendulum angle and tolerate disturbance with higher magnitude compared to the LQR.

REFERENCES

L. Behera. Query based model learning and stable tracking of a robot arm using radial basis function network. *Computers and Electrical Engineering*, 29:553–573, 2003.

L. Behera and K. K. Anand. Guaranteed tracking and regulatory performance of nonlinear dynamic systems using fuzzy neural networks. *IEE Proc. Control Theory Appl.*, 146, 1999.

L. Behera, M. Gopal, and S. Chaudhury. Inversion of rbf networks and applications to adaptive control of nonlinear systems. *IEE Proceedings On Control Theory and Applications*, 142(6):617–624, 1995.

L. Behera, M. Gopal, and S. Chaudhury. On adaptive control of a robot manipulator using inversion of its neural emulator. *IEEE Trans Neural Networks*, 7(6):1401–1414, 1996.

S. G. Cao, N. W. Rees, and G. Feng. Lyapunov- like stability theorems for continuous-time fuzzy control systems. *Int. J . Control*, 69(1):49–64, 1998.

G. Feng, S.G. Cao, N.W. Rees, and C.K. Chak. Design of fuzzy control systems with guaranteed stability. *Fuzzy Sets and Syst.*, 85:1–10, 1997.

D. A. Hoskins, J. N. Hwang, and J. Vagners. Ijerative inversion of neural networks and its applications to adaptive control. *IEEE Trans. Neural Net*, 3(2), 292-301 1992.

T. A. Johansen and B. A. Foss. Constructing narmax models using armax models. *Int. J. Control*, 58:1125–1153, 1993.

A. Linden and J. Kindermann. Inversion of multilayer nets. In *IJCNN, Washington, DC*, June 1989.

P. Premkumar, Indrani Kar, and Laxmidhar Behera. Variable gain controllers for nonlinear systems using t-s fuzzy model. *IEEE Tran. SMC-B*, 36:1442–1449, 2006.

T. Takagi and M. Sugeno. Fuzzy identification of systems and its application to modelling and control. *IEEE Tran. SMC*, 15:116–132, 1985.

K. Tanaka. Stability and stabilizability of fuzzy-neural-linear control systems. *IEEE Tran. Fuzzy Syst.*, 3:438–447, 1995.

K. Tanaka and M. Sugeno. Stability analysis and design of fuzzy control systems. *Fuzzy Sets and Systems*, 45, 1992.

K. Tanaka, T. Ikeda, and H. O. Wang. Fuzzy regulators and fuzzy observers: Relaxed stability conditions and lmi based designs. *IEEE Tran. on Fuzzy Systems*, 6(2), 1998.

S. H. Zak. *Systems and control*. Oxford University Press, 2003.

Stainslaw H. Zak. Stabilizing fuzzy system models using linear controllers. *IEEE Tran. Fuzzy Systems*, 7:236–240, 1999.

J. Zhang and A. J. Morris. Fuzzy neural networks for nonlinear systems modeling. *IEE Proc. Control Theory Appl.*, 142: 551–561, 1995.